HW - 1 solutions

Q.1)

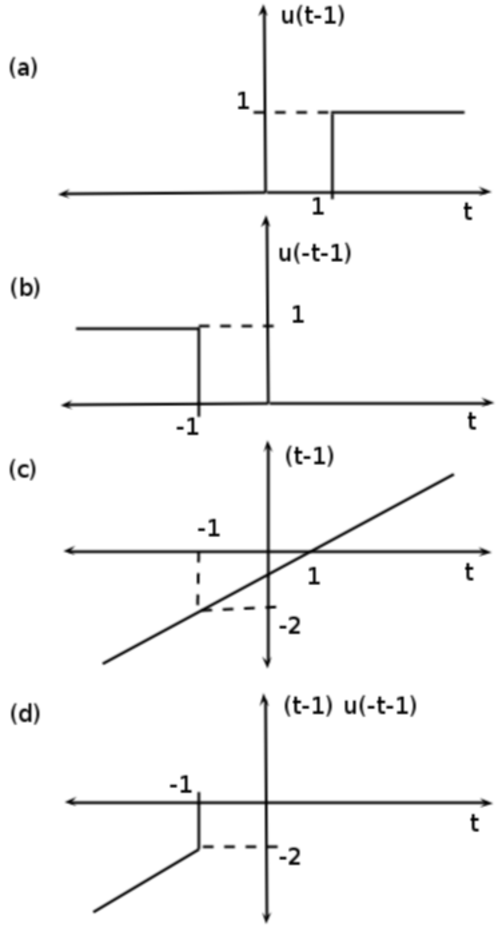


Figure 1: Sketches for question-1.

$$
\begin{aligned}
g(t) = &\quad tu(-t-1) - u(-t-1) \\
= &\quad (t-1)u(-t-1) \qquad (1)
\end{aligned}
$$

The given $g(t)$ can also be written as shown in eqn. 1, i.e., $g(t)$ is the product of the signals $(t-1)$ and $u(-t-1)$. The figures drawn alongside are explained below:

(a) $u(t)$ is shifted by 1 unit to the right to give $u(t-1)$.

(b) $u(t-1)$ in (a) is inverted to give $u(-t-1)$

(c) the signal $y(t) = t$ is shifted to the right by 1 unit to give $(t-1)$.

(d) The signals in (b) and (c) are then multiplied to give the required $g(t)$.

Q.2)

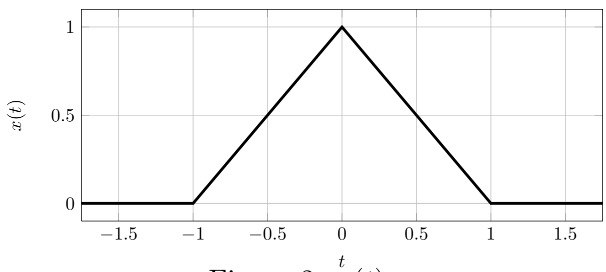


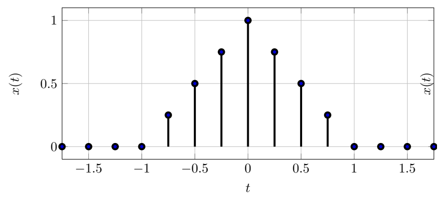
Figure 2: $x(t)$

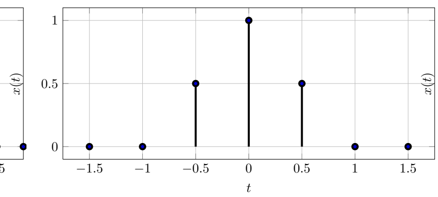
Sampling points are

(a)-1, -0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75, 1s

(b)-1, -0.5, 0, 0.5, 1s

(c)-1, 0, 1s





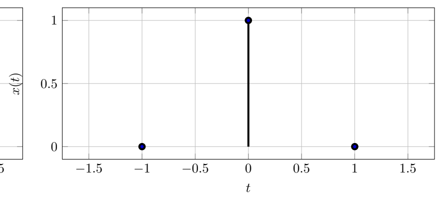


Figure 3: Q2.a)

Figure 4: Q2.b)

Figure 5: Q2.c)

Q.3)

Question 3a) translates the signal $x(t)$ to the right by 4 units.

