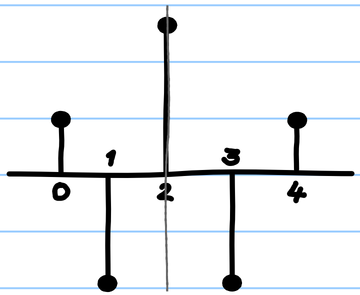
EE5330 Oct. 10, 2013

Note Title 10-10-2013

Since $2\tau_g = N-1$, for an FIR filter with real-valued coefficients,

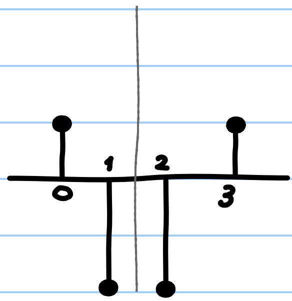
$$h[N-1-n] = \pm h[n]$$



Type I

$N = 5$

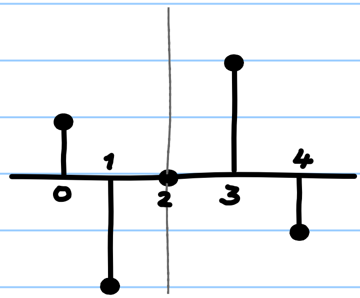
$\tau_g = \dfrac{5-1}{2} = 2$



Type II

$N = 4$

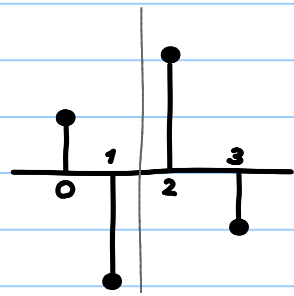
$\tau_g = \dfrac{4-1}{2} = 1.5$



Type III

$N = 5$

$\tau_g = \dfrac{5-1}{2} = 2$



Type IV

$N = 4$

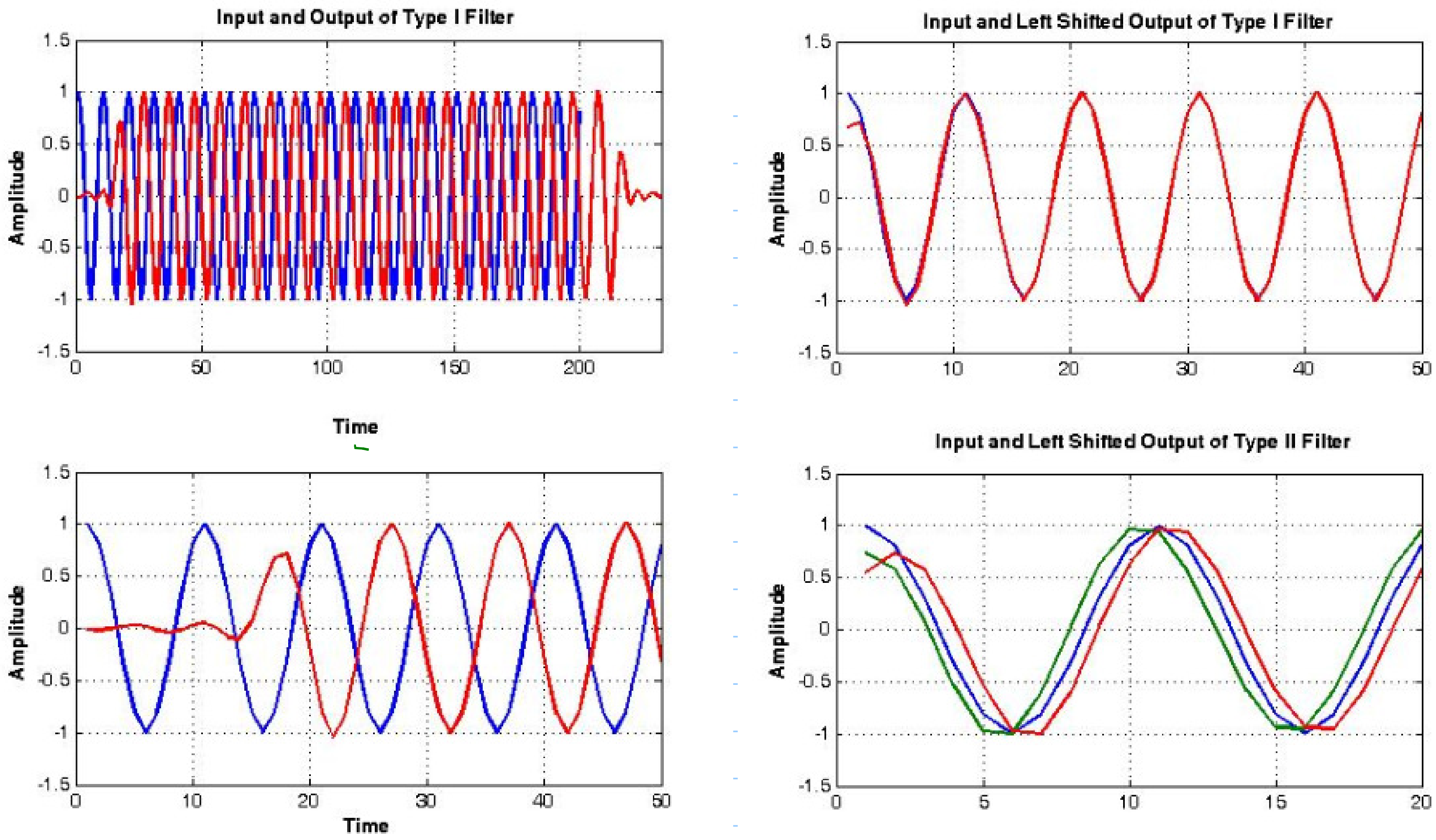
$\tau_g = \dfrac{4-1}{2} = 1.5$

If a filter produces integer delay, the output can be shifted to the left by that many samples and the original and filtered signals can be time-aligned.

If the filter introduces half-sample delays, the time-alignment is not possible since the shift can only be an integer.

Types I and III introduce integer delays

Types II and IV introduce half-sample delays.



Integer shift equal to 0.5 x (N-1) produces exact alignment with the input

Neither of the integer shifts closest to 0.5 x (N-1) produces exact alignment with the input

For Types I and III, the centre of symmetry falls on a sample

For Types II and IV the centre of symmetry falls midway between samples.

Symmetry is both necessary and sufficient for an FIR filter to be linear phase.

