Note Title 01-08-2016

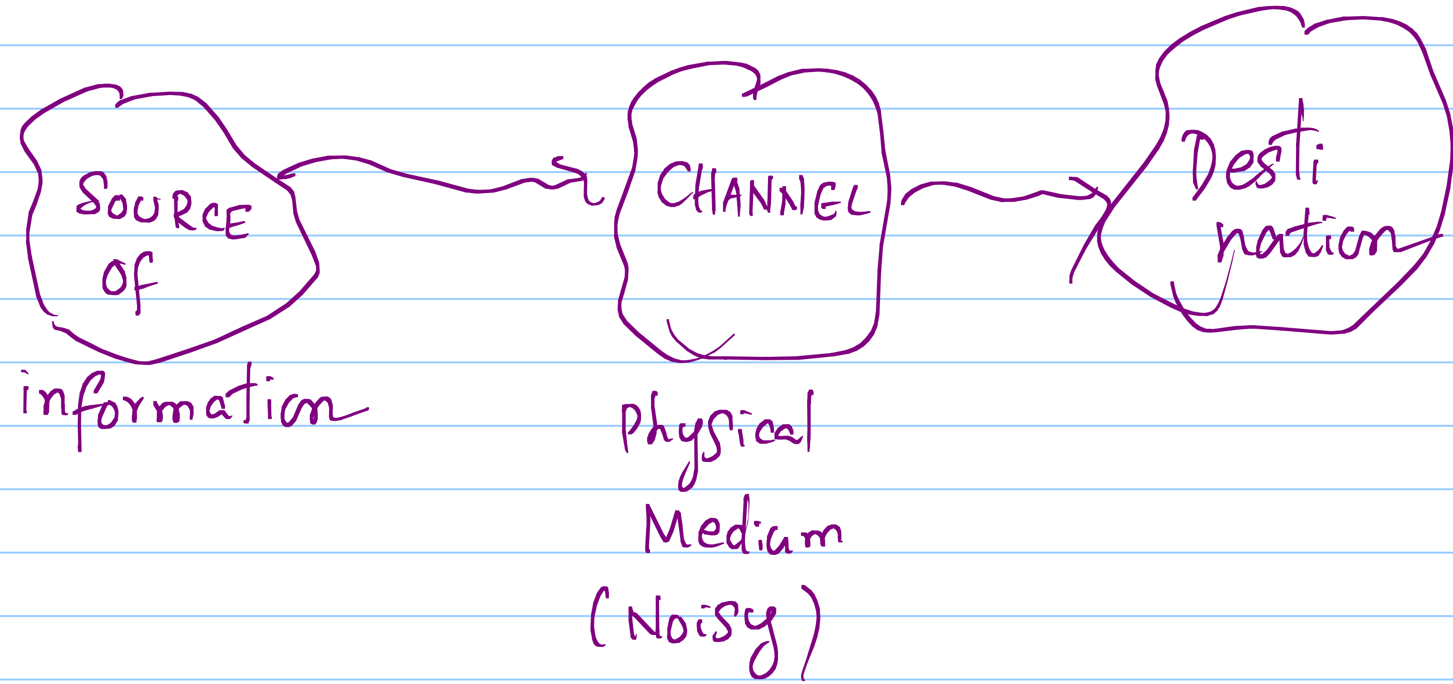
EE 5140 Digital Modulation & Coding

Prereq: Probability

Contents:
- Complex baseband Representation
- Digital Modulation
- Demodulation
- ISI channels & Equalization

Evaluation:
- Quiz 1 & 2 : 25 % each
- End Sem : 50 %

Comm. System



* Info has to go from Source to destination via noisy medium
* Want reliable communication

