Interferer Identification in HetNets using Compressive Sensing Framework

Niranjan M Gowda and Arun Pachai Kannu

Abstract—We consider heterogeneous cellular networks (HetNet) where each base station (BS) sends unique training signal based on its physical layer cell identity. Received signal at mobile terminal (MT) is superposition of training signals from different BS. Neither BS identities nor their channel response is known apriori at MT. For this scenario, we consider the problem of finding constituent BS identities from superimposed components in received signal. Though number of BS with unique identities can be quite large in a HetNet, in any given scenario, actual number of BS interfering at MT is relatively few. By exploiting this sparseness, we show that our problem can be solved using block sparse signal reconstruction algorithms under compressive sensing framework where sensing matrix is block-matrix with circulant-blocks (BCB). We apply convex programming based \(\ell_2/\ell_1\) mixed norm minimization and greed based subspace matching pursuits approaches to recover interfering BS identities. We characterize block restricted isometry property, mutual subspace incoherence of BCB matrices with i.i.d. Rademacher distributed entries and establish certain recovery guarantees. Our proposed approaches give significant improvements over conventional successive interference cancellation approach over both randomly generated and 3GPP-LTE training signals.

Index Terms—block sparse signal reconstruction, block restricted isometry property, support recovery, cell search, cellular networks, interference cancellation.

I. INTRODUCTION

A CELLULAR NETWORK comprises of base stations (BS) serving users in their respective coverage areas. With growing demand for high data rates, heterogeneous cellular networks (HetNets) are being designed by deploying small low powered base stations within the coverage area of a conventional cellular base station [1], [2]. The components in a HetNet are macro cells (conventional BS), pico cells (BS with smaller coverage than macro) and femto cells (user deployed BS with home coverage). The following are the key features of interference in HetNets.

- **Unplanned** interference which is caused due to the femto cells which are user deployed and uncoordinated.
- **Strong** interference which is caused due to near-far effects and differences in power levels of different BS.
- **Multiple** sources of interference (for instance, multiple femto BS interferers in the neighborhood of a mobile).

These low powered base stations are already deployed in hot spots like hotels and stadiums. According to the estimates [3], in 2012, about 33 percent of global mobile data traffic was offloaded using low-powered nodes such as femto, pico BS and this offloading is projected to increase rapidly in the future.

Interference is a key challenge in the HetNet deployment. The received signal at the mobile terminal (MT) is the superposition of signals from various BS in its vicinity. It is important to develop suitable signal processing architectures for basic physical layer operations such as synchronization and channel estimation for the mobile MT operating in HetNets. The performance of physical layer signal processing depends heavily on the how well the interfering signals from various BS can be eliminated from the observed signal. Successful elimination of interference involves, identifying the set of interfering BS, estimating the interference and subsequent cancellation of interference.

In this paper, we consider the problem of finding the identities of constituent BS from the superimposed components in the received signal of MT. We consider the case where the channels between MT and BS are frequency selective. We consider the identification problem at the “start-up” during which there is no prior information available about the interferers. We address this problem using the framework of compressive sensing [4], [5]. Compressive sensing (CS) considers solving an under determined system of linear equations when the unknown vector is sparse (i.e., has only very few non-zero entries). Several convex-programming-based and greedy-based algorithms are studied in the CS literature. Performance of the sparse signal recovery depends heavily on the nature of the sensing matrix (i.e., the coefficient matrix in the linear system of equations). Specifically, restricted isometry property (RIP) and mutual incoherence property of the sensing matrix play a critical role in the recovery guarantees.

In our HetNet model, the number of BS with unique identities can be quite large. But the actual number of BS interfering in a MT in any given scenario is relatively few. Exploiting the sparseness of the interferers, we solve our problem using the block sparse signal reconstruction approaches [6] under CS framework. Our methods give significant performance gains over the conventional successive interference cancellation algorithm. A related problem for flat fading channels is addressed in [7], [8].

Main contributions from our work are

- We establish the connection between the problem of identifying interferers in HetNets and the problem of block sparse signal recovery in compressive sensing where the
sensing matrix is shown to be of the structure block-matrix with circulant blocks (BCB).

- We characterize the block-RIP and mutual subspace incoherence of BCB matrices.
- We study convex programming based mixed norm minimization algorithm for our problem and establish its recovery guarantees for BCB sensing matrices in the bounded noise scenario.
- We study greedy based subspace matching pursuit algorithm and establish its recovery guarantees with BCB sensing matrices in the bounded noise scenario.

**Notation:** matrices/vectors are denoted by bold upper-case/lowercase letters, cardinality of the set denoted by \(| \cdot |\), \(l_p\) norm by \(\| \cdot \|_p\), inner product by \((\cdot, \cdot)\), transpose by \((\cdot)^t\) and hermitian by \((\cdot)^*\), sets denoted by mathcal font \(\mathcal{A}\) and set minus operation by \(\mathcal{A}\setminus \mathcal{B}\).

