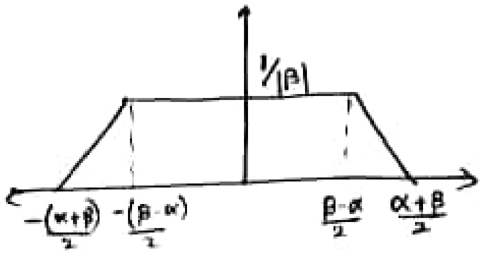
2) $p(t) = \mathrm{sinc}(\alpha t)\,\mathrm{sinc}(\beta t)$

a) $\mathrm{sinc}(\alpha t) \xrightarrow{\mathcal{F}} \frac{1}{|\alpha|}\,\mathrm{rect}\left(\frac{f}{\alpha}\right)$

$\mathrm{sinc}(\beta t) \xrightarrow{\mathcal{F}} \frac{1}{|\beta|}\,\mathrm{rect}\left(\frac{f}{\beta}\right)$

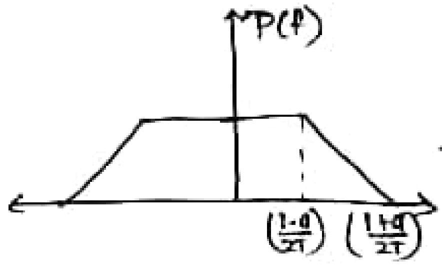
$$p(t) \xrightarrow{\mathcal{F}} P(f) = \frac{1}{|\alpha|}\,\mathrm{rect}\left(\frac{f}{\alpha}\right) * \frac{1}{|\beta|}\,\mathrm{rect}\left(\frac{f}{\beta}\right)$$



(Assume $\alpha<\beta$) —— (1)

In terms of excess Bandwidth,

$p(t) = \mathrm{sinc}\left(\frac{at}{T}\right)\mathrm{sinc}\left(\frac{t}{T}\right) \longleftrightarrow$



—— (2)

~~[illegible]~~

Given, $a = .5$

$$\text{Symbol rate} = \frac{\text{bit rate}}{\text{No. of bits/symbol}} = \frac{40\times10^6 \text{ bps}}{\log_2 16}$$

$$= 10\times10^6 \text{ symbols per sec}$$

ie, $\frac{1}{T} = 10\times10^6$ symbols per sec

~~[illegible]~~ Comparing (1) & (2),

$$\left.\begin{aligned} \alpha+\beta &= \frac{1+a}{T} \\ \beta-\alpha &= \frac{1-a}{T} \end{aligned}\right\} \Rightarrow \beta = \frac{1}{T},\ \alpha = \frac{a}{T}.$$

$\therefore$ $\boxed{\begin{aligned} \alpha &= .5\times10\times10^6 = 5\times10^6 \text{ symbols per sec} \\ \beta &= \frac{1}{T} = 10\times10^6 \text{ symbols per sec.} \end{aligned}}$

$BW = 2\cdot\left(\frac{1+a}{2T}\right) = \frac{1+a}{T} = 1.5 \times 10 \times 10^6$ symbols per sec.

$= 15$ Mega symbols per sec

b) Given two symbol rates.

Symbol rate 1, $S_1 = \frac{40 \times 10^6}{\log_2 16} = 10 \times 10^6$ symbols per sec. $= \frac{1}{T}$

Symbol rate 2, $S_2 = \frac{8 \times 10^6}{\log_2 8} = \frac{8}{3} \times 10^6$ symbols per sec. $= \frac{a}{T}$

$BW = \frac{1+a}{T} = \left(10 + \frac{8}{3}\right)$ Mega symbols per sec

$= 12.67$ Mega symbols per sec.

3). a) $p(t) \rightarrow$ Nyquist at symbol rate K symbols per sec.

a) $p(t)$ is Nyquist at symbol rate $K \Rightarrow p(t)\big/_{n/K} = P(n/K) = \begin{cases} 1; & n=0 \\ 0; & n\neq 0 \end{cases}$ ——— (1)

sampling at multiples of $1/K$.

Suppose $K' = K/M$.

Sampling at multiples of $1/K' \Rightarrow p(t)\big/_{n/K'} = P(n/K') = P(nM/K)$

Also $P\left(\frac{nM}{K}\right) = \begin{cases} 1; & nM=0 \\ 0; & nM\neq 0 \end{cases}$ from (1).

$\because$ M is a positive integer; $nM=0 \Rightarrow n=0$; $nM \neq 0 \Rightarrow n \neq 0$.