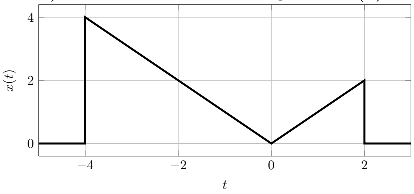


Figure 6: Q3) $x(t)$

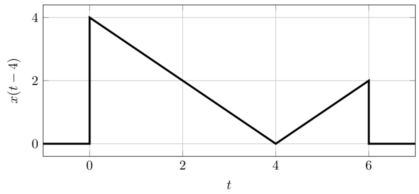


Figure 7: Q3.a) $x(t-4)$

Question 3b) expands the signal in time domain by a factor of 1.5 and 3c) is time reversal of the signal.

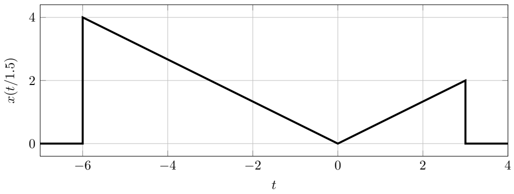


Figure 8: Q3.b) $x(t/1.5)$

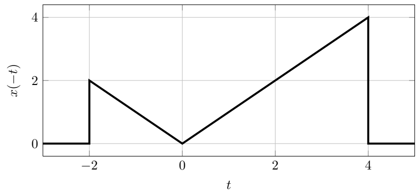


Figure 9: Q3.c) $x(-t)$

3d) is a scaled version of the translated signal in 3a). 3e) is a translated version of solution from 3c) by (-2) units.

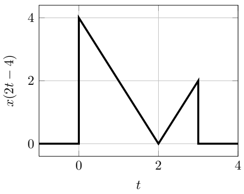


Figure 10: Q3.d) $x(2t-4)$

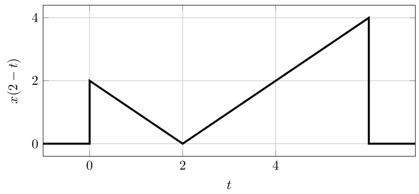


Figure 11: Q3.e) $x(2-t)$

Q.4)

$$y(t) = \frac{1}{5}x(-2t-3)$$

Changing the variable $(-2t-3)$ with τ,

$$x(\tau) = 5 \times y\left(-\frac{\tau}{2} - \frac{3}{2}\right)$$

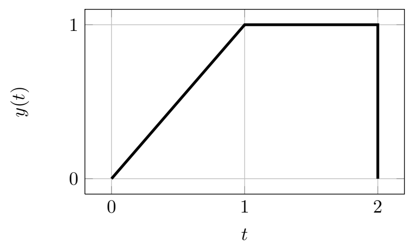


Figure 12: Q4) $y(t)$

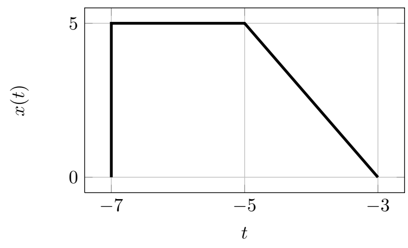


Figure 13: Q4.a) $x(t) = 5 \times y\left(-\frac{t}{2} - \frac{3}{2}\right)$

Odd portion of $y(t)$, $y_o(t) = \frac{y(t)-y(-t)}{2}$

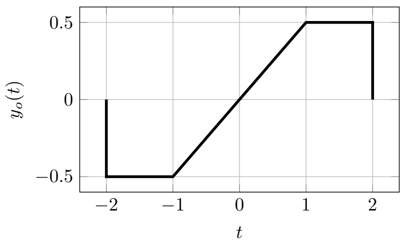


Figure 14: Q4.b) $y_o(t)$

Q.5)

$$f(t) = Ae^{st}$$

In the above equation A and s can be complex numbers. s is complex frequency because it has dimension $/sec$.