**II. System Model and Problem Statement**

We consider the downlink scenario in a HetNet, where the received signal at a mobile terminal is a superposition of signals transmitted by different base stations in the neighborhood of MT. Each BS in a cellular network has a physical layer cell identity and let \(I = \{1, \cdots, C\}\) denote the set of cell identities available in a cellular system. For instance, for 3GPP-LTE system, the total number of cell identities available in a cellular system. For instance, for 3GPP-LTE system, the total number of cell identities available is \(C = 504\). The identity \(q \in I\) assigned to the base station uniquely determines the training signals sent by BS. These training signals facilitate various operations such as synchronization and channel estimation at the MT. We assume that the frames from different base stations in the given area are aligned in time. This can be facilitated by enforcing the femto and pico BS to track the macro BS timing by observing the timing synchronization signals present in the frame transmitted by macro BS.

Consider the scenario where a given mobile terminal receives superposition of signals from different base stations with identities denoted in the set \(B = \{q_1, \cdots, q_s\} \subset I\). We consider a cyclic prefixed transmission and the channel’s impulse response (CIR) length \(L\) is assumed to be less than cyclic prefix, is given by, for \(i \in \mathcal{I}\) and \(q \in B\)

\[
y_q(n) = \sum_{i=0}^{L-1} \sqrt{P_q h_q(i)} x_q(n-i) + w(n),
\]

where \(h_q \in \mathbb{C}^{L \times 1}\) is CIR vector of BS with identity \(q\), \(w \in \mathbb{C}^{N \times 1}\) is noise vector, and \(X_q \in \mathbb{C}^{N \times L}\) is a circulant matrix with its generating vector (first column) being \(\{x_q(0), \cdots, x_q(N-1)\}\).

In this paper, we consider the problem of finding all the interfering base stations, essentially the set \(\mathcal{B}\), based on the received observation \(y\). The receiver knows the training signal set \(\{x_q(n), \forall q \in I\}\). However, MT does not know the CIR vectors \(h_q\) and the transmit power levels \(P_q\) for any identity \(q\). If \(\sqrt{P_q h_q}\) is known for some \(q\), then those components can be “subtracted out” from the observation without any loss of generality. Also, we assume that the identities \(q_i \in \mathcal{B}\) are distinct. Identification of the set of interfering BS plays an important role in cellular networks. Interferer identification helps in subsequent cancellation of interference. It also helps in hand-off related issues. In the following section, we show that CS framework can be employed to solve our problem of identifying the set of interfering BS.

**III. Compressive Sensing Framework**

In this section, we establish the connection between our problem of identifying interfering BS in a HetNet to the sparse signal reconstruction problem in compressive sensing. Towards that, we rewrite our observation model in (2) as follows. For all \(q \notin \mathcal{B}\), let us define \(h_q = [0, \cdots, 0]^t \in \mathbb{C}^{L \times 1}\) and \(P_q = 0\). Now, defining \(x_q = \sqrt{P_q h_q}\), we can write (2) as

\[
y = \sum_{q \in \mathcal{I}} X_q z_q + w, \quad \text{(3)}
\]

\[
X \hat{z} = \begin{bmatrix} z_1 \\ \vdots \\ z_C \end{bmatrix} + w, \quad \text{(4)}
\]

where the dimensions after concatenation are \(X \in \mathbb{C}^{N \times LC}\) and \(z \in \mathbb{C}^{LC \times 1}\). We introduce the following notation for use in subsequent sections. For any \(A \subset \mathcal{I}\), we define \(\hat{z}_A\) as the (vertical) concatenation of blocks \(z_i\) for all \(i \in A\) in the ascending order of indices. Similarly, \(X_A\) denotes (horizontal) concatenation of blocks \(X_i\) for \(i \in A\).

In any given HetNet scenario, the number of interfering base stations is relatively small compared to the total number cell identities available, i.e., \(S \ll C\). In the CS terminology, \(z\) is called a \(S\)-block sparse signal, which is precisely defined as

**Definition 1.** A vector \(u \in \mathbb{C}^{CL \times 1}\), which is obtained by concatenation of \(C\) vectors \(\{u_q \in \mathbb{C}^{L \times 1}, 1 \leq q \leq C\}\), is called \(S\)-block sparse signal if \(\|u_q\|_2 > 0\) for at most \(S\) indices.

In our model, the unknown channel vector \(z\) is a \(S\)-block sparse signal. Only \(S\) sub-blocks of \(z\), each of length \(L\), contribute for the observation \(y\). It is possible that the length of the training sequence \(N \ll LC\), in which case, (4) is an under-determined system (with additive noise). In order to identify the set of interfering base stations from the superimposed observation \(y\), we first consider the problem of recovering the complete CIR vector \(z\) from \(y\). Based on
the reconstructed CIR \( \hat{z} \), we subsequently detect the set \( B \). Our problem comes under the typical problem of recovering a sparse signal from the under-determined set of observations [4] addressed in the CS literature. Specifically, our problem comes under the recovery of block sparse signals, previously studied in [6], [9], [10]. The training signal matrix \( \bar{X} \) in (4), which we shall also refer as sensing matrix using the CS terminology, plays a critical role on the performance of recovering \( \hat{z} \) from \( y \). Towards that, we define the notion of block restricted isometry property (block-RIP) of a matrix.