Comm. System Design Principle

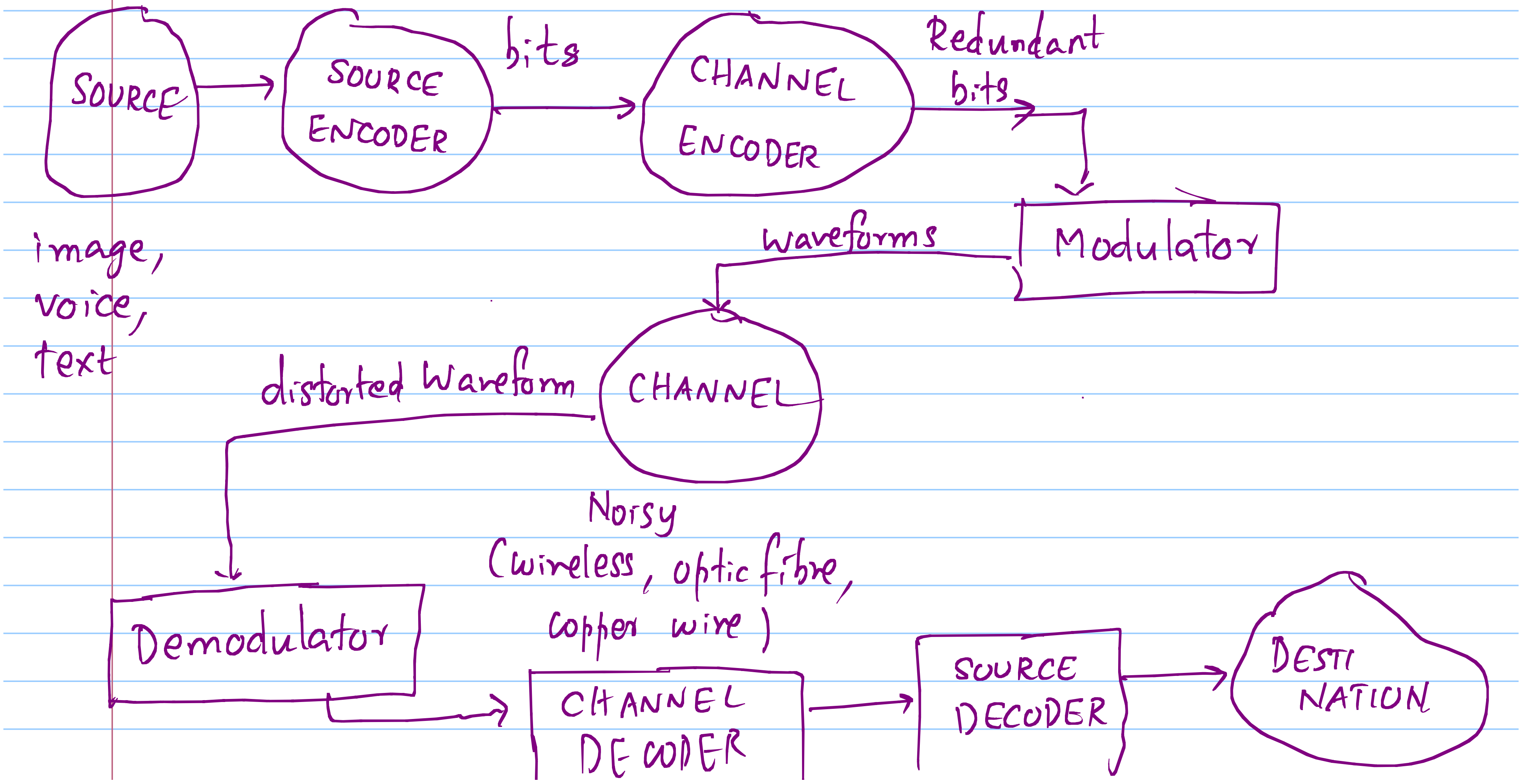
Resource → Design tools → Performance Metrics

Power, Bandwidth

Encoders
Decoders
Signal Processor

Error Rate
Data Rate

Components of Digital Comm. System



Reasons for using digital Communication systems are from Shannon's theorems in information theory

1. Channel Coding theorem

   We can build (almost) error-free digital Communication systems across noisy medium with finite power & finite bandwidth

2. Source - channel Separation theorem

   Source coding & channel coding can be done separately.

