1) $s(t) = \text{sinc}(t)\,\text{sinc}(2t)$

a) Fourier transform properties.

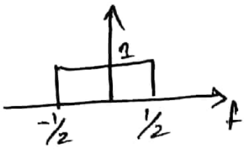
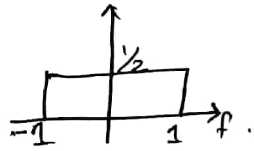
$$\mathcal{F}(g(t)\cdot h(t)) = G(f) * H(f) \quad \text{—— ①}$$

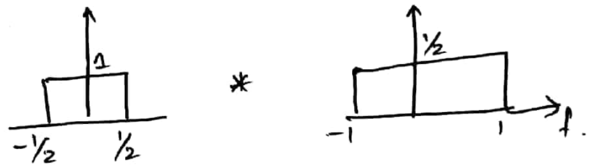
$$\mathcal{F}(g(t)) = G(f) \Rightarrow \mathcal{F}(G(t)) = g(-f) \quad \text{—— ②}$$

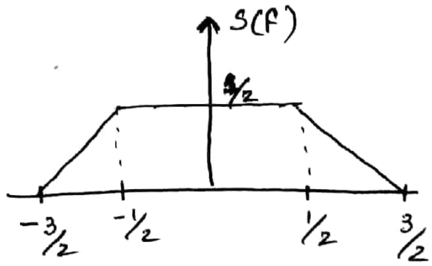
Fourier transform,

$$\mathcal{F}(\text{rect}(t/T)) = T\,\text{sinc}(fT)$$

$$\Rightarrow \text{sinc}(tT) \xleftrightarrow{\mathcal{F}} \frac{1}{T}\text{rec}\left(\frac{f}{T}\right) \quad (\text{from ②})$$

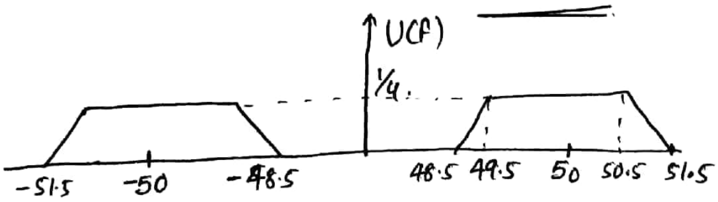
- $\text{sinc}(t) \xrightarrow{\mathcal{F}}$  ; $\text{sinc}(2t) \xrightarrow{\mathcal{F}}$ 

- From ① $s(t) = \text{sinc}(t)\cdot\text{sinc}(2t)$. $\xrightarrow{\mathcal{F}}$ 

= 

b) $u(t) = s(t)\cos(100\pi t) \xrightarrow{\mathcal{F}} U(f) = S(f) * \frac{1}{2}(\delta(f-50) + \delta(f+50))$

ie, $U(f) = \frac{1}{2}S(f-50) + \frac{1}{2}S(f+50)$



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4) a) $s(t) = s_c(t) + j\, s_s(t)$ → Complex Baseband signal

$s_p(t) = \mathrm{Re}\left(\sqrt{2}\, s(t)\, e^{j2\pi f_c t}\right)$ → Passband signal

- $u_p(t) = \sqrt{2}\,\mathrm{sinc}(2t)\cos 100\pi t$

$$= \mathrm{Re}\Big(\sqrt{2}\,\underbrace{\mathrm{sinc}(2t)}_{u(t)}\, e^{j2\pi 50t}\Big)$$

Comparing with the Passband s/g representation, $u(t) = \mathrm{sinc}(2t)$

- $v_p(t) = \sqrt{2}\,\mathrm{sinc}(t)\sin\left(101\pi t + \frac{\pi}{4}\right)$

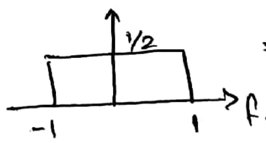
$$= \mathrm{Re}\left((-j)\sqrt{2}\,\mathrm{sinc}(t)\, e^{j2\pi\left(50t + \frac{1}{2}t + \frac{1}{8}\right)}\right)$$

$$= \mathrm{Re}\Big(\sqrt{2}\,\mathrm{sinc}(t)\cdot e^{j\pi t}\cdot e^{j\pi/4}\cdot \underbrace{(-j)}_{e^{-j\pi/2}}\; e^{j2\pi\cdot 50t}\Big)$$

$$= \mathrm{Re}\Big(\sqrt{2}\cdot \underbrace{\mathrm{sinc}(t)\, e^{j\pi t}\cdot e^{-j\pi/4}}_{v(t)}\cdot e^{j2\pi\cdot 50t}\Big)$$

$\therefore\ v(t) = \mathrm{sinc}(t)\, e^{j(\pi t - \pi/4)}$

b) $u(t) = \mathrm{sinc}\, 2t \Rightarrow U(f) =$



⇒ Bandwidth of $U(f)$ = Bandwidth of $u_p(t) = 2\,Hz$.

