EE5330: Digital Signal Processing

Tutorial 4

(September 23, 2013)

1. Consider the non-causal rectangular window $x[n] = 1$ for $-M \leq n \leq M$. Derive the expression for its DTFT $X(e^{j\omega})$. Now make the window causal, i.e., $x[n] = 1$ for $0 \leq n \leq 2M$, and compute its DTFT. Can you express it in the form $e^{-jK\omega}A(\omega)$, where $A(\omega)$ is a purely real-valued function? What is the value of K? Note: Since $A(\omega)$ is *real-valued*, it can take on both positive and negative values.

2. Consider the pair $x[n] \overset{\text{DTFT}}{\longleftrightarrow} X(e^{j\omega})$. Find $x[n]$ corresponding to (a) $X(e^{j\omega}) = j(4 + 2\cos\omega + 3\cos 2\omega) \cdot \sin(\omega/2) \cdot e^{j\omega/2}$, (b) $X(e^{j\omega}) = \cos^N(\omega)$ where $N > 0$, and (c) $X(e^{j\omega}) = \cos^N(\omega/2)$ where $N > 0$ is an *even* integer.

3. The 2π-periodic function $X(e^{j\omega})$ is defined as follows, where $A > 0$:

$$X(e^{j\omega}) = \begin{cases} A + A\dfrac{\omega}{\pi} & -\pi \leq \omega < 0 \\ -A + A\dfrac{\omega}{\pi} & 0 \leq \omega < \pi \end{cases}$$

   (a) Derive the inverse DTFT of $X(e^{j\omega})$.

   (b) Using the above result derive the DTFT of $x[n] = 1/n$ for $n \geq 1$ and zero otherwise.

   (c) Apply Parseval's theorem and find the value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

4. Evaluate $\sum_{n=-\infty}^{\infty} \frac{\sin(\pi n/4)\sin(\pi n/6)}{\pi^2 n^2}$. The value of the summand at $n = 0$ is $1/24$.

5. Consider the real, positive function $G(e^{j\omega}) = \frac{1}{A - B\cos\omega}$ where $0 < B < A$. We wish to find an *absolutely summable* sequence $x[n] = \frac{1}{\sqrt{\alpha}} c^n u[n]$ such that $G(e^{j\omega}) = |X(e^{j\omega})|^2$.

   (a) By setting up a pair of equations, solve for α and c in terms of A and B.

   (b) Using Parseval's relation, obtain an expression for $\frac{1}{2\pi}\int_{-\pi}^{\pi} G(e^{j\omega})\,d\omega$ in terms of A and B.

6. The input to a cascade of two LTI systems is the signal $x[n] = \cos(0.6\pi n) + 3\delta[n-5] + 2$. The first system's impulse response has the DTFT given by

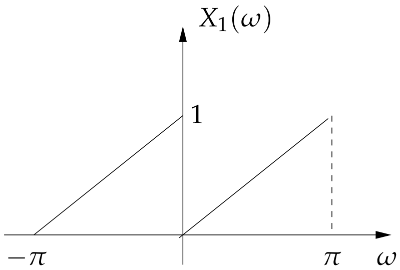
$$H_1(e^{j\omega}) = \begin{cases} 1 & |\omega| \leq 0.5\pi \\ 0 & 0.5\pi < |\omega| \leq \pi \end{cases}$$

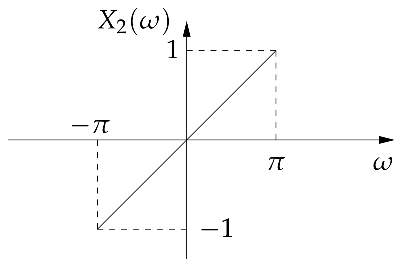
The second system is governed by the following input-output relationship: $y[n] = w[n] - w[n-1]$. Find the output $y[n]$.

7. Let $h[n] = h_R[n] + jh_I[n]$ have DTFT $H(e^{j\omega}) = H_R(e^{j\omega}) + jH_I(e^{j\omega})$, where the subscripts R and I denote the real and imaginary parts. Let H_{ER} and H_{OR} denote the even and odd parts of H_R; similarly, H_{EI} and H_{OI} are the even and odd parts of H_I. If the transform of $h_R[n]$ is $H_a + jH_b$ and that of $h_I[n]$ is $H_c + jH_d$, express H_a, H_b, H_c, and H_d in terms of H_{ER}, H_{OR}, H_{EI}, and H_{OI}.

8. Make a comparative table of continuous-time and discrete-time Fourier transforms of various commonly encountered CT signals and their discrete-time counterparts: (a) $\delta(t) \bowtie \delta[n]$, (b) $u(t) \bowtie u[n]$, (c) $\cos(\Omega_0 t) \bowtie \cos(\omega_0 n)$, (d) $\text{sgn}(t) \bowtie \text{sgn}[n]$ ($\text{sgn}[0] = 1$), (e) $e^{-at}u(t) \bowtie a^n u[n]$, (f) $u(t+T) - u(t-T) \bowtie u[n+M] - u[n-M]$.

9. For the $X_i(\omega)$ given below, find the corresponding $x_i[n]$. Note that the notation $X_i(\omega)$ has been used in this problem, rather than the usual $X_i(e^{j\omega})$





10. Let $X_1(e^{j\omega}) = 1$ for $|\omega| \leq \pi/4$ and $X_1(e^{j\omega}) = X_2(e^{j\omega})$. Plot the result of *circularly convolving* $X_1(e^{j\omega})$ and $X_2(e^{j\omega})$. Now if the cutoff frequency of the ideal LPF $X_1(e^{j\omega})$ is $\pi/2$ and that of $X_2(e^{j\omega})$ is $3\pi/4$, how will the above convolution result change? Let $X_1(\Omega)$ and $X_2(\Omega)$ be the *aperiodic* versions of $X_1(e^{j\omega})$ and $X_2(e^{j\omega})$. For both cases, *linearly* convolve $X_1(\Omega)$ and $X_2(\Omega)$, and take its result to form a periodic signal with period 2π. This can be done by shifting the linear convolution result by $2\pi k$ and adding over all k. How does this compare with the result of circular convolution? Can you now relate linear and circular convolution? Commonly used notation for circular convolution is $\circledast$.