Chapter 1: Complex Baseband Representation

Textbook:
Digital Comm.
by Madhow

a) Signals

→ Power, Energy, bandwidth

(FOURIER TRANSFORM)

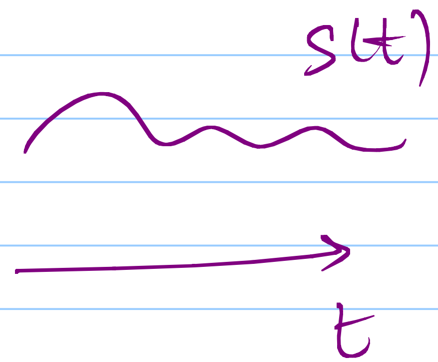
b) Baseband & Passband Signals

c) Complex baseband Representation

Signals

Signal $s(t)$ is a function of time t

t is continuous



1) Energy of signal $s(t)$

$$E_s = \int_{-\infty}^{\infty} |s(t)|^2 \, dt$$

2) Power of Signal $s(t)$

$$P_s = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |s(t)|^2 \, dt$$

Note:
If P_s is finite then $E_s = \infty$

If E_s is finite, $P_s = 0$

Dc Value of Signal

$$\overline{s(t)} = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} s(t)\, dt$$

1) Real Sinusoid $s(t) = A \sin(2\pi f_0 t + \theta)$

Amplitude frequeny phase

2) Complex exponential $s(t) = A e^{j(2\pi f_0 t + \theta)}$

$$= A\left[\cos(2\pi f_0 t + \theta) + j \sin(2\pi f_0 t + \theta)\right]$$

$$= A e^{j\theta} \left[e^{j 2\pi f_0 t} \right]$$

Energy $E_s = \int_{-\infty}^{\infty} |Ae^{j\theta} e^{j2\pi f_0 t}|^2 \, dt$

$= \int_{-\infty}^{\infty} A^2 \, dt$

$= \infty$

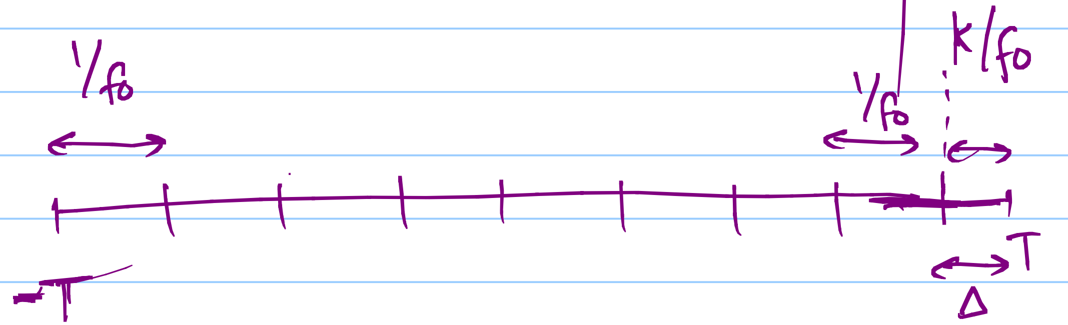
Power $P_s = A^2$ Since $|s(t)|^2 = A^2$ for all t

Dc Value of $Ae^{j\theta} e^{j2\pi f_0 t} = ?$

Need to find

$$\int_{-T}^{T} A e^{j\theta} e^{j2\pi f_0 t} \, dt$$

Time period is $1/f_0$



$$\int_{-T}^{T} A e^{j\theta} e^{j2\pi f_0 t} \, dt = \underbrace{\int_{-T}^{-T+k/f_0} A e^{j\theta} e^{j2\pi f_0 t}}_{0} + \int_{-T+k/f_0}^{T} A e^{j\theta} e^{j2\pi f_0 t} \, dt$$

$$= \int_{-T+k/f_0}^{T} A e^{j\theta} e^{j2\pi f_0 t} \, dt$$

$$\leq \int_{-T+k/f_0}^{T} \left| A e^{j\theta} e^{j2\pi f_0 t} \right| dt$$

$$\leq \int_{-T+k/f_0}^{T} |A| \, dt$$

$$\Delta \leq 1/f_0$$

$$\leq |A| \cdot \Delta$$

$$\leq \frac{|A|}{f_0}$$

Similarly we can show this integral $\geq -\dfrac{|A|}{f_0}$

So $\left| \int_{-T}^{T} A e^{j\theta} e^{j2\pi f_0 t} \, dt \right| \leq \frac{|A|}{f_0}$ for any T

dc Value

$$\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} A e^{j\theta} e^{j2\pi f_0 t} \, dt \leq \underbrace{\lim_{T\to\infty} \frac{1}{2T} \frac{|A|}{f_0}}$$

$$= 0$$

Example: Real Sinusoid

$$s(t) = A \sin(2\pi f_0 t + \theta) = \text{Imag}\left\{ A e^{j\theta} e^{j2\pi f_0 t} \right\}$$

