Note Title — EE5330 Sep. 17, 2013 — 17-09-2013

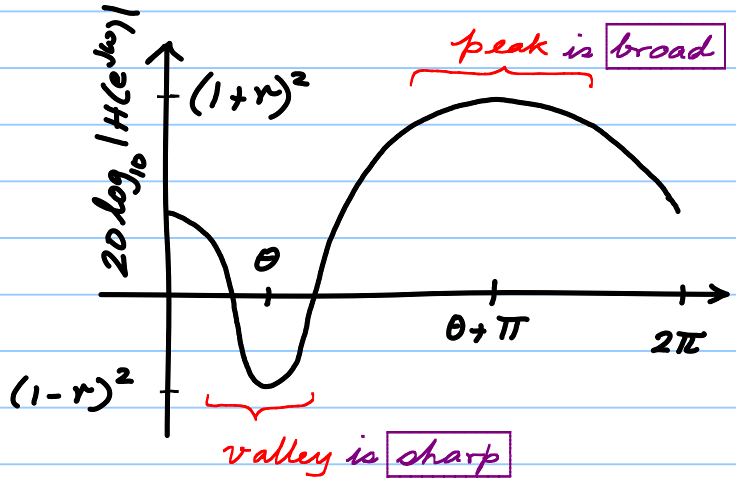
Response of a single complex zero:

Let $H(z) = 1 - re^{j\theta}z^{-1}$

$$|H(e^{j\omega})|^2 = |1 - re^{j\theta}e^{-j\omega}|^2$$

$$= 1 + r^2 - 2r\cos(\omega - \theta)$$

replacing 'ω' by '$-\omega$' gives a different response as $h[n]$ is complex-valued, except when $\omega = 0$ and $\omega = \pi$



Minimum occurs at $\omega = \theta$; $|H(e^{j\omega})|^2_{min} = (1-r)^2$

Maximum occurs at $\omega = \theta + \pi$; $|H(e^{j\omega})|^2_{max} = (1+r)^2$

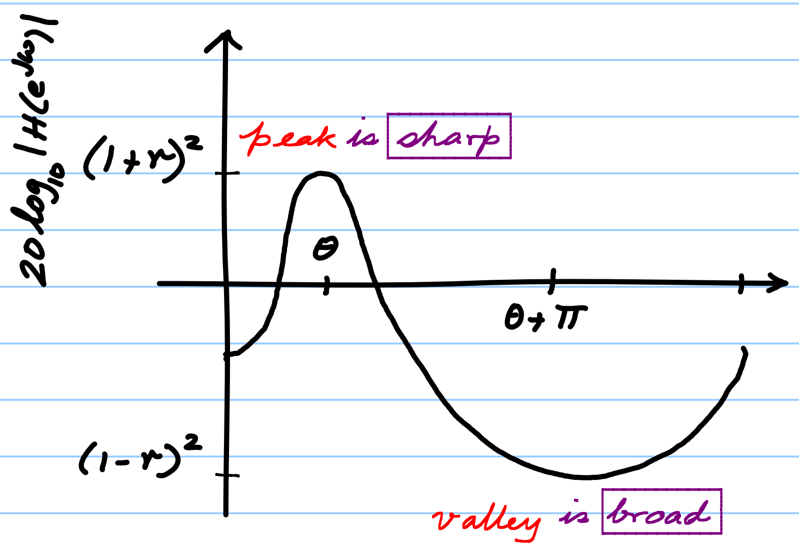
For $r = 0.9$, $|H(e^{j\omega})|^2_{min} = 0.01$

$$|H(e^{j\omega})|^2_{max} = 3.61$$

For a single complex pole,

$$H(z) = \frac{1}{1 - re^{j\theta}z^{-1}}$$

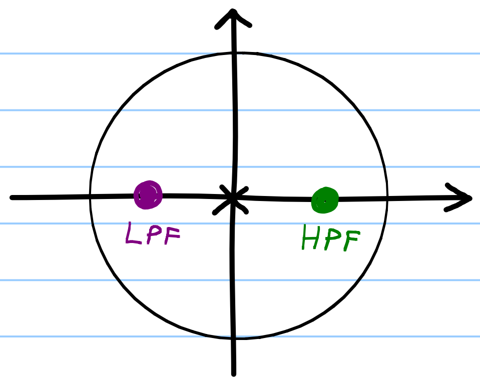
$\Rightarrow$ the log plot of $|H(e^{j\omega})|^2$ is the negative of the previous plot

