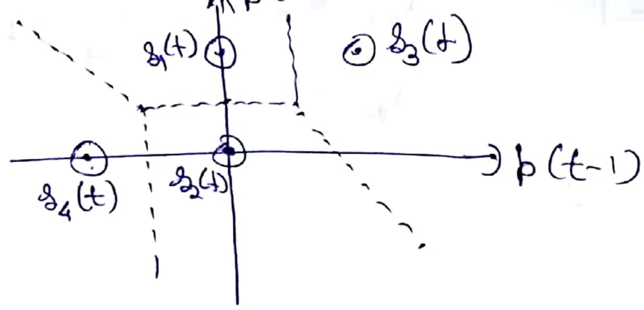
(1)
(2)




$p(t)$, and $p(t-1)$ are orthogonal (normal), since $\int_{-\infty}^{\infty} p(t)\,p(t-1)\,dt = 0$

$s_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $s_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $s_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\cancel{s_4 = \begin{bmatrix} 0 & -1 \\ & 0 \end{bmatrix}}$ $s_4 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

(3)

$$y(t) = \begin{cases} 1-t, & 0 \le t \le 1.5 \\ 0 & \text{otherwise} \end{cases}$$

Coordinate along $\psi_2(t)$ i.e. $p(t) = \int_{-\infty}^{\infty} y(t)\,p(t)\,dt$

$$= \int_0^1 (1-t)\cdot 1\,dt = \left(t - \frac{t^2}{2}\right)_0^1 = \frac{1}{2}$$

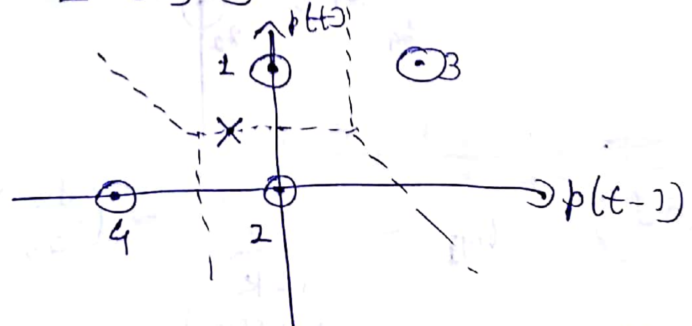
Coordinate along $\varphi_1(t)$ i.e. $p(t-1) = \int_{-\infty}^{\infty} y(t)\,p(t-1)\,dt$

$$= \int_1^{1.5} (1-t)\cdot 1\,dt = \left(t - \frac{t^2}{2}\right)_1^{3/2} = \left(\frac{3}{2} - \frac{9}{8}\right) - \left(\frac{1}{2}\right)$$

$$= 1 - \frac{9}{8} = -\frac{1}{8} = -0.125$$

So, $y(t) = \frac{1}{2} p(t) + (-0.125)\, p(t-1)$

'OR' $y = \begin{bmatrix} -0.125 \\ 0.5 \end{bmatrix}$ in Signal Space Representation




'y' lies on the boundary of $T_1$ and $T_2$, So o/p of ML demodulator is either $s_1$ or $s_2$

(4) $\cancel{E_b} = \cancel{\dfrac{\|s_1\|^2+\|s_2\|^2+\|s_3\|^2+\|s_4\|^2}{4}} = \cancel{\dfrac{1+0+2+1}{4}} = \cancel{1}$

$$P_{e|i} \leq \sum_{\substack{i \neq k \\ k \in \text{nearest neighbours}}} Q\left(\frac{d_{ik}}{2\sigma}\right) = \sum_{\substack{i \neq k \\ k \in \text{nearest neighbours}}} Q\left(d_{ik}\sqrt{\frac{E_b}{N_0}}\right)$$

$\boxed{\text{See next page}}$ $\leftarrow \dfrac{1}{2\sigma} = \sqrt{\dfrac{E_b}{N_0}}$ ~~(since $E_b = 1$)~~

$\boxed{\text{* bottom}}$

$d_{12} = d_{21} = 1$ $\quad d_{23} = d_{32} = \sqrt{2}$ $\quad d_{34} = d_{43} = \sqrt{5}$