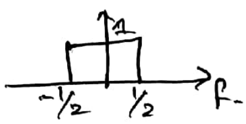
$$v(t) = \mathrm{sinc}(t)\, \underbrace{e^{j(\pi t - \pi/4)}}$$

~~correspo~~

- $e^{-j\pi/4}$ corresponds to amplitude scaling.
- $e^{j\pi t}$ corresponds to frequency shift

} Hence won't affect the bandwidth

$\mathrm{sinc}(t) \overset{\mathcal{F}}{\longleftrightarrow}$



$\therefore$ Bandwidth of $v_p(t)$ = bandwidth of $\mathrm{sinc}(t)$ = $1\ Hz$.

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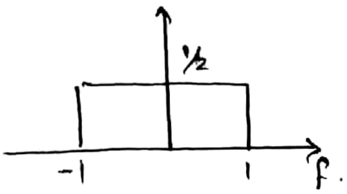
c) $\langle u_p(t), v_p(t) \rangle = Re(\langle u(t), v(t) \rangle)$ —— 1

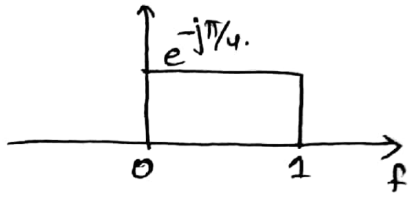
$= \langle u_c(t), v_c(t) \rangle + \langle u_s(t), v_s(t) \rangle$.

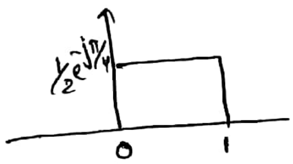
Parseval's theorem $\Rightarrow$ $\langle u(t), v(t) \rangle = \langle U(f), V(f) \rangle$.

So from 1, $\langle u_p(t), v_p(t) \rangle = Re(\langle U(f), V(f) \rangle)$

$= Re\left[\int_{-\infty}^{\infty} U(f) V^*(f)\, df \right]$.

$U(f) =$ 

$V(f) =$ 

$\Rightarrow$ $U(f) \times V^*(f) =$  —— 2.

Integrating 2, $\langle u_p(t), v_p(t) \rangle = Re\left[\frac{1}{2} e^{-j\pi/4} \int_0^1 df. \right]$

$= \frac{1}{2\sqrt{2}}$

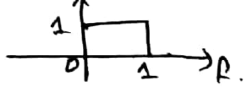
d) Passband signal, $Y_p(f) = S_p(f) H_p(f)$

$\Rightarrow$ Baseband ~~signal, Y(f)~~ complex envelope,

$Y(f) = \frac{1}{\sqrt{2}} S(f) H(f)$.

Here $y_p(t) = u_p(t) * v_p(t)$ $\Rightarrow$ $Y_p(f) = U_p(f) V_p(f)$

ie, $Y(f) = \frac{1}{\sqrt{2}} U(f) V(f) = \frac{1}{2\sqrt{2}} e^{-j\pi/4} . I_{[0,1]}(f)$ (from 2 in part (c))

where $I_{[0,1]}(f) =$ 

$\Rightarrow$ $y(t) = \frac{e^{-j\pi/4}}{2\sqrt{2}} . e^{j\pi t} . \text{sinc}(t)$

$\therefore$ $y_p(t) = Re(\sqrt{2} . y(t) e^{j2\pi(50t)}) = Re\left(\frac{\text{sinc}(t)}{2} . e^{j\pi(101t - \pi/4)} \right)$

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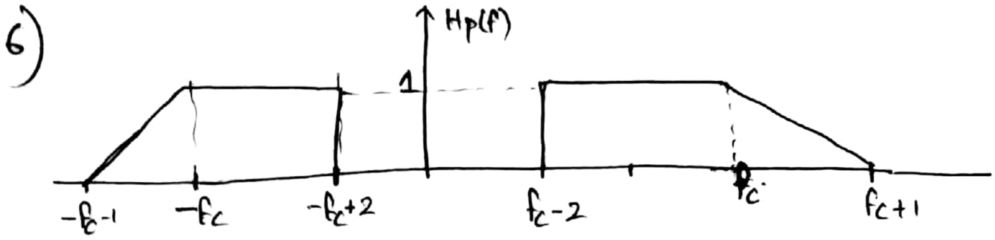
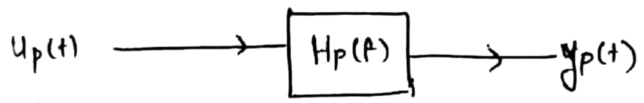
$$= \frac{1}{2}\operatorname{sinc}(t)\cos\left(101\pi t - \pi/4\right)$$

or

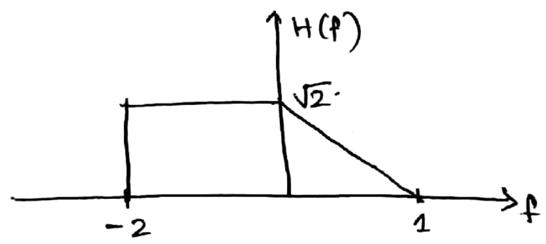
$$\tfrac{1}{2}\operatorname{sinc}(t)\sin\left(101\pi t + \pi/4\right)$$

6)

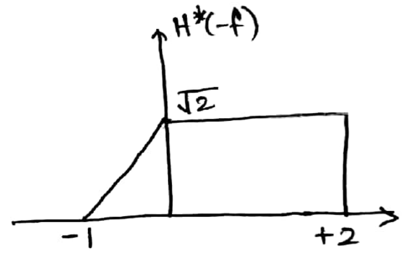




Complex envelope, $H(f) = \sqrt{2}\,H^{+}(f+f_c)$.

