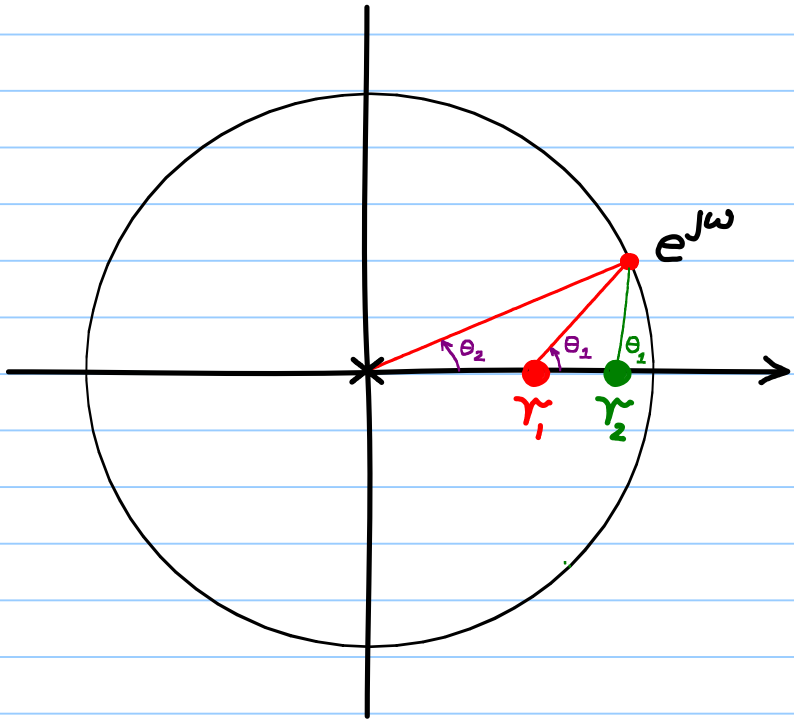
Note Title 24-09-2013

EE5330 Sep. 24, 2013



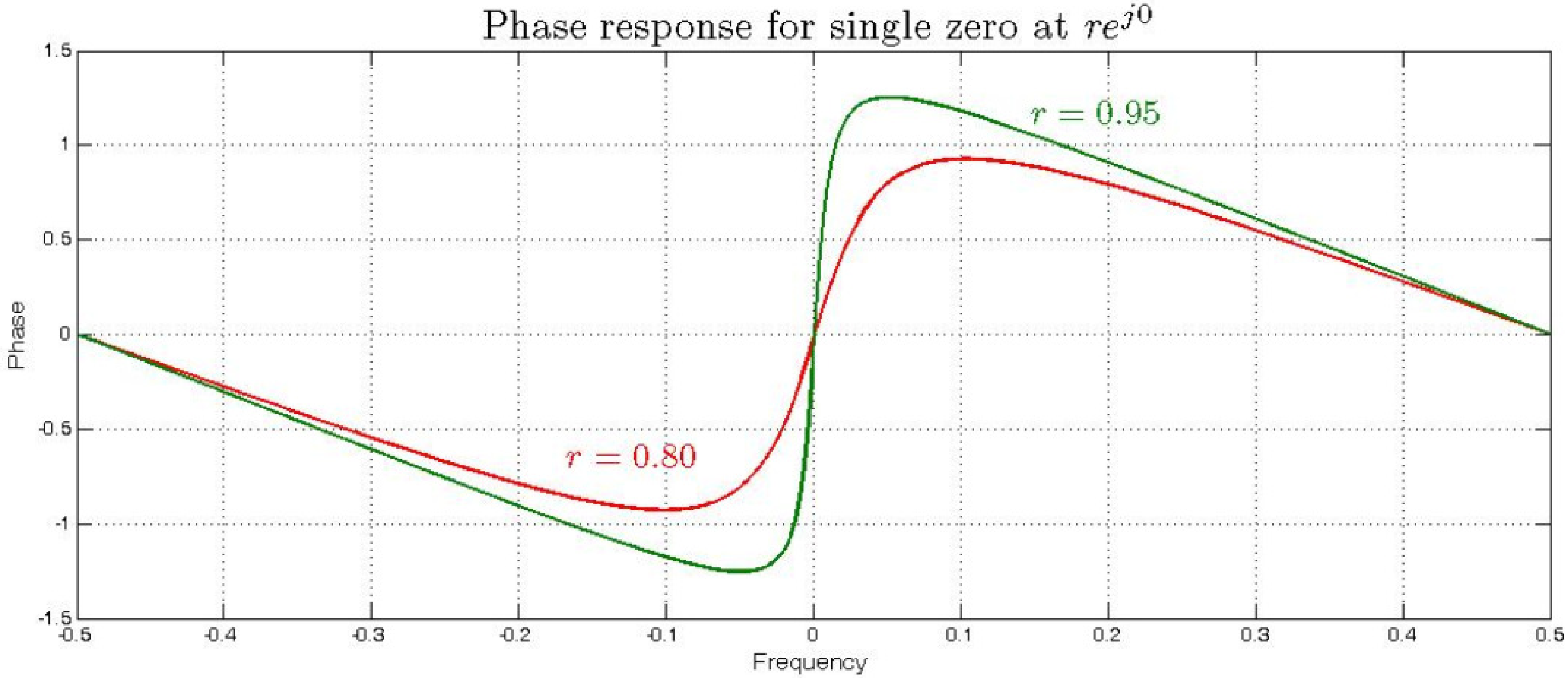
For $0 < r_1 < r_2 < 1$ the shape of the phase response is shown below.

