Note Title

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Cepstral Analysis:

In 1963, Bogert, Healy, & Tukey published a paper with an unusual title:

B. P. Bogert, M. J. R. Healy, and J. W. Tukey: "The Quefrency Alanysis of Time Series for Echoes: Cepstrum, Pseudo Autocovariance, Cross-Cepstrum and Saphe Cracking". Proceedings of the Symposium on Time Series Analysis (M. Rosenblatt, Ed) Chapter 15, 209-243. New York: Wiley, 1963.

In that paper they coined many unusual terms, such as,

Cepstrum (Spectrum), Alanysis (Analysis), Liftering (Filtering),

Quefrency (Frequency), Rahmonics (Harmonics), Saphe (Phase)

Some of these have stood the test of time.

They were studying the problem of signal + echoes and observed that the log spectrum contained the log signal spectrum + a periodic component. Hence the Fourier analysis of the log spectrum could highlight the periodic component present and could lead to a new indicator of occurrence of an echo. In that paper they remark,

> "We find ourselves operating on the frequency side in ways customary on the time side and vice-versa."

Example:

Consider the simple single echo case:

$$x[n] = v[n] + \alpha\, v[n-n_0]$$

$$= v[n] * \left(\delta[n] + \alpha\,\delta[n-n_0]\right)$$

$$X(e^{j\omega}) = V(e^{j\omega}) \cdot \left[1 + \alpha e^{-j\omega n_0}\right]$$

$$|X(e^{j\omega})| = |V(e^{j\omega})| \sqrt{1+\alpha^2 + 2\alpha \cos \omega n_0}$$

$$\log|X(e^{j\omega})| = \log|V(e^{j\omega})| + \frac{1}{2}\log\left(1+\alpha^2+2\alpha\cos\omega n_0\right)$$

Since $\omega = 2\pi f$, we get

$$\underbrace{\log|X(e^{j2\pi f})|}_{C_x(e^{j2\pi f})} = \underbrace{\log|V(e^{j2\pi f})|}_{\text{solely due to signal}} + \underbrace{\frac{1}{2}\log\left(1+\alpha^2+\cos 2\pi f n_0\right)}_{\text{due to echo}}$$

$C_x(e^{j2\pi f})$: continuous signal with independent variable 'f'

The component due to the echo is periodic in 'f' with period $\frac{1}{n_0}$

[Note that, since $X(e^{j2\pi f})$ is the DTFT of a sequence, it is periodic in 'f' with period 1 also.]