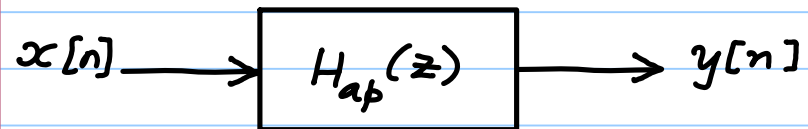


An all-pass filter preserves signal energy.



$$\| \underline{x} \|_2^2 = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$\| \underline{y} \|_2^2 = \sum_{n=-\infty}^{\infty} |y[n]|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |Y(e^{j\omega})|^2 d\omega \quad [\text{Parseval's Theorem}]$$

$$Y(e^{j\omega}) = X(e^{j\omega}) H_{ap}(e^{j\omega})$$

$$|Y(e^{j\omega})|^2 = |X(e^{j\omega})|^2 \cdot |H_{ap}(e^{j\omega})|^2 = |X(e^{j\omega})|^2$$

Hence, since  $|X(e^{j\omega})|^2 = |Y(e^{j\omega})|^2$ ,

$$\|\underline{x}\|_2^2 = \|\underline{y}\|_2^2 \quad \text{i.e.,} \quad \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=-\infty}^{\infty} |y[n]|^2$$

That is, the all-pass filter preserves energy.

We will now prove the following stronger result:

$$\sum_{n=-\infty}^{n_0} |x[n]|^2 \geq \sum_{n=-\infty}^{n_0} |y[n]|^2$$

i.e., the running sum of the output energy of an all-pass filter is always less than or equal to the corresponding i/p energy sum.