1. For s(t) = sinc(t)sinc(2t)
   a) Find and sketch the Fourier transform S(f).
   b) Find and sketch the Fourier transform U(f) of u(t) = s(t) cos(100πt)
   (Sketch real and imaginary parts separately if U(f) is complex-valued)

2. Consider a real-valued passband signal $x_p(t)$ whose Fourier transform for positive frequencies is given by

$$Re\left(X_p(f)\right) = \begin{cases} \sqrt{2}, & 20 \le f \le 22, \\ 0, & 0 \le f \le 20, \\ 0, & 22 < f < \infty \end{cases}$$

$$Im\left(X_p(f)\right) = \begin{cases} \frac{1}{\sqrt{2}}(1 - |f - 22|), & 21 \le f \le 23, \\ 0, & 0 \le f \le 21, \\ 0, & 23 < f < \infty \end{cases}$$

   (a) Sketch the real and imaginary parts of $X_p(f)$ for both positive and negative frequencies.
   (b) Specify the time domain waveform that you get when you pass $\sqrt{2}\, x_p(t)\cos(40\pi t)$ through a low pass filter

3. Let $v_p(t)$ denote a real denote a real passband signal, with Fourier transform $V_p(f)$ specified as follows for negative frequencies:

$$V_p(f) = \begin{cases} f + 101, & -101 \le f \le -99, \\ 0, & f < -101 \text{ or } -99 < f \le 0. \end{cases}$$

   a) Sketch $V_p(f)$ for both positive and negative frequencies.
   b) Without explicitly taking the inverse Fourier transform, can you say whether $v_p(t)$=$v_p(-t)$ or not?
   c) Choosing f_0 = 100, find real baseband waveforms $v_c(t)$ and $v_s(t)$ such that

$$v_p(t) = \sqrt{2}\,(v_c(t)cos2\pi f_0 t - v_s(t)sin2\pi f_0 t)$$

   d) Repeat (c) for f_0 = 101

4. Consider the following two passband signals:

$$u_p(t) = \sqrt{2}\text{sinc}(2t)\cos 100\pi t$$

$$v_p(t) = \sqrt{2}\text{sinc}(t)\sin(101\pi t + \frac{\pi}{4})$$

a) Find the complex envelopes u(t) and v(t) for u_p and v_p, respectively, with respect to the frequency reference fc= 50.
b) What is the bandwidth of $u_p(t)$? What is the bandwidth of $v_p(t)$?
c) Find the inner product $\langle u_p(t), v_p(t)\rangle$ using the result in (a).
d) Find the convolution $y_p(t)= u_p(t)*v_p(t)$,using the result in (a).

5. Let $u(t)$ denote a real baseband waveform with Fourier transform for $f > 0$ specified by

$$U(f) = \begin{cases} e^{j\pi f} & 0 < f < 1, \\ 0 & f > 1. \end{cases}$$

a) Sketch $\mathrm{Re}(U(f))$ and $\mathrm{Im}(U(f))$ for both positive and negative frequencies.
b) Find $u(t)$

Now, consider the bandpass waveform $v(t)$ generated from $u(t)$ as follows:

$$v(t) = \sqrt{2}u(t)cos200\pi t$$

c) Sketch $\mathrm{Re}(V(f))$ and $\mathrm{Im}(V(f))$ for both positive and negative frequencies.
d) Let $y(t) = (v * h_{hp}(t))$ denote the result of filtering $v(t)$ using a high pass filter with transfer function

$$H_{hp}(f) = \begin{cases} 1 & |f| \geq 100, \\ 0 & else \end{cases}$$

Find real baseband waveforms y_c, y_s such that

$$y(t) = \sqrt{2}\,(y_c(t)cos200\pi t - y_s(t)sin200\pi t\ ).$$

e) Finally, pass $y(t)cos200\pi t$ through an ideal low pass filter with transfer function

$$H_{hp}(f) = \begin{cases} 1 & |f| \leq 1, \\ 0 & else \end{cases}$$

How is the result related to (t) ?

6. Consider a passband filter whose transfer function for f > 0 is specified by

$$H_p(f) = \begin{cases} 1 & f_c - 2 \leq f \leq f_c \\ 1 - f + f_c & f_c \leq f \leq f_c + 1 \quad (f_c \gg 1) \\ 0 & \text{else.} \end{cases}$$

Let $y_p(t)$ denote the output of the filter when fed by a passband signal $u_p(t)$. We would like to generate $y_p(t)$ from $u_p(t)$ using baseband processing in the system shown in Figure 1.

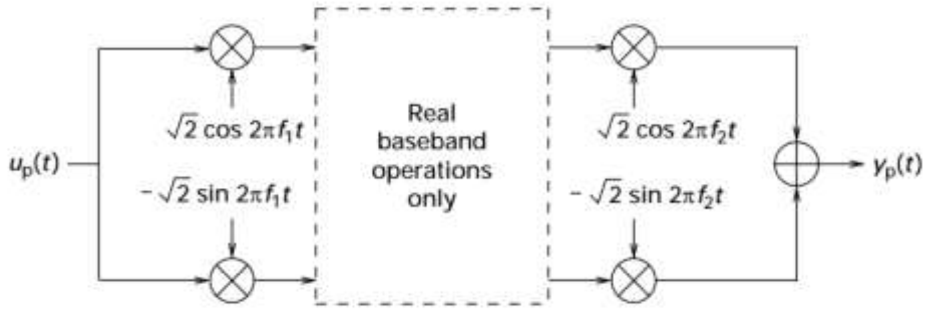

Fig 1: Implementation of a passband filter using downconversion, baseband operations and upconversion

a) For $f_1= f_2= f_c$, sketch the baseband processing required, specifying completely the transfer function of all baseband filters used. Be careful with signs.

b) Repeat (a) for $f_1= f_c+1/2$ and $f_2= f_c-1/2$.

Hint: The inputs to the black box are the real and imaginary parts of the complex baseband representation for u(t) centered at f_1. Hence, we can use baseband filtering to produce the real and imaginary parts for the complex baseband representation for the output y(t) using f_1 as center frequency. Then use baseband processing to construct the real and imaginary parts of the complex baseband representation for y(t) centered at f_2. These will be the output of the black box.