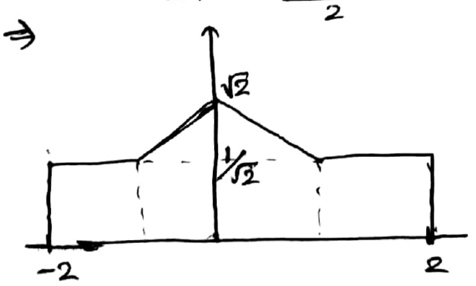


$\Rightarrow$

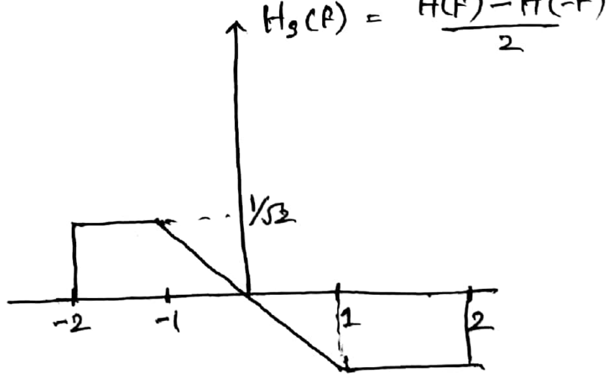


$$H_I(f) = \frac{H(f) + H^*(-f)}{2}$$

$\Rightarrow$



$$H_Q(f) = \frac{H(f) - H^*(-f)}{2}$$



a) $f_1 = f_2 = f_c$

When $f_1 = f_2 = f_c$, the i/ps to the ~~real~~ base-band processing unit are the I & Q components of the complex baseband signal $u(t)$. (The highfrequency term corresponding to $2f_c$ ~~is~~ ~~also~~ gets filtered by the baseband filters used in the box). The output of the baseband processing unit corresponds to the I & Q components of

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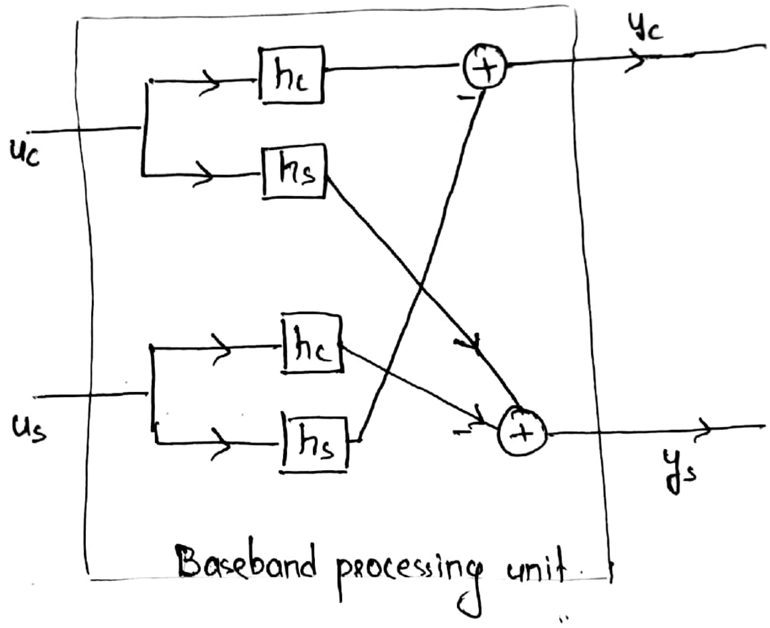
the baseband signal $y(t)$

$$y(t) = y_c(t) + j y_s(t) = u(t) * h(t)$$
$$= (u_c + j u_s) * (h_c + j h_s)$$
$$= (u_c * h_c - u_s * h_s) + j(u_c * h_s + u_s * h_c)$$

i.e, $y_c = u_c * h_c - u_s * h_s$

$y_s = u_c * h_s - u_s * h_c.$



b) $f_1 = f_c + 1/2$; $f_2 = f_c - 1/2$

Let $u_p(t) = Re\left(\sqrt{2}\, u_1(t)\, e^{j2\pi(f_c + 1/2)t}\right)$

$$= Re\left(\sqrt{2}\, \underbrace{u_1(t)\, e^{j\pi t}}\, .\, e^{j2\pi f_c t}\right)$$

∴, s/g at the i/p of Baseband unit ; $u(t) = u_1(t) e^{j\pi t}$

o/p $y_p(t) = Re\left(\sqrt{2}\, y_1(t)\, e^{j2\pi(f_c - 1/2)t}\right)$

$$= Re\left(\sqrt{2}\, \underbrace{y_1(t)\, e^{-j\pi t}}_{y(t)}\, e^{j2\pi f_c t}\right)$$

s/g at the o/p of Baseband processing unit; $y(t) = y_1(t) e^{-j\pi t}$.