So dc value of $s(t) = 0$, $E_s = \infty$, $P_s = |A|^2/2$ (Verify)

Inner Product between two Signals $s(t)$ & $r(t)$ is defined as

$$\langle s(t), r(t) \rangle = \int_{-\infty}^{\infty} s(t)\, r^*(t)\, dt$$

$* \rightarrow$ denotes Conjugation

Two signals are called orthogonal if their inner product is zero

Bandwidth of a Signal

We need to know frequeny components in a Signal to understand bandwidth

FOURIER TRANSFORM

Let $x(t)$ be a <u>finite Energy</u> Signal

FOURIER TRANSFORM (FT) of signal $x(t)$ defined as

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

→ function of f

$X(f)$ is called spectrum of $x(t)$; $f \in (-\infty, \infty)$

$X(f)$ is in general Complex valued
(even if $x(t)$ is real)

$|X(f)| \rightarrow$ Magnitude spectrum

$\angle X(f) \rightarrow$ Phase spectrum

From $X(f)$, we get $x(t)$ as

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{j2\pi ft}\, df$$

We have both +ve & −ve frequencies in spectrum

Complex exponentials

$e^{j2\pi f_0 t}$ & $e^{-j2\pi f_0 t}$

(+ve) (−ve)

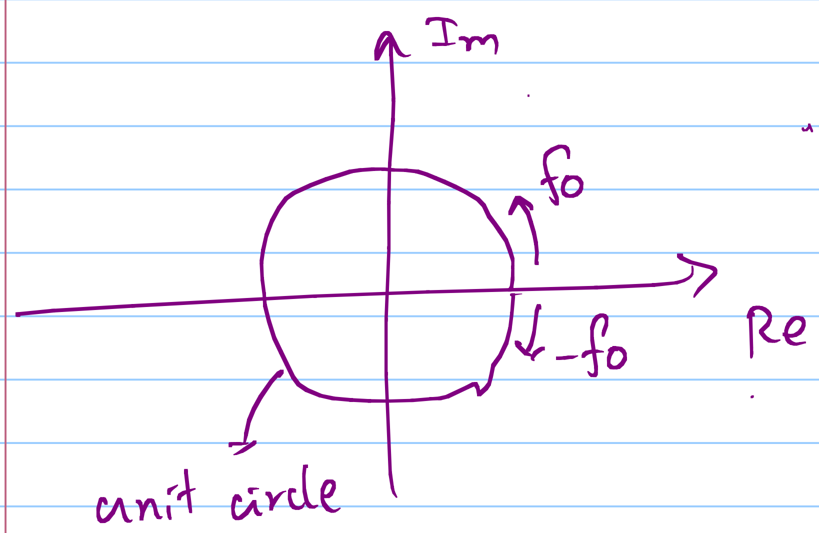
are different

$\cos(2\pi(-f_0)t)$

$= \cos(2\pi f_0 t)$

$\sin(2\pi(-f_0)t)$

$= -\sin(2\pi f_0 t)$



$|e^{j2\pi f_0 t}| = 1 = |e^{-j2\pi f_0 t}|$

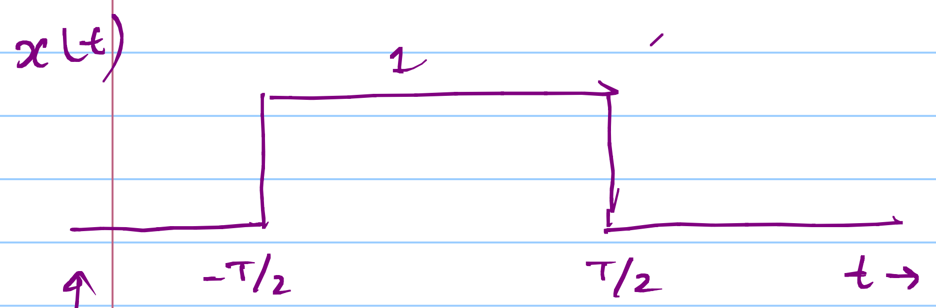
Phasors $e^{j2\pi f_0 t}$ & $e^{-j2\pi f_0 t}$

