EE5330: Digital Signal Processing

Tutorial 3

(September 2, 2013)

1. Let $X(z)$ be a rational function that does not have a pole at $z = \infty$. Show that $dX(z)/dz$ has a zero of order at least 2 at $z = \infty$. What can you say about the function $z\frac{dX(z)}{dz}$?

2. The *correlation* between two sequences $x[n]$ and $y[n]$ is denoted as $r_{xy}[n]$ and defined as

$$r_{xy}[n] = \sum_{k=-\infty}^{\infty} x[k]\, y[k-n]$$

   Let $S_{xy}(z)$ denote the z-transform of $r_{xy}[n]$.

   (a) Express $S_{xy}(z)$ in terms of $X(z)$ and $Y(z)$.

   (b) Let $x[n] = a^n\, u[n]$, where $|a| < 1$. Find the simplified expression for $S_{xx}(z)$.

3. An LTI system is described by the system function

$$H(z) = \frac{\left(1 - \frac{1}{2}z^{-2}\right)}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{4}z^{-1}\right)}, \qquad |z| > \frac{1}{2}$$

   Determine the impulse response of the system and the difference equation that relates the input and output.

4. Find the causal inverse z-transform of the following functions:

$$X_1(z) = \frac{z^3}{z-1} \qquad X_2(z) = \frac{4z^2 + 8z}{4z^2 - 5z + 1} \qquad X_3(z) = \frac{4}{z^3(2z-1)}$$

5. Find all inverse transforms of the function

$$X(z) = \frac{z}{(z-1)^2(z-2)}$$

6. (a) If $x[n] \overset{\mathcal{Z}}{\longleftrightarrow} X(z)$, $x_1[n] \overset{\mathcal{Z}}{\longleftrightarrow} X_1(z)$, and $X_1(z) = X(z^N)$, express $x_1[n]$ in terms of $x[n]$.

   (b) Use the result from the previous part to find the causal inverse pz-transform of the function $X_1(z) = z^N/(z^N - 1)$.

7. Determine the inverse z-transform using the specified method for each of the following: (a) $\dfrac{1 - \frac{1}{3}z^{-1}}{1 + \frac{1}{3}z^{-1}}$; right-sided sequence, long-division, (b) $\dfrac{3}{z - \frac{1}{4} - \frac{1}{8}z^{-1}}$; stable sequence, partial-fraction expansion, (c) $\ln(1 - 4z)$, $|z| < \frac{1}{4}$, power-series method.

8. The following facts are given about and LTI system characterized by $h[n] \overset{\mathcal{Z}}{\longleftrightarrow} H(z)$: (i) $h[n]$ is real-valued, (ii) $h[n]$ is right-sided, (iii) $\lim_{z\to\infty} H(z) = 1$, (iv) $H(z)$ has two zeros, and (v) $H(z)$ has a complex-valued pole on the circle defined by $|z| = 3/4$. Is the system causal? Is it stable?

9. Consider the following z-transform pairs:

$$u[n] \overset{\mathcal{Z}}{\longleftrightarrow} \frac{1}{1-z^{-1}} \qquad e^{-|n|} \overset{\mathcal{Z}}{\longleftrightarrow} \frac{1-e^{-2}}{(1-e^{-1}z^{-1})(1-e^{-1}z)}$$

along with their associated RoCs. Find the z-transform of $e^{-|n|}\,u[n]$ using the product theorem. That is, evaluate

$$\frac{1}{2\pi j}\oint_c \frac{1}{1-\tau^{-1}} \frac{1-e^{-2}}{\left(1-e^{-1}\frac{\tau}{z}\right)\left(1-e^{-1}\frac{z}{\tau}\right)} \frac{d\tau}{\tau}$$

In what region does the contour of integration C lie?

10. If $X(z) = \log(1+2z^{-1})$ with RoC $|z| > 2$, find $x[n]$.

11. Find the inverse z-transform of the following functions: (i) $X(z) = \sin(z^2)$ with RoC $z \in \mathbb{C}$, (ii) $X(z) = e^{1/z}$ with RoC $z \in \mathbb{C}\backslash\{0\}$.

12. Find all inverse transforms of the function

$$X(z) = \frac{-1+\frac{5}{2}z^{-1}-\frac{5}{4}z^{-2}}{(1-z^{-1})(1-\frac{1}{2}z^{-1})^2}$$

13. Let $x[n] \overset{\mathcal{Z}}{\longleftrightarrow} X(z)$. The sequence $y[n]$ is defined as

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

What is $Y(z)$ and its associated RoC ?

14. Let $x[n] = \sum_{k=-n}^{n} a^{|k|}$ for $n \geq 0$; $y[n] = 0$ for $n < 0$. It is further given that $|a| < 1$. Find $X(z)$.

15. Consider $x[n] = \sum_{k=1}^{n} w[k]$. Verify that $x[n]$ can be written as $(w[n]\cdot u[n-1]) * u[n]$. Let $w[n] = a^n$ where $|a| < 1$. Using the product theorem, find the z-transform of $w[n]\cdot u[n-1]$. Using this result, find $X(z)$.