At $\omega = 0$, $\theta_1 = \theta_2 = 0 \Rightarrow \theta_1 - \theta_2 = 0$

At $\omega = \pi$, $\theta_1 = \theta_2 = \pi \Rightarrow \theta_1 - \theta_2 = 0$

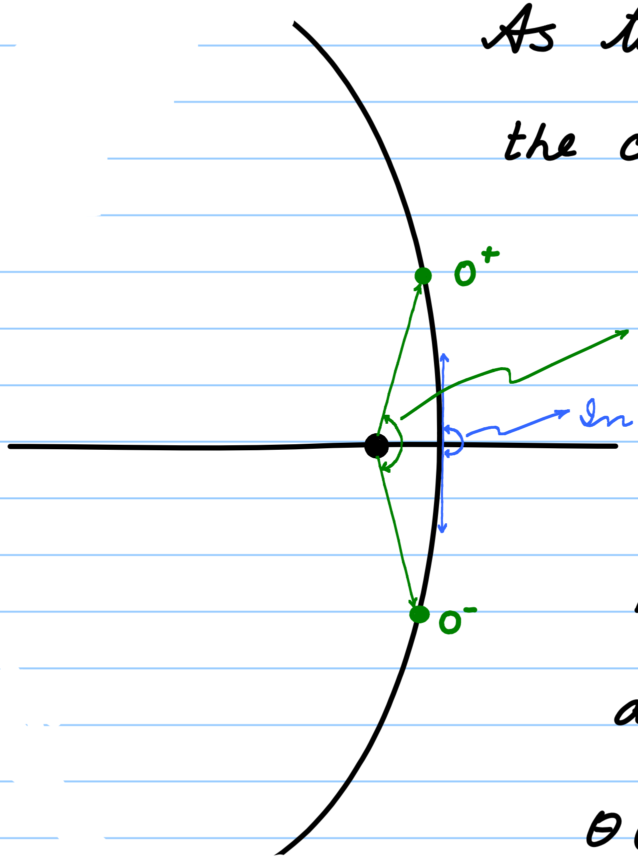
Just beyond $\omega = 0$, θ_1 increases more rapidly than $\theta_2 \Rightarrow \theta_1 - \theta_2 > 0$.

Hence the final shape of the phase response is as follows:



If $\theta \neq 0$, the curve will be centred at $\omega = \theta$ rather than at 0.

As the zero tends towards the unit circle, the change in angle tends towards π :



As r tends to 1, the change in angle tends to π

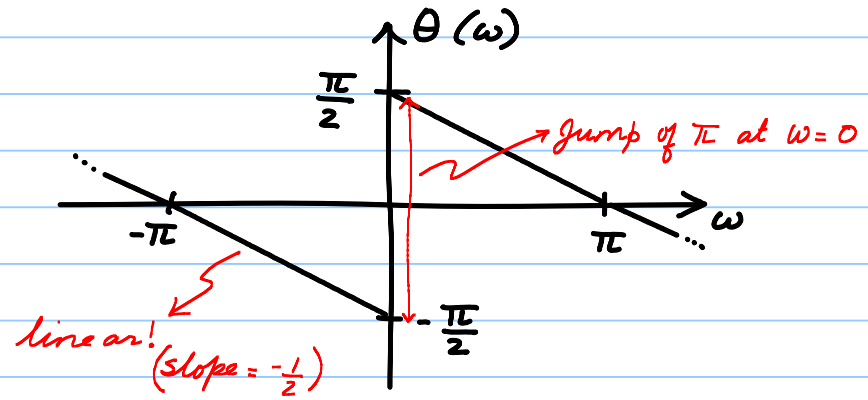
In the limit as $r \to 1$ the change in angle equals $\boxed{\pi}$

