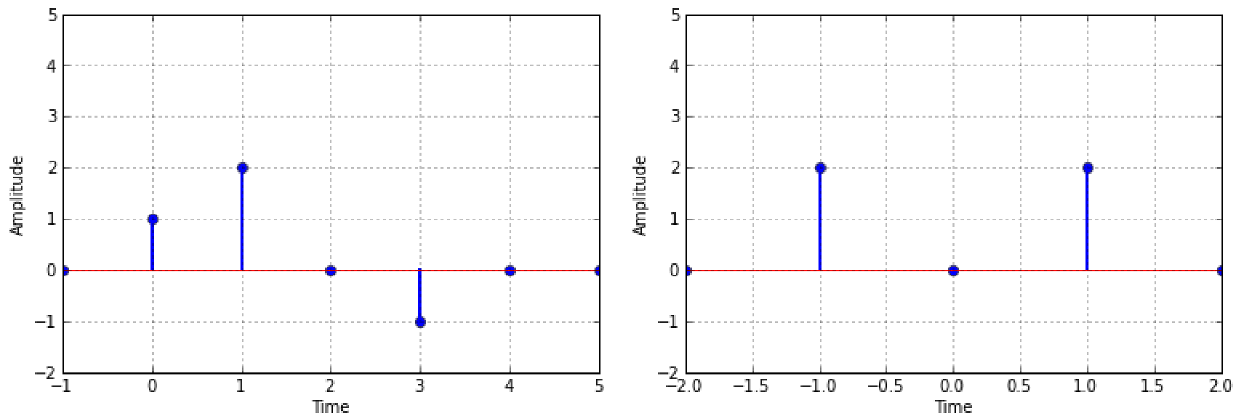
HW - 4 solutions (draft)

Q1)



(a) $x[n]$ (b) $h[n]$

Figure 1: Signals $h[n]$ and $x[n]$

a)

If $y[n]$ is the ouput of the system, we get

$$y[n] = \sum_{k=\infty}^{k=-\infty} x[k]h[n-k]$$

This gives us the following terms for the output

n	$y[n]$[1]
-1	$x[0]h[-1] = 2$
0	$x[1]h[-1] = 4$
1	$x[0]h[1] = 2$
2	$x[1]h[1] + x[3]h[-1] = 2$
3	0
4	$x[3]h[1] = -2$

Plotting this gives

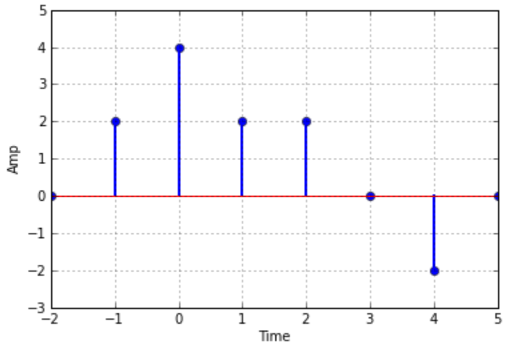


Figure 2: $x[n] * h[n] = y[n]$

b) The system here is a time invariant system hence on advancing/delaying the input, will give a output advanced/delayed by the same amount.

c) Since we know, $x[n] * h[n] = h[n] * x[n]$, we can consider either of them(x or h) to be the impulse response of the system. This gives us $x[n+2] * h[n] = x[n] * h[n+2]$. Hence the plot of this output and the previous sections output will be the same, i.e, advanced by 2.

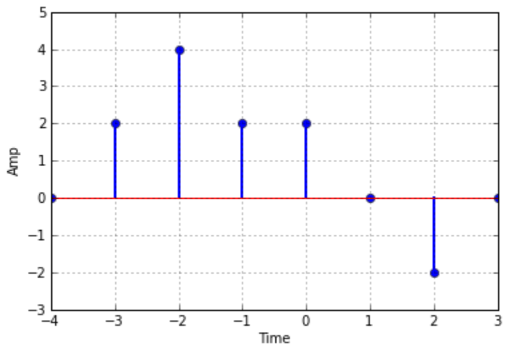


Figure 3: $x[n] * h[n+2] = x[n+2] * h[n] = y[n+2]$

Q2)

