EE 5150: Math Methods for Signal Processing
Assignment 3

1. Consider the augmented matrices $[\boldsymbol{A} \mid \underline{\boldsymbol{b}}]$ given in first Homework set ($\boldsymbol{A}$ is the coeffe-cient matrix and the last column is RHS vector $\underline{\boldsymbol{b}}$) which are reproduced below.

   (a) Number of unknowns equal to the number of equations

$$\begin{bmatrix} 2 & 3 & 1 & 10 \\ 1 & 1 & 1 & 5 \\ 4 & 2 & 2 & 12 \end{bmatrix}, \quad \begin{bmatrix} 3 & 4 & 2 & 1 \\ 1 & 1 & 1 & 1 \\ 2 & 0 & 4 & 2 \end{bmatrix}, \quad \begin{bmatrix} 3 & 2 & 4 & 5 \\ 2 & 4 & 0 & 6 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

   (b) Number of unknowns less than the number of equations

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & 1 & 0 \\ 3 & 2 & 0 & 1 \\ 4 & 2 & 1 & 3 \end{bmatrix}, \quad \begin{bmatrix} 2 & 1 & 3 & 4 \\ 1 & 0 & 1 & 2 \\ 2 & 4 & 1 & 8 \\ 2 & 3 & 0 & 4 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 2 & 1 \\ 2 & 3 & 10 & 5 \\ 1 & 2 & 6 & 3 \end{bmatrix}$$

   (c) Number of unknowns greater than the number of equations

$$\begin{bmatrix} 1 & 2 & 0 & 1 & 3 \\ 1 & 0 & 2 & 3 & 1 \\ 2 & 1 & 3 & 5 & 0 \end{bmatrix}, \quad \begin{bmatrix} 2 & 1 & 3 & 5 & 6 \\ 1 & 0 & 2 & 3 & 3 \\ 1 & 2 & 0 & 1 & 3 \end{bmatrix}, \quad \begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix}$$

   (a) Find a basis for column space of $\boldsymbol{A}$

   (b) Find a basis for null space of $\boldsymbol{A}$

   (c) Find out about the existence/uniquencess/multiplicy of solutions for $\boldsymbol{A}\underline{\boldsymbol{x}} = \underline{\boldsymbol{b}}$ using the basis you found for the column and null spaces of $\boldsymbol{A}$.

2. Determine wheter the set $\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \right\}$ is a basis for the space spanned by the set $\mathcal{C} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 8 \\ 7 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} \right\}$

3. Find a basis for $U \cap W$ where $U = \texttt{span}\left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 2 \end{bmatrix} \right\}$ and

$$W = \texttt{span}\left\{ \begin{bmatrix} 2 \\ 8 \\ -4 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 0 \\ 6 \end{bmatrix} \right\}$$