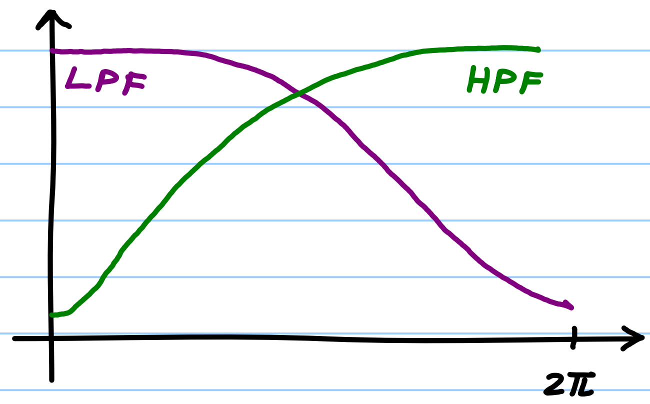


Pole near the unit circle boosts the frequency response

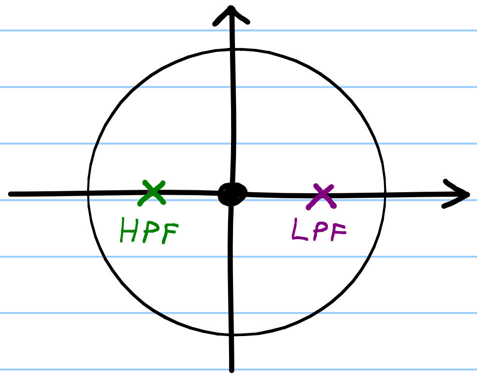
Zero near the unit circle attenuates the frequency response

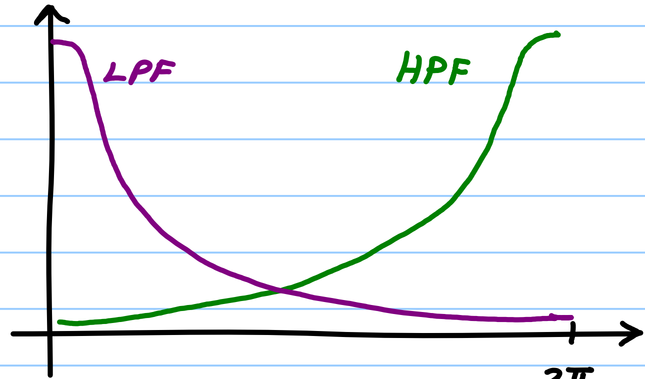
Lowpass and Highpass filters realized using single complex zero :





Lowpass and Highpass filters realized using single complex pole:





The main difference between an LPF realized using a pole versus another realized using a zero is the narrowness of the passband.

Zeros cause sharp valleys and broad peaks in the frequency response
Poles cause sharp peaks and broad valleys in the frequency response

Consider the following two LPFs:

(1) Realized Using a Pole:

$$H(z) = \frac{1}{1 - a z^{-1}} \qquad 0 < a < 1$$

$$|H(e^{j0})|^2 = \frac{1}{(1-a)^2}$$

$$= 100 \ (20\,dB)$$

if $a = 0.9$

$$|H(e^{j\frac{\pi}{2}})|^2 = \frac{1}{1+a^2} = 0.55$$

(ii) Realized Using a zero:

