Note Title 10-09-2013

EE5330 Sep. 10, 2013

Typical 2nd Order Section

If $H(z) = \dfrac{B(z)}{A(z)} = \dfrac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_M z^{-M}}{1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_N z^{-N}}$

where $a_k, b_k \in \mathbb{R}$. The Following is a typical pair, assuming simple poles:

$$\frac{A_k}{1 - p_k z^{-1}} + \frac{A_k^*}{1 - p_k^* z^{-1}}$$

$$= \frac{(A_k + A_k^*) - z^{-1}(A_k^* p_k + A_k p_k^*)}{1 - (p_k + p_k^*) z^{-1} + |p_k|^2 z^{-2}}$$

$$= \frac{p_0 + p_1 z^{-1}}{1 + q_1 z^{-1} + q_2 z^{-2}} \qquad p_k, q_k \in \mathbb{R}$$

The above is a typical second order section that shows up in practice in the parallel form implementation of digital filters.

Another popular form:

$$H(z) = \frac{B(z)}{A(z)} = \frac{b_0 \prod_{\ell=1}^{M} (1 - z_\ell z^{-1})}{\prod_{k=1}^{N} (1 - p_k z^{-1})}$$

Typical 2^{nd} order section: $\dfrac{c_0 + c_1 z^{-1} + c_2 z^{-2}}{1 + d_1 z^{-1} + d_2 z^{-2}} \qquad c_k, d_k \in \mathbb{R}$

where two complex-conjugate roots have been combined – Cascade Form section.

One-sided Z-Transform:

The two-sided z-transform cannot be used for solving LCCDE with initial conditions. The one-sided z-transform is naturally equipped to do so.

$$X_+(z) = \sum_{n=0}^{\infty} x[n]\, z^{-n} \qquad \text{RoC: } |z| > r_{max}$$

The time-shift property behaves differently when compared with its two-sided counterpart.

Let $k \geq 0$ and let $y[n] = x[n-k]$. It is easy to see that $Y_+(z) = z^{-k} X_+(z)$. This is identical to the result of the two-sided counterpart.

OTOH, consider $y[n] = x[n+k]$ where $k > 0$. Then,

$$x : \{ \ldots 0, 0, 0, \underset{\uparrow}{x[0]}, x[1], x[2], x[3], \ldots \}$$

$$y : \{ \ldots 0, 0, 0, x[0], x[1], \ldots, x[k-1], \underset{\uparrow}{x[k]}, x[k+1], x[k+2], \ldots \}$$

$$X_+(z) = x[0] + x[1] z^{-1} + x[2] z^{-2} + \cdots$$

$$Y_+(z) = x[k] + x[k+1]z^{-1} + x[k+2]z^{-2} + \cdots$$

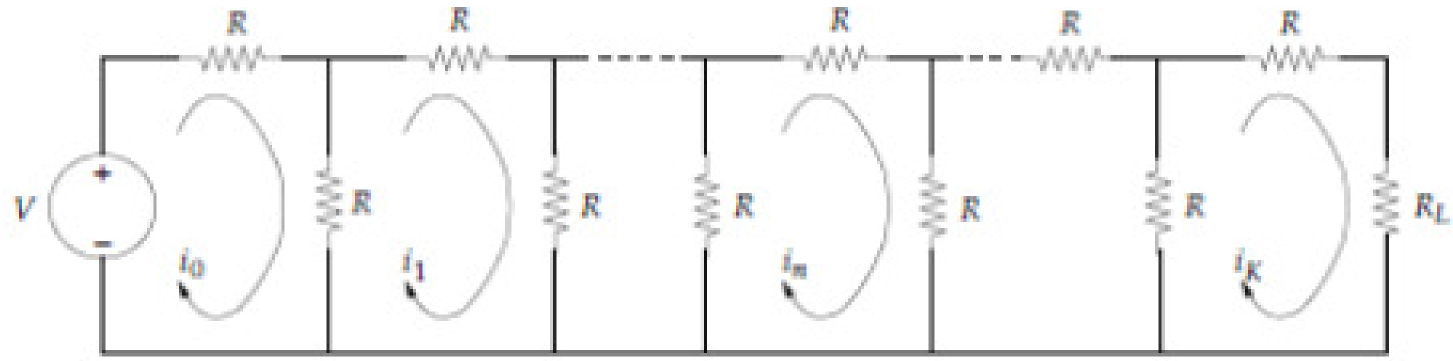
$$X_+(z) - \sum_{n=0}^{k-1} x[n]z^{-n} = x[k]z^{-k} + x[k+1]z^{-(k+1)} + \cdots$$

Thus,

$$Y_+(z) = z^k\left[X_+(z) - \sum_{n=0}^{k-1} x[n]z^{-n}\right]$$

Note that if all the initial conditions are zero, the above reduces to the earlier result.

Example The one-sided transform can be used for solving the currents in the circuit shown below:



The difference equation that relates the loop currents i_n, i_{n+1}, i_{n+2} can easily be verified to be the following:

$$i_n - 3i_{n+1} + i_{n+2} = 0$$

Transforming the above, we get,

$$I(z) - 3z\left[I(z) - i_0\right] + z^2\left[I(z) - i_0 - i_1 z^{-1}\right] = 0$$

$$\Rightarrow \quad I(z) = \frac{z(i_0 z - 3i_0 + i_1)}{z^2 - 3z + 1}$$

We can eliminate i_1 from the equation related to the first loop:

$$V = 2Ri_0 - i_1 R \Rightarrow i_1 = 2i_0 - \frac{V}{R}$$

$$i_n = i_0\left[\cosh \omega_0 n + \frac{\frac{1}{2} - (V/Ri_0)}{\sqrt{5}/2} \sinh \omega_0 n\right] \quad \text{where} \quad \cosh \omega_0 = \frac{3}{2} \quad \sinh \omega_0 = \frac{\sqrt{5}}{2}$$

Note on the convergence condition of the DTFT

Recall the following definition:

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n] e^{-j\omega n}$$

If $|H(e^{j\omega})| < \infty$, then $\left| \sum_{n=-\infty}^{\infty} h[n] e^{-j\omega n} \right| \leq \sum_{n=-\infty}^{\infty} |h[n]|$

Thus, the DTFT exists if the sequence is absolutely summable.

This condition is sufficient but not necessary. Sequences such as $u[n]$ are not absolutely summable but yet possess DTFT.

If the sequence is absolutely summable, the DTFT will be a continuous function of ω. [Why?]