

EE5330: Digital Signal Processing

Tutorial 1

(August 05, 2013)

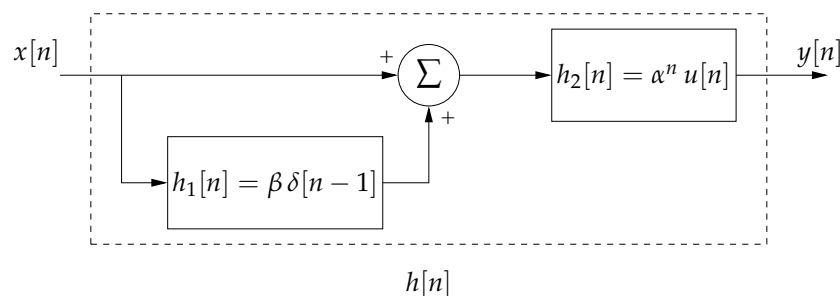
1. Consider $x[n] = \cos(\omega_0 n)$ for various values of ω_0 in the set $\{0, 0.1\pi, 0.2\pi, 0.8\pi, 0.9\pi, \pi, 1.1\pi, 1.2\pi, \sqrt{2}\}$. What is the period of $x[\cdot]$ in each case?
2. Determine the period of $x[n] = \sin(0.08\pi n)$. List two other *distinct* sinusoids that have the same period as $x[n]$.
3. Consider the harmonically related periodic signals $x_k[n] = \cos(2\pi kn/N)$, where $0 \leq k \leq N-1$. Since $x_k[n]$ is real-valued, the highest value of k can be limited to the integer that is less than or equal to $N/2$. What is the period of $x_k[n]$ for each k when $N = 11$ and $N = 12$? Is there any difference in behaviour when compared with its continuous-time counterpart?
4. Determine whether or not the system defined below is additive and/or homogeneous. Justify your answers by providing a proof for each property if it holds, or a counter-example if it does not.

$$y[n] = \begin{cases} \frac{x[n]x[n-2]}{x[n-1]} & x[n-1] \neq 0 \\ 0 & x[n-1] = 0 \end{cases}$$

5. Can you think of a system that is additive but not homogeneous?
6. A system may or may not be (a) memoryless, (b) time-invariant, (c) linear, (d) causal, (e) stable. Determine which of these properties hold and which do not hold for each of the systems whose input-output relationships are given below. Justify your answers. In each example, $x[n]$ denotes system input and $y[n]$ denotes system output. (i) $y[n] = \text{Ev}\{x[n-1]\}$ (where $\text{Ev}\{\cdot\}$ represents the even part),
(ii) $y[n] = \begin{cases} x[n] & n \geq 1 \\ 0 & n = 0 \\ x[n+1] & n \leq -1 \end{cases}$, (iii) $y[n] = x[2n]$, (iv) $y[n] = x[n^2]$.
7. Given an example of a cascade of two nonlinear systems that is linear overall. Repeat for two time-variant systems that is overall time-invariant. How about two non-causal systems that are overall causal?
8. Consider the difference equation $y[n] - \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = 3x[n]$. Both a causal and an anticausal LTI system can be characterized by this difference equation. Find the impulse responses of the two systems. Comment about system stability in each case.
9. A periodic signal $\tilde{x}[n]$ is applied to an LTI system with impulse response $h[n]$. Is the output $y[n]$ periodic? If it is, what is its period?
10. $y[n] = x[n] * h[n]$, where “*” denotes convolution. Let $h[n] = 0$ outside $0 \leq n \leq N-1$, which reduces the convolution summation to a finite one. For each n , the output $y[n]$ can be thought of as an *inner product* of two N -dimensional vectors, i.e., $\mathbf{a}^T \mathbf{b}$. Let \mathbf{y}

represent in vector form the output from 0 to $N - 1$. The convolution can be expressed as $\mathbf{y} = \mathbf{X}\mathbf{h}$. What are the entries of \mathbf{X} and \mathbf{h} ? What can you say about the diagonal entries of \mathbf{X} ?

11. Suppose $y = x * h$. $\{x\} = \{2, 2, 3, 3, 4\}$ and $\{y\} = \{2, 4, 9, 10, 13, 10, 8\}$. Find $\{h\}$. For the given sequences, the first sample corresponds to $n = 0$.
12. If you multiply two polynomials, the coefficients of the result can be obtained by considering each polynomial's coefficients as a sequence and convolving the two. Consider $\{1, 3, 1\} * \{1, 2, 2\}$. Use this result to compute 131×122 . See the paper, "A Novel Method for Calculating the Convolution Sum of Two Finite Length Sequences" by John W. Pierre (IEEE Trans. on Education, vol. 39, no. 1, Feb. 1996).
13. Consider the difference equation $y[n] = ay[n - 1] + x[n]$. Let $x[n] = \delta[n]$. To get a causal response we need $y[-1] = 0$. That is, the initial condition is specified at a location that is just to the left of $n = 0$. To get an anticausal response, we need to specify the "final rest condition", i.e., $y[n_0] = 0$. Guess the value of n_0 . Now rewrite the difference equation in the form required for backward recursion. What should n_0 be for you to be able to compute the anticausal $y[n]$ recursively? Was your earlier guess correct? At what time index does the first output sample appear?
14. Consider the system shown in the figure below. (a) Find the overall impulse response $h[n]$. (b) Specify the difference equation that relates the output $y[n]$ and the input $x[n]$. (c) Is the system causal? (d) Under what condition would the system be stable?



