EE 5150: Math Methods for Signal Processing
Assignment 5

1. If U and W are orthogonal subspaces ($\underline{\boldsymbol{u}} \perp \underline{\boldsymbol{w}}$ for every $\underline{\boldsymbol{u}} \in U$, $\underline{\boldsymbol{w}} \in W$) then show that $U \cap W = \{\underline{\mathbf{0}}\}$.

2. Let S be a subspace of $\mathbb{R}^4$ containing vectors such that $x_1 + x_2 + x_3 + x_4 = 0$. Find a basis for $S^\perp$ which contains all the vectors orthogonal to S.

3. Show that $\underline{\boldsymbol{x}} - \underline{\boldsymbol{y}}$ is orthogonal to $\underline{\boldsymbol{x}} + \underline{\boldsymbol{y}}$ if and only if $\|\underline{\boldsymbol{x}}\| = \|\underline{\boldsymbol{y}}\|$.

4. Let $W = \text{span}\{\begin{bmatrix} j \\ 0 \\ 1 \end{bmatrix}\}$ in $\mathbb{C}^3$ with standard inner product. Find orthonormal bases for W and $W^\perp$.

5. Consider second degree polynomilals $V = \mathbb{P}_2$ in the interval $[0, 1]$ with inner product $< f, g >= \int_0^1 f(x)g(x)dx$. Find an orthonormal basis for V.

6. Let $W = \text{span}\{\begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ 4 \\ -3 \end{bmatrix}\}$ be a subspace of $\mathbb{R}^4$. Let $W^\perp$ be orthogonal complement of W, that is $W^\perp = \{\underline{\boldsymbol{x}} \in \mathbb{R}^4 \text{ such that } < \underline{\boldsymbol{x}}, \underline{\boldsymbol{y}} >= 0, \ \forall\ \underline{\boldsymbol{y}} \in W\}$.

   (a) Find orthonormal basis for $W^\perp$.

   (b) For $\underline{\boldsymbol{v}} = \begin{bmatrix} 4 \\ 0 \\ 1 \\ 2 \end{bmatrix}$, find projection of $\underline{\boldsymbol{v}}$ onto subspace W.

7. Let V be the vector space of all $n \times n$ matrices with real entries. `trace` of a matrix is the sum of its diagonal entries, i.e., $\text{trace}(\boldsymbol{A}) = \sum_{i=1}^n a_{i,i}$ where $a_{m,k}$ is the entry in m^{th} row, k^{th} column of $\boldsymbol{A}$.

   (a) Show that $< \boldsymbol{A}, \boldsymbol{B} >= \text{trace}(\boldsymbol{A}^T\boldsymbol{B})$ is a valid inner product in V.

   (b) Let W be a subspace of V containing only diagonal matrices. Find the orthogonal complement $W^\perp$ based on the above inner product.