EE 6110: Assignment 4 (RLS problems)

1. Consider an IIR LTI System with input $u(n)$ and the output $z(n) = \alpha z(n-1) + \beta z(n-2) + \gamma u(n)$. The values of α, β, γ are unknown. The measured output is given by $y(n) = z(n) + v(n)$ where $v(n)$ is additive noise. Define $\underline{\theta} = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$ and $\underline{y} = \begin{bmatrix} y(1) \\ \vdots \\ y(N) \end{bmatrix}$.

   (a) Write $\underline{y} = H\underline{\theta} + \tilde{\underline{v}}$ such that matrix H depends only on sequences $\{y(n), u(n)\}$ and vector $\tilde{\underline{v}}$ depends only on noise sequence $\{v(n)\}$.

   (b) If noise $v(n)$ has zero mean, what is the mean of $\tilde{\underline{v}}$.

   (c) Given $\underline{y}$ and H, what is the least squares estimate of $\underline{\theta}$.

2. Consider the linear observation model: $\underline{y} = H\underline{\theta} + \underline{v}$. Given that H is full rank $M \times M$ square matrix.

   (a) Find the least squares estimate $\hat{\underline{\theta}}_{LS}$ given $\underline{y}, H$. (Note that the standard LS formula will simplify in this case.)

   (b) Suppose $\underline{v}$ is i.i.d. Gaussian with variance σ^2. Note that $\hat{\underline{\theta}}_{LS}$ is random (even if $\underline{\theta}$ is not random). Find the covariance matrix of the estimation error $\tilde{\underline{\theta}} = \underline{\theta} - \hat{\underline{\theta}}_{LS}$.

   (c) Suppose $\underline{v}$ is i.i.d. Gaussian with variance σ^2 and suppose H is a unitary matrix. Find the value of $\mathrm{E}\,\|\underline{\theta} - \hat{\underline{\theta}}_{LS}\|^2$.