**Definition 2.** A matrix \( A \in \mathbb{C}^{N \times LC} \) is said to satisfy block-RIP of order \( S \) with block-RIP constant \( \delta_S \in [0, 1) \) if, for all \( S \) block sparse signals \( u \in \mathbb{C}^{LC \times 1} \), we have

\[
(1 - \delta_S) \| u \|_2^2 \leq \| Au \|_2^2 \leq (1 + \delta_S) \| u \|_2^2
\]

Theorem 1. When the training sequences are generated using i.i.d. Rademacher distribution, the BCB matrix \( \bar{X} \) in (4) with

\[
N \geq \frac{6(LS)^2 \log(6L^2C)}{\delta^2}
\]

satisfies block-RIP of order \( S \) with block-RIP constant \( \delta_S \leq \delta \) with probability at least \( 1 - \beta \), where \( \beta \in (0, 1] \) and \( \delta \in (0, 1) \).

**Proof:** See Appendix B.

Theorem 1 characterizes block-RIP of our BCB matrix. Our result suggests that the length of the training sequence \( N \) sufficient to guarantee block-RIP scales quadratically with sparsity level \( LS \). Block-RIP of unstructured matrices with random Gaussian entries has been characterized in [6]. The block restricted isometry property introduced in [11] is a relaxed requirement compared to the restricted isometry property (RIP) introduced in [12]. RIP of matrices without any structure [13], [14] and RIP of structured matrices such as toepitiz/circulant matrices [15], [16] have been studied previously.

Another property of sensing matrix \( \bar{X} \) which is relevant for block sparse signal reconstruction is the mutual subspace incoherence - \( \mu \) which is defined as [17]

\[
\mu = \max_{i,j \in \mathcal{I}} \left\{ \frac{\max_{a_i \in X_i} | \langle a_i, a_j \rangle |}{\|a_i\|_2 \|a_j\|_2} \right\}
\]

where \( X_i \) is column space of \( X_i \), \( \mu \) is a measure of the smallest angle between any two column spaces of the blocks \( \{X_i, i \in \mathcal{I} \} \). Small value of \( \mu \) is desirable for good performance in the sparse signal recovery. The following theorem characterizes the parameter \( \mu \) of our HetNet sensing matrix \( \bar{X} \).

**Theorem 2.** When the training sequences are generated using i.i.d. Rademacher distribution, the parameter \( \mu \) of BCB matrix \( \bar{X} \) satisfies \( \mu \leq \frac{\delta}{\sqrt{S}} \) for some \( \delta \in (0, 1) \) with probability at least \( 1 - \beta \), if \( N \) satisfies (7).

**Proof:** See Appendix C.

We have established the connection between the problem of identification of interfering base stations in HetNets and the block sparse signal reconstruction problem considered in the CS framework. We have also characterized the block-RIP property and mutual subspace incoherence of our BCB matrix \( \bar{X} \) with generating vectors of each block being i.i.d. Rademacher distributed entries. In the subsequent sections, we will see the role these properties of \( \bar{X} \) in guaranteeing the recovery of \( \hat{z} \) from the observation \( y \). Specific reconstruction algorithms and their guarantees on the recovery are discussed in the following section.

**IV. BLOCK SPARSE SIGNAL RECONSTRUCTION**

In this section, we present different algorithms for block sparse signal recovery and subsequent identification of interfering base stations. We also provide some guarantees on the recovery.

**A. \( l_2/l_1 \) mixed norm minimization algorithm**

The \( l_2/l_1 \) mixed norm minimization (MMN) [6] is the convex optimization program

\[
\min_{\hat{z}} \sum_{i=1}^{C} \|\hat{z}_i\|_2
\]

subject to \( \|\bar{X} \hat{z} - y\|_2 \leq \sigma \),

where \( \sigma \) is a constant. The recovery guarantees of the \( l_2/l_1 \) mixed norm estimator for real vectors was established in [6]. In our model, the unknown CIR vector is complex. To establish the recovery guarantees for the complex case, we use the following lemma, which is an extension of Lemma 2.1 in [18].

**Lemma 1.** Let \( \bar{z}_1, \bar{z}_2 \in \mathbb{C}^{CL \times 1} \) be \( S \)-block sparse with disjoint supports, \( B_1 \cap B_2 = \emptyset \). If \( \bar{X} \) satisfies block-RIP of order \( 2S \), then

\[
|\langle \bar{X} \bar{z}_1, \bar{X} \bar{z}_2 \rangle| \leq 2\delta_{2S} \|\bar{z}_1\|_2 \|\bar{z}_2\|_2
\]

**Proof:** Let \( x_1 = \frac{\bar{z}_1}{\|\bar{z}_1\|_2}, x_2 = \frac{\bar{z}_2}{\|\bar{z}_2\|_2} \). From the parallelogram identity for complex vectors,

\[
|\langle x_1, x_2 \rangle| \leq \frac{1}{4} \left[ \|x_1 + x_2\|^2 + \|x_1 - x_2\|^2 - \|ix_1 + x_2\|^2 - \|ix_1 - x_2\|^2 \right]
\]