For $r = 1$, the expression for the phase angle becomes

$$\theta(\omega) = \tan^{-1} \frac{\sin \omega}{1 - \cos \omega} = \tan^{-1}\left[\tan\left(\frac{\pi}{2} - \frac{\omega}{2}\right)\right]$$

Hence,

$$\theta(\omega) = \begin{cases} \frac{\pi}{2} - \frac{\omega}{2} & \omega > 0 \\ -\frac{\pi}{2} - \frac{\omega}{2} & \omega < 0 \end{cases}$$



Important Features:

(a) Phase jump of π at $\omega = 0$

(b) Phase is linear

(c) Slope of the linear phase part is $-\frac{1}{2}$

If $r = 1$ and $\theta \neq 0$, jump of π will occur at $\omega = \theta$

Slope will still be linear with value unchanged from $-\frac{1}{2}$

Any collection of zeros on the unit circle will give rise to an overall phase response that is LINEAR with jumps of π occuring at the locations of the zeros. The slope of the linear region equals $-\frac{N}{2}$. If these zeros occur in complex conjugate pairs, the overall response will be odd symmetric.

Let there be a zero at $\omega = \theta$ on the unit circle.

Let the frequency response be equal to H_1 at $\omega = \theta^-$

Consider the frequency response at $\omega = \theta^+$; let it be H_2.

The change in $H(e^{j\omega})$ due to all the other poles and zeros will be negligible because of the negligible change in both the distances as well as angles. The only change will be contributed by the zero at $\omega = \theta$. This zero contributes a phase change of π. Hence, $H_2 = H_1 \cdot e^{j\pi} = -H_1$

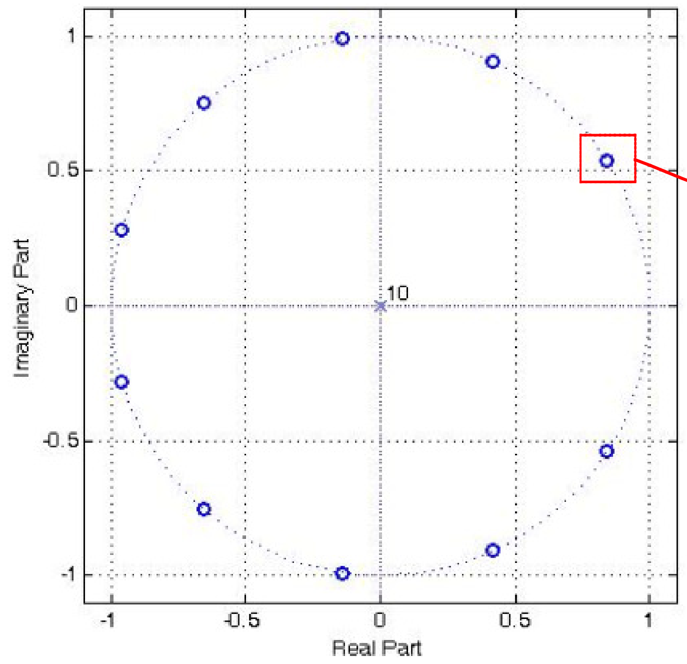
Hence, crossing a first order zero on the unit circle causes a sign change in the frequency response.