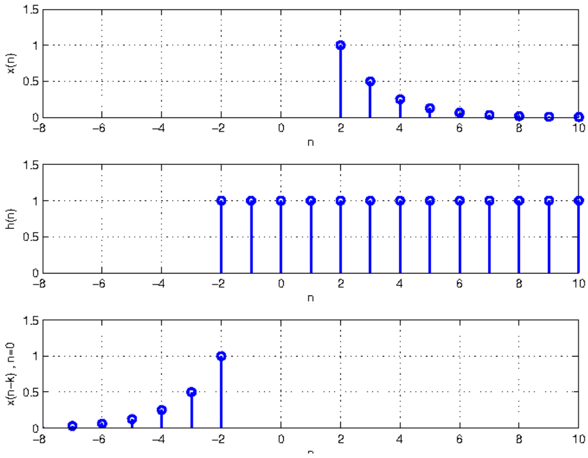


Figure 4: Part of the signals $x(n)$ and $h(n)$

Given,

$$x(n) = \left(\tfrac{1}{2}\right)^{(n-2)} u(n-2)$$

$$\text{and, } h(n) = u(n+2)$$

Then, output $y(n)$ is:

$$y(n) = \textstyle\sum_{-\infty}^{+\infty} x(k)h(n-k)$$

$$= \textstyle\sum_{-\infty}^{+\infty} h(k)x(n-k)$$

For $\mathbf{n < 0}$:

No overlap between the two signals. Therefore, $y(n) = 0$

For $\mathbf{n \geq 0}$:

The overlap is equal to the amount of shift. Say, the current shift $= n$.

$$y(n) = \textstyle\sum_0^n \left(\tfrac{1}{2}\right)^n = 1 + \left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 + \left(\tfrac{1}{2}\right)^3 + ... + \left(\tfrac{1}{2}\right)^n$$

Above is a geometric series, and sum is given by:

$$y(n) = \frac{a(1 - r^{(n+1)})}{1 - r}$$

Substituting, $a = 1$ and $r = \frac{1}{2}$:

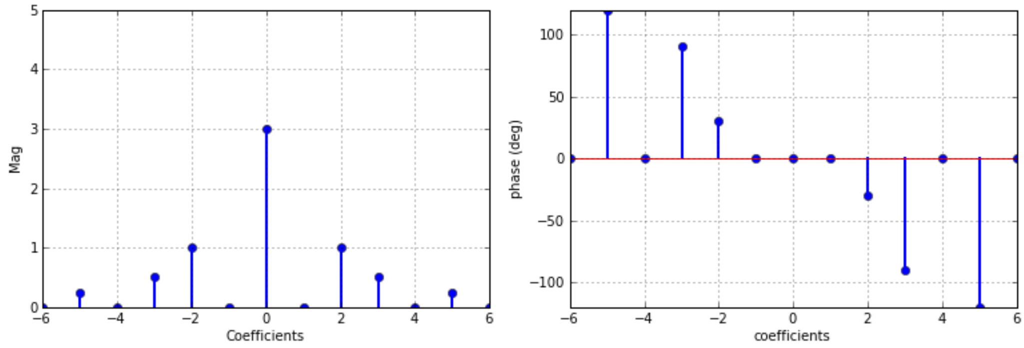
$$y(n) = \frac{1-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}} = \frac{2^{n+1} - 1}{2^n}$$

$$\implies y(n) = \begin{cases} 0, & n < 0 \\ \frac{2^{n+1} - 1}{2^n}, & n \geq 0 \end{cases} \tag{1}$$

Q3)

$$x(t) = 3 + \sqrt{3}cos(2t) + sin(2t) + sin(3t) - \frac{1}{2}cos(5t + \frac{\pi}{3})$$

$$= 3 + 2\left(\frac{\sqrt{3}}{2}cos(2t) + \frac{1}{2}sin(2t)\right) + sin(3t) - \frac{1}{2}cos(5t + \frac{\pi}{3})$$



(a) Magnitude Spectrum (b) Phase Spectrum

Figure 5: Magnitude and Phase plots

$$= 3 + 2cos\left(2t - \frac{\pi}{6}\right) + cos(3t - \frac{\pi}{2}) + \frac{1}{2}cos(5t - \frac{2\pi}{3})$$

The time period of this signal will be 2πs. The following will be amplitudes and phase values at various coefficients

Coefficient	Amplitude	Phase (deg)
0	3	0
2	1	-$\frac{\pi}{6}$
-2	1	$\frac{\pi}{6}$
3	$\frac{1}{2}$	$-\frac{\pi}{2}$
-3	$\frac{1}{2}$	$\frac{\pi}{2}$
5	$\frac{1}{4}$	$-\frac{2\pi}{3}$
-5	$\frac{1}{4}$	$\frac{2\pi}{3}$

Q4)

(a) : Finding fourier series coefficients of saw tooth waveform ($x(t)$).