rotate in opposite directions

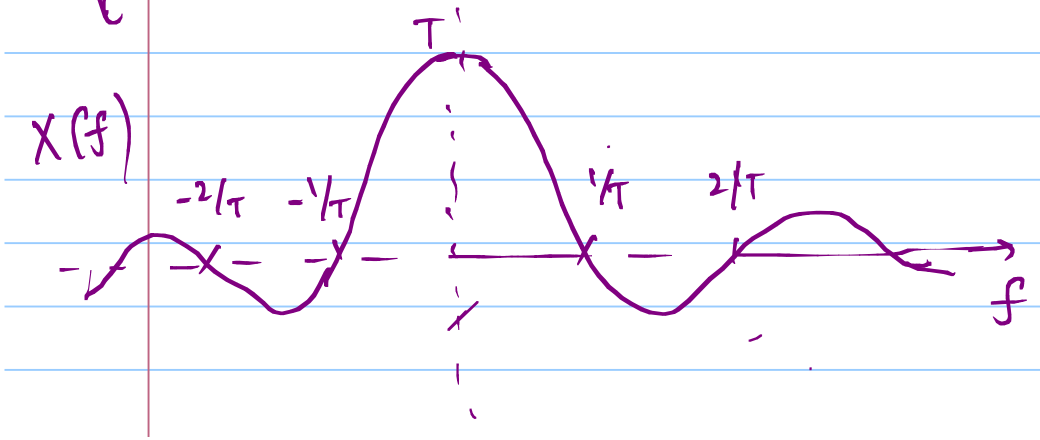
Example

Notation $\quad x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(f)$

$$x(t) = \begin{cases} 1 & \text{if } |t| \le T/2 \\ 0 & \text{else} \end{cases}$$



$\updownarrow \mathcal{F}$



$$X(f) = \int_{-T/2}^{T/2} e^{-j2\pi f t}\, dt$$

$$= \left. \frac{e^{-j2\pi f t}}{-j2\pi f} \right|_{-T/2}^{T/2}$$

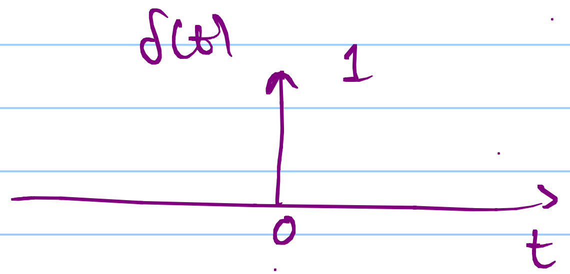
$$= \frac{\sin(\pi f T)}{\pi f}$$

$$= T \operatorname{Sinc}(fT)$$

$$\mathrm{Sinc}(x) = \frac{\sin \pi x}{\pi x}$$

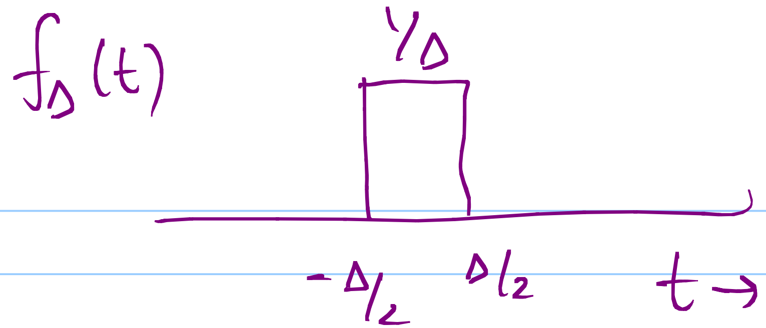
Extension of FOURIER TRANSFORM to signals with infinite energy using impulse function.

