EC2102 Networks and Systems – HW 4

September 6, 2012

1. Let $x[n] = \delta[n] + 2\delta[n-1] - \delta[n-3]$ and $h[n] = 2\delta[n+1] + 2\delta[n-1]$. Compute and plot each of the following convolutions.
   (a) $y_1[n] = x[n] \star h[n]$
   (b) $y_2[n] = x[n+2] \star h[n]$
   (c) $y_3[n] = x[n] \star h[n+2]$

2. Consider an input $x[n]$ and a unit impulse response $h[n]$ given by,

$$x[n] = \left(\frac{1}{2}\right)^{n-2} u[n-2]$$
$$h[n] = u[n+2]$$

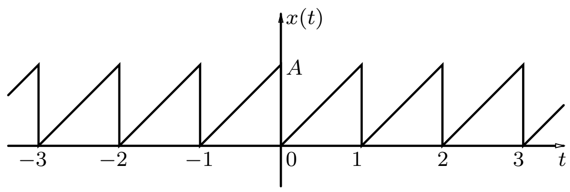
   Determine and plot the output $y[n] = x[n] \star h[n]$.

3. The trigonometric Fourier series of a certain periodic signal is given by

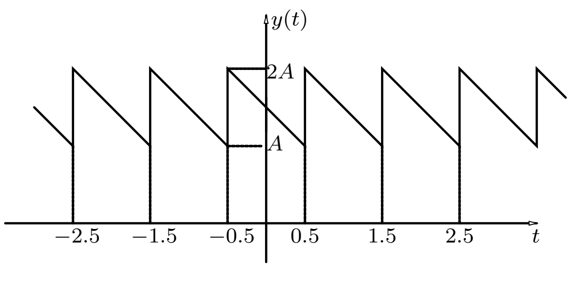
$$x(t) = 3 + \sqrt{3}\cos(2t) + \sin(2t) + \sin(3t) - \frac{1}{2}\cos(5t + \frac{\pi}{3})$$

   Sketch the amplitude and phase spectra

4. A periodic signal $x(t)$ is given below.



   (a) Determine its Fourier Series coefficients in the exponential form. Sketch its magnitude and phase spectrum.
   (b) Using the results in part (a) above and without doing elaborate integrations, determine the coefficients of the Fourier series of the periodic signal $y(t)$ shown below. Sketch the magnitude and phase spectrum.



5. Determine the coefficients of the Fourier series of the periodic signals $x_1(t)$ and $x_2(t)$ with period T_0 and defined in the interval $[-T_0/2, T_0/2)$ as follows.

$$x_1(t) = \begin{cases} A & |t| < d/2 \\ 0 & \text{otherwise} \end{cases}$$

$$x_2(t) = \begin{cases} A\sin(2\pi t/T_0) & 0 \le t < T_0/2 \\ 0 & -T_0/2 \le t < 0 \end{cases}$$

   Sketch the magnitude and phase spectrum in each case.

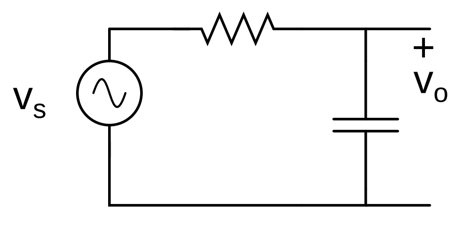
6. A 2π periodic signal $x(t)$ is specified over one period as

$$\begin{aligned} x(t) &= \frac{t}{A}, & 0 \le t < A \\ &= 1, & A \le t < \pi \\ &= 0, & \pi \le t \le 2\pi \end{aligned}$$

   Represent the function as an exponential Fourier series

7. If $x(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi nt}$ and $y(t) = \sum_{n=-\infty}^{\infty} (-1)^n c_n e^{j2\pi nt}$, express $y(t)$ in terms of $x(t)$.

8. In the figure shown, $v_s(t)$ is a half-wave rectified sine wave with amplitude $100V$, $R = 1k\Omega$, $\omega_o = 100\pi$ rads/sec, and $C = \frac{50}{\pi}\mu F$. Find the output $v_o(t)$ as a trigonometric Fourier series. Find the numerical value of the magnitudes of the first three harmonics.



9. An LTI system has an impulse response $h(t) = e^{-4t}u(t)$. The input $x(t) = \sin(4\pi t) + \cos(6\pi t + \frac{\pi}{4})$. Find the Fourier series representation of the output.

10. An ideal lowpass filter with cutoff frequency 500 Hz is an LTI system H_l whose frequency response is:

$$H_l(\omega) = \begin{matrix} 1 & |\omega| \leq 2\pi.500\text{rad/s} \\ 0 & \text{otherwise} \end{matrix}$$

   (a) What is the response of this filter to the input signal $x(t) = \cos(2\pi.750t) + \sin(2\pi.1500t)$?

   (b) What is th eresponse of this filter for a periodic square wave with period 4.5 ms? The square wave oscillates between +1 V and −1 V with 50% duty cycle and is an even function of time.

11. An LTI system has an impulse response $h(t) = \delta(t) - e^{-t}u(t)$. For the following signals input to the system, find the output using the Fourier series analysis.

   (a) $x(t) = \cos 3\pi t + \frac{\pi}{3}$.

   (b) $x(t) = \sum_{n=-\infty}^{\infty} \delta(t-n)$.

   (c) $x(t) = \sum_{n=-\infty}^{\infty} (-1)^n \delta(t-2n)$.