∴, after converting $u_1(t)$ to $u(t)$, $u(t)$ can be fed as i/p to the base band processing unit. The o/p at the baseband unit $y(t)$, then

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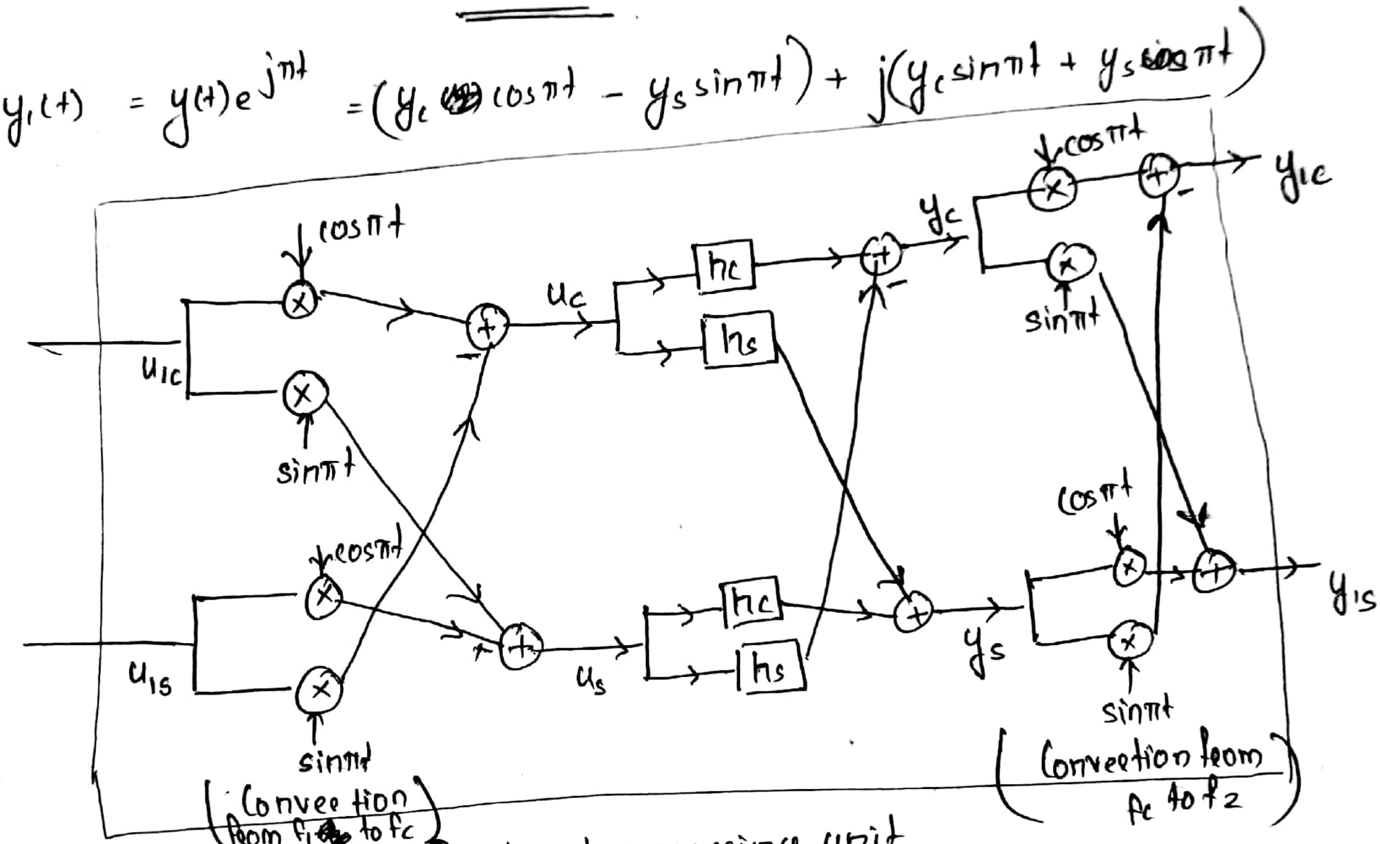
should be converted to $y_1(t)$

- $u(t) = u_c + ju_s = u_1(t)e^{j\pi t} = (u_{1c} + ju_{1s})(\cos\pi t + j\sin\pi t)$
  $= (u_{1c}\cos\pi t - u_{1s}\sin\pi t) + j(u_{1c}\sin\pi t + u_{1s}\cos\pi t)$

- $y_1(t) = y(t)e^{j\pi t} = (y_c\cos\pi t - y_s\sin\pi t) + j(y_c\sin\pi t + y_s\cos\pi t)$



Baseband processing unit

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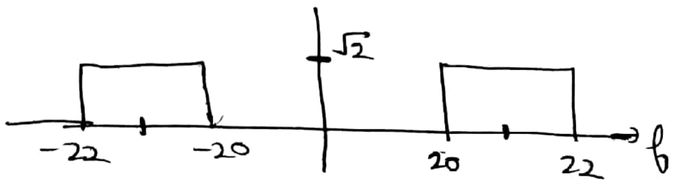
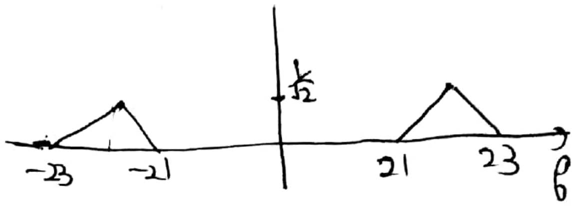
(2)

(a)

