EC2102 Networks and Systems – HW 3

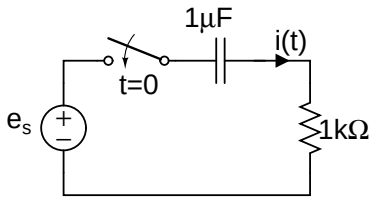
August 23, 2012

1. Perform the following convolutions where $\star$ indicates convolution.
   (a) For $u(t)$ a unit step function, find $r(t) = u(t) \star u(t)$.
   (b) Find $x(t) \star h(t)$, where $h(t) = (-e^{-t} + 2e^{-2t})u(t)$ and $x(t) = 10e^{-3t}u(t)$.
   (c) Find the output $y(t)$ of an LTI system with impulse response $h(t) = 2e^{-2t}u(t)$ when excited with an input $x(t)$ given by
   $$x(t) = \begin{cases} 1 & 2 \le t \le 4 \\ 0 & \text{otherwise} \end{cases}$$
   (d) Sketch $y(t) = [u(t) \star u(t-2)]\, u(4-t)$.
   (e) Determine graphically $h(t) = f(t) \star g(t)$, where (1) $f(t) = u(-t)$ and $g(t) = 2(u(t) - u(t-1))$, and (2) $f(t) = r(t) - r(t-2)$ and $g(t) = u(t-3) - u(t-6)$.

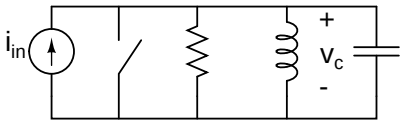
2. Let $y(t) = x(t) \star h(t)$. $x(t)$ is non-negative for $t \in (2,3)$ and zero elsewhere, and is symmetric about $t = 5/2$. $h(t) = 1$ for $t \in (3,4)$ and zero elsewhere.
   (a) During what times will the values $y(t)$ be non-zero?
   (b) At what time(s) will $y(t)$ achieve its maximum value.

3. Consider a system with input $x(t)$ and output $y(t)$ related by:
   $$y(t) = \int_{-\infty}^{t+1} \sin(t-\tau)x(\tau)\, d\tau.$$
   (a) What is the system impulse response?
   (b) Is the system time-invariant? Prove.
   (c) Is the system causal?

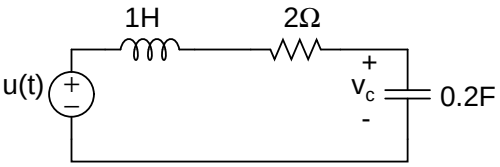
4. In the circuit shown below $v_c(0^-) = 1V$ and $e_s(t) = 30\cos 2\pi 10^3 t$ V is applied at $t = 0$. The output is $i(t)$. Find the zero-state and zero-input response of the circuit.



5. For the parallel RLC circuit shown below the switch is opened at $t = 0$. The natural frequency $\omega_o = 10 rad/s$, $Q = 1$ and $C = 1F$. The output of the circuit is v_c and $i_{in}(t) = 2A$. The initial conditions are $V_c(0) = 2V$ and $i_L(0) = 5A$. Find the zero-input and zero-state response of the circuit.



6. For the circuit shown below, find the step and impulse response. The output is $v_c(t)$.



7. The impulse response to an LTI circuit is given as
   $$h(t) = 2e^{-t}, \quad 0 \le t < 3$$
   $$= 0 \quad t \ge 3$$
   Find the zero state response to an input
   $$i(t) = 4u(t), \quad 0 \le t < 2$$
   $$= 0 \quad t \ge 2$$

8. The impulse response of a circuit is $h(t) = e^{-2t}u(t)$. Find the response to $x(t) = \cos 4\pi t$.