$\therefore p(t)\big/_{n/K'} = \begin{matrix} 1; & n=0 \\ 0; & n\neq 0 \end{matrix}$

This means $p(t)$ is nyquist at symbol rate $K' = \frac{K}{M}$ where M is any positive integer.

b) $p(at)\ ;\ a>0$

Suppose $x(t) = p(at)\ ;\ a>0$

Sampling $x(t)$ at $k' = \cancel{\ }ak$,

$$x(t)\Big/_{\frac{n}{k'}} = x(n/k') = x(n/ak).$$

Also $x(t) = p(at) \Rightarrow x(n) = p(an)$ (when sampled).

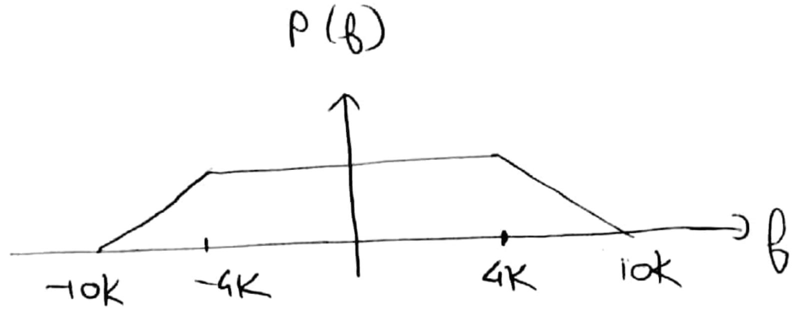
$$\therefore\ x(t)\Big/_{n/k'} = p\left(\frac{an}{ak}\right) = p\left(\frac{n}{k}\right) = \begin{cases} 1 ; & n=0 \\ 0 ; & n \neq 0 \end{cases}$$

(since $p(t)$ is nyquist at sampling rate k symbols per sec)

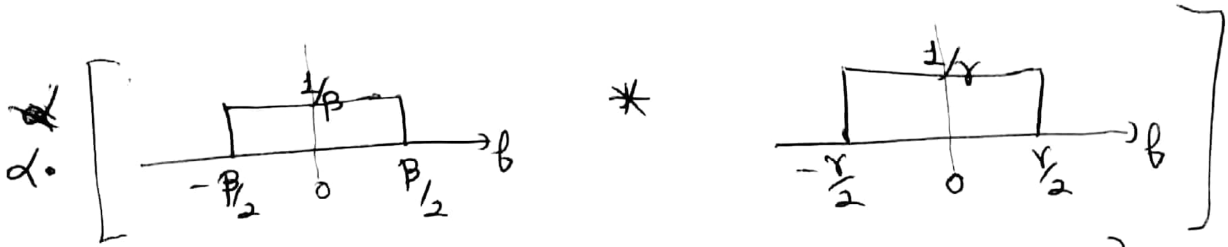
$x(t)\Big/_{n/k'} = \begin{cases} 1 ; & n=0 \\ 0 ; & \text{o.w} \end{cases}$ means that $x(t)$ is nyquist at sampling rate $k' = \cancel{\ }ak$.

1

(a)



Using Basic convolution properties,



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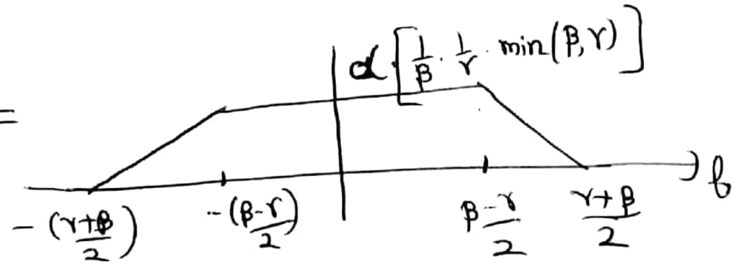


Assuming $\beta > \gamma$

Taking IFT,

$$\alpha \ \mathrm{sinc}(\beta t) \cdot \mathrm{sinc}(\gamma t) = p(t)$$

~~considering~~

$$\frac{\gamma+\beta}{2} = 10K, \quad \frac{\beta-\gamma}{2} = 4K,$$

we get $\beta = 14K$, $\gamma = 6K$

Considering the amplitude,

$$\alpha \cdot \left(\frac{1}{\beta} \cdot \frac{1}{\gamma} \cdot \gamma\right) = 1.$$

$$\therefore \alpha = \beta = 14K$$

$$\therefore p(t) = (14K)\ \mathrm{sinc}(14K\,t)\ \mathrm{sinc}(6K\,t)$$

(b)

$$p(t) = \text{sinc}(14K\,t)\ \text{sinc}(6K\,t)$$

~~p(t) satisfies~~ Nyquist criterion in Freq. Domain

$$\frac{1}{T}\sum_{k=-\infty}^{\infty} P\left(f-\frac{k}{T}\right) = \text{constant}$$

~~shift~~

If we shift $P(f)$ by 14K, ~~we~~ ~~get~~ and add, we get constant.

