EE5330: Digital Signal Processing

Tutorial 7

(November 12, 2013)

1. Let $\phi(t)$ be a continuous-time aperiodic signal with CTFT $\Phi(\Omega)$. Let $\tilde{\phi}(t)$ be the periodic signal defined as $\tilde{\phi}(t) = \sum_n \phi(t + nT)$. Prove the following:

$$\tilde{\phi}(t) = \frac{1}{T} \sum_{k=-\infty}^{\infty} \Phi(k\Omega_0)\, e^{jk\Omega_0 t}$$

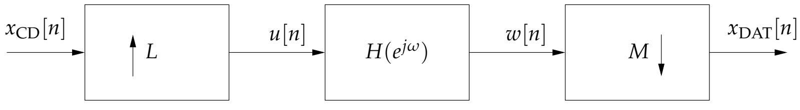
   where $\Omega_0 = 2\pi/T$. Note that the above is nothing but the Fourier series expansion of the periodic signal $\tilde{\phi}(t)$.

2. A finite-energy continuous-time lowpass signal $x_c(t)$ is sampled at a rate that satisfies the condition $F_s \geq 2F_c$, where F_c is the highest frequency component in $x_c(t)$. Let $x[n]$ denote the sampled signal. Develop a relationship between the energy of the continuous-time signal and that of its discrete-time counterpart.

3. A continuous-time signal $x_c(t)$ is composed of a linear combination of sinusoidal signals with frequencies F_1 Hz, F_2 Hz, F_3 Hz, and F_4 Hz. The sampling frequency F_s is 10 kHz, and the sampled signal is passed through an ideal LPF with cutoff frequency 4 kHz. The reconstructed signal is found to contain sinusoids with frequencies 350 Hz, 575 Hz, and 815 Hz. What are the possible values of F_1, F_2, F_3, and F_4 ? Is your answer unique? If not, can you specify another set of possible values of these frequencies?

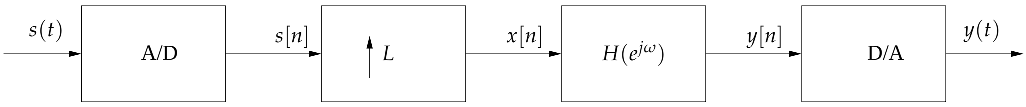
4. The left and right channels of an analog stereo audio signal are sampled at a 45-kHz rate, with each channel the being converted to a digital bit stream using a 12-bit A/D converter. Determine the overall bitrate after sampling and quantization.

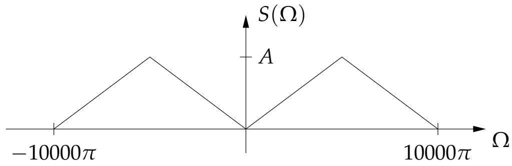
5. Digital Audio Tape (DAT) drives have a sampling frequency of 48 kHz, whereas a Compact Disk (CD) player operates at a rate of 44.1 kHz. In order to record directly from a CD onto a DAT, it is necessary to convert the sampling rate from 44.1 to 48 kHz. Therefore, consider the following system for performing this sample rate conversion:



   Find the smallest possible values for L and M and find the appropriate filter $H(e^{j\omega})$ to perform this conversion.

6. Suppose that we would like to slow a segment of speech to one-half its normal speed. The speech signal $s(t)$ is assumed to have no energy outside of 5 kHz, and is sampled at a rate of 10 kHz, yielding the sequence $s[n] = s(nT)$. The following system is proposed to create the slowed-down speech signal. Assume that $S(\Omega)$ is as shown in the figure.





(a) Find the spectrum of $x[n]$.

(b) Suppose that the discrete-time filter is described by the difference equation

$$y[n] = x[n] + \frac{1}{2}\left(x[n+1] + x[n-1]\right)$$

Find the frequency response of the filter and describe its effect on $x[n]$.

(c) What is $Y(\Omega)$ in terms of $S(\Omega)$? Does $y(t)$ correspond to slowed-down speech?

7. Consider the system whose input/output relationship is given by $y[n] = x[Mn]$. Which of the following signals can be downsampled by a factor of 2 without any loss of information? (a) $x[n] = \delta[n - n_0]$ for some unknown integer n_0, (b) $x[n] = \cos(\pi n/4)$, (c) $x[n] = \cos(\pi n/4) + \cos(3\pi n/4)$, (d) $x[n] = \sin(\pi n/3)/(\pi n/3)$, and (e) $x[n] = (-1)^n \sin(\pi n/3)/(\pi n/3)$.

8. A certain system has three blocks in cascade. The input/output relationship for the first block is $x_d[n] = x[3n]$. For the second block, the output $x_e[n]$ is obtained by taking $x_d[n]$ and inserting 2 zeros between every sample. The final block is an ideal lowpass filter with cutoff frequency $\omega = \pi/3$ and gain 3. The LPF's output is denoted by $x_r[n]$. For which of the following signals is $x_r[n] = x[n]$? (a) $\cos(\pi n/4)$, (b) $\cos(\pi n/2)$, and (c) $\sin^2(\pi n/8)/(\pi^2 n^2)$.

9. Consider the usual discrete-time processing of continuous-time signals, consisting of the A/D converter, discrete-time filter, and the reconstruction filter to get the continuous-time output. The input $x(t)$ has a CTFT that is triangular in shape, centred at $\Omega = 0$, with cutoff frequency $\Omega_0 = 2\pi(1000)$. The discrete-time filter is an ideal LPF with cutoff frequency ω_c. For $\omega_c = \pi/2$, what is the minimum sampling frequency needed for $y(t) = x(t)$?

10. (a) Let $x_c(t)$ be a lowpass signal whose spectrum is such that $X(F) = 0$ for $|F| > F_m/2$ Hz. What is the minimum sampling frequency needed for sampling $x^2(2t)$ to avoid aliasing?

(b) The continuous-time signal $x_c(t) = \sin(20\pi t) + \cos(40\pi t)$ is sampled with a sampling period T to obtain the discrete-time signal $x[n] = \sin\left(\frac{\pi n}{5}\right) + \cos\left(\frac{2\pi n}{5}\right)$.

i. Determine a choice for T consistent with this information.

ii. If your choice of T is unique, explain why. If not, specify another choice of T consistent with this information.

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