$Re(X_p(f))$ $\qquad$ $Im(X_p(f))$





(b)

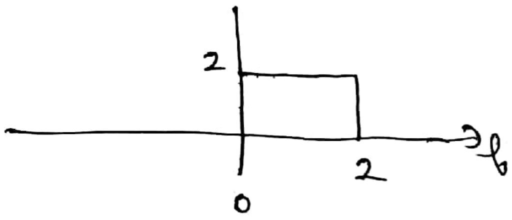
$$x_p(t) \longrightarrow \boxed{\sqrt{2}\cdot x_p(t)\cos(40\pi t)} \longrightarrow \boxed{LPF} \longrightarrow x(t)$$

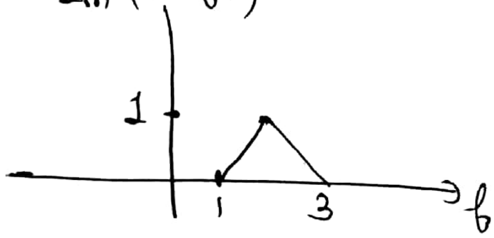
$f_c = 20$

[shift by ± 20]

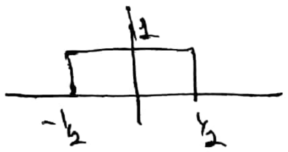
[High frequency components removed]

$Re(X(f))$ $\qquad$ $Im(X(f))$

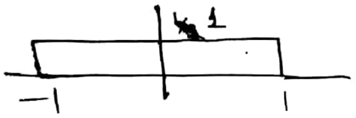




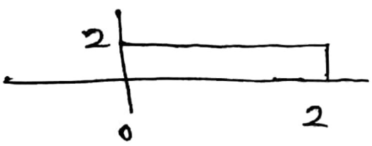
$\updownarrow$



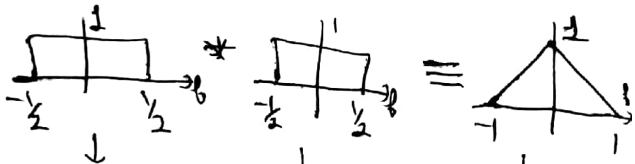
$\longrightarrow \text{sinc}(t)$



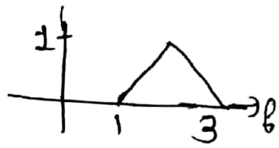
$\longrightarrow 2\,\text{sinc}(2t)$



$\rightarrow \underline{4\,\text{sinc}(2t)\,e^{j2\pi t}}$



$\text{sinc}(t)$ $\quad$ $\text{sinc}(t)$ $\quad$ $\text{sinc}^2(t)$



$\longrightarrow \underline{\text{sinc}^2(t)\,e^{j4\pi t}}$

$$\therefore X(f) = Re(X(f)) + j\,Im(X(f)) \Leftrightarrow$$

$$x(t) = 4\,\text{sinc}(2t) + j\,\text{sinc}^2(t)\,e^{j4\pi t}$$

$$= 4\,\text{sinc}(2t) - \text{sinc}^2(t)\sin(4\pi t) + j\,\text{sinc}^2(t)\cos(4\pi t)$$

$$x_c(t) = Re(x(t)) = 4\,\text{sinc}(2t) - \text{sinc}^2(t)\sin(4\pi t)$$

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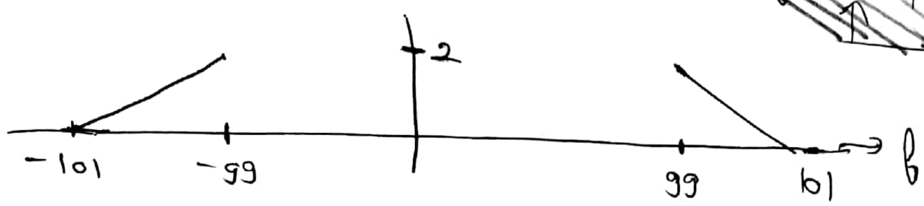
③ (a) $V_p(f)$

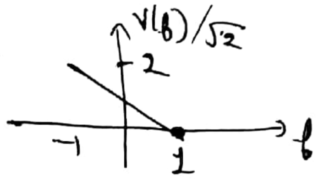


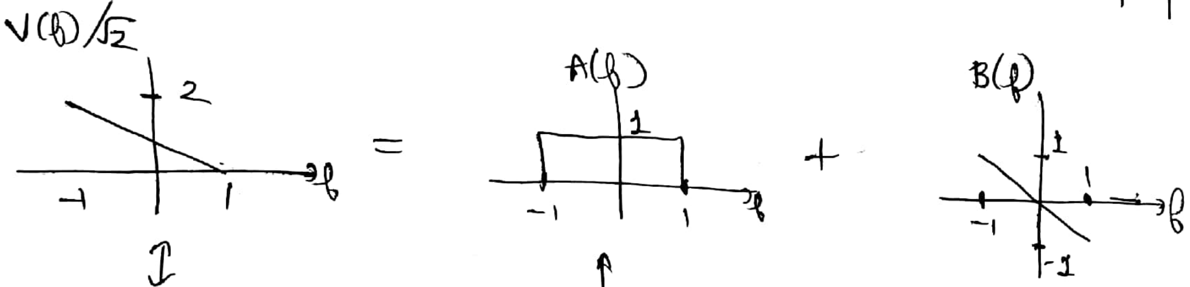
Since $v_p(t)$ is real valued, $V_p(f) = V_p^*(-f)$ — ①

