EE 5140: Digital Modulation and Coding: QUIZ 2

1. (10 Marks) Let $p(t) = I_{[0,1]}(t)$ denote a rectangular pulse of unit height and unit duration (ranging from $t = 0$ to $t = 1$). A digital communication system uses the following waveforms for signaling:
   $s_1(t) = p(t) + p(t-2)$, $s_2(t) = p(t-1) + p(t-3)$, $s_3(t) = p(t-1) + p(t-2)$, $s_4(t) = p(t-1) - p(t-3)$.

   (a) Find the smallest distance ($d_{\min}$) between the above set of signal waveforms, that is,

   $$d_{\min} = \min_{\substack{p,q \\ p \neq q}} \|s_p(t) - s_q(t)\|$$

   (b) Assume that the prior probabilities are $\pi_2 = \pi_3 = \pi_4 = \frac{1}{3}$ and $\pi_1 = 0$. With AWGN of variance $\sigma^2 = 10$, let the received waveform be $y(t) = 3p(t) + 3p(t-1) - 2p(t-2) + p(t-3)$. Find the output of the minimum probability of error (MPE) decoder.

2. (10 Marks) For a BPSK signalling scheme ($b = \pm 1$), consider the following discrete time observation model (real case)

$$\begin{aligned} y_1 &= 2\,b + w_1 \\ y_2 &= -4\,b + w_2 \end{aligned}$$

where w_1 and w_2 are zero-mean independent Gaussian random variables with variances σ^2 and $4\sigma^2$ respectively. Show that maximum likelihood rule to detect b from observations y_1 and y_2 is of the form

$$\hat{b} = \begin{cases} 1 & \text{if } y_1 + \alpha\, y_2 > 0 \\ -1 & \text{if } y_1 + \alpha\, y_2 < 0 \end{cases}$$

and find the value of constant α.