Demodulator Implementation

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done initially

- Signal Waveforms
  $$\{ s_1(t), s_2(t), \ldots, s_M(t) \}$$
  are given

- Orthonormal basis → obtain
  $$\{ \psi_1(t), \psi_2(t), \ldots, \psi_N(t) \}$$
  using Gram-Schmidt procedure

- Basis Expansion
  $$s_i(t) = \sum_{k=1}^{N} a_i(k)\, \psi_k(t)$$
  Signal vectors $\underline{s_i} = \begin{bmatrix} a_i(1) \\ a_i(2) \\ \vdots \\ a_i(N) \end{bmatrix}$

  $a_i(k) = \langle s_i(t), \psi_k(t) \rangle$

- Get the received waveform $y(t)$

- Find projection coefficients
  $$Y(k) = \langle y(t), \psi_k(t) \rangle$$
  Observation vector
  $$\underline{Y} = \begin{bmatrix} Y(1) \\ \vdots \\ Y(N) \end{bmatrix}$$

- $$\delta_{ML} = \arg\min_i \| \underline{Y} - \underline{s_i} \|^2$$
  $$= \arg\max_i \langle \underline{Y}, \underline{s_i} \rangle - \frac{\| \underline{s_i} \|^2}{2}$$

- $$\delta_{MAP} = \arg\min_i \| \underline{Y} - \underline{s_i} \|^2 - 2\sigma^2 \log \pi_i$$

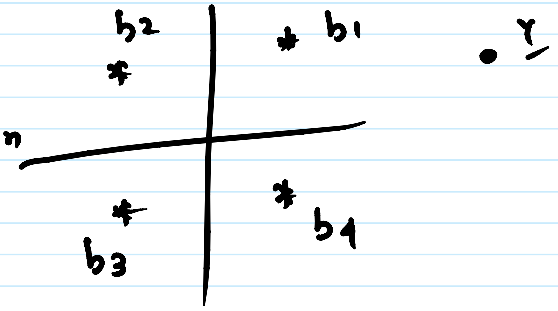
$$\delta_{MAP} = \arg\min_i \|\underline{r} - \underline{s}_i\|^2 - 2\sigma^2 \log \pi_i$$

$\pi_i \rightarrow$ Prior probability of sending i^{th} signal waveform $s_i(t)$

Optimal in avg. minimizing Prob. of error

Recall linear modulation QPSK

$p(t) \rightarrow$ pulse shape for modulation



$b_i \rightarrow i^{th}$ constellation point. $1 \leq i \leq M$

$b_i\, p(t) \rightarrow$ complex baseband representation of i^{th} signal waveform

$\downarrow$

Corresponding passband representation

$b_i = b_{ci} + j\, b_{si}$

Passband signal $s_i(t) = b_{ci}\, p(t) \cos 2\pi f_c t - b_{si}\, p(t) \sin 2\pi f_c t$.

Signal space $\mathcal{S} = \{ s_1(t), \ldots, s_M(t) \}$

We already know $p(t) \cos 2\pi f_c t$ (In phase) and $p(t) \sin (2\pi f_c t)$ (quad) are orthogonal

Suppose, $p(t)$ is normalized such that

$p(t) \cos 2\pi f_c t$ & $p(t) \sin 2\pi f_c t$

are unit norm

We can choose $\psi_1(t) = p(t) \cos 2\pi f_c t$

$\psi_2(t) = -p(t) \sin 2\pi f_c t$

Given $y(t)$ find

$$Y_c = \langle y(t), \psi_1(t) \rangle = \int_{-\infty}^{\infty} y(t)\, p(t) \cos 2\pi f_c t \, dt$$

$$Y_s = \langle y(t), \psi_2(t) \rangle = \int_{-\infty}^{\infty} y(t)\, p(t) \sin 2\pi f_c t \, dt$$

Under H_i

$$\begin{bmatrix} Y_c \\ Y_s \end{bmatrix} = \begin{bmatrix} b_{ci} \\ b_{si} \end{bmatrix} + \begin{bmatrix} W_c \\ W_s \end{bmatrix}$$

Signal space is two dimensional

$\underline{Y} \rightarrow$ find closest constellation point to the observed vector $\underline{Y}$