15. Let $p[n] = \alpha^n$, $q[n] = \delta[n] - \alpha \delta[n - 1]$, $r[n] = \alpha^n u[n]$, where $\alpha \neq 0$ is a complex number. Show that the following are well-defined: (i) $p[n] * q[n]$, (ii) $(p[n] * q[n]) * r[n]$, (iii) $q[n] * r[n]$, and (iv) $p[n] * (q[n] * r[n])$. Verify whether or not $(p[n] * q[n]) * r[n]$ and $p[n] * (q[n] * r[n])$ are the same.
16. **Computer assignment** Let $x(t) = \cos(2\pi f_0 t)$. Form $x[n] = x(nT)$, where T is called the "sampling period". Let $f_0 = 50$ Hz. Plot the signal $x[n]$ versus nT for a duration of 40ms for various values of T : $125\mu\text{s}$, 1 ms, 2 ms, and 10 ms using the default option (which joins the individual points by straight line); repeat to get "stem" plots. Observe what happens when you plot $x[n]$ versus n in all these cases.
17. **Computer assignment** Let $x(t) = f_c \text{sinc}(f_c t)$, where $\text{sinc}(t) = \frac{\sin \pi t}{\pi t}$. Form $x[n] = x(nT)$, where T is sampling period. Let $f_c = 5$ Hz. Plot the signal $x[n]$ versus nT by

sampling $x(t)$ over the interval $-2 \leq t \leq 2$ with $T = 0.02$. Change f_c to 10, 20, 30, 40, and 50 repeat the plots. Observe what happens when you plot $x[n]$ versus n in all these cases. What is the value of $x[0]$?

18. **Computer assignment** The general form of an LCCDE is $\sum_{l=0}^N a_l y[n-l] = \sum_{m=0}^M b_m x[n-m]$. In MATLAB, a routine called `filter` is available for solving it. Study the `filter` command and its various options. Find the output when the input is (a) unit impulse, and (b) unit step function. Look up the `impz` function and try out some examples. Next consider $x_1(t) = \cos(2\pi 50t)$ sampled at $T = 1$ ms and $x_2(t) = \cos(2\pi 100t)$ sampled at $T = 0.5$ ms. Plot $x_1[n]$ and $x_2[n]$ versus n and comment on the result.
19. **Computer assignment** Consider the convolution of $\{1, 2, 3, 4\}$ and $\{3, 2, 1\}$. Determine the 6×4 matrix \mathbf{H} such that $\mathbf{y} = \mathbf{H}\mathbf{x}$. Since convolution is commutative, obtain \mathbf{y} by using the equation $\mathbf{X}\mathbf{h}$. What are the entries of \mathbf{X} ? A convolution matrix can be quickly constructed using the `toeplitz` command; see also `convmtx`.
20. **Computer assignment** Let $\mathbf{y} = \mathbf{X}\mathbf{h}$, where the matrices are as defined in the previous problem. Consider the sequences given in Problem 11. Let $\mathbf{h} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{y}$. Does this result agree with what you obtained earlier? Look up material on the least-squares solution of $\mathbf{A}\mathbf{x} = \mathbf{y}$. Can you obtain \mathbf{h} by solving $\mathbf{y} = \mathbf{X}\mathbf{h}$ recursively?
21. **Computer assignment** Consider the convolution of $x[n]$ and $y[n]$, where $y[n] = x[-n]$. The result is called the *autocorrelation* of $x[n]$. Let $x[n] = \text{rand}(100, 1)$. Find the autocorrelation of $x[n]$. Normalize the peak value to unity. At what time index does it occur? Replace `rand` by `randn` and repeat. Do you see any qualitative change in the result? Study the function `xcorr`. If the samples of a sequence are uncorrelated, the resulting autocorrelation sequence will be impulsive (or approximately so). For example, consider N successive digits of π . Replace an even digit by 1 and odd digit by -1 . Find the autocorrelation of the resulting sequence. The first ten thousand digits of π can be found at <http://www.joyofpi.com/pi.html>.
22. **Computer assignment** In the previous experiment, if you replaced each odd digit by 0 instead of -1 , how does the plot change? Can you explain the result? \diamond