By block-RIP property of \( \bar{X} \),

\[
(1 - \delta_{2S})\|x_1 + x_2\|^2 \leq \|\bar{X}(x_1 + x_2)\|^2 \leq (1 + \delta_{2S})\|x_1 + x_2\|^2
\]

Since \( B_1 \cap B_2 = \emptyset \),

\[
2(1 - \delta_{2S}) \leq \|\bar{X}(x_1 + x_2)\|^2 \leq 2(1 + \delta_{2S})
\]
and hence \(|\|X(x_1 + x_2)\|^2 - |X(x_1 - x_2)\|^2| \leq 2(1 - \delta_{2S}) - 2(1 + 2\delta_{2S}) = 4\delta_{2S}\). Similarly, \(|X(iw_1 - x_2)\|^2 - |X(iw_1 + x_2)\|^2| \leq 4\delta_{2S}\). Using these in (10), we have
\[ |\langle X_{x_1}, X_{x_2} \rangle| \leq \frac{4\delta_{2S}}{4} + \frac{4\delta_{2S}}{4} = 2\delta_{2S}. \]

Using the above lemma, the recovery guarantees of MNM algorithm for the complex vector case can be derived along the same lines as that of Theorem 2 in [6], and the result is stated below.

**Theorem 3.** For our block sparse model (4), if \(X\) satisfies block-RIP of order \(2S\) with block-RIP constant \(\delta_{2S} \leq \delta_{2S} \leq 0.2612\), then the solution of optimization problem (9) satisfies the following inequality,
\[ |\|z - \hat{z}\|_2^2 \leq \frac{4\sqrt{1 + \delta_{2S}}}{1 - (1 + 2\sqrt{2})\delta_{2S}} \|X\|_F \] (12)
when the noise vector is bounded as \(|u|_2 \leq \sigma\).

With small \(\delta_{2S}\), the \(l_2/l_1\) mixed norm estimator behaves as if it has the knowledge of \(B\). For the noiseless case \(\sigma = 0\), recovery is perfect. From the reconstructed \(\hat{z}\), we have the reconstructed CIR of every BS \(\hat{z}_q\) so that \(\hat{z} = [\hat{z}_1 \cdots \hat{z}_C]^{\top}\) from which we identify the set of interferers as
\[ \hat{B} = \{q, \text{ such that } |\hat{z}_q|_1 \geq |\hat{z}_i|_1 \forall i \in A\} \] (13)
for some \(A \subset I\) with \(|A| = C - S\). Essentially, \(\hat{B}\) is formed by selecting the top \(S\) identities based on the \(l_1\) norm of reconstructed CIR of each BS.

**B. Subspace Matching Pursuit Algorithm**

Subspace matching pursuit (SMP) [17] is a greedy block sparse signal reconstruction algorithm. Let \(\Pi_j(r)\) be the projection of a vector \(r\) onto the subspace \(X_j\). Subspace matching pursuit algorithm to recover \(z\) from \(y\) proceeds as follows.

**Step 1.** Set \(r_0 = y\), \(k = 1\), \(\hat{B} = \emptyset\), \(\hat{z}_0 = 0\).
**Step 2.** Find the index \(i_k\) such that
\[ i_k = \arg \min_{j \in I \setminus \hat{B}} \|r_{k-1} - \Pi_j(r_{k-1})\|_2 \]
**Step 3.** \(\hat{B} \leftarrow \hat{B} \cup \{i_k\}\)
**Step 4.** \(\hat{x} = \arg \min_{x} \|y - X_{\hat{B}}x\|_2\)
**Step 5.** \(r_k \leftarrow y - X_{\hat{B}}\hat{x}\) and \(k \leftarrow k + 1\)
**Step 6.** If \(k \leq S\) and \(|r_k| > \sigma\) go to Step 2.
**Step 7.** \(\hat{z}_{\hat{B}} \leftarrow \hat{x}\)

When the algorithm stops, \(\hat{B}\) gives the set of detected base stations and \(\hat{z}_{\hat{B}}\) is their corresponding reconstructed CIR.

**Theorem 4.** For the block sparse model (4), with the training signals being generated using i.i.d. Rademacher distribution, in the absence of noise, when \(N\) satisfies
\[ N \geq \frac{6(2LS)^2 \log(B^2L^C)}{\delta^2}, \delta \in (0, 1), \]
SMP recovers the set of interferers \(B\) exactly with probability at least \(1 - \beta\).

**Proof:** It has been shown in [17] that if \(S < \frac{1}{2} \left(1 + \frac{1}{\mu}\right)\), then SMP recovers support set \(B\) perfectly. Now, from Theorem 2, if \(N\) satisfies (14), for \(\delta \in (0, 1)\), we have \(\mu \leq \frac{\delta_{2S}}{2S - \delta}\), with probability greater than \(1 - \beta\). Putting these results together, we obtain the desired result.

