Note Title 14-10-2013

EE5330 Oct. 14, 2013

Zero Locations of Linear Phase FIR Filters:

Recall that linear phase imposes the following condition:

$h[n] = \pm h^*[M-n]$ where $M = N-1$

Hence

$$H(z) = \pm z^{-M} H^*(1/z^*)$$

Suppose z_0 is a zero of $H(z)$. That is, $H(z_0) = 0$.

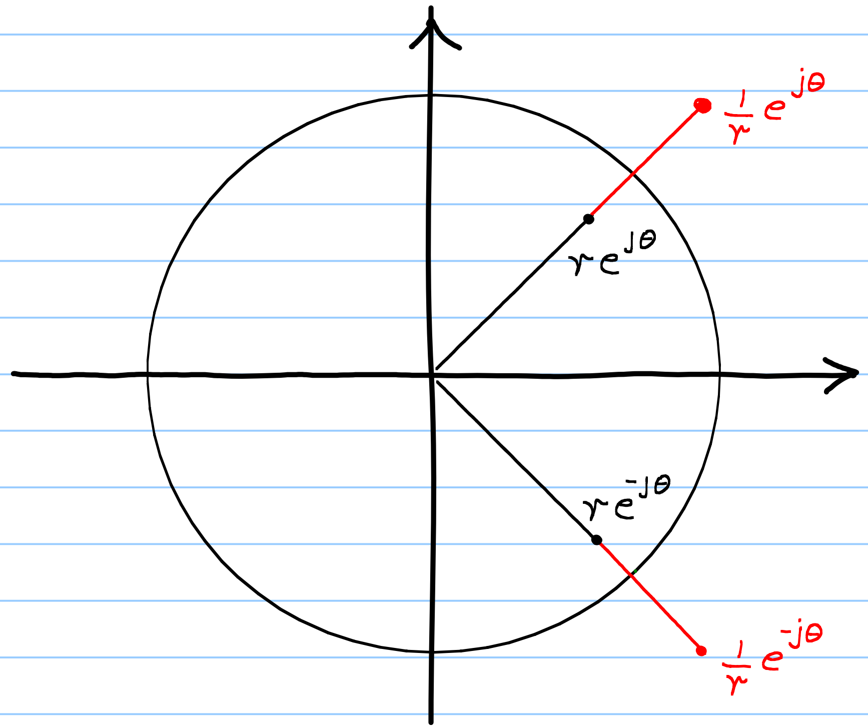
This means that

$$H(z_0) = 0 = z_0^{-M} H^*(1/z_0^*) \Rightarrow 1/z_0^* \text{ is also a zero}$$

Thus, if $re^{j\theta}$ is a zero, then $\frac{1}{r}e^{j\theta}$ is also a zero.

If $h[n] \in \mathbb{R}$, then $re^{-j\theta}$ will also be a zero $\Rightarrow$ $\frac{1}{r}e^{-j\theta}$ will be a zero too. Thus, a complex zero that is not on the unit circle must occur in $\boxed{\text{sets of } 4}$ for a linear phase FIR filter with real-valued impulse response.

If $r = 1$, the $\boxed{\text{same zero}}$ satisfies both $H(z_0) = 0$ and the $H(1/z_0^*) = 0$.



$h[n] \in \mathbb{R}$ and linear phase mean that, if $re^{j\theta}$ is a zero, then the set of related zeros is $\{ re^{\pm j\theta}, \frac{1}{r}e^{\pm j\theta} \}$

Linear phase filters also have constrained zeros.

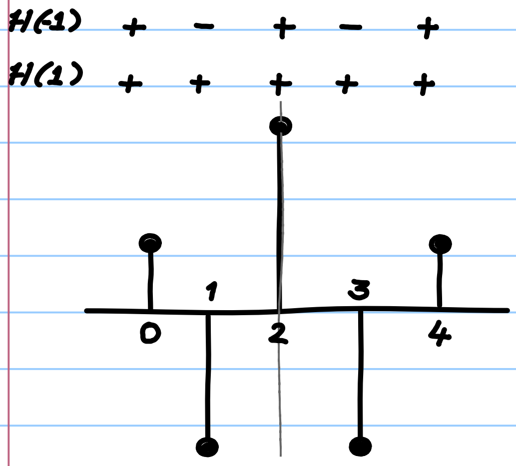
For an FIR filter $h[n]$,

$$H(z) = \sum_{n=0}^{N-1} h[n] z^{-n}$$

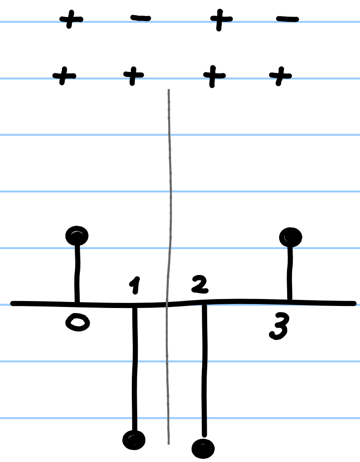
We will examine $H(1)$ and $H(-1)$.

