EE 5140: Tutorial: Digital Modulation

1. With $A = 4$ kHZ and $B = 10$ kHz, define the trapezoidal spectrum

$$P(f) = \begin{cases} 1, & |f| \leq A \\ \frac{B-|f|}{B-A} & A \leq |f| \leq B \\ 0 & \text{Otherwise.} \end{cases}$$

   (a) Show that the time domain pulse is of the form $p(t) = \alpha \text{sinc}(\beta t)\text{sinc}(\gamma t)$ and find the values of α, β and γ explicitly. `Hint:` Trapeziod is obtained by convolving two rectangles.

   (b) What is the maximum symbol rate for which the pulse $p(t)$ satisfies the Nyquist criterion?

   (c) Can the pulse $p(t)$ be used for Nyquist signalling for bit rate of 18 kbps with 8-PSK constellation?

   (d) Can the pulse $p(t)$ be used for Nyquist signalling for bit rate of 25 kbps with QPSK constellation?

   (e) Can the pulse $p(t)$ be used for Nyquist signalling for bit rate of 21 kbps with 8-PSK constellation?

2. Consider the pulse $p(t) = \text{sinc}(\alpha t)\text{sinc}(\beta t)$ where α and β have to be determined.

   (a) How should α and β be chosen such that $p(t)$ satisfies Nyquist criterion with excess bandwidth parameter 0.5 for the data rate of 40 Mbps using 16-QAM constellation. Specify the bandwidth occupied for this case.

   (b) How should α and β be chosen such that $p(t)$ satisfies Nyquist criterion for both the following cases: 40 Mbps using 16-QAM and 8 Mbps using 8-PSK. Specify the bandwidth occupied in this case.

3. Suppose the pulse $p(t)$ satisfies Nyquist criterion at symbol rate K symbols per second.

   (a) Verify/show that the same pulse $p(t)$ will satisfy Nyquist criterion for $\frac{K}{M}$ symbols per second where M is any positive integer.

   (b) Verify/show that the scaled pulse $p(at)$ for $a > 0$ satisfies Nyquist criterion at symbol rate aK symbols per second.

4. `Raised Cosine Pulse:` Consider the two spectrums $R(f)$ and $C(f)$ defined as

$$R(f) = \begin{cases} 1, & -\frac{1}{2} \leq f \leq \frac{1}{2} \\ 0, & \text{Otherwise.} \end{cases}$$

$$C(f) = \begin{cases} \frac{\pi}{2a}\cos(\frac{\pi}{a}f), & -\frac{a}{2} \leq f \leq \frac{a}{2} \\ 0, & \text{Otherwise.} \end{cases}$$

(a) Sketch the spectrums $R(f)$ and $Cf)$ assuming $0 < a < 1$

(b) Compute the spectrum $P(f) = R(f) * C(f)$ where $*$ denotes convolution. This corresponds to the raised cosine spectrum.

(c) Find the time domain pulse as $p(t) = r(t)c(t)$ where $r(t)$ and $c(t)$ are inverse fouriere transforms of $R(f)$ and $C(f)$ respectively

(d) Where are the zeros of $p(t)$? What is the highest symbol rate for which the scaled pulse $p(t/T)$ is ISI-free, satisfying the Nyquist condition?