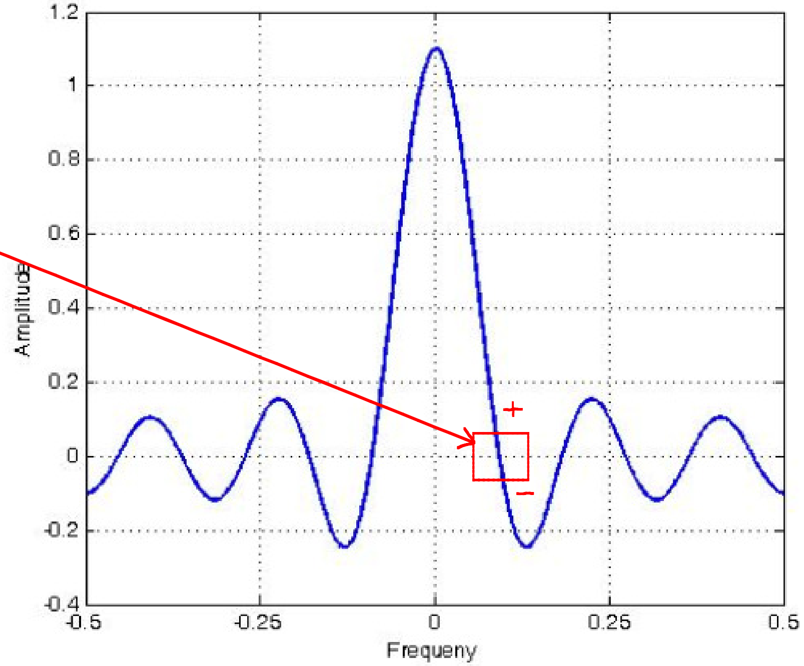
Example

$$h[n] = 1 \qquad -N \leq n \leq N$$

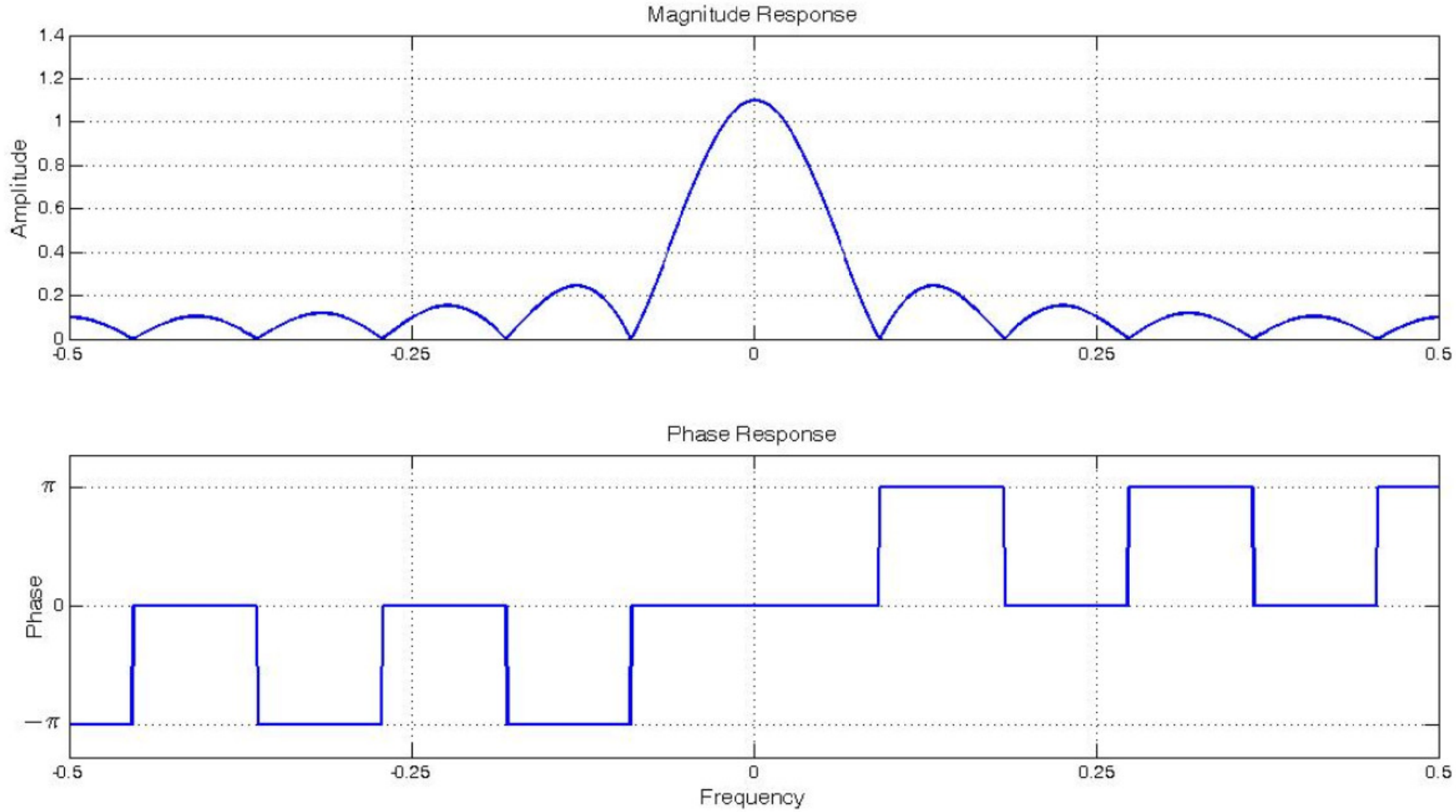
$$H(e^{j\omega}) = \frac{\sin (2N+1)\omega/2}{\sin \omega/2}$$

Crossing each zero introduces a sign change!