Time period of the given signal, $T = 1\,$S.

$$
\begin{aligned}
a_k &= A\int_0^1 t\,e^{-jk(2\pi)t}dt \\
&= A\left(\frac{-t\,e^{-jk(2\pi)t}}{jk(2\pi)} + \frac{e^{-jk(2\pi)t}}{k^2(2\pi)^2}\right)_0^1 \\
&= \frac{Aj}{2\pi k},\ \ (for\ \ k \neq 0) \\
|a_k| &= \frac{A}{2\pi k},\ \ (for\ \ k \neq 0) \\
\angle a_k &= \pi/2,\ \ (for\ \ k > 0) \\
\angle a_k &= -\pi/2,\ \ (for\ \ k < 0) \\
a_0 &= A\int_0^1 tdt \\
&= A\left(\frac{t^2}{2}\right)_0^1 \\
&= \frac{A}{2} \\
|a_0| &= \frac{A}{2} \\
\angle a_0 &= 0
\end{aligned}
\tag{2}
$$

The magnitude and phase of the fourier series coefficients is shown in Fig.6a and 6b respectively.

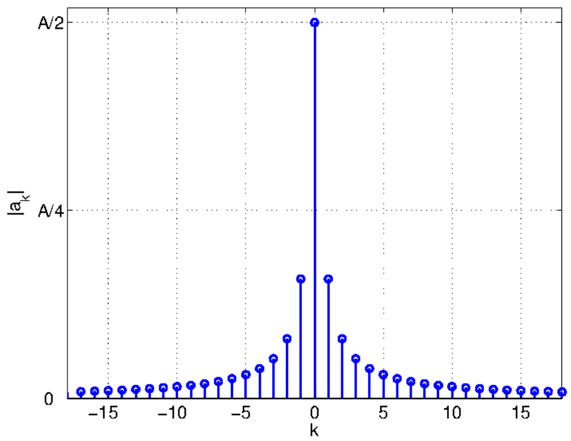
(b) Finding fourier coefficients of $y(t)$.

$$y(t) = x(-t - 0.5) + A$$

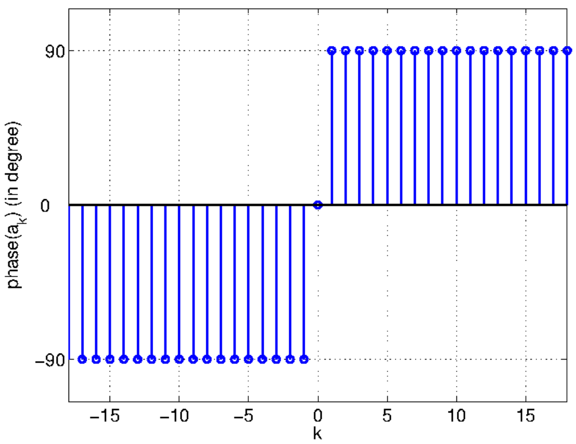
If a_k are the fourier coefficients of $x(t)$, then the fourier coefficients of,

$$
\begin{aligned}
x(t - t_o) &\rightarrow e^{-jk(2\pi/T)t_o}a_k \\
x(-t) &\rightarrow a_{-k}
\end{aligned}
\tag{3}
$$