For the noisy observation model with \(|u|_2 \leq \sigma\), the following theorem (theorem 3 in [17]) gives recovery guarantees.

**Theorem 5.** For the block sparse model (4) with \(\hat{z}_{\hat{B}}\) as SMP recovered CIR vector and \(\hat{z}_X = [\|X_1 z_1\|_2 \cdots \|X_C z_C\|_2]^T\), an upper bound on reconstruction error is given by,
\[ |\|\hat{z}_{\hat{B}} - \hat{z}_X\|_2 \leq \frac{\sigma^2}{1 - \mu(S - 1)} \]
if the following inequality holds,
\[ S \leq \frac{1}{2} + \frac{1}{\mu} \left(\frac{1}{2} - \frac{\sigma}{c}\right) \text{ where } c = \min_{q \in B} \{\|X_q z_q\|_2\}. \] (15)

To understand the implication of the above theorem, suppose \(\mu\) satisfies the bound in Theorem 2 (assuming \(N\) satisfies (14), we have
\[ \frac{1}{2} + \frac{2S - \delta}{\delta} \left(\frac{1}{2} - \frac{\sigma}{c}\right) \leq \frac{1}{2} + \frac{1}{\mu} \left(\frac{1}{2} - \frac{\sigma}{c}\right) \]
or\[ \frac{2S}{\delta} \left(\frac{1}{2} - \frac{\sigma}{c}\right) + \frac{\sigma}{c} \leq \frac{1}{2} + \frac{1}{\mu} \left(\frac{1}{2} \frac{\sigma}{c}\right) \]
Hence, if \(\sigma^2 > 4\sigma^2\) and \(\delta \leq 1 - 2\sigma/c\), then (15) holds. Hence, when \(\mu\) satisfies the bound in Theorem 2 with \(\delta \leq 1 - 2\sigma/c\) and the least received power from a BS is at least 6dB above noise power, recovery guarantee in Theorem 5 holds. Since SMP is a greedy algorithm, its computational complexity is less compared to the mixed norm estimator.

**C. Successive Interference Cancellation Algorithm**

The successive interference cancellation (SIC) is a conventional algorithm for the model SIC is implemented as follows.

**Step 1.** Set \(r_0 = y\), \(k = 1\), \(\hat{B} = \emptyset\), \(\hat{z}_0 = 0\).
**Step 2.** Find \(i_k\) such that
\[ i_k = \arg \max_{j \in I \setminus \hat{B}} \|X^*_j r_{k-1}\|_\infty \]
**Step 3.** \(\hat{B} \leftarrow \hat{B} \cup \{i_k\}\)
**Step 4.** \(\hat{x} = \arg \min_{x} \|y - X_{\hat{B}}x\|_2\)
**Step 5.** \(r_k \leftarrow y - X_{\hat{B}}\hat{x}\) and \(k \leftarrow k + 1\)
**Step 6.** If \(k \leq S\) and \(|r_k| > \sigma\) goto step 2.
**Step 7.** \(\hat{z}_{\hat{B}} \leftarrow \hat{x}\)
When the algorithm stops, \( \hat{\mathcal{B}} \) gives the set of detected base stations and \( \hat{\mathbf{x}}_{\mathcal{B}} \) is the corresponding reconstructed CIR. Here, we correlate the observation with training signals of different BS and choose the BS based on the correlation metric (and cancel the components from previously detected BS and proceed successively).

D. Complexity of Algorithms
Convex optimization based algorithms are in general more complex than greedy algorithms [19]. MNM can be cast as second order cone programming whose complexity is much higher than SMP though it can be solved in polynomial time [20]. In SMP, evaluation of the projection of residual for each block involves three matrix multiplications, a matrix inverse and a matrix-vector multiplication, while in SIC there is only one matrix-vector multiplication.

V. SIMULATION RESULTS
We study the performance of algorithms presented in the previous section using Monte-Carlo simulations. CVX package has been used for MNM simulations. The channel coefficients of interfering BS are i.i.d. complex Gaussian across both BS and taps with tap variance \( \frac{1}{\sqrt{2}} \). The additive noise is generated as a complex white Gaussian noise with variance \( \sigma^2 \).

The number of correctly identified base stations is given by \( N_d = |\mathcal{B} \cap \hat{\mathcal{B}}| \). We say that at least \( k \) base stations are detected if \( N_d \geq k \). We plot the various detection probabilities with respect to inverse of noise power \((-20 \log \sigma)\).

A. Performance of Algorithms
We average the detection performance over 500 trials wherein each trial the set of interfering BS \( \mathcal{B} \) is randomly chosen with \(|\mathcal{B}| = S\) and the results are plotted in Figures 1-3. The parameters \( N, C, L, S \) are indicated in the title of each plot.

Figures 1 and 2 correspond to the case where the training sequences are randomly generated with the entries being \( \{\pm 1\} \) i.i.d, a scaled version of Rademacher distribution. Figure 1 shows the performance of equal interference case (with power levels \( P_q \) of all interferers at \(-8 \) dB) and Figure 2 corresponds to unequal interference (with power levels \(-2, -5, -8, -8, -11, -14 \) dB).

