EE 5150: Math Methods for Signal Processing
Assignment 4: Submission date 5/10/12 (Friday)

1. (a) Find the matrix representation $\boldsymbol{A}$ for the *right shift* transformation from $\mathbb{R}^3$ to $\mathbb{R}^4$, i.e., vector $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ is transformed into $\begin{bmatrix} 0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$.

   (b) Find the matrix representation $\boldsymbol{B}$ for the *left shift* transformation from $\mathbb{R}^4$ to $\mathbb{R}^3$, i.e., vector $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ is transformed into $\begin{bmatrix} x_2 \\ x_3 \\ x_3 \end{bmatrix}$.

   (c) Find the products $\boldsymbol{AB}$ and $\boldsymbol{BA}$.

2. Let V be a vector space of all 2×2 matrices. Define the transformation $L : V \to \mathbb{R}^2$ as

$$L\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} a + 2d \\ b - c \end{bmatrix}$$

   (a) Show that L is a linear transformation

   (b) Find a basis for null space of L

   (c) Find a basis for range of L

3. Let $L : \mathbb{P}_2 \to \mathbb{P}_3$ be defined as $L(p) = x^3p'' - x^2p' + 3p$. Let basis for $\mathbb{P}_2$ be $\mathcal{B} = \{1, x, x^2\}$ and let basis for $\mathbb{P}_3$ be $\mathcal{C} = \{3, 3x - x^2, 3x^2, x^3\}$. Find matrix representation of L with respect to the bases $\mathcal{B}$ and $\mathcal{C}$. (Note that $'$ denotes derivative.)

4. Let $L_1 : V \to W$ be a linear transformation from V to W and $L_2 : W \to V$ be a linear transformation from W to V. Consider the composition $\tilde{L} = L_2L_1$ so that $\tilde{L}$ is a linear transformation from V to V.

   (a) Show that if $\dim(W) < \dim(V)$ then $\tilde{L}$ is *not* invertible.

   (b) `Corollary`: For any matrices $\boldsymbol{A}_{n\times m}$ and $\boldsymbol{B}_{m\times n}$, if $m < n$, then product $\boldsymbol{AB}$ is not invertible.

   (c) Give an example such that $\tilde{L}$ is invertible when $\dim(W) > \dim(V)$.

5. Let L be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ defined as

$$L\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3x_1 \\ x_1 - x_2 \\ 2x_1 + x_2 + x_3 \end{bmatrix}$$

   (a) Is L invertible? If so, find L^{-1}.

   (b) Show that $(L^2 - I)(L - 3I) = 0$. (I is identity transformation and 0 is zero transformation)

6. Let L be a transformation from $\mathbb{P}_3$ to 2×2 matrices such that

$$L(p) = \begin{bmatrix} p(1) & p(2) \\ p(3) & p(4) \end{bmatrix}$$

   where p is a polynomial in $\mathbb{P}_3$, i.e., $p(x) = \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3$.

   (a) Show that L is linear

   (b) Is L invertible? Why?