Using these properties,



(a) Magnitude

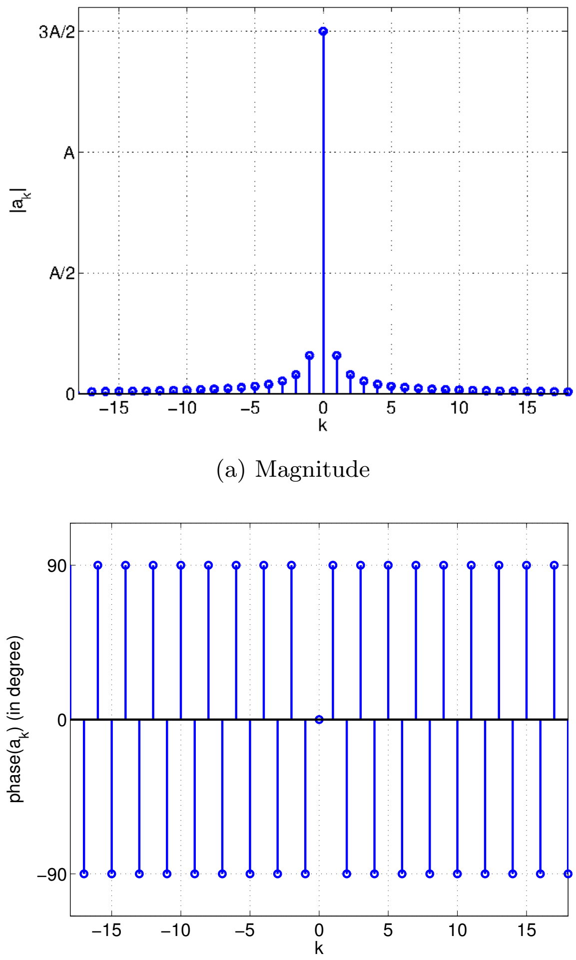


(b) Phase

Figure 6: Magnitude and phase of the fourier coefficients of Q4.(a)

$$\begin{aligned} x(t-0.5) &\rightarrow a_k e^{-jk\pi} \\ x(-t-0.5) &\rightarrow a_{-k} e^{jk\pi} \\ &\rightarrow \frac{-Aj(-1)^k}{2\pi k}, \;\; (for\;\; k \neq 0) \end{aligned} \tag{4}$$

$y(t)$ has a DC shift of A compared to $x(t)$. So, $a_0 = A/2 + A$. The magnitude and phase of the fourier coefficients of $y(t)$ is shown in Fig.7a and 7b respectively.



(a) Magnitude

(b) Phase

Figure 7: Magnitude and phase of the fourier coefficients of Q4.(b)

Q5

a)

The fourier series coefficients, a_k of a periodic signal $x(t)$ whose fundamental period is T_0 is given by, the equation

$$a_k = \frac{1}{T_0}\int_{<T_o>} x(t)e^{\frac{-j2\pi kt}{T_0}}\mathrm{d}t \tag{5}$$

where the integral is over one time period T_0. In the problem, the coefficients can be computed by,

$$\begin{aligned} a_K &= \frac{1}{T_0}\int_{-d/2}^{d/2} Ae^{\frac{-j2\pi kt}{T_0}}\mathrm{d}t \\ &= \frac{A}{\pi k}\sin(\frac{\pi kd}{T_0}) \end{aligned} \tag{6}$$

The above equation is true for $k \neq 0$. For $k = 0$,

$$\begin{aligned} a_0 &= \frac{1}{T_0}\int_{\frac{-d}{2}}^{\frac{d}{2}} A\mathrm{d}t \\ &= \frac{Ad}{T_0}. \end{aligned} \tag{7}$$

Since the given signal is even, the fourier series coefficients are all purely real.

b)

Using 5, the coefficients for $x_2(t)$ is given by, Solving for $k = 1$

$$\begin{aligned} a_1 &= \int_0^{T_0/2} A\sin(\frac{2\pi t}{T_0})e^{\frac{-j2\pi t}{T_0}}\,\mathrm{d}t. \\ &= \frac{A}{4j}. \end{aligned} \tag{8}$$

Solving for $k = -1$

$$\begin{aligned} a_{-1} &= \int_0^{T_0/2} A\sin(\frac{2\pi t}{T_0})e^{\frac{j2\pi t}{T_0}}\,\mathrm{d}t. \\ &= \frac{-A}{4j}. \end{aligned} \tag{9}$$

Solving for other values of k

$$\begin{aligned} a_k &= \frac{1}{T_0}\int_{<T_0>} A.\sin(\frac{2\pi t}{T_0})e^{\frac{-j2\pi kt}{T_0}}\,\mathrm{d}t \\ &= \frac{A}{2jT_0}\int_0^{T_0/2}\left[e^{\frac{j2\pi t(1-k)}{T_0}} - e^{\frac{-j2\pi t(1+k)}{T_0}}\,\mathrm{d}t\right] \\ &= \frac{A}{2j}\left[\frac{e^{j\pi(1-k)}-1}{j2\pi(1-k)} + \frac{e^{-j\pi(1+k)}-1}{j2\pi(1+k)}\right] \\ &= \frac{A}{4\pi}\left[\frac{1-e^{-j\pi(1+k)}}{k+1} + \frac{e^{j\pi(1-k)}-1}{k-1}\right] \\ &= \begin{cases} 0 & \text{k is odd} \\ \frac{A}{\pi(K^2-1)} & \text{k is even} \\ \frac{A}{4j} & \text{k=1} \\ \frac{-A}{4j} & \text{k=-1} \end{cases} \end{aligned} \tag{10}$$

The fourier series coefficients are all real except for $k = 1$ and $k = -1$.

