

EE 5150: Assignment 1

1. For the augmented matrices given below, find the *reduced* row echelon form. Check for consistency and find the general form of solution (if consistent).

(a) Number of unknowns equal to the number of equations

$$\begin{bmatrix} 2 & 3 & 1 & 10 \\ 1 & 1 & 1 & 5 \\ 4 & 2 & 2 & 12 \end{bmatrix}, \quad \begin{bmatrix} 3 & 4 & 2 & 1 \\ 1 & 1 & 1 & 1 \\ 2 & 0 & 4 & 2 \end{bmatrix}, \quad \begin{bmatrix} 3 & 2 & 4 & 5 \\ 2 & 4 & 0 & 6 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

(b) Number of unknowns less than the number of equations

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & 1 & 0 \\ 3 & 2 & 0 & 1 \\ 4 & 2 & 1 & 3 \end{bmatrix}, \quad \begin{bmatrix} 2 & 1 & 3 & 4 \\ 1 & 0 & 1 & 2 \\ 2 & 4 & 1 & 8 \\ 2 & 3 & 0 & 4 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 2 & 1 \\ 2 & 3 & 10 & 5 \\ 1 & 2 & 6 & 3 \end{bmatrix}$$

(c) Number of unknowns greater than the number of equations

$$\begin{bmatrix} 1 & 2 & 0 & 1 & 3 \\ 1 & 0 & 2 & 3 & 1 \\ 2 & 1 & 3 & 5 & 0 \end{bmatrix}, \quad \begin{bmatrix} 2 & 1 & 3 & 5 & 6 \\ 1 & 0 & 2 & 3 & 3 \\ 1 & 2 & 0 & 1 & 3 \end{bmatrix}, \quad \begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix}$$

2. Using your results from the above problem, can you identify the connection between the number of pivots, number of unknowns and the uniqueness of solution?
3. Say, \mathbf{A} is an *invertible* $n \times n$ matrix. Consider the augmented matrix (of size $n \times 2n$) $\mathbf{B} = [\mathbf{A} \ \mathbf{I}]$ where \mathbf{I} is an $n \times n$ identity matrix. Denote the reduced row echelon form of \mathbf{B} to be $[\mathbf{C} \ \mathbf{D}]$ where \mathbf{C} and \mathbf{D} are $n \times n$ matrices. What can you say about \mathbf{C} and \mathbf{D} ? (Hint: There is a connection to inverse of \mathbf{A}).