(b) Also, $V_p(f)$ is real, $\therefore$ ~~$V_p(f)$~~ $= V_p^*(-f) = V_p(-f)$

$\therefore$ From ①, $V_p(f) = V_p(-f) \xrightarrow{IFT} v_p(t) = v_p(-t)$

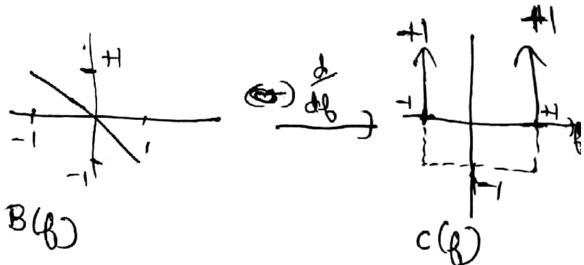
(c) $f_0 = 100$, Baseband representation $V(f)$:





$\updownarrow$ $\qquad$ $\updownarrow$

$v(t)/\sqrt{2}$ $\qquad$ $2\,\mathrm{sinc}(2t)$



$$C(f) = \frac{d}{df}(B(f))$$

$$c(t) = -j2\pi t \cdot b(t)$$

$$c(t) = e^{j2\pi t} + e^{-j2\pi t} - 2\,\mathrm{sinc}(2t)$$

$$\cancel{b(t)}\ \cancel{c(t)} = 2\cos(2\pi t) - 2\,\mathrm{sinc}(2t)$$

$$\therefore b(t) = \frac{2\cos(2\pi t) - 2\,\mathrm{sinc}(2t)}{-j2\pi t}$$

$$\therefore \frac{v(t)}{\sqrt{2}} = 2\,\mathrm{sinc}(2t) + j\frac{\cos(2\pi t)}{\pi t} - j\frac{\mathrm{sinc}(2t)}{\pi t}$$

$$\therefore v_c(t) = \mathrm{Re}(v(t)) = 2\sqrt{2}\,\mathrm{sinc}(2t)$$

$$v_s(t) = \mathrm{Im}(v(t)) = \sqrt{2}\left[\frac{\cos(2\pi t)}{\pi t} - \frac{\mathrm{sinc}(2t)}{\pi t}\right]$$

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(d) $f_0 = 101$

$\rightarrow$ In (c) P.B. B.B.

$$v_p(t) \leftrightarrow v(t) = v_c(t) + j\, v_s(t)$$

$$v_p(t) = v_c(t)\cos(2\pi 100t) - v_s(t)\sin(2\pi 100t)$$

$$= \text{Re}\{ v(t)\, e^{j2\pi 100t} \} \quad -(1)$$

$\rightarrow$ In (D)

$$v_p(t) \leftrightarrow \tilde{v}(t) = \tilde{v}_c(t) + j\,\tilde{v}_s(t)$$

$$v_p(t) = \text{Re}\{ \tilde{v}(t)\, e^{j2\pi 101t} \}$$

$$= \text{Re}\{ \tilde{v}(t)\, e^{j2\pi t}\, e^{j2\pi 100t} \} \quad -(2)$$

(1) = (2) $\therefore$ $v(t) = \tilde{v}(t)\, e^{j2\pi t}$

$$\therefore \tilde{v}(t) = v(t)\, e^{-j2\pi t}$$

WKT. $v(t) = \sqrt{2} \cdot 2\,\text{sinc}(2t) + j\sqrt{2}\left[\dfrac{\cos(2\pi t)}{\pi t} - j\dfrac{\text{sinc}(2t)}{\pi t} \right]$

from (c)

Thus, we can find $\tilde{v}(t)$ and corresponding $\tilde{v}_c(t)$ and $\tilde{v}_s(t)$

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(5) Baseband w/f

$$U(f) = \begin{cases} e^{j2\pi f} = \cos(2\pi f) + j\sin(2\pi f), & 0 < f < 1 \\ 0 & \text{else} \end{cases}$$

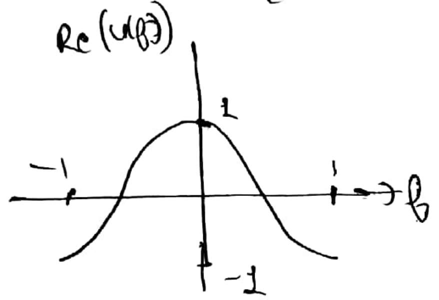
(a)

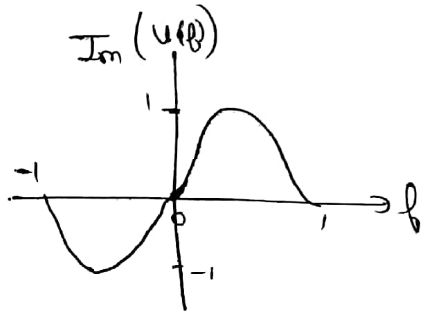
$u(t)$ is Real, $\therefore u(t) = u^*(t)$