$d_{13} = d_{31} = 1$ $\quad d_{24} = d_{42} = 1$

$d_{14} = d_{41} = \sqrt{2}$

For, $s_1$ $\xrightarrow{}$ $k = 2,3,4$

$$P_{e|1} \leq Q\left(d_{12}\sqrt{\frac{E_b}{N_0}}\right) + Q\left(d_{13}\sqrt{\frac{E_b}{N_0}}\right) + Q\left(d_{14}\sqrt{\frac{E_b}{N_0}}\right)$$
$$= 2Q\left(\sqrt{\frac{E_b}{N_0}}\right) + Q\left(\sqrt{2}\sqrt{\frac{E_b}{N_0}}\right)$$

For $s_2$, $k = 1,3,4$,

$$P_{e|2} \leq Q\left(d_{21}\sqrt{\frac{E_b}{N_0}}\right) + Q\left(d_{23}\sqrt{\frac{E_b}{N_0}}\right) + Q\left(d_{24}\sqrt{\frac{E_b}{N_0}}\right)$$
$$= 2Q\left(\sqrt{\frac{E_b}{N_0}}\right) + Q\left(\sqrt{2}\sqrt{\frac{E_b}{N_0}}\right)$$

For $s_3$, $k = 1,2$

$$P_{e|3} \leq Q\left(d_{31}\sqrt{\frac{E_b}{N_0}}\right) + Q\left(d_{32}\sqrt{\frac{E_b}{N_0}}\right)$$
$$= Q\left(\sqrt{\frac{E_b}{N_0}}\right) + Q\left(\sqrt{2}\sqrt{\frac{E_b}{N_0}}\right)$$

For $s_4$, $k = 1,2$

$$P_{e|4} \leq Q\left(d_{41}\sqrt{\frac{E_b}{N_0}}\right) + Q\left(d_{42}\sqrt{\frac{E_b}{N_0}}\right)$$
$$= Q\left(\sqrt{2}\sqrt{\frac{E_b}{N_0}}\right) + Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$P_e = \frac{1}{4}\left(6Q\left(\sqrt{\frac{E_b}{N_0}}\right) + 4Q\left(\sqrt{2}\sqrt{\frac{E_b}{N_0}}\right)\right)$$
$$= \frac{3}{2}Q\left(\sqrt{\frac{E_b}{N_0}}\right) + Q\left(\sqrt{2}\sqrt{\frac{E_b}{N_0}}\right)$$

(5) $\eta_p = \frac{d_{min}^2}{E_b}$

$E_s = \frac{\sum_i \|s_i\|_2^2}{\text{No. of points}}$

$E_b = \frac{E_s}{\log_2(\cdot)}$

Remove $s_1$, $d_{min} = 1 = d_{24}$

$E_b = \frac{0+2+1}{3\log_2(3)} = \frac{1}{1.5850}$ $\eta_p = 1.5850$

Remove $s_2$, $d_{min} = d_{13} = 1$

$E_b' = \frac{1+2+1}{3\log_2(3)} = \frac{4}{3(1.5850)}$, $\eta_p = \frac{3}{4} = 1.1887$

Remove $s_3$, $d_{min} = d_{12} = 1$

$E_b = \frac{1+0+1}{3\log_2(3)} = \frac{2}{3(1.5850)}$ $\boxed{\eta_p = 2.3774}$

Remove $s_4$, $d_{min} = d_{12} = 1$

$E_b = \frac{1+0+2}{3\log_2(3)} = \frac{1}{1.5850}$ $\eta_p = 1.5850$

$\frac{d}{2\sigma} = \sqrt{\frac{d^2}{4\sigma^2}}$ $\sqrt{\frac{d^2}{E_b} \cdot \frac{E_b}{2N_0}}$ $\sqrt{\frac{d^2}{2N_0}}$

$\therefore$ we remove $s_3$ to get maximum power efficiency

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$E_s = \frac{d^2+d^2+2d^2}{4} = d^2$, So, $E_b = \frac{E_s}{2} = \frac{d^2}{2}$

$\sigma^2 = \frac{N_0}{2}$,

$\frac{E_b}{N_0} = \frac{d^2/2}{2\sigma^2} = \frac{d^2}{4\sigma^2}$, OR $\frac{d}{2\sigma} = \sqrt{\frac{E_b}{N_0}}$

Here $d = 1$, $\frac{1}{2\sigma} = \sqrt{\frac{E_b}{N_0}}$

OR $\frac{d_{ik}}{2\sigma} = d_{ik}\sqrt{\frac{E_b}{N_0}}$

(u1)

anyway $i$ $\langle y(t), s_i(t)\rangle - \frac{\|s_i(t)\|^2}{2}$