$$H(z) = 1 + a z^{-1} \qquad 0 < a < 1$$

$$|H(e^{j0})|^2 = (1+a)^2$$

$$= 3.61 \ (5.58\,dB)$$

if $a = 0.9$

$$|H(e^{j\frac{\pi}{2}})|^2 = 1 + a^2$$

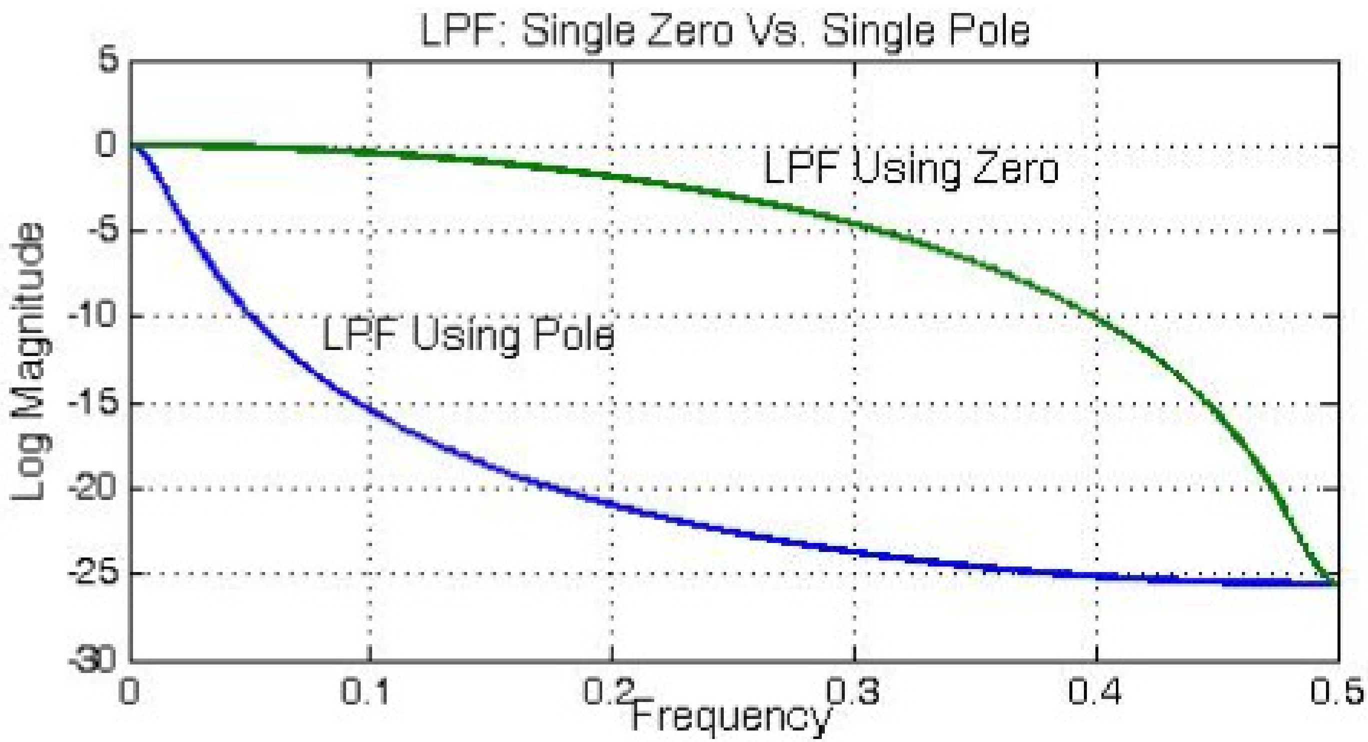
$$= 1.81 \ (2.58\,dB)$$

Thus, the 3-dB Bandwidth for an LPF realized using a single zero is $\frac{\pi}{2}$ if $a = 0.9$.

The 3-dB BW for the LPF realized using a single pole can be shown to be $\frac{\pi}{30}$, i.e., fifteen times narrower.

<u>Exercise</u>: Derive the 3-dB bandwidth of $H(z) = \frac{1}{1 - az^{-1}}$ $\quad -1 < a < 1$

Poles are more powerful in shaping the frequency response than zeros.

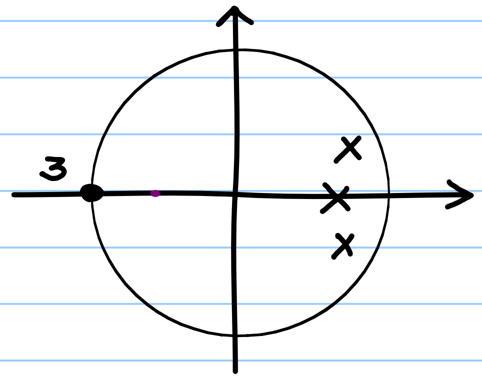


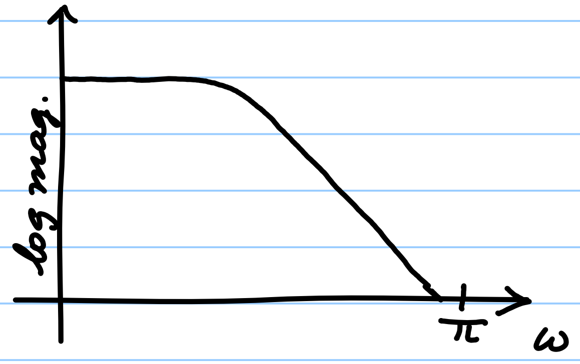
3 dB Bandwidth of LPF realized using pole is 15 times smaller!

$H_{LPF}(z) = \dfrac{1}{1-0.9z^{-1}}$ Vs. $H_{LPF}(z) = 1+0.9z^{-1}$

Peak gain has been normalized to unity.

One can add more poles to get a flatter passband:





Systematic procedures for designing filters will be taught in the Digital Filter Design course.

Classical Analog Filters: Butterworth, Chebychev, Elliptic, Bessel