EE 6110: Assignment 3 (Matlab exercises)
Due date - October 31, 2014

1. Let $Z(n)$ be an AR process with $Z(n) = -0.6Z(n-1) + 0.1Z(n-2) + V(n)$ where $V(n)$ is an i.i.d. Gaussian noise with variance $\sigma_v^2 = 0.2$. Let the desired signal be $X(n) = Z(n)$ and the input to the adaptive filter being $\underline{Y}(n) = \begin{bmatrix} Z(n-1) \\ Z(n-2) \end{bmatrix}$. Hence, we are interested in *predicting* $Z(n)$ using the past samples $\{Z(n-1), Z(n-2)\}$.

   (a) For the above model, find σ_x^2, R_Y, R_{YX}.

   (b) Find the optimal filter $\underline{w}_{\text{opt}}$ which minimizes $\mathrm{E}\,|X(n) - \underline{w}^*\underline{Y}(n)|^2$ and find the correponding minimum MSE $J_{\min}$.

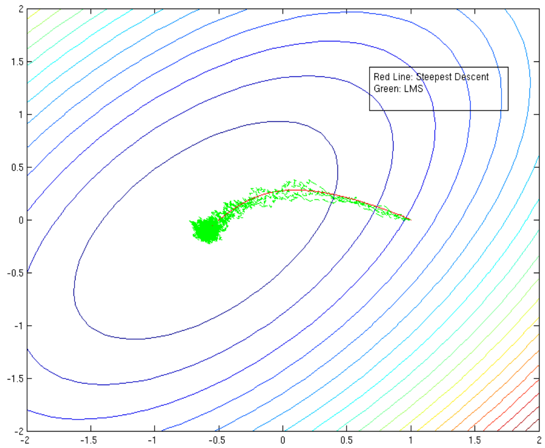
   (c) With $\underline{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$, consider the MSE cost $J(\underline{w}) = \mathrm{E}\,|X(n) - \underline{w}^*\underline{Y}(n)|^2$. For w_1, w_2 in the range [-2 to 2], plot the MSE contour $J(\underline{w})$. A sample of the plot is shown in Figure 1. Useful matlab command `contour`.

   (d) Consider the steepest descent update $\underline{w}(n+1) = \underline{w}(n) + \mu[R_{YX} - R_Y\underline{w}(n)]$. Consider two different initial conditions $\underline{w}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\underline{w}(0) = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$ and two different step sizes $\mu = 0.02, 0.2$. For each combination of step size and initial condition, plot the trajectory of $\underline{w}(n)$ for $n = 1, \cdots, 5000$. Superimpose the trajectory on the MSE contour (as shown in Fig.1).

   (e) Generate $Z(n)$ by filtering white noise $V(n)$ as per the AR model given above. Taking $X(n) = Z(n)$ and $\underline{Y}(n) = \begin{bmatrix} Z(n-1) \\ Z(n-2) \end{bmatrix}$, run the LMS update $\underline{w}(n+1) = \underline{w}(n) + \mu\underline{Y}(n)[X^*(n) - \underline{Y}^*(n)\underline{w}(n)]$, for $n = 1, \cdots, 5000$. For all the combinations of step sizes and initial conditions given in part (d), plot the LMS trajectory superimposed as in Fig.1. Useful matlab commands `randn, plot`.

   (f) Run the LMS with $\mu = 0.02$ and the initial condition $\underline{w}(0) = \underline{w}_{\text{opt}}$ for $n = 1, \cdots, 50000$. Consider the corresponding error sequence $d(n) = X(n) - \underline{w}^*(n)\underline{Y}(n)$. Let us estimate the excess MSE as $\hat{\eta} = \left(\frac{1}{50000}\sum_{n=1}^{50000} |d(n)|^2\right) - J_{\min}$. Compare this value with theoretical result obtained in the class $\eta = \frac{\mu}{2}\sigma_v^2 \,\mathrm{Tr}(R_Y)$.