Symmetry is sufficient but not necessary for an IIR filter to be linear phase

$$h[n] = \frac{\sin w_c(n-\alpha)}{\pi(n-\alpha)}$$ is linear phase for any α.

However $h[n]$ is symmetric around $n = \alpha$ only if α is integer or integer $+ \frac{1}{2}$.

Frequency Response of Linear Phase FIR Filters

$$H(\omega) = \sum_{n=0}^{N-1 \rightarrow M} h[n] e^{-j\omega n}.$$ It is usual to let $M = N-1$.

For Type I, $h[0] = h[M]$, $h[1] = h[M-1]$, and so on. Hence,

$$H(\omega) = h[0] + h[1]e^{-j\omega} + \dots + h[M-1]e^{-j(M-1)\omega} + h[M]e^{-jM\omega}$$

$$= h[0] + h[1]e^{-j\omega} + \dots + h[1]\, e^{-j(M-1)\omega} + h[0]\, e^{-jM\omega}$$

$$= e^{-j\omega M/2} \underbrace{\left\{ h\left[\frac{M}{2}\right] + \sum_{n=0}^{\frac{M}{2}-1} 2h[n] \cos\left(\left(\frac{M}{2}-n\right)\omega\right) \right\}}_{A(\omega)}$$

Similarly,

$$H(\omega) = e^{-j\omega M/2} \left\{ \sum_{n=0}^{\frac{M-1}{2}} 2h[n] \cos\left(\left(\frac{M}{2}-n\right)\omega\right) \right\}$$

$$H(\omega) = \boxed{j}\, e^{-j\omega M/2} \left\{ \sum_{n=0}^{\frac{M}{2}-1} 2h[n] \sin\left(\left(\frac{M}{2}-n\right)\omega\right) \right\}$$

$$e^{j\overbrace{\boxed{\frac{\pi}{2}}}^{\beta}}$$

$$H(\omega) = \boxed{j}\, e^{-j\omega M/2} \left\{ \sum_{n=0}^{\frac{M-1}{2}} 2h[n] \sin\left(\left(\frac{M}{2}-n\right)\omega\right) \right\}$$