

EE 6340: Information Theory

Quiz II - Mar/22/2012

1. **Source Coding:** Consider a source X with 6 symbols.
 - (a) Find the minimum value of D so that we can construct a uniquely decodable code for X using D -ary alphabets with codeword lengths $\{\ell_1 = 1, \ell_2 = 1, \ell_3 = 2, \ell_4 = 2, \ell_5 = 2, \ell_6 = 3\}$. [2 pts]
 - (b) Using the value of D found in part (a), construct a uniquely decodable code with codeword lengths given in part (a). [3 pts]
 - (c) Choose a suitable value for D and construct a uniquely decodable code for X so that the average codeword length is minimal. [2 pts]
2. **Multiplicative Channel:** Let X takes values $\{1, 2, \dots, n\}$ with corresponding probabilities $\{q_1, q_2, \dots, q_n\}$ and Z be a binary random variable taking values 0 or 1 with $P\{Z = 0\} = p$. Let random variable Y be defined as

$$Y = ZX$$

- (a) Find entropy of Y in terms of entropies of X and Z . [3 pts]
 - (b) Find the values of p and $\{q_1, q_2, \dots, q_n\}$ so that $H(Y)$ is maximum. [3 pts]
 - (c) Assume that X and Y are input and output of a discrete memoryless channel. For a given value of p , find the channel capacity $C(p)$. Find the value of p which maximizes the capacity $C(p)$. [3 pts]
3. **Z Channel:** Find the capacity of binary Z channel given below. [4 pts]