Q6)

$$
\begin{aligned}
D_n &= \frac{1}{T_0}\int_{T_0} f(t)e^{-jn\omega_0 t}\mathrm{d}t \\
D_n &= \frac{1}{2\pi}\left(\frac{1}{A}\int_0^A te^{-jn\omega_0 t}\mathrm{d}t + \int_A^\pi e^{-jn\omega_0 t}\mathrm{d}t\right) \\
&= \frac{1}{2\pi}\left(\frac{1}{A}\left\{\left.\frac{te^{-jn\omega_0 t}}{-jn\omega_0}\right|_0^A - \left.\frac{e^{-jn\omega_0 t}}{(-jn\omega_0)^2}\right|_0^A\right\} + \left.\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right|_A^\pi\right) \\
&= \frac{1}{2\pi}\left(\frac{1}{A}\left\{\frac{Ae^{-jn\omega_0 A}}{-jn\omega_0} + \frac{e^{-jn\omega_0 A}-1}{n^2\omega_0^2}\right\} + \frac{e^{-jn\omega_0 \pi}-e^{-jn\omega_0 A}}{-jn\omega_0}\right) \\
&= \frac{1}{2\pi}\left(\frac{e^{-jnA}-1}{An^2} - \frac{e^{-jn\pi}}{jn}\right) \\
&= \frac{1}{2\pi}\left(\frac{e^{-jnA}-1}{An^2} - \frac{(-1)^n}{jn}\right) \qquad \text{for } n \neq 0 \\
D_0 &= \frac{1}{2\pi}\left(\frac{1}{A}\int_0^A t\mathrm{d}t\right) = \frac{A}{4\pi}
\end{aligned}
$$

Q7)

$$
y\left(t\right) = \sum_{-\infty}^{n=\infty}\left(-1\right)^n c_n e^{j2\pi nt} = \sum_{-\infty}^{n=\infty}\left(e^{j\pi}\right)^n c_n e^{j2\pi nt} = \sum_{-\infty}^{n=\infty} c_n e^{j2\pi nt + jn\pi} = \sum_{-\infty}^{n=\infty} c_n e^{j2\pi n\left(t+\frac{1}{2}\right)} = x\left(t+\frac{1}{2}\right)
$$

Q8)

Given Amplitude=100V, R= 1kΩ, $\omega_0 = 100\pi$ and C=$\frac{50}{\pi}\mu F$ The transfer function of half wave rectifier is

$$
\begin{aligned}
\frac{V_o}{V_s} &= \frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}} \\
\frac{V_o}{V_s} &= \frac{1}{1+j\omega RC} \\
H(\omega) &= \frac{1}{1+\frac{j\omega}{20\pi}}
\end{aligned}
$$

First we find the exponential series representation of half wave rectified output

Calculating the coefficient c_0

$$
\begin{aligned}
c_0 &= \frac{1}{T}\int_{-T/2}^{T/2} 100sin(100\pi t) \ \ \mathrm{d}t \\
&= 100 \times 50 \int_{0}^{T/2} sin(100\pi t) \ \ \mathrm{d}t \\
&= 100 \times 50 \frac{-1}{100\pi} \ cos(100\pi t)|_0^{T/2} \\
&= \frac{100}{\pi}
\end{aligned}
$$

$$
\begin{aligned}
c_k &= \frac{1}{T}\int_{-T/2}^{T/2} 100sin(100\pi t) \ \ e^{-j100k\pi t} \ \ \mathrm{d}t \\
&= 50 \times 100 \int_{0}^{T/2} \frac{e^{j100\pi t} - e^{-j100\pi t}}{2j} \ \ e^{-j100k\pi t} \ \ \mathrm{d}t \\
&= -50j \times 50 \int_{0}^{T/2} e^{j100\pi(1-k)t} - e^{-j100\pi(1+k)t} \ \ \mathrm{d}t \\
&= -50j \times 50 \left[\left. \frac{e^{j(1-k)100\pi t}}{j(1-k)100\pi} \right|_0^{\frac{1}{100}} + \left. \frac{e^{-j(k+1)100\pi t}}{j(k+1)100\pi} \right|_0^{\frac{1}{100}} \right]
\end{aligned}
$$

$$
c_k = \begin{cases} \frac{100}{\pi(1-k^2)} \text{ for even k} \\ 0 \text{ for odd k} \end{cases}
$$

