EE 5330 Oct. 31, 2013

$X[k]$ = "k^{th} DFT bin value"

How can we translate bin index to true analog frequency?

$$X[k] = X[k+N]$$

$$X_s(F) = X_s(F+F_s)$$

Hence, the k^{th} bin maps to $\frac{k}{N} \cdot F_s$ (zero-based index)

$k=0$ maps to 0 Hz

$k=1$ maps to $\frac{F_s}{N}$ Hz

$\vdots$

$k=N-1$ maps to $\frac{N-1}{N} F_s$ Hz

Effects of Zero Padding

Consider the N-point sequence $x[n]$ (defined over $n = 0, 1, \ldots, N-1$) and its N-point transform $X[k]$.

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi k n}{N}} \qquad k = 0, 1, \ldots, N-1.$$

Now consider the following L-point ($L > N$) sequence $y[n]$:

$$y[n] = \begin{cases} x[n] & n = 0, 1, \ldots, N-1 \\ 0 & n = N, N+1, \ldots, L-1 \end{cases}$$

$y[n]$ is the zero-padded version of $x[n]$.

Consider the following L-point DFT of $y[n]$:

$$Y[k] = \sum_{n=0}^{L-1} y[n] e^{-j\frac{2\pi k n}{L}} \qquad k = 0, 1, \ldots, L-1$$

$$= \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi k n}{L}} \qquad k = 0, 1, \ldots, L-1$$

Note that $Y(e^{j\omega}) = \sum_{n=0}^{L-1} y[n] e^{-j\omega n}$

$$= \sum_{n=0}^{N-1} x[n] e^{-j\omega n}$$

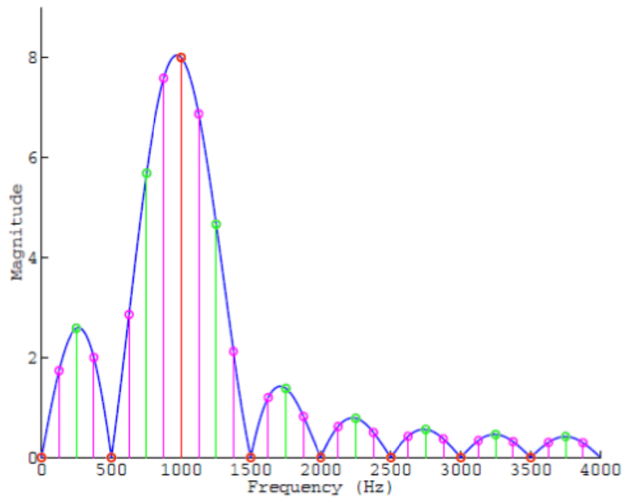
$$= X(e^{j\omega})$$

That is, the underlying DTFT remains the same.

However, since the DFT can be interpreted as sampling the DTFT, zero-padding enables us to sample the underlying DTFT at a finer set of points.

$$x[n] = \sin \frac{2\pi n}{8} \quad n = 0, 1, \ldots, 15$$

Blue: DTFT
Red: 16-point DFT
Green: 32-point DFT (contains the red samples as a subset)
Magenta: 64 point DFT
(contains the red & green samples as a subset)



DTFT and 64-pt DFT