Note Title 05-09-2013

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What modes are present in the output when an input is applied?

Let $X(z) = \frac{P(z)}{Q(z)}$. Assume, for illustrative purposes, only simple poles are present in $X(z)$. Then,

$$X(z) = \frac{P(z)}{Q(z)} = \sum_{k=1}^{Q} \frac{A_k}{1-\xi_k z^{-1}} \longleftrightarrow \sum_{k=1}^{Q} A_k (\xi_k)^n u[n]$$

(assuming causality)

$(\xi_k)^n u[n]$ are called as the input modes.

Similarly, let $H(z) = \dfrac{B(z)}{A(z)} = \displaystyle\sum_{k=1}^{N} \frac{B_k}{1 - p_k z^{-1}} \longleftrightarrow \sum_{k=1}^{N} B_k (p_k)^n u[n]$

(again assuming causality)

$(p_k)^n u[n]$ are called as the natural modes (also called system modes)

$$X(z) \longrightarrow \boxed{H(z)} \longrightarrow Y(z) = \frac{P(z)}{Q(z)} \, \frac{B(z)}{A(z)}$$

$$= \sum_{\ell=1}^{Q} \frac{C_\ell}{1 - \xi_\ell z^{-1}} + \sum_{k=1}^{N} \frac{D_k}{1 - p_k z^{-1}}$$

Hence, assuming causality,

$$y[n] = \underbrace{\sum_{\ell=1}^{Q} C_\ell (\xi_\ell)^n u[n]}_{\text{input modes}} + \underbrace{\sum_{k=1}^{N} D_k (p_k)^n u[n]}_{\text{natural modes}}$$

The output consists of input modes and natural modes.

Input modes are the particular solution

Natural modes are the homogeneous solution

Similar arguments apply for CT systems governed by LCCDE

$$Y(s) = \frac{P(s)}{Q(s)} \cdot \frac{B(s)}{A(s)} \leftrightarrow y(t) = \text{input modes} + \text{natural modes}$$

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

$$h[n] = a^n u[n]$$

$$x[n] = b^n u[n] \qquad b \neq a$$

$$Y(z) = X(z)\, H(z)$$

$$= \frac{1}{1 - az^{-1}} \; \frac{1}{1 - bz^{-1}}$$

$$= \frac{1}{a-b} \left[\frac{a}{1 - az^{-1}} - \frac{b}{1 - bz^{-1}} \right]$$

$$\longleftrightarrow \frac{a}{a-b} a^n u[n] - \frac{b}{a-b} b^n u[n]$$

natural mode

input mode

If $x[n] = a^n u[n]$, $y[n] = (n+1) a^n u[n]$

RESONANCE !