EE5330 Oct. 21, 2013

An all-pass filter preserves signal energy.

$$x[n] \longrightarrow \boxed{H_{ap}(z)} \longrightarrow y[n]$$

$$\|\underline{x}\|_2^2 = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$\|\underline{y}\|_2^2 = \sum_{n=-\infty}^{\infty} |y[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |Y(e^{j\omega})|^2 d\omega$$ [Parseval's Theorem]

$$Y(e^{j\omega}) = X(e^{j\omega})\, H_{ap}(e^{j\omega})$$

$$|Y(e^{j\omega})|^2 = |X(e^{j\omega})|^2 \cdot \cancel{|H_{ap}(e^{j\omega})|^2} = |X(e^{j\omega})|^2$$

Hence, since $|X(e^{j\omega})|^2 = |Y(e^{j\omega})|^2$,

$$\|\underline{x}\|_2^2 = \|\underline{y}\|_2^2 \quad \text{i.e.,} \quad \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=-\infty}^{\infty} |y[n]|^2$$

That is, the $\boxed{\text{all-pass filter preserves energy.}}$

We will now prove the following stronger result:

$$\sum_{n=-\infty}^{n_0} |x[n]|^2 \geq \sum_{n=-\infty}^{n_0} |y[n]|^2$$

i.e., the running sum of the output energy of an all-pass filter is always less than or equal to the corresponding i/p energy sum.