$\therefore U(f) = U^*(-f)$

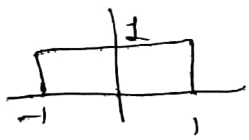
$\therefore$ for $-1 < f < 0$, $U(f) = U^*(-f) = \left(e^{j2\pi(-f)}\right)^* = e^{j2\pi f}$

$$\therefore \mathrm{Re}(U(f)) = \begin{cases} \cos(2\pi f), & -1 < f < 1 \\ 0, & \text{else} \end{cases} \qquad \mathrm{Im}(U(f)) = \begin{cases} \sin(2\pi f), & -1 < f < 1 \\ 0, & \text{else} \end{cases}$$

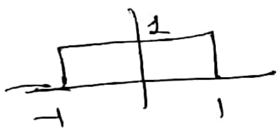




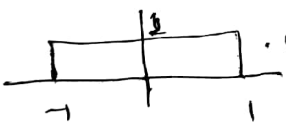
(b) $U(f) = $



$\times\ e^{j2\pi f}$



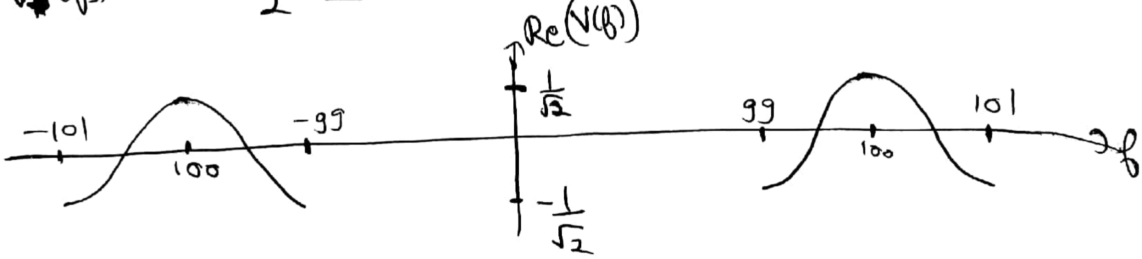
$\leftrightarrow 2\,\mathrm{sinc}(2t)$

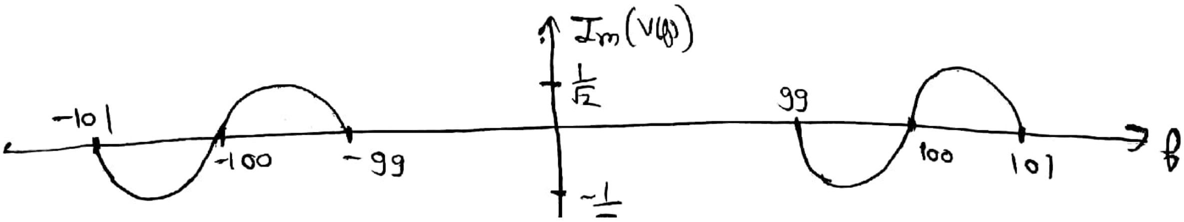


$\cdot\ e^{j2\pi f} \leftrightarrow 2\,\mathrm{sinc}(2t+1) = 2\,\mathrm{sinc}\left(2\left(t+\frac{1}{2}\right)\right)$

(c)
$$v(t) = \sqrt{2}\, u(t) \cos(200\pi t)$$
$$= \sqrt{2}\, u(t) \left(\frac{e^{j2\pi 100t} + e^{-j2\pi 100t}}{2}\right)$$

Pass band
$$V_p(f) = \frac{\sqrt{2}}{2}\left[U(f-100) + U(f+100)\right]$$





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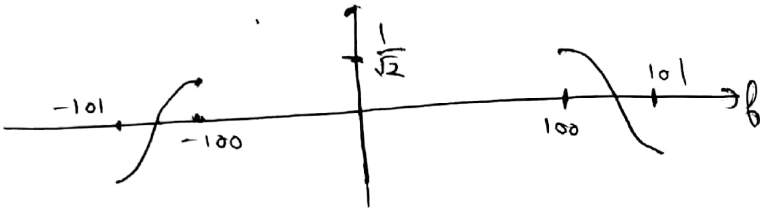
(d) $y(t) = v(t) * h_{hp}(t)$

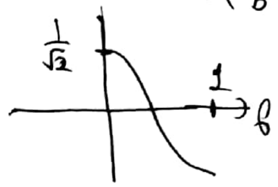
$Y(f) = V(f) \cdot H_{hp}(f)$

$$H_{hp}(f) = \begin{cases} 1, & |f| \geq 100 \\ 0, & \text{else} \end{cases}$$

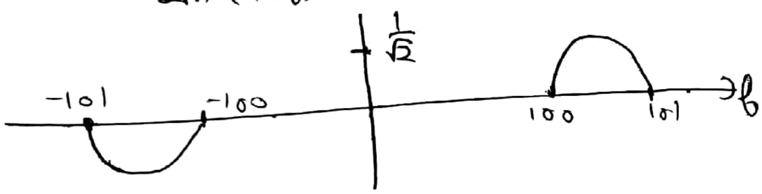
$Re(Y(f))$ (Pass band) $\qquad$ (Base band) $Re(Y_B(f))$