11. Let

$$y[n] = \left(\frac{\sin \frac{\pi}{4} n}{\pi n}\right)^2 * \left(\frac{\sin \omega_c n}{\pi n}\right)$$

where $*$ denotes convolution and $|\omega_c| \leq \pi$. Determine a stricter constraint on ω_c which ensures that

$$y[n] = \left(\frac{\sin \frac{\pi}{4} n}{\pi n}\right)^2$$

12. Let the inverse Fourier transform of $Y(e^{j\omega})$ be

$$y[n] = \left(\frac{\sin \omega_c n}{\pi n}\right)^2$$

where $0 < \omega_c < \pi$. Determine the value of ω_c which ensures that $Y(e^{j\pi}) = 1/2$.

13. The DTFT of a particular signal is

$$X(e^{j\omega}) = \sum_{k=0}^{3} \frac{(1/2)^k}{1 - \frac{1}{4}e^{-j(\omega - k\pi/2)}}$$

It can be shown that $x[n] = g[n]q[n]$, where $g[n]$ is of the form $\alpha^n u[n]$ and $q[n]$ is a periodic signal with period N. (a) Determine the value of α. (b) Determine the value of N. (c) Is $x[n]$ real-valued?

14. Determine and sketch the DTFT $X_i(\omega)$ of $x_i[n]$:

    (a) $x_1[n] = \{1, 1, \underset{\uparrow}{1}, 1, 1\}$

    (b) $x_2[n] = \{1, 0, 1, 0, \underset{\uparrow}{1}, 0, 1, 0, 1\}$

    (c) $x_3[n] = \{1, 0, 0, 1, 0, 0, \underset{\uparrow}{1}, 0, 0, 1, 0, 0, 1\}$

    Is there any relation between $X_1(\omega)$, $X_2(\omega)$, and $X_3(\omega)$?

15. Consider and LTI system with frequency response $H(e^{j\omega}) = \dfrac{1 - e^{-j2\omega}}{1 + r^2 e^{-j2\omega}}$ where $r = 0.9$. The input to this system is $\cos n\pi/8 + \cos n\pi/2$. The output can be expressed as $A_1 \cos(n\pi/8 + \theta_1) + A_2 \cos(n\pi/2 + \theta_2)$. Determine A_i and θ_i for $i = 1, 2$.

16. Consider a sequence $x[n]$ with DTFT $X(e^{j\omega})$. The sequence $x[n]$ is real-valued and causal, and

$$\Re\left\{X(e^{j\omega})\right\} = 2 - 2a \cos \omega$$

Determine $\Im\left\{X(e^{j\omega})\right\}$. Note: $\Re\{\cdot\}$ denotes "real-part" and $\Im\{\cdot\}$ denotes "imaginary-part".

17. Consider the DTFT pair $x[n] \overset{\text{DTFT}}{\longleftrightarrow} X(e^{j\omega})$. The following are known: (i) $x[n]$ is real-valued and causal, and (ii) $\Re\left\{X(e^{j\omega})\right\} = \frac{5}{4} - \cos \omega$. Determine a sequence $x[n]$ that is consistent with the given information.

18. The imaginary part of $X(e^{j\omega})$ is

$$X_I(e^{j\omega}) = 2 \sin \omega - 3 \sin 4\omega$$

It is known that the corresponding time-domain sequence is causal and real-valued. Furthermore, $X(e^{j\omega})\big|_{\omega=0} = 6$. Find $x[n]$.

19. (a) $x[n]$ is a real-valued causal sequence with the imaginary part of its DTFT being given by

$$X_I(e^{j\omega}) = \sin\omega + 2\sin 2\omega$$

Determine a choice of $x[n]$.

(b) Is your answer to part (a) unique? If so, explain why. If not, determine a second, distinct choice for $x[n]$ satisfying the relationship given in part (a)

20. **Computer assignment** Plot the real/imaginary parts, as well as magnitude/phase for various values of r and θ of the following DTFT:

$$G(e^{j\omega}) = \frac{1}{1 - 2r(\cos\theta)\, e^{-j\omega} + r^2\, e^{-j2\omega}}$$

$\theta, \omega \in [0, 2\pi)$. In particular, observe what happens when (a) r is close to unity versus well away, and (b) θ is close to 0 (or π) versus $\pi/2$ (or $3\pi/2$). If you consider the denominator as a polynomial in $e^{j\omega}$, where are its roots? Can you relate the peak locations of the DTFT and the root locations? Although ω is a continuous variable, on a computer you will have to discretize it, which is typically done by taking N uniformly spaced points in $[-\pi, \pi)$.

21. **Computer assignment** Study the MATLAB function `freqz`. Consider the $G(e^{j\omega})$ given in the previous problem. Let `G = freqz(1, [1, -2*r*cos(theta), r*r], 4000)`. Plot the magnitude and phase of `G` and compare with the result of the previous experiment. Also plot `20*log10(abs(G))` and compare it with it linear-scale counterpart.

22. **Computer assignment** Study carefully and implement MATLAB Examples 3.1–3.16 in the book “DSP Using MATLAB” by Proakis and Ingle (PWS Publishing Company, 1997). These are important and illuminating examples. Note that this early edition uses an old version of MATLAB. ◇