For $k = 1$

$$
\begin{aligned}
c_k &= \frac{1}{T}\int_{-T/2}^{T/2} 100sin(100\pi t) \ \ e^{-j100k\pi t} \ \ \mathrm{d}t \\
c_1 &= 50 \times 100 \int_{0}^{T/2} \frac{e^{j100\pi t} - e^{-j100\pi t}}{2j} \ \ e^{-j100\pi t} \ \ \mathrm{d}t \\
&= 50 \times 100 \int_{0}^{T/2} \frac{1 - e^{-j200\pi t}}{2j} \ \ \mathrm{d}t \\
&= 50 \times -50j \int_{0}^{T/2} (1 - e^{-j200\pi t}) \ \ \mathrm{d}t \\
&= 50 \times -50j \left. \left(t - \frac{e^{-j200\pi t}}{-j200\pi} \right) \right|_0^{T/2} \\
&= -25j
\end{aligned}
$$

Similarly for $k = -1$, $c_{-1} \;=\; 25j$.

As $v_s(t)$ has only first harmonic in the odd harmonics, it can be written as

$$\begin{aligned} v_s &= c_0 + c_1 e^{j100\pi t} + c_{-1} e^{-j100\pi t} + \sum_{m=-\infty}^{\infty} c_k e^{j100(2m)\pi t} \\ &= \frac{100}{\pi} + (-25j) e^{j100\pi t} + 25j e^{-j100\pi t} + \sum_{m=-\infty}^{\infty} \frac{100}{\pi(1-(2m)^2)} e^{j100(2m)\pi t} \end{aligned}$$

If $H(\omega)$ is the transfer function, response for the input $e^{j\omega t}$ is

$$e^{j\omega t} \longrightarrow H(\omega) \times e^{j\omega t}$$

Response to the input $\frac{100}{\pi}$ is $\frac{100}{\pi}$ as $H(0) \;=\; 1$

Response to the input $-25j \times e^{j100\pi t}$ is

$$\begin{aligned} -25j e^{j100\pi t} &\longrightarrow H(100\pi) \times -25j \; e^{j100\pi t} \\ &\longrightarrow \left(\frac{1}{1+j5}\right) \times -25j \; e^{j100\pi t} \\ &\longrightarrow \left(\frac{25(-5-j)}{26}\right) \times e^{j100\pi t} \end{aligned}$$

Response to the input $25j \times e^{-j100\pi t}$ is

$$\begin{aligned} 25j e^{-j100\pi t} &\longrightarrow H(-100\pi) \times 25j \; e^{-j100\pi t} \\ &\longrightarrow \left(\frac{1}{1-j5}\right) \times 25j \; e^{-j100\pi t} \\ &\longrightarrow \left(\frac{25(-5+j)}{26}\right) \times e^{-j100\pi t} \end{aligned}$$

Response to the input $\frac{100}{\pi(1-k^2)} \times e^{j100k\pi t}$

$$\begin{aligned} e^{j100\pi t} &\longrightarrow H(100k\pi) \times \frac{100}{\pi(1-k^2)} \; e^{j100\pi t} \\ &\longrightarrow \left(\frac{1}{1+j5k}\right) \times \frac{100}{\pi(1-k^2)} \; e^{j100k\pi t} \\ &\longrightarrow \left(\frac{100(1-j5k)}{\pi(1-k^2)(1+25k^2)}\right) \times \; e^{j100k\pi t} \end{aligned}$$

Fourier's series representation of $V_0(t)$ with $k = 2m$

$$= \frac{100}{\pi} + \frac{25(5-j)}{26} \times e^{j100\pi t} + \sum_{m=-\infty}^{\infty} \frac{100(1-j10m)}{\pi(1-4m^2)(1+100m^2)} \times e^{j200m\pi t}$$

Magnitude of the 1^{st} harmonic is magnitude of $e^{j100\pi t}$ is

$$\left|\frac{25(5-j)}{26}\right| = \frac{25}{\sqrt{26}}$$

Magnitude of the 2^{nd} harmonic is magnitude of $e^{j100k\pi t}$, when $k = 2$

$$\left|\frac{100(1-j5k)}{\pi(1-k^2)(1+25k^2)}\right| = \frac{100}{3\pi\sqrt{101}}$$

Magnitude of the 3^{rd} harmonic is 0

Q9)

Given $h(t) = e^{-4t}u(t)$

Formula: $H(j\omega) = \int\limits_{-\infty}^{\infty} h(t)e^{-j\omega t}\mathrm{d}t$

