EE5330: Digital Signal Processing

Tutorial 2

(August 14, 2013)

1. Determine the z-transform for each of the following sequences. Sketch the pole-zero plot and indicate the RoC. Also indicate whether or not the discrete-time Fourier transform exists, and if it does, whether it can be obtained by evaluating the z-transform on the unit circle. (a) $\delta[n]$, (b) $\delta[-n]$, (c) $\delta[n-1]$, (d) $\delta[n+1]$, (e) $-(1/2)^n u[-n-1]$, (f) $(1/2)^n u[-n]$, (g) 1, (h) $7 \cdot (1/3)^n \cos(2\pi n/6 + \pi/4) u[n]$, (i) $7 \cdot (1/3)^n \cos(2\pi n/6 + \pi/4)$, (j) a^n.

2. The transfer function $H(z)$ of an LTI system can be determined to within a scale factor given its pole/zero locations. Let the zeros be at c_1, c_2, and 0, with the poles at d_i, $i = 1, 2, 3$. Write the expression for $H(z)$ with factors of the form (i) $(z - a)$, and (ii) $(1 - az^{-1})$. Next, suppose the zeros are at c_1 and c_2, and the poles remain unchanged. Write down $H(z)$ in both forms. Note carefully the difference between these two examples. It is customary to use powers of z^{-1} in DSP literature because they correspond to causal delay elements. Note that the vector `c` in the expression `roots(c)` corresponds to the roots of the polynomial `C(1)*X^N + ··· + C(N)*X + C(N+1)`, i.e., the first coefficient corresponds to the highest power of `X`. In MATLAB be careful about whether a coefficient vector corresponds to a polynomial in z or z^{-1}.

3. Determine the RoC of the z-transform of the following sequences without explicitly finding $X(z)$; in each case specify whether or not the DTFT exists.
   (a) $x[n] = \left[\left(\frac{1}{2}\right)^n - \left(-\frac{4}{3}\right)^n\right] u[n-10]$
   (b) $x[n] = \left(-\frac{10}{9}\right)^n u[-n+4]$
   (c) $x[n] = \left(\frac{e}{\pi}\right)^n u[n+4]$
   (d) $x[n] = \left(\frac{e}{-\pi}\right)^n u[n] + (2+3j)^{n-2} u[-n-1]$

4. Determine $X(z)$ and the corresponding RoC for $x[n] = |n| a^{|n|}$ where $|a| < 1$.

5. The input to an LTI system is $x[n] = 2^n u[-n-1] + 0.5^n u[n]$, which produces the output $y[n] = 6 \cdot (0.5^n - 0.75^n) u[n]$.
   (a) Determine the system function $H(z)$, its poles, zeros, and RoC.
   (b) Write down the difference equation that relates $x[n]$ and $y[n]$.

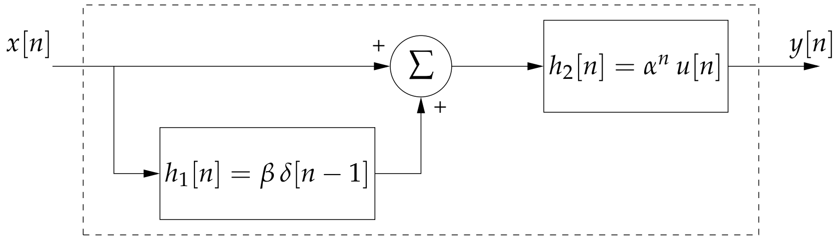
6. The input to a causal LTI system $H(z)$ is the signal $x[n] = -4 \cdot 2^n u[-n-1] - 0.5^n u[n]$. The output signal's z-transform is given by

$$Y(z) = \frac{1 - z^{-2}}{(1 - 0.5z^{-1})(1 - 2z^{-1})}$$

Determine (a) $H(z)$ and its RoC, and (b) the RoC of $Y(z)$.

7. Consider the rational function $X(z) = \dfrac{1+z^{-1}}{1+z^{-1}+z^{-2}}$. Expand $X(z)$ into a power series such that the series is convergent at $z = 0$. Identify the three samples of the corresponding inverse closest to the origin. Is the sample at the origin nonzero?

8. Consider the system shown in the figure below. (a) Find the overall transfer function $H(z)$. (b) From $H(z)$ find the difference equation that relates the input and output. Finding the difference equation using the z-transform approach is much easier than obtaining the answer using the time-domain approach.



$h[n]$

9. Consider
$$Y(z) = \frac{1-a^2}{1+a^2-a(z+z^{-1})}$$
Using long-division, find (i) causal inverse z-transform (quotient should be in powers of z^{-1}), and (ii) anti-causal inverse z-transform (quotient should be in powers of z).

10. Let $x[n] = u[-n-1] + (1/2)^n u[n]$. It is convolved with $h[n]$, where $h[n] = 0$ for $n < 0$, and results in $y[n]$ whose z-transform is give by
$$Y(z) = \frac{-0.5z^{-1}}{(1-0.5z^{-1})(1-\alpha z^{-1})}$$
where $0.5 < |\alpha| < 1$. Determine $H(z)$ and its RoC. Also specify the RoC of $Y(z)$.

11. Consider the filter $H(z)$ that corresponds to the system characterized by the difference equation $y[n] - y[n-2] = x[n] - x[n-6]$.
    (a) Determine the poles and zeros of $H(z)$. Is the filter causal? stable?
    (b) Derive and sketch the magnitude frequency response.

    Refer to the material on pp. 388–389 of S.K. Mitra's "Digital Signal Processing" (3rd edition, Tata McGraw-Hill, 2006).

12. **Computer assignment** Consider the command `roots(poly(a))`, where `a` is a vector containing roots. Consider a root at 0.9 with multiplicity ranging from 1 to 6, and observe the accuracy of the answer (use `format double`) to observe more digits than the default display precision. "`roots(poly(1:20))` generates Wilkinson's famous example" (see also `http://www.ima.umn.edu/~arnold/455.f97/labs/lab02.ps`).

13. **Computer assignment** Carry out the partial fraction expansion of the transforms in P4.9 (p. 113 of the book "DSP Using MATLAB" by Proakis and Ingle, Brooks/Cole, 2000). Use the `residue` command.

14. **Computer assignment** Using MATLAB try problem M 6.1 on p. 351 of S.K. Mitra's "Digital Signal Processing" (3rd edition, Tata McGraw-Hill, 2006). Also lookup the `zplane` command. ◇