EE 6340: Information Theory
Quiz II - Apr/10/2014

Remarks

- You are allowed to bring one formula sheet (hand-written).
- If anything is not clear, make/state your assumptions and proceed.

Problems

1. (5 pts) Let source X has four outcomes $\{1, 2, 3, 4\}$ with p_i being probability of i^{th} outcome. Let a binary Huffman code is constructed for X and ℓ_i denote the length of codeword for i^{th} outcome. It is given that $p_1 > p_2 = p_3 = p_4$.

   (a) Show that if $p_1 > 0.4$ then $\ell_1 = 1$.

   (b) Show (by example) that if $p_1 = 0.4$ then a Huffman code exists with $\ell_1 > 1$.

2. (5 pts) Let X be a random variable with K outcomes. Let $H_3(X)$ denote the entropy of X in ternary units. An instantaneous ternary code is found for this source with expected code length $L = H_3(X)$. Show that K is odd.

3. (10 pts) Consider the 3-input 3-output discrete memoryless channel illustrated below.

   (a) What is the channel capacity for $p = 1$?

   (b) What is the channel capacity for $p = 0.5$?

   (c) Show that, for any value of p, channel capacity $C \geq 1$ (bit per channel use).

   (d) Find the channel capacity for a general value of p.