$H(j\omega) = \frac{1}{4+j\omega}$

Given $x(t) = \sin(4\pi t) + \cos(6\pi t + \frac{\pi}{4})$

$$x(t) = \left(\frac{e^{j4\pi t} - e^{-j4\pi t}}{2j}\right) + \left(\frac{e^{j(6\pi t+\pi/4)} + e^{-j(6\pi t+\pi/4)}}{2}\right)$$

$$\text{for } x(t) = \frac{e^{j4\pi t}}{2j} \longrightarrow y(t) = \frac{H(j4\pi)}{2j}e^{j4\pi t} = \frac{1}{2(4+j4\pi)}e^{j4\pi t} = \frac{1-j\pi}{8(1+\pi^2)}e^{j4\pi t}$$

$$\text{for } x(t) = \frac{-e^{-j4\pi t}}{2j} \longrightarrow y(t) = \frac{-H(-j4\pi)}{2j}e^{-j4\pi t} = \frac{-1}{2(4-j4\pi)}e^{-j4\pi t} = \frac{-1-j\pi}{8(1+\pi^2)}e^{-j4\pi t}$$

$$\text{for } x(t) = \frac{e^{j\pi/4}e^{j6\pi t}}{2} \longrightarrow y(t) = \frac{e^{j\pi/4}H(j6\pi)}{2}e^{j6\pi t} = \frac{e^{j\pi/4}}{2(4+j6\pi)}e^{j6\pi t} = \frac{(2+3\pi)+j(2-3\pi)}{4\sqrt{2}(4+9\pi^2)}e^{j6\pi t}$$

$$\text{for } x(t) = \frac{e^{-j\pi/4}e^{-j6\pi t}}{2} \longrightarrow y(t) = \frac{e^{-j\pi/4}H(-j6\pi)}{2}e^{-j6\pi t} = \frac{e^{-j\pi/4}}{2(4-j6\pi)}e^{-j6\pi t} = \frac{(2+3\pi)-j(2-3\pi)}{4\sqrt{2}(4+9\pi^2)}e^{-j6\pi t}$$

Complete Fourier series expansion of $y(t)$ for $x(t) = \sin(4\pi t) + \cos(6\pi t + \frac{\pi}{4})$:

$$y(t) = \frac{1-j\pi}{8(1+\pi^2)}e^{j4\pi t} + \frac{-1-j\pi}{8(1+\pi^2)}e^{-j4\pi t} + \frac{(2+3\pi)+j(2-3\pi)}{4\sqrt{2}(4+9\pi^2)}e^{j6\pi t} + \frac{(2+3\pi)-j(2-3\pi)}{4\sqrt{2}(4+9\pi^2)}e^{-j6\pi t}$$

Q10)

$$H_l(\omega) = \begin{cases} 1 & |\omega| \leq 2\pi.500 \text{ rad/s} \\ 0 & \text{otherwise} \end{cases}$$

a)

$$x(t) = cos(2\pi 750t) + sin(2\pi 1500t)$$

$x(t)$ comprises of two frequences 750Hz and 1500Hz which are greater than the cut off frequency(500Hz). Hence the response will be 0.

b) $x(t)$ is a square wave oscillating between +1V and -1V with period 4.5ms and 50% duty cycle. Using the fourier series expansion, $x(t)$ can be written as

$$x(t) = \sum_k C_k e^{jk\omega_o t}$$

where $\omega_o = \frac{2\pi}{T} = 2\pi 222.22$ rad/s.

$$\begin{aligned}
C_0 &= \frac{1}{T}\int_{-T/4}^{3T/4} x(t)\mathrm{d}t = 0 \\
C_k &= \frac{1}{T}\int_{-T/4}^{3T/4} x(t)e^{-jk\omega_o t}\mathrm{d}t \\
&= \frac{1}{T}\int_{-T/4}^{T/4} e^{-jk\omega_o t}\mathrm{d}t + \frac{1}{T}\int_{T/4}^{3T/4} -e^{-jk\omega_o t}\mathrm{d}t \\
&= \frac{1}{T}\left[\frac{e^{jk\omega_o T/4} - e^{-jk\omega_o T/4}}{jk\omega_o} + \frac{e^{-jk\omega_o 3T/4} - e^{-jk\omega_o T/4}}{jk\omega_o}\right] \\
&= \frac{e^{jk\pi/2} - e^{-jk\pi/2}}{jk2\pi} + \frac{e^{-jk3\pi/2} - e^{-jk\pi/2}}{jk2\pi} \\
&= \frac{e^{jk\pi/2} - e^{-jk\pi/2}}{jk\pi} = \frac{2}{k\pi}\sin(k\pi/2)
\end{aligned}$$