(a) $\cos 3t = \frac{1}{2}(e^{j3t} + e^{-j3t})$ the complex frequencies are $+j3, -j3$

(b) $-3 + j3, -3 - j3$

(c) $2 + j3, 2 - j3$

(d) -2

(e) 2

(f) $5 = 5e^{j0t}$ the complex frequency is 0

Q.6)

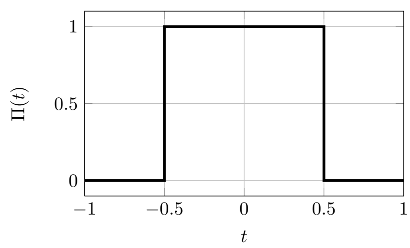


Figure 15: Q6) $\Pi(t)$

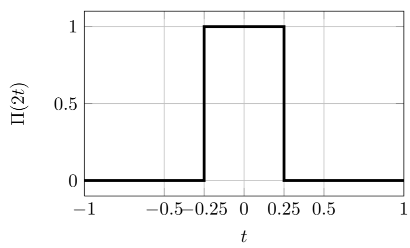


Figure 16: Q6.a) $\Pi(2t)$

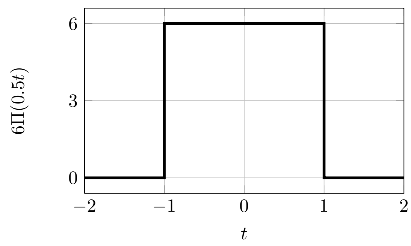


Figure 17: Q6.b) $6\Pi(0.5t)$

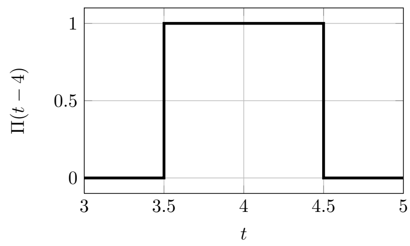


Figure 18: Q6.c) $\Pi(t-4)$

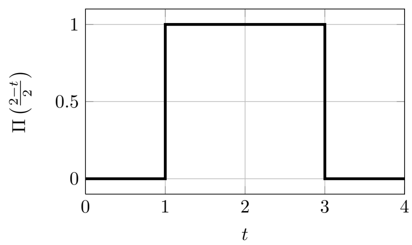


Figure 19: Q6.d) $\Pi\left(\frac{2-t}{2}\right)$

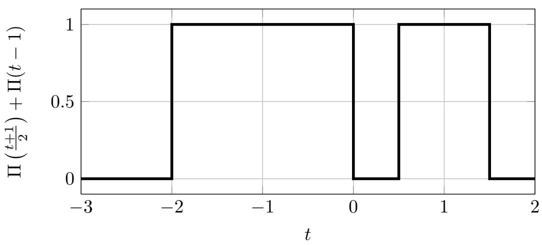


Figure 20: Q6.e) $\Pi\left(\frac{t+1}{2}\right)+\Pi(t-1)$

Q.7)

For a sinusoidal signal with amplitude α the average power in the signal is $\alpha^2/2$.

(a) Fundamental period $= 2\,\text{s}$, Average power $= 0.5\,\text{W}$

(b) Fundamental period $= 0.2\,\text{s}$, Average power $= A^2/2\,\text{W}$

(c) Fundamental period $= 2/\sqrt{3}\,\text{s}$, Average power $= 0.5\,\text{W}$

(d) $\exp jt = cos(t) + jsin(t)$

Fundamental period $= 2\pi\,\text{s}$, Average power $= 1\,\text{W}$

(e) Fundamental period $= 0.5\,\text{s}$, Average power $= A^2/2\,\text{W}$. (The π given in the sine argument is only a phase shift.)

f At any instant of time, there are 5 rectangular pulses being added up. The summation gives a DC signal with 5 being its amplitude. Average power $= 25\,\text{W}$.

g The signal is shown in Fig.. This signal has a fundamental period of $2\,\text{s}$ and an average power of $0.5\,\text{W}$.

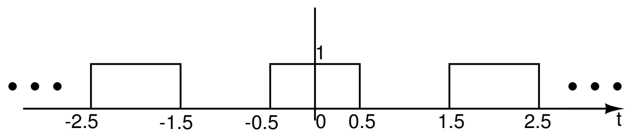


Figure 21: Q7.g