If the above frequency response is plotted as two separate plots, i.e., as magnitude and phase plots, the plots will be as shown below:



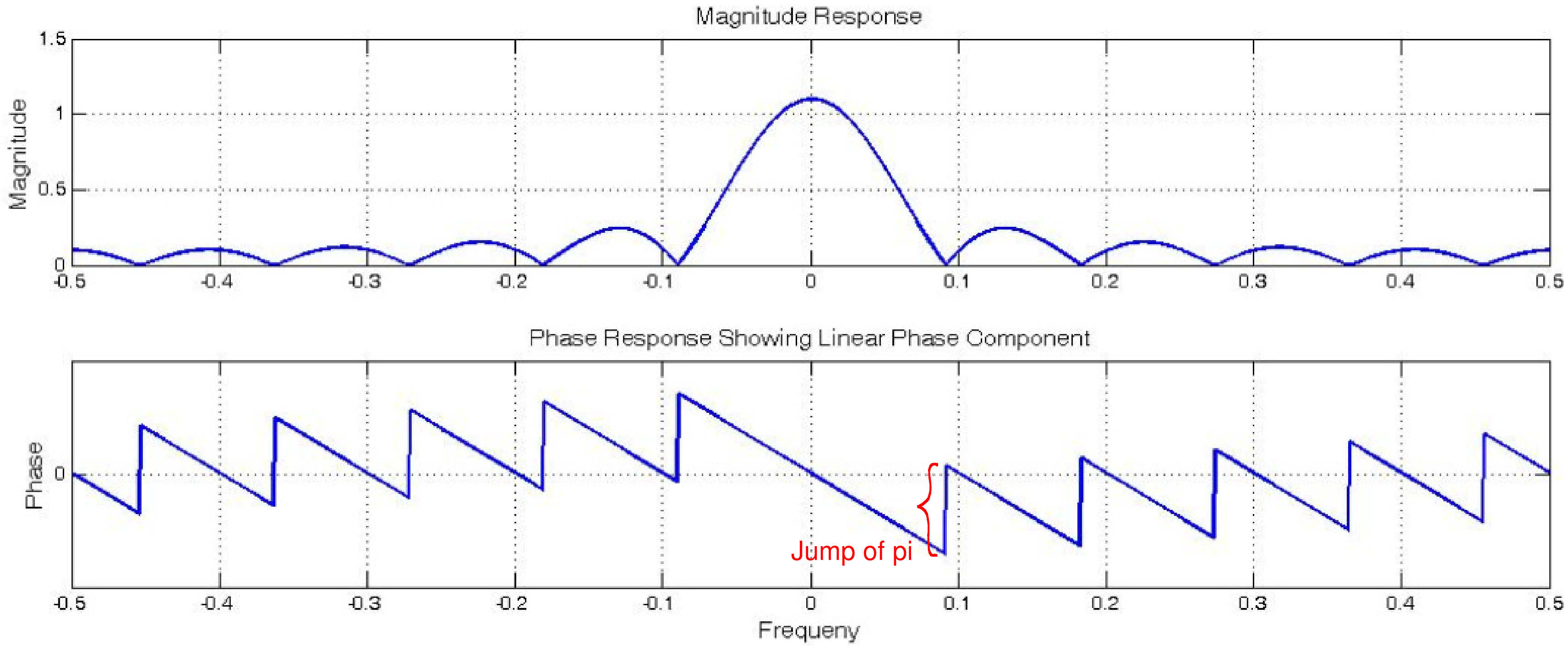
Magnitude Response
Amplitude
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.5
-0.25
0
0.25
0.5
Phase Response
Phase
π
0
$-\pi$
-0.5
-0.25
0
0.25
0.5
Frequency

By convention, a sign change is shown as a phase change of π (rather than $-\pi$) for $\omega > 0$.

If $h[n] = 1 \quad 0 \le n \le 2N$, $\quad H(e^{j\omega}) = e^{-j\omega N/2} \dfrac{\sin (2N+1)\omega/2}{\sin \omega/2}$

The magnitude plot remains unchanged.

The phase plot acquires a linear phase term with slope equals $-\frac{N}{2}$. The new magnitude and phase plots are shown below.


Magnitude Response
Magnitude
1.5
1
0.5
0
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
Phase Response Showing Linear Phase Component
Phase
0
Jump of pi
Frequeny

When we cross a first order zero on the unit circle, we acquire a phase change of π. If we cross an N^{th} order zero, we acquire a phase change of $N\pi$.

Hence, crossing a 2^{nd} order zero causes a phase change of 2π, which causes no sign change!

Exercise:

Plot the magnitude and phase plots of $g[n] = h[n] * h[n]$ where $h[n]$ is as shown before. Examine behaviour around zero crossings. What is the slope of the frequency response at the zero locations?