ML receiverd

M-ary FSK Modulation

$s_1(t) = \cos(2\pi f_1 t) \qquad 0 \le t \le T$

$\vdots$

$s_M(t) = \cos(2\pi f_M t)$ $1/T$ Symbol rate

Suppose $f_1, f_2, \ldots, f_M$ are chosen

Such that $s_1(t), \ldots, s_M(t)$ are

mutually orthogonal

Orthonormal basis

$$\alpha = \sqrt{2/T}$$

$$\psi_1(t) = \alpha \, S_1(t)$$

$$\psi_2(t) = \alpha \, S_2(t)$$

$$\vdots$$

$$\psi_M(t) = \alpha \, S_M(t)$$

$$\underline{S_1} = \begin{bmatrix} \alpha \|S_1\|^2 \\ 0 \\ 0 \\ \vdots \end{bmatrix}$$

$$\underline{S_2} = \begin{bmatrix} 0 \\ \alpha \|\underline{S_2}\|^2 \\ 0 \\ 0 \end{bmatrix}$$

Signal Space is M-dimensional $\Rightarrow$ Proceed with projecting $y(t)$ into $\psi_k(t)$

x ———— x

Alternative receiver implementation

Recall, $\langle S_i(t), S_i(t) \rangle = \| S_i(t) \|^2$

$$= \| \underline{S_i} \|^2$$

Signal $S_i(t) \longrightarrow \underline{S_i}$ ← vector

$y(t) \longrightarrow \underline{Y}$

It can be shown that

$$\langle \underline{Y}, \underline{S_i} \rangle = \langle y(t), S_i(t) \rangle$$

$$\langle \underline{y}(t), S_i(t) \rangle = \langle y_s(t) + y^{\perp}(t), S_i(t) \rangle$$

$$= \langle y_s(t), S_i(t) \rangle + \underbrace{\langle y^{\perp}(t), S_i(t) \rangle}_{0}$$

$$= \langle y_s(t), s_i(t) \rangle$$

$$= \langle \underline{Y}, \underline{s_i} \rangle$$

Theorem

$$\delta_{ML}(y(t)) = \underset{1 \le i \le M}{\arg\max} \; \langle y(t), s_i(t) \rangle - \frac{\|s_i(t)\|^2}{2}$$

$$\delta_{MAP}(y(t)) = \underset{1 \le i \le M}{\arg\max} \; \langle y(t), s_i(t) \rangle - \frac{\|s_i(t)\|^2}{2} + \sigma^2 \log \pi_i$$

called correlator or matched filter

Called Coherent demodulation

Requires perfect alignment of carrier frequency and phase between transmitter & receiver

x ——————→ x

Above result applies for (real) passband signals/waveforms

Explicitly

$$y_p(t) = s_{i,p}(t) + w_p(t)$$

$p \to$ denotes passband

$$\delta_{ML} = \underset{1 \le i \le M}{\arg\max} \; \langle y_p(t), s_{i,p}(t) \rangle - \frac{\| s_{i,p}(t) \|^2}{2}$$

$$\delta_{MAP} = \underset{1 \le i \le M}{\arg\max} \; \langle y_p(t), s_{i,p}(t) \rangle - \frac{1}{2} \| s_{i,p}(t) \|^2 + \sigma^2 \log \pi_i$$

Let complex baseband representation be

$$y_p(t) \longrightarrow \tilde{y}(t)$$

$$s_{i,p}(t) \longrightarrow \tilde{s}_i(t)$$

$$w_p(t) \longrightarrow \tilde{w}(t)$$

Real Noise with PSD $\frac{N_0}{2}$ Variance σ^2

complex noise with PSD N_0 & Variance $2\sigma^2$

Optimal demodulation in baseband

(Recall) We have $\langle y_p(t), s_{i,p}(t) \rangle$

$$= \frac{1}{2} \mathrm{Re} \{ \langle \tilde{y}(t), \tilde{s}_i(t) \rangle \}$$

Also $\quad \| s_{i,p}(t) \|^2 = \frac{1}{2} \| \tilde{s}_i(t) \|^2$

Optimal Demodulation in Baseband

$$\delta_{MAP} = \underset{1 \leq i \leq M}{\arg\max} \; Re\{\langle \tilde{y}(t), \tilde{s}_i(t) \rangle\} - \frac{\|\tilde{s}_i(t)\|^2}{2} + 2\sigma^2 \log \pi_i$$

$$\delta_{ML} = \underset{1 \leq i \leq M}{\arg\max} \; Re\{\langle \tilde{y}(t), \tilde{s}_i(t) \rangle\} - \frac{1}{2}\|\tilde{s}_i(t)\|^2$$

x ———— x