2. Consider the tracking of a time varying system. Let $Z(n)$ be an AR process with $Z(n) = -0.6Z(n-1) + 0.1Z(n-2) - 0.1Z(n-3) - 0.1Z(n-4) + V(n)$ where $V(n)$ is i.i.d. Gaussian with variance $\sigma_v^2 = 0.5$. Now, $Z(n)$ is passed thru a time varying filter $\underline{h}(n)$ with 3 taps, and the noise corrupted output of the system is obtained as $X(n) = \underline{h}^*(n)\underline{Y}(n) + U(n)$ where $\underline{Y}(n) = \begin{bmatrix} Z(n) \\ Z(n-1) \\ Z(n-2) \end{bmatrix}$ and $U(n)$ is i.i.d. Gaussian



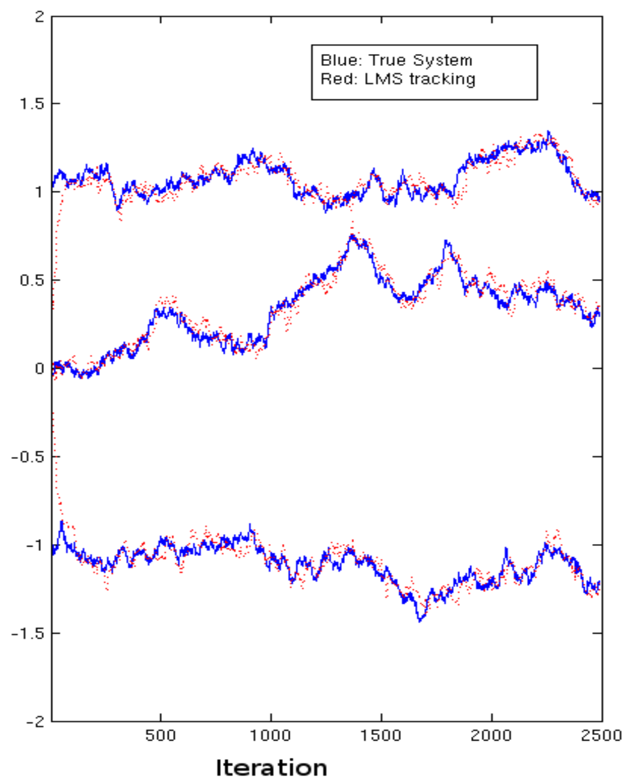
noise with variance $\sigma_u^2 = 0.1$. The time variation of the system follows as $\underline{h}(n) = \underline{h}(n-1) + \underline{q}(n)$ where $q(n)$ is i.i.d. Gaussian with covariance R_Q. Using $X(n)$ and $\underline{Y}(n)$ we are interested in adaptively tracking the filter response $\underline{h}(n)$.

(a) We want to generate sequence i.i.d. $\underline{q}(n)$ with covariance $R_Q = 10^{-4} \times \begin{bmatrix} 1.1 & 0.1 & 0.1 \\ 0.1 & 1.1 & 0.1 \\ 0.1 & 0.1 & 1.1 \end{bmatrix}$. Towards that we generate the i.i.d. sequence $\underline{p}(n)$ with covariance $R_P = I$ and obtain $\underline{q}(n) = A\underline{p}(n)$. Find the 3×3 matrix A. (Hint: Matrix A is related to the eigen decomposition of R_Q).

(b) For our model, with R_Q given in part (a), find the optimal step size μ_{opt} which will minimize the Excess MSE in the steady state, using the formula derived in the class (using small μ approximation).

(c) As per the specified model and R_Q given in part (a), generate the sequences $Z(n), X(n), \underline{Y}(n), \underline{h}(n)$ with the intial conditions $\underline{h}(0) = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$. With the initial conditions $\underline{w}(0) = \underline{0}$, run the LMS algorithm for $n = 1, \cdots, 2500$. Plot the time evaluation of all the three taps of $\underline{h}(n)$ and $\underline{w}(n)$. A sample plot is shown in Fig.2.

(d) Plot the theoretial EMSE approximation (small step size) as a function of μ in the range $\mu = 0.001$ to 0.1. Now, for $\mu = \{0.1, \mu_{\text{opt}}, 0.01, 0.002\}$, compute the EMSE of LMS using simulations by averaging $|d(n)|^2$ (with more than 50000 samples for averaging) with the initialization $\underline{w}(0) = \underline{h}(0)$. Superimpose these EMSE values on the theoretical EMSE curve and write your interpretations.


Blue: True System
Red: LMS tracking
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
500
1000
1500
2000
2500
Iteration