Delta function $\delta(t)$ denotes impulse function



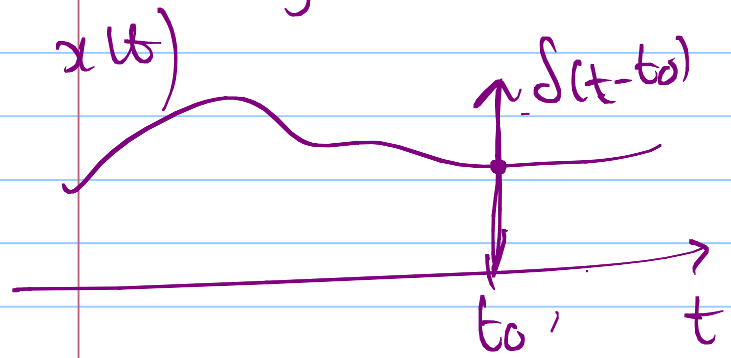
$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

$$\delta(t) = \lim_{\Delta \to 0} f_\Delta(t)$$



Sifting Property:

Say $x(t)$ is a Continuous function



$$x(t_0) = \int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt$$

1) $\delta(t) \overset{\mathcal{F}}{\longleftrightarrow} 1$ (Constant for all f)

2) $Ae^{j\theta} e^{j2\pi f_0 t} \overset{\mathcal{F}}{\longleftrightarrow} Ae^{j\theta} \delta(f - f_0)$

3) Linearity

$\alpha\, x(t) + \beta\, y(t) \overset{\mathcal{F}}{\longleftrightarrow} \alpha X(f) + \beta Y(f)$

Real Sinusoid

$$A\left(\frac{e^{j(2\pi f_0 t+\theta)} - e^{-j(2\pi f_0 t+\theta)}}{2j}\right) = A \sin(2\pi f_0 t + \theta) \overset{\mathcal{F}}{\longleftrightarrow} \frac{e^{j\theta} A}{2j}\delta(f - f_0) - \frac{e^{-j\theta} A}{2j}\delta(f + f_0)$$

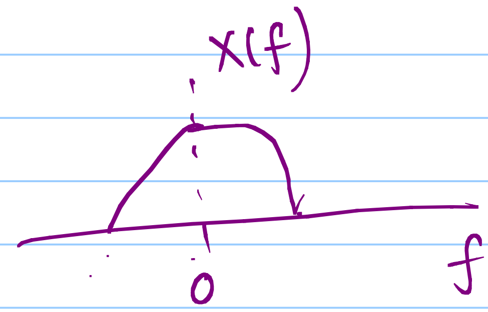
4) Time delay $\Leftrightarrow$ linear phase shift in frequeny domain

$$x(t) \xleftrightarrow{\mathcal{F}} X(f)$$

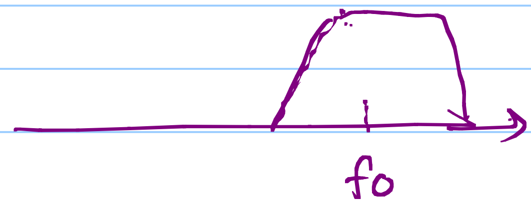
$$x(t-t_0) \xleftrightarrow{\mathcal{F}} X(f)\, e^{-j2\pi f t_0}$$

5) Modulation Property

$$x(t) \xleftrightarrow{\mathcal{F}} X(f)$$



$$x(t)\, e^{j2\pi f_0 t} \xleftrightarrow{\mathcal{F}} X(f-f_0)$$



6) Parseval's Theorem

$$\int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_{-\infty}^{\infty} |X(f)|^2 \, df$$

Energy in time domain = Energy in frequency domain

7) Suppose $x(t)$ is Real valued Signal

$$x(t) \xrightarrow{F} X(f)$$

then $X(f) = X^*(-f)$

that is

real part $\mathrm{Re}\{X(f)\} = \mathrm{Re}\{X(-f)\}$

imag. part $\mathrm{Im}\{X(f)\} = -\mathrm{Im}\{X(-f)\}$

For real signals, spectrum on positive frequencies alone is sufficient to reconstruct the signal

