EE 5150: Math Methods: Quiz II - Oct/18/2012

- **You are allowed to bring one hand-written formula sheet.**
- **If something is not clear, make/state your assumptions and proceed.**

1. Prove that, if there exists a one-to-one linear transformation from vector space V to vector space W, then $\dim(V) \leq \dim(W)$. [2 pts]

2. Define the linear transformation $L : \mathbb{R}^2 \to \mathbb{R}^2$ such that $L \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ -y \end{bmatrix}$. Find the matrix representation of L with respect to the basis $\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \}$. [2 pts]

3. Consider linear transformation $L : \mathbb{R}^2 \to \mathbb{R}^3$ such that $L \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$ and $L \begin{bmatrix} 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$. Find $L \begin{bmatrix} -2 \\ 3 \end{bmatrix}$. [2 pts]

4. Find two linear transformations $L_1 : \mathbb{R}^2 \to \mathbb{R}^2$ and $L_2 : \mathbb{R}^2 \to \mathbb{R}^2$ such that $L_1 L_2 = \mathbf{0}$ but $L_2 L_1 \neq \mathbf{0}$. (Here $\mathbf{0}$ denotes the Zero transformation) [2 pts]

5. In $\mathbb{R}^2$ with standard inner product, find the orthogonal projection of the vector $\begin{bmatrix} a \\ b \end{bmatrix}$ onto the subspace defined by the straight line $x = y$ [2 pts]

6. Prove or disprove the following statement: For the vectors $\underline{\boldsymbol{u}}, \underline{\boldsymbol{v}}, \underline{\boldsymbol{w}}$ in an inner product space, if $\underline{\boldsymbol{u}} \perp \underline{\boldsymbol{v}}$ and $\underline{\boldsymbol{v}} \perp \underline{\boldsymbol{w}}$ then it follows that $\underline{\boldsymbol{u}} \perp \underline{\boldsymbol{w}}$. [2 pts]

7. Prove that for any $\underline{\boldsymbol{x}}, \underline{\boldsymbol{y}}$ in a real inner product space, $\|\underline{\boldsymbol{x}} + \underline{\boldsymbol{y}}\|^2 + \|\underline{\boldsymbol{x}} - \underline{\boldsymbol{y}}\|^2 = 2(\|\underline{\boldsymbol{x}}\|^2 + \|\underline{\boldsymbol{y}}\|^2)$. This is called parallelogram law. [2 pts]

8. Let V be a real inner product space of dimension n. Let W be a subspace of V with dimension k. Let $L : V \to V$ be a transformation defined as the projection onto subspace W, that is, for $\underline{\boldsymbol{v}} \in V$, $L(\underline{\boldsymbol{v}}) = \underline{\boldsymbol{w}}$ where $\underline{\boldsymbol{w}}$ is orthogonal projection of $\underline{\boldsymbol{v}}$ onto W.

   (a) Show that L is linear. [2 pts]

   (b) Show that $L^2 = L$, that is, for any $\underline{\boldsymbol{v}} \in V$ we have $L^2(\underline{\boldsymbol{v}}) = L(\underline{\boldsymbol{v}})$. [2 pts]

   (c) A basis for V is chosen and let $\boldsymbol{P}$ be matrix representation of L. Let $\tilde{L} : V \to V$ be the projection onto subspace $W^{\perp}$. What is the matrix representation of $\tilde{L}$? [2 pts]