$x(t)$ contains only odd harmonics. Frequency of third harmonic of $x(t)$ = 666.67Hz. As the cutoff frequency is 500Hz, only the first harmonic remains on filtering. The response of the filter will be

$$\begin{aligned}
y(t) &= \frac{2}{\pi}e^{-j2\pi 222.22\times t} + \frac{2}{\pi}e^{j2\pi 222.22\times t} \\
&= \frac{4}{\pi}\cos(2\pi 222.22 \times t)
\end{aligned}$$

Q11)

$$h(t) = \delta(t) - e^{-t}u(t)$$

$$\begin{aligned}
H(\omega) &= \int_{-\infty}^{\infty} h(\tau)e^{-j\omega\tau}d\tau \\
&= \int_{-\infty}^{\infty} \delta(\tau)e^{-j\omega\tau}d\tau - \int_{0}^{\infty} e^{-\tau}e^{-j\omega\tau}d\tau \\
&= 1 - \frac{1}{1+j\omega} = \frac{j\omega}{1+j\omega}
\end{aligned}$$

a.

$$x(t) = cos\,(3\pi t) + \frac{\pi}{3}$$

By Eigen function property

$$
\begin{aligned}
e^{jn\omega_o t} &\longrightarrow H(n\omega_o)e^{jn\omega_o t} \\
\frac{\pi}{3} &\longrightarrow H(0) = 0 \\
cos(3\pi t) &= \frac{1}{2}\left(e^{j3\pi t} + e^{-j3\pi t}\right) \\
e^{j3\pi t} &\longrightarrow H(3\pi)e^{j3\pi t} \\
&\longrightarrow \frac{j3\pi}{1+j3\pi}e^{j3\pi t} \\
e^{-j3\pi t} &\longrightarrow H(-3\pi)e^{-3\pi t} \\
&\longrightarrow \frac{-j3\pi}{1-j3\pi}e^{-j3\pi t} \\
cos(3\pi t) &\longrightarrow \frac{1}{2}\left(\frac{j3\pi}{1+j3\pi}e^{j3\pi t} + \frac{-j3\pi}{1-j3\pi}e^{-j3\pi t}\right) \\
x(t) &\longrightarrow \frac{3\pi}{1+9\pi^2}\left(3\pi\cos(3\pi t) - \sin(3\pi t)\right) \\
y(t) &= \frac{3\pi}{1+9\pi^2}\left(3\pi\cos(3\pi t) - \sin(3\pi t)\right)
\end{aligned}
$$

b

$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

$$T = 1, \qquad \omega_o = 2\pi$$

$$
\begin{aligned}
x(t) &= \sum_{k=-\infty}^{\infty} c_k e^{-jk\omega_o t} \\
c_k &= \frac{1}{T}\int_{-T/2}^{T/2} \delta(t)e^{-jk\omega_o t}dt \\
&= \frac{1}{T} = 1
\end{aligned}
$$

$$
\begin{aligned}
\Longrightarrow y(t) &= \sum_{k=-\infty}^{\infty} c_k \, \frac{jk\omega}{1+jk\omega} \, e^{jk\omega t} \\
&= \sum_{k=-\infty}^{\infty} \frac{jk2\pi}{1+jk2\pi} \, e^{jk2\pi t}
\end{aligned}
$$

c.

$$
\begin{aligned}
x(t) &= \sum_{n=-\infty}^{\infty} (-1)^n \delta(t-2n), \qquad \omega = \frac{2\pi}{4} \\
c_k &= \frac{1}{T} \int_{-0.5}^{3.5} \sum_{n=-\infty}^{\infty} (-1)^n \delta(t-2n) e^{-jk\omega t} \mathrm{d}t \\
&= \frac{e^{-jk\omega\times 0} - e^{-jk\omega\times 2}}{4} \\
&= \frac{1-e^{-jk\pi}}{4}
\end{aligned}
$$

$$
\begin{aligned}
c_k &= \begin{cases} 0, & k \text{ is even} \\ 0.5, & k \text{ is odd} \end{cases} \\
H(j\omega) &= \frac{j\omega}{1+j\omega}
\end{aligned}
$$

$$
\implies y(t) = \sum_{k=-\infty}^{\infty} c_k \, \frac{jk\omega}{1+jk\omega} \, e^{jk\omega t}
$$

As c_k is 0 for even k,

$$
y(t) = \sum_{m=-\infty}^{\infty} \frac{j(2m+1)\pi/4}{1+j(2m+1)\pi/2} e^{j(2m+1)\pi t/2}
$$