In Fig. 3, we use the training signals specified in the 3GPP-LTE standard. Specifically, we use the secondary synchronization signals (SSS) specified in the LTE standard [21]. SSS are 64 length sequences generated by taking IFFT of 62 length binary sequences (with 2 tones punctured) which convey partial cell identity information taking 168 possibilities. For the LTE training signals, we consider 4 interferers all at power levels \(-5 \)dB. Following remarks can be made about the plots.

- MNM and SMP perform significantly better than the conventional SIC algorithm with randomly generated training signals as well as LTE training signals. MNM and SMP shows good detection performance even at very low signal to interference plus noise ratio levels. We also note that the gains of the proposed approaches (MNM and SMP) over SIC is more pronounced with the random training signals. Our results conform to the well known fact that CS based algorithms perform better than SIC when the unknown signal is sparse [5].
- Figures 1 and 2 show that SIC detection performance in the unequal interference case is relatively better than the equal interference case. This is in agreement with the conventional wisdom that SIC is good in the strong or weak interference regimes. On the contrary, MNM detection performance is better in the equal interference regime (Fig. 1) compared to the unequal interference regime (Fig. 2). This is not surprising as MNM recovers all the interferers jointly.
- SIC performance is interference-limited because even at low noise levels, the probability of detecting all six interferers is negligible as shown in Figure 1. MNM and SMP does not appear to be interference-limited as their performance improves steadily with reduction of noise variance. Recall that we have established recovery guarantees for MNM and SMP (for both noiseless and noisy cases), as long as the training signals guarantee desirable properties for sensing matrix \( \mathbf{X} \).

Figure 4 compares the average reconstruction error of detected channels recovered by the algorithms when received power of all the six interferers is at \(-8 \)dB. MNM has superior MSE performance while SMP and SIC have similar MSE performance.
and number of interferers to be order to declare that an interferer is present. We kept the true B. Effect of Thresholding

In Figure 5, we study the effect of having a threshold in order to declare that an interferer is present. We kept the true number of interferers to be $S = 6$. The recovery algorithms do not apriori know the value of $S$, but known the maximum number of possible interferers $S_{\text{max}}$ (whose value was set as 12). For SMP and SIC, the iterations stop based on the following condition, $|\tilde{B}| = S_{\text{max}}$ or residual norm $\|r_k\|_2 < \sigma \sqrt{N}$. For MNM we choose smallest $J$ such that $J \leq S_{\text{max}}$ and

$$\|X_{B_j}z_{B_j} - y\|_2 < \sigma \sqrt{N}, \quad \forall S_{\text{max}} \geq j \geq J$$

where $B_j$ is defined in (13) and it holds the indices of 'top' $j$ blocks of $z$. We take $\hat{B}_j$ as the recovered blocks (detected interferers).

In each simulation trial, the algorithms come up with a set of interferers (whose size is random) out of which some are correctly identified interferers and some are falsely identified interferers (false alarms). We find the average number of correctly identified cells and the average number of falsely identified cells over 1000 trials and the results are given in Figure 5. We note that MNM and SMP have higher detection and lower false alarm rates than SIC. False hits of SMP, MNM are almost zero irrespective of noise variance while detections depend on noise. MNM, SMP with the thresholds we have used tend to give near optimal false alarm rates. To better detection performance, threshold needs to be increased but it also increases false hits.

C. Effect of number of interferers

In Figure 6, we plot detection probability of the algorithms by varying the number of interferers $S$ from 1 to 10. We have set the power levels of interferers at 0 dB and noise variance also at 0 dB. We see that MNM and SMP are superior to SIC. We also note that, for the chosen parameters, if $S > 6$ then $SL > N$ and hence $X_B$ will have non-empty null space. So, after 6th iteration for all $S > 6$, Step 4 in SMP and SIC would be non-unique and hence can not recover more than 6 blocks. On the other hand, MNM is able to recover more than 6 blocks correctly. However, the reconstructed CIR will not be faithful.

VI. CONCLUSION

We solved the problem of identifying the set of interfering BS in HetNets by establishing the connection with the block sparse signal reconstruction problem in the CS framework. The new algorithms proposed based on this connection perform significantly better than conventional SIC algorithm. Performance studies of other block sparse signal recovery algorithms [5] and establishing theoretical guarantees of their recovery for our interferer identification problem is a future work. Another future work will focus on using training signals from multiple frames to improve the detection performance, which will come under the multiple measurement vector problems in the CS literature.
APPENDIX A
BOUNDING OFF DIAGONAL GRAM MATRIX ENTRIES

The off-diagonal entries of the Gram matrix $X^\dagger X$ are the inner products between the columns of $X$. The methods we have chosen to establish b-RIP and bounds on subspace incoherence for $X$ require bounding these inner products. Towards that we give the following lemma,

**Lemma 2.** If $N$ satisfies (7), then for all the pairs of columns $\{x_i, x_j\} \forall i \neq j$ chosen from $X$, the inner product can be bounded as $P[|\langle x_i, x_j \rangle| \leq \frac{\delta}{SL}] \geq 1 - \beta$, for $\delta \in (0, 1)$.

