Note Title 01-08-2013

EE 5330 Aug. 1, 2013

Systems
- Finite Impulse Response (FIR)
- Infinite Impulse Response (IIR)

$$y[n] = -\sum_{k=1}^{N} a_k\, y[n-k] + \sum_{\ell=0}^{M} b_\ell\, x[n-\ell]$$

If the system is IIR, then at least one a_k is non-zero

If the system is FIR, then $a_k = 0$ for $k = 1, 2, \ldots, N$.

Consider the following system:

$$y[n] = \frac{1}{N} \sum_{k=n-N+1}^{n} x[n] \quad \text{——} \quad (1) \qquad (\text{FIR system})$$

The above is the same as

$$y[n] = y[n-1] + \frac{1}{N} x[n] - \frac{1}{N} x[n-N] \quad \text{——} \quad (11)$$

Eqn. (11) is a recursive implementation of a non-recursive equation, i.e., even though a_1 is non-zero, the system is not IIR

Z-Transform

The z-transform of a sequence $x[n]$ is defined as

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$$

One way of arriving at this is by applying the eigensignal z_0^n as the input to an LTI system:

$$z_0^n \longrightarrow \boxed{\begin{matrix} h[n] \\ (LTI) \end{matrix}} \longrightarrow y[n] = z_0^n * h[n]$$

$$= \sum_{k=-\infty}^{\infty} z_0^{n-k}\, h[k]$$

$$= z_0^n \sum_{k=-\infty}^{\infty} h[k]\, z_0^{-k}$$

$$= z_0^n\, H(z_0)$$

where $H(z_0) = \sum_{k=-\infty}^{\infty} h[k]\, z_0^{-k}$

The z-transform is a complex function of a complex variable and hence requires 4 dim. to plot: 2 for the indep. var. and 2 for the dep. variable

Refer to CONFORMAL MAPPING

Examples: $x[n] = a^n u[n]$

$$X(z) = \sum_{n=0}^{\infty} a^n z^{-n}$$

$$= \frac{1}{1-az^{-1}} \quad \text{if } |az^{-1}| < 1$$

$$\text{i.e., } |z| > |a|$$

Example

$$x[n] = -a^n u[-n-1]$$

$$X(z) = -\sum_{n=-\infty}^{-1} a^n z^{-n}$$

$$= \frac{1}{1-az^{-1}} \quad \text{if. } |z| < |a|$$

The algebraic expression for $X(z)$ is the same as before but the region over which it is valid is different

The range of 'z' over which the z-transform expression is valid is called Region of Convergence (RoC)

Examples

$\{-\underset{\uparrow}{1}, 2, 4, \pi\} \longleftrightarrow -1 + 2z^{-1} + 4z^{-2} + \pi z^{-3}$ $\quad 0 \notin RoC$

right sided, $n \geq 0$

$\{-1, \underset{\uparrow}{2}, 4, \pi\} \longleftrightarrow -z + 2 + 4z^{-1} + \pi z^{-2}$ $\quad 0, \infty \notin RoC$

two-sided

$\{-1, 2, 4, \underset{\uparrow}{\pi}\} \longleftrightarrow -z^{3} + 2z^{2} + 4z + \pi$ $\quad \infty \notin RoC$

left sided, $n \leq 0$

Example

$$x[n] = \left(-\tfrac{1}{2}\right)^n u[n] + \left(\tfrac{1}{3}\right)^n u[n]$$

$$\left(-\tfrac{1}{2}\right)^n u[n] \longleftrightarrow \frac{1}{1+\frac{1}{2}z^{-1}} \qquad |z| > \tfrac{1}{2}$$

$$\left(\tfrac{1}{3}\right)^n u[n] \longleftrightarrow \frac{1}{1-\frac{1}{3}z^{-1}} \qquad |z| > \tfrac{1}{3}$$

$$X(z) = \frac{1}{1+\frac{1}{2}z^{-1}} + \frac{1}{1-\frac{1}{3}z^{-1}} = \frac{2-\frac{1}{6}z^{-1}}{\left(1+\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)}$$