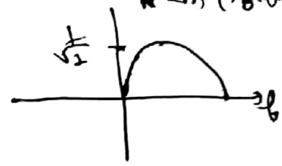
$\xrightarrow{\quad}$ $f_c = 100$



$Im(Y(f))$ (Pass band) $\qquad$ $Im(Y_B(f))$



$\xrightarrow{\quad}$ $f_c = 100$



$$y(t) = \sqrt{2}\left(y_c(t)\cos(2\pi\, 100t) - y_s(t)\sin(2\pi\, 100t)\right)$$

$$= \sqrt{2}\, Re\{y_b(t)\, e^{j2\pi 100t}\}$$

where $y_b(t) = y_c(t) + j\, y_s(t)$

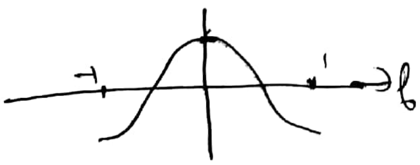
$$y_c(t) = Re\{y_b(t)\} = \frac{y_b(t) + y_b^*(t)}{2}$$

$$y_s(t) = Im\{y_b(t)\} = \frac{y_b(t) - y_b^*(t)}{2j}$$

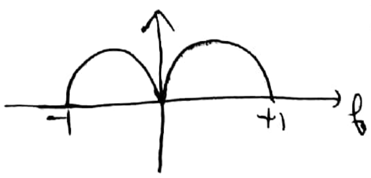
$$\therefore Y_c(f) = \frac{Y_B(f) + Y_B^*(-f)}{2}$$

$$Y_s(f) = \frac{Y_B(f) - Y_B^*(-f)}{2j}$$

$$Re(Y_c(f)) = \frac{Re(Y_B(f)) + Re(Y_B(-f))}{2}$$



$$Re(Y_s(f)) = \frac{Im(Y_B(f)) + Im(Y_B(-f))}{2} \; *$$



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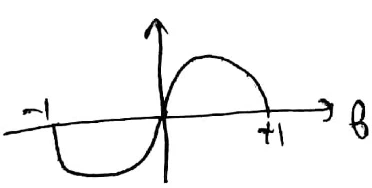
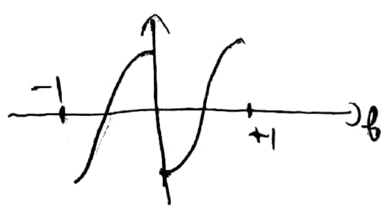
$$Im(Y_c(f)) = \frac{Im(Y_B(f)) - Im(Y_B(-f))}{2}$$



$$Im(Y_s(f)) = \frac{-Re(Y_B(f)) + Re(Y_B(-f))}{2} \; *$$



* Ignoring the scaling factors

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(e) Now,

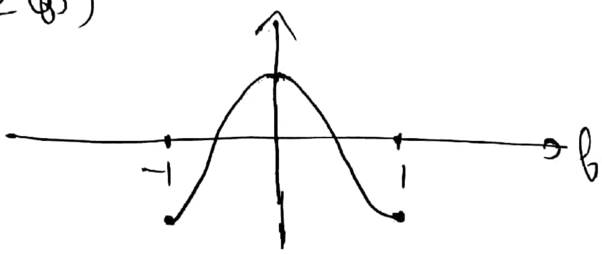
$$z(t) = y(t)\cdot\cos(200\pi t) * h_{lp}(t)$$

$$= y(t)\left(\frac{e^{j2\pi 100t} + e^{-j2\pi 100t}}{2}\right) * h_{lp}(t)$$

i.e. ~~y(t) shifted~~ $Y(f)$ shifted by ± 100, then passed through a LPF

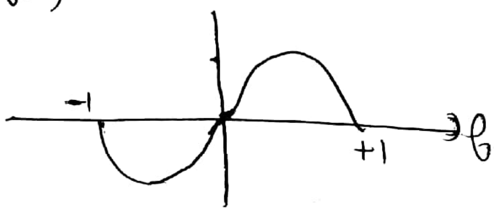
$$H_{lp}(f) = \begin{cases} 1 & |f| \le 1 \\ 0 & \text{else} \end{cases}$$

$\therefore$ $Re(Z(f))$



$\equiv Re(U(f)) \cdot k_R$

$Im(Z(f))$



$\equiv Im(U(f)) \cdot K_I$

where K_R, K_I are some constants

$\therefore$ $z(t) = k\,u(t)$

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* $$Y_s(f) = \frac{Y_B(f) - Y_B^*(-f)}{2j}$$

So, $$Re(Y_s(f)) = Re\left(\frac{Y_B(f)}{2j}\right) - Re\left(\frac{Y_B^*(-f)}{2j}\right)$$

$$= Re\left[\frac{Re(Y_B(f)) + j\,Im(Y_B(f))}{2j}\right] - Re\left[\frac{Re(Y_B(-f)) - j\,Im(Y_B(-f))}{2j}\right]$$

$$= \left[Im(Y_B(f)) + Im(Y_B(-f))\right]/2$$

$$Im(Y_s(f)) = Im\left[\frac{Re(Y_B(f)) + j\,Im(Y_B(f))}{2j}\right] - Im\left[\frac{Re(Y_B(-f)) - j\,Re(Y_B(-f))}{2j}\right]$$

$$= \left[-Re(Y_B(f)) + Re(Y_B(-f))\right] /_{2}$$

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