**Proof:** Above lemma gives a sufficient lower bound on $N$ such that all the possible pairs of columns from $X$ have inner product less than a small value $(\frac{\delta}{SL})$ with relatively high probability $(1 - \beta)$. Consider the matrix $B = X^\dagger X$ which is composed of blocks such that blocks along the diagonal are of the form $X_i^\dagger X_i$ with $i \in I$ and off-diagonal blocks are $X_i^\dagger X_j$ with $i, j \in I$ and $i \neq j$. To establish the lemma, we need to bound all the off-diagonal entries of $B$. First, let us focus on the off-diagonal blocks of the form $X_i^\dagger X_j$ with $i \neq j$. Since the generating vectors (i.e., first columns) of all blocks are independent of each other, any two columns not from the same block are independent. Hence the entries in the off-diagonal-blocks of $B$ are inner products of random vectors with i.i.d. Rademacher distributed entries and can be bounded by Hoeffding’s inequality as follows. Let $x_i$ denote a column of $X_i$ and $x_j$ be a column of $X_j$ with $i \neq j$, then for any $\delta > 0$ the inner product $\langle x_i, x_j \rangle$ can be bounded using Hoeffding’s inequality as,

$$P[|\langle x_i, x_j \rangle| \geq \frac{\delta}{SL}] \leq 2 \exp \left( -\frac{\delta^2 N}{2(SL)^2} \right).$$

The number of possible combinations for the off-diagonal blocks are $C(C-1)$ and the number of possible unique entries in the off-diagonal blocks are $L^2C(C-1)/2$. For establishing the lemma, we need to bound each of the unique entries which can be done using union bound as

$$P[|\langle x_i', x_j' \rangle| \geq \frac{\delta}{SL} : \forall x_i' \in X_i, x_j' \in X_j, \forall i \neq j \in I] \leq L^2C(C-1) \exp \left( -\frac{\delta^2 N}{2(SL)^2} \right) \leq L^2C^2 \exp \left( -\frac{\delta^2 N}{2(SL)^2} \right).$$

Now, we consider the diagonal blocks of $B$ which are of the form $X_i^\dagger X_i$ with $i \in A$. To get a bound on the off-diagonal entries in the diagonal blocks which are inner products of a vector and its circularly shifted version, a method similar to [15] is employed. For any such $b_{ij} = \langle x_i, x_j \rangle = \sum_{k=1}^{N} x_{ik}x_{jk}$ with $x_i$ and $x_j$ are circular shifts of each other, the set $V = \{x_{ik}x_{jk}, k = 1, \cdots, N\}$ can be partitioned into three sets $V_1, V_2$ and $V_3$ such that entries in different sets are independent. Now, as detailed in [15], using Haznalszemeredi theorem on equitable coloring we conclude that $|N/3| \leq |V_1| \leq |N/3|$. Now,

$$b_{ij} = \sum_{k=1}^{N} x_{ik}x_{jk} = \sum_{k=1}^{N} \sum_{i=1}^{\lfloor N/3 \rfloor} v_{ki}, \text{ where } v_{ki} \in V_k.$$

By Hoeffding’s inequality,

$$P\left[|\sum_{i=1}^{V_k} v_{ki}| \geq \frac{\delta}{3SL}\right] \leq 2 \exp \left( -\frac{\delta^2 N^2}{18|V_k|(SL)^2} \right).$$

Now, for $x_i, x_j \in X_m$, and $i \neq j$,

$$P\left[|\langle x_i, x_j \rangle| \geq \frac{\delta}{SL}\right] \leq 3 \max_k \left\{ P\left[|\sum_{i=1}^{V_k} v_{ki}| \geq \frac{\delta}{3SL}\right] \right\} \leq 6 \exp \left( -\frac{\delta^2 N^2}{18|N/3|(SL)^2} \right).$$

We need to bound the off-diagonal entries of all the possible diagonal blocks by applying union bound over $L(L-1)/2$ unique off diagonal entries of diagonal blocks. Assuming $N$ is divisible by 3 for convenience, we have

$$P\left[|\langle x_i, x_j \rangle| \geq \frac{\delta}{SL} : \forall x_i, x_j \in X_m, \forall m \in I \text{ and } i \neq j \right] \leq 3L^2C \exp \left( -\frac{\delta^2 N}{6(SL)^2} \right) \leq 2L^2C \exp \left\{ 3 \exp \left( -\frac{\delta^2 N}{6(SL)^2} \right) \right\} \exp \left( -\frac{\delta^2 N}{2(SL)^2} \right).$$

