

What modes are present in the output when an input is applied?

Let  $X(z) = \frac{P(z)}{Q(z)}$ . Assume, for illustrative purposes, only simple poles are present in  $X(z)$ . Then,

$$X(z) = \frac{P(z)}{Q(z)} = \sum_{k=1}^Q \frac{A_k}{1 - z_k z^{-1}} \longleftrightarrow \sum_{k=1}^Q A_k (z_k)^n u[n]$$

(assuming causality)

$(z_k)^n u[n]$  are called as the *input modes*.

Similarly, let 
$$H(z) = \frac{B(z)}{A(z)} = \sum_{k=1}^N \frac{B_k}{1 - p_k z^{-1}} \leftrightarrow \sum_{k=1}^N B_k (p_k)^n u[n]$$

(again assuming causality)

$(p_k)^n u[n]$  are called as the *natural modes* (also called *system modes*)

$X(z)$   $\rightarrow$   $\boxed{H(z)}$   $\rightarrow$   $Y(z) = \frac{P(z)}{Q(z)} \frac{B(z)}{A(z)}$

$$= \sum_{l=1}^Q \frac{C_l}{1 - z_l z^{-1}} + \sum_{k=1}^N \frac{D_k}{1 - p_k z^{-1}}$$

Hence, assuming causality,

$$y[n] = \underbrace{\sum_{l=1}^Q C_l (z_l)^n u[n]}_{\text{input modes}} + \underbrace{\sum_{k=1}^N D_k (p_k)^n u[n]}_{\text{natural modes}}$$

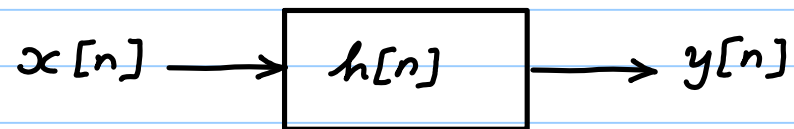
The output consists of input modes and natural modes.

Input modes are the particular solution

Natural modes are the homogeneous solution

Similar arguments apply for CT systems governed by LCCDE

$$Y(s) = \frac{P(s)}{Q(s)} \cdot \frac{B(s)}{A(s)} \leftrightarrow y(t) = \text{input modes} + \text{natural modes}$$



$$h[n] = a^n u[n]$$

$$x[n] = b^n u[n] \quad b \neq a$$

$$Y(z) = X(z) H(z)$$

$$= \frac{1}{1 - az^{-1}} \frac{1}{1 - bz^{-1}}$$

$$= \frac{1}{a-b} \left[ \frac{a}{1 - az^{-1}} - \frac{b}{1 - bz^{-1}} \right]$$

$$\longleftrightarrow \frac{a}{a-b} a^n u[n] - \frac{b}{a-b} b^n u[n]$$

natural mode  $\uparrow$

$\uparrow$  input mode

If  $x[n] = a^n u[n]$ ,  $y[n] = (n+1)a^n u[n] \leftarrow \text{RESONANCE!}$