$\therefore \frac{1}{T}$ = symbol rate = 14K symbols/sec

(c) ~~p(t)~~ R_b = 18 Kbps

$$\frac{R_b}{\log_2 M} = \frac{18K}{\log_2 8} = 6K$$

Since '6K' is one of the two possible symbol rates, it can be used

(d) R_b = 25 Kbps

$$\frac{R_b}{\log_2 M} = \frac{25K}{\log_2 4} = 12.5K$$

12.5K is not one of the two symbol rates,
$\therefore$ Cannot be used

(e) R_b = 21 Kbps

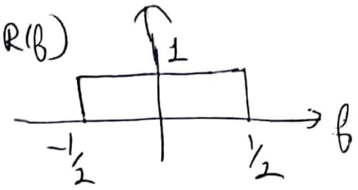
$$\frac{R_b}{\log_2 M} = \frac{21K}{3} = 7K$$, which is also one of the symbol rate $\therefore$ can be used *

~~7K is not one of the two possible symbol rates.~~
~~$\therefore$ Cannot be used~~

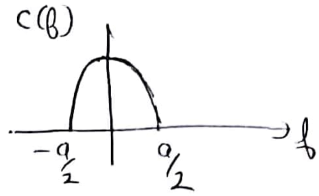
*
Since $p\left(\frac{n}{K}\right) = \begin{cases} 0, & n\neq 0 \\ 1, & n=0 \end{cases}$, $p\left(n\frac{m}{K}\right) = \begin{cases} 0, & n\neq 0 \\ 1, & n=0 \end{cases}$, where m is any integer

rate = $\boxed{K}$ rate = $\boxed{\frac{K}{m}}$, Put $m=2$ in 2(e)

④ (a) $R(f) = \begin{cases} 1, & -\frac{1}{2} \le f \le \frac{1}{2} \\ 0, & \text{o.w} \end{cases}$



$C(f) = \begin{cases} \frac{\pi}{2a} \cos\left(\frac{\pi}{a} f\right), & -\frac{a}{2} \le f \le \frac{a}{2} \\ 0, & \text{o.w.} \end{cases}$



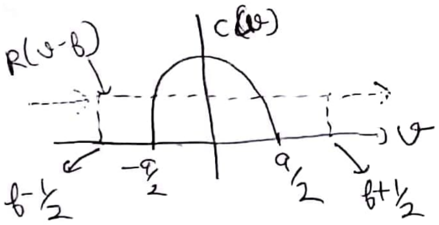
$0 < a < 1$

(b) $P(f) = R(f) * C(f)$

We will compute this using direct convolution

Since $R(f)$, $C(f)$ both are symmetric, $P(f)$ will also be symmetric. $\therefore$ we evaluate for $f > 0$

$$P(f) = \int R(f-v)\, C(v)\, dv = \int R(v-f)\, C(v)\, dv$$

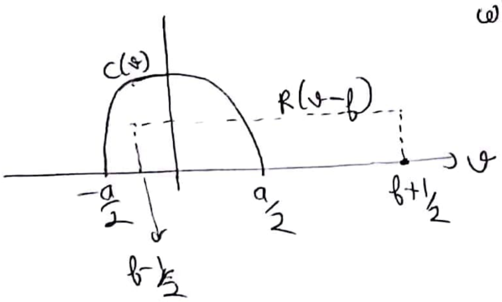


when, $f - \frac{1}{2} < -\frac{a}{2}$ or $0 < f < \left(\frac{1-a}{2}\right)$

; $P(f) = \int_{-a/2}^{a/2} c(v)\, dv = 1$

when, $-\frac{a}{2} < f - \frac{1}{2} < \frac{a}{2}$

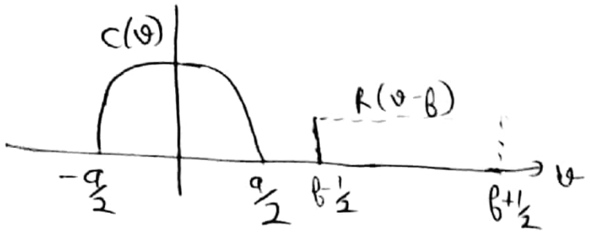
or $\frac{1-a}{2} < f < \frac{1+a}{2}$



$$P(f) = \int_{f-\frac{1}{2}}^{a/2} c(v)\, dv$$

$$= \frac{\pi}{2a} \int_{f-\frac{1}{2}}^{a/2} \cos\left(\frac{\pi}{a} v\right) dv$$