Using (18) and (16), all the off-diagonal entries $b_{ij}$ of $B$ can be bounded as

$$P\left[|b_{ij}| \geq \frac{\delta}{SL} : \forall i \neq j \right] \leq L^2C \left( 3 \exp \left( -\frac{\delta^2 N}{6(SL)^2} \right) + C \exp \left( -\frac{\delta^2 N}{2(SL)^2} \right) \right) \leq 2L^2C \exp \left\{ 3 \exp \left( -\frac{\delta^2 N}{6(SL)^2} \right) \right\} \exp \left( -\frac{\delta^2 N}{2(SL)^2} \right).$$

If $N$ satisfy (7), the first term inside the braces dominates in the above expression and the upper bound is less than 1. So we have,

$$P\left[|b_{ij}| \geq \frac{\delta}{SL} : \forall i \neq j \right] \leq 6L^2C \exp \left( -\frac{\delta^2 N}{6(SL)^2} \right) \leq 1 - \beta,$$

or equivalently $P[|\langle x_i, x_j \rangle| \leq \frac{\delta}{SL} : \forall i \neq j] \geq 1 - \beta$.

**APPENDIX B
PROOF OF BLOCK-RIP OF BCB MATRIX**

Real matrix $X$ would satisfy block-RIP of order $S$ with $\delta_S \leq \delta$, where $\delta \in (0, 1)$, if for every $A \subset I$ with $|A| \leq S$, the eigenvalues of the matrix $X_A^\dagger X_A$, lie in $[1 - \delta, 1 + \delta]$. Let $\Lambda_A$ be the set of all eigenvalues of the matrix $A = X_A^\dagger X_A$ for any $A \subset I$ with $|A| \leq S$. By Gershgorin circle theorem, every $\lambda \in \Lambda_A$ satisfies the inequality,

$$|\lambda - 1| \leq \sum_{k=1}^{SL} |a_{jk}| \text{ for some } j \in \{1, 2, \cdots, SL\},$$

where $a_{jk}$ denotes the $(j, k)^{th}$ element of $A$. We use Lemma 2 proved in Appendix A to bound $|\lambda - 1|$. When $N$ satisfies (7), then by Lemma 2, $|\lambda - 1| \leq \delta$ with probability at least $1 - \beta$. 


APPENDIX C
PROOF OF MUTUAL SUBSPACE INCOHERENCE BOUND

We rewrite subspace incoherence (8) as,

$$\mu = \max_{i,j \in I, i \neq j} \left\{ \max_{w, u \in \mathbb{C}^{L}} \left\{ \frac{\left| \langle X_i w, X_j u \rangle \right|}{\|X_i w\|_2 \|X_j u\|_2} \right\} \right\} \quad (20)$$

Let \{x_{i_k}\}, \{x_{j_k}\} be the columns of matrices \(X_i, X_j\) respectively, then

$$\langle X_i w, X_j u \rangle = \sum_{k=1}^{L} u_k^* \sum_{m=1}^{L} x_{i_m} \langle x_{i_m}, x_{j_k} \rangle w_m.$$  

From Lemma 2 proved in Appendix A, for all pairs of columns of \(X\), we have \(\left| \langle x_i, x_j \rangle \right| \leq \frac{\delta}{\sqrt{L}}\) with probability at least \(1 - \beta\). Hence, with probability greater than \(1 - \beta\),

$$|\langle X_i w, X_j u \rangle| \leq \frac{\delta}{S L} \sum_{k=1}^{L} |u_k| \sum_{m=1}^{L} |w_m|$$

$$= \frac{\delta}{S L} \|u\|_1 \|w\|_1,$$

$$\leq \frac{\delta}{S} \|u\|_2 \|w\|_2,$$  

(21)

where we have used the norm equivalence, \(\|w\|_1 \leq \sqrt{L} \|w\|_2\).

to get (22) from (21).

Using Gershgorin circle theorem as in the proof of Theorem (1), with probability greater than \(1 - \beta\), the eigen values of the matrices \(X_i^* X_i, \forall i \in I\) lie in \([1 - \delta/S, 1 + \delta/S]\).

From (22) and bounds of singular values of \(X_i\)'s,

$$\frac{\left| \langle X_i w, X_j u \rangle \right|}{\|X_i w\|_2 \|X_j u\|_2} \leq \frac{\delta}{S} \frac{\|X_i w\|_2 \|X_j u\|_2}{\|X_i w\|_2 \|X_j u\|_2} \leq \frac{\delta}{S} \leq 1 - \frac{\delta}{S}.$$

Therefore, \(\mu \leq \frac{\delta}{S}\) with probability greater than \(1 - \beta\) when \(N\) satisfies (7) and \(\delta \in (0, 1)\).

REFERENCES


Niranjan M. Gowda is an MS student in the department of Electrical Engineering at the Indian Institute of Technology, Madras. He obtained his BE degree from SJCE, Mysore. He is working in Saankhya Labs, Bangalore, India.

Arun Pachai Kannu received the M.S. and Ph.D. degrees in Electrical and Computer Engineering from The Ohio State University in 2004 and 2007, respectively. From 2007 to 2009, he worked as a Senior Engineer in Qualcomm Research Center, San Diego, USA. Since 2009, he is an Assistant Professor in the department of Electrical Engineering at the Indian Institute of Technology, Madras.