RoC : $|z| > \frac{1}{2} \;\cap\; |z| > \frac{1}{3} = |z| > \frac{1}{2}$

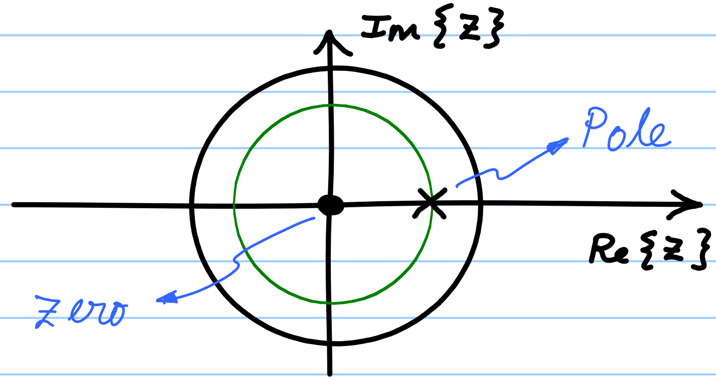
The final RoC is the INTERSECTION of the individual RoCs.

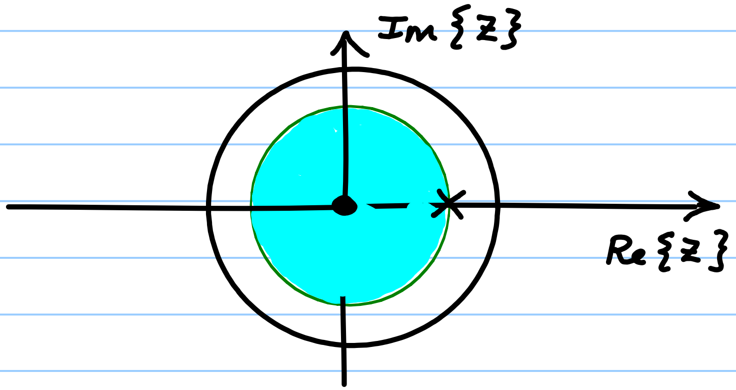
Poles and Zeros

$$X(z) = \frac{1}{1 - a z^{-1}} = \frac{z}{z - a}$$

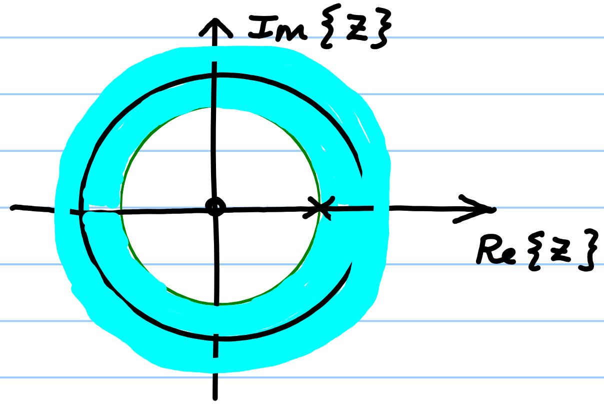
Pole: $z = a$

Zero: $z = 0$





RoC: $|z| < |a| \Rightarrow x[n] = -a^n u[-n-1]$



RoC: $|z| > |a| \Rightarrow x[n] = a^n u[n]$

Suppose $e^{j\omega} \in$ RoC. We can then evaluate $H(z)$ at $z = e^{j\omega}$

$$H(z)\Big|_{z=e^{j\omega}} = \sum_{n=-\infty}^{\infty} h[n]\, e^{-j\omega n}$$

$$= H(e^{j\omega}) \quad \text{Discrete-Time Fourier Transform}$$

DTFT is a complex function of a single real variable ω. Hence can be plotted in $\boxed{\text{one 3-D plot}}$.

Typically two 2-D plots are shown: mag. vs. ω & phase vs. ω