$$= \frac{1}{2}\left(1 - \sin\left(\frac{\pi}{a}\left(f - \frac{1}{2}\right)\right)\right)$$



when $f-\frac{1}{2} > \frac{a}{2}$ or $f > \frac{1+a}{2}$

$$P(f) = 0$$

$$\therefore P(f) = 1 \quad , \text{ for } 0 \le |f| < \left(\frac{1-a}{2}\right)$$
$$= \frac{1}{2}\left[1 - \sin\left(\frac{\pi}{a}\left(f-\frac{1}{2}\right)\right)\right] , \text{ for } \frac{1-a}{2} < |f| < \frac{1+a}{2}$$
$$= 0 \quad , \text{ for } f > \frac{1+a}{2}$$

→ Consider $\frac{\pi}{a}\left(f - \frac{1-a}{2}\right) = \tilde{f}$,

then, $\frac{\pi}{a}\left(f-\frac{1}{2}\right) = \tilde{f} - \frac{\pi}{2}$

So, now for $\frac{1-a}{2} < f < \frac{1+a}{2}$, we have $0 < \tilde{f} < \frac{\pi}{2}$

Also, $P(\tilde{f}) = \frac{1}{2}\left[1 - \sin\left(\tilde{f} - \frac{\pi}{2}\right)\right]$

$$= \frac{1}{2}\left[1 + \cos(\tilde{f})\right] \quad \text{... Raised Cosine Shape}$$

(c) $p(t) = r(t) \cdot c(t)$

$$r(t) = \text{sinc}(t)$$

$$c(t) = \int c(f)\, e^{j2\pi f t}\, df = \int_{-a/2}^{a/2} \frac{\pi}{2a} \cos\left(\frac{\pi}{a} f\right) e^{j2\pi f t}\, df$$

$$= \frac{\pi}{2a} \int_{-a/2}^{a/2} \frac{1}{2}\left(e^{j\pi \frac{f}{a}} + e^{-j2\pi \frac{f}{a}}\right) e^{j2\pi f t}\, df$$

$$= \frac{\pi}{4a}\left[\frac{e^{j\left(\frac{\pi}{a}+2\pi t\right)f}}{j\left(\frac{\pi}{a}+2\pi t\right)} + \frac{e^{j\left(-\frac{\pi}{a}+2\pi t\right)f}}{j\left(-\frac{\pi}{a}+2\pi t\right)}\right]_{f=-\frac{a}{2}}^{f=\frac{a}{2}}$$

$$= \frac{\pi}{4a}\left\{ \frac{e^{j\left(\frac{\pi}{2}+\pi a t\right)} - e^{j\left(-\frac{\pi}{2}-\pi a t\right)}}{j\left(\frac{\pi}{a}+2\pi t\right)} + \frac{e^{j\left(-\frac{\pi}{2}+\pi a t\right)} - e^{j\left(\frac{\pi}{2}-\pi a t\right)}}{j\left(-\frac{\pi}{a}+2\pi t\right)} \right\}$$

$$= \frac{\pi}{4a}\left\{ \frac{2j\cos(\pi a t)}{j\left(\frac{\pi}{a}+2\pi t\right)} + \frac{-2j\cos(\pi a t)}{j\left(-\frac{\pi}{a}+2\pi t\right)} \right\}$$

$$= \frac{\cos(\pi a t)}{1-4a^2t^2}$$

$$\therefore\ p(t) = \operatorname{sinc}(t)\cdot\frac{\cos(\pi a t)}{1-4a^2t^2}$$

(d) $p(t) = 0$ when $t = \ldots -3, -2, -1, \cancel{0}, 1, 2, \ldots$ ~~or~~ $\cancel{t \in \{z\}}$ or $t \in Z \setminus \{0\}$

and when $t = \frac{(2n+1)}{2a}$

$$p\left(\frac{t}{T}\right) = \operatorname{sinc}\left(\frac{t}{T}\right)\left(\frac{\cos\left(\pi a \frac{t}{T}\right)}{1-4a^2\frac{t^2}{T^2}}\right)$$

Say, $k(t) = p\left(\frac{t}{T}\right)$

$k(t) = 0$ when $t = \ldots -2T, -T, +T, +2T, \ldots$

and $t = \ldots -\frac{3T}{2a}, -\frac{T}{2a}, \frac{T}{2a}, +\frac{3T}{2a}, \ldots$

Nyquist criterion $k(t) = 1$ for $t = 0$
$= 0$ for $t = nT$

and we need highest rate for which $k(t)$ is ISI free

$\therefore$ we select 'n' as 1 or $\frac{1}{2a}$

$\therefore$ Symbol rate $\frac{1}{T}$ or $\frac{2a}{T}$ depending on value of 'a'