$$H(1) = \sum_{n=0}^{N-1} h[n]$$

$$H(-1) = \sum_{n=0}^{N-1} h[n](-1)^{n}$$

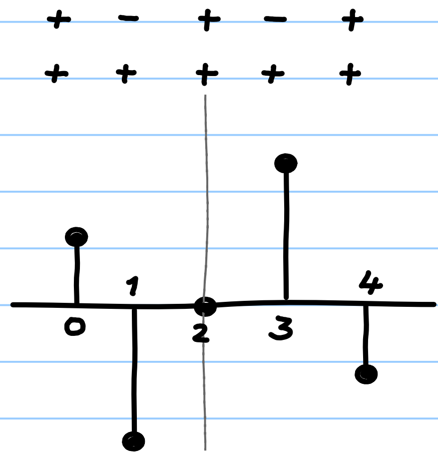


Type I



Type II

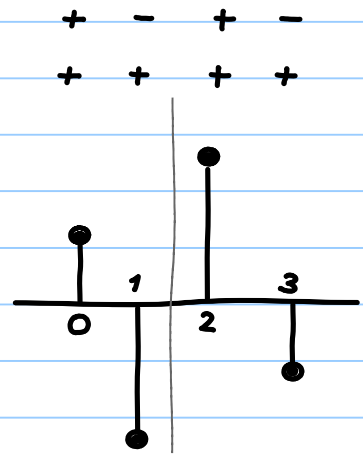
$H(-1) = 0$
always



Type III

$H(1) = 0$
always

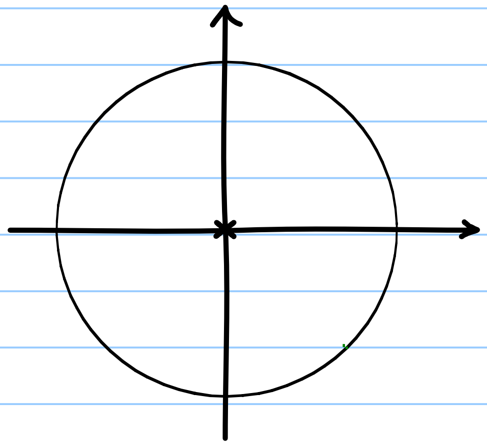
$H(-1) = 0$
always



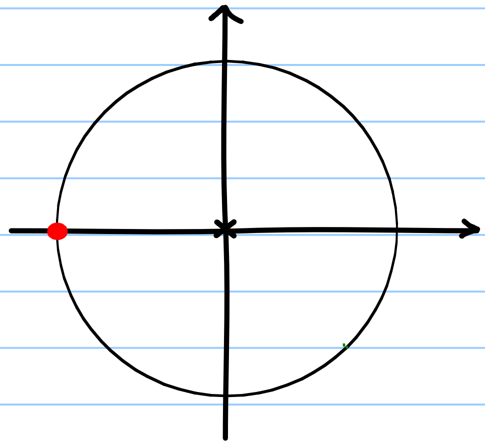
Type IV

$H(1) = 0$
always

Type I

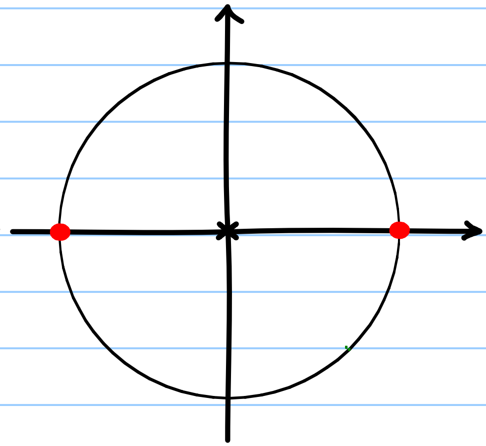


Type II



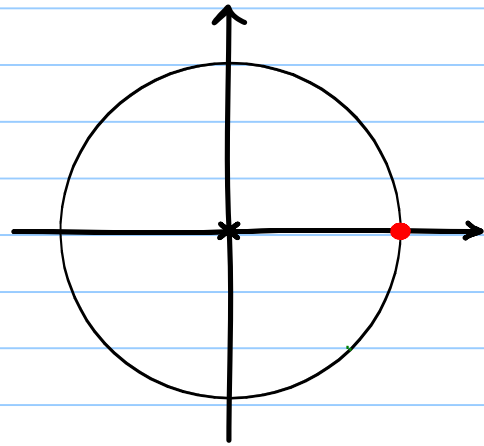
Cannot be used for building HPF

Type III



Cannot be used for building LPF, HPF

Type IV



Cannot be used for building LPF