8) Differentiation

$$x(t) \longrightarrow X(f)$$

$$\frac{d}{dt} x(t) \longrightarrow j2\pi f \, X(f)$$

linear gain with frequency

Higher gain for high frequency

(High Pass filter)

9) Integration $\quad x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(f)$

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

$$Y(f) = \frac{X(f)}{j2\pi f} + \bar{x}\,\delta(f)$$

$$\bar{x} = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(\tau)\, d\tau$$

low pass filter

LTI Systems

$$x(t) \xrightarrow{\text{input}} \boxed{\text{System}} \xrightarrow{\text{output}} y(t)$$

$x(t) \longrightarrow y(t)$ (shorthand notation)

1) linearity

Suppose $\begin{cases} x_1(t) \longrightarrow y_1(t) \\ x_2(t) \longrightarrow y_2(t) \end{cases}$

if , $\alpha x_1(t) + \beta x_2(t) \longrightarrow \alpha y_1(t) + \beta y_2(t)$

then, system is linear

2) time invariance

if $x(t) \longrightarrow y(t)$

then $x(t-t_0) \longrightarrow y(t-t_0)$

system is time invariant

LTI is both linear & time invariant system

Properties of LTI System

1) $\delta(t) \longrightarrow \boxed{\text{System}} \longrightarrow h(t)$

$h(t) \rightarrow$ impulse response of System

$h(t)$ completely characterizes the system

2) For input $x(t)$, output $y(t)$ is given by

$$\underbrace{y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau}_{\text{Convolution}}$$

Proof: We know

$$\delta(t) \longrightarrow h(t)$$

(Time invariance) $\delta(t-\tau) \longrightarrow h(t-\tau)$

(linearity) $x(\tau)\,\delta(t-\tau) \longrightarrow x(\tau)\,h(t-\tau)$

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau \longrightarrow \underbrace{\int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau}_{=\,y(t)}$$

Sifting Property

$$y(t) = x(t) * h(t)$$

$\downarrow$ convolution

Properties of Convolution

Commutative 1) $x(t) * h(t) = h(t) * x(t)$

Distributive 2) $x(t) * \left(h_1(t) + h_2(t)\right) = x(t) * h_1(t) + x(t) * h_2(t)$

Associative 3) $x(t) * \left(h_1(t) * h_2(t)\right) = \left(x(t) * h_1(t)\right) * h_2(t)$

$$x(t) \longrightarrow \overset{\text{LTI}}{\boxed{h_1(t)}} \longrightarrow \overset{\text{LTI}}{\boxed{h_2(t)}} \longrightarrow y(t)$$

Equivalent $\Updownarrow$

$$y(t) = \big(x(t) * h_1(t)\big) * h_2(t)$$

$$x(t) \longrightarrow \overset{\text{LTI}}{\boxed{h_1(t) * h_2(t)}} \longrightarrow y(t)$$

$\Updownarrow$ Commutative

$$x(t) \longrightarrow \boxed{h_2(t) * h_1(t)} \longrightarrow y(t)$$

$\Updownarrow$ Associativity

$$\Downarrow$$

$$x(t) \longrightarrow \boxed{h_2(t)} \longrightarrow \boxed{h_1(t)} \longrightarrow y(t)$$

Fourier Transforms & LTI Systems

$$x(t) \longrightarrow X(f)$$

$$h(t) \longrightarrow H(f)$$

$$x(t) * h(t) \longrightarrow Y(f) = X(f)H(f)$$

Convolution in time domain $\Leftrightarrow$ Multiplication in frequency domain

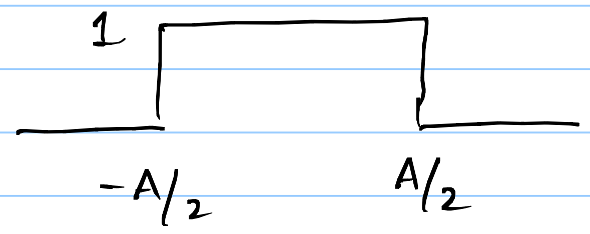
$x(t) \cdot h(t) \iff X(f) * H(f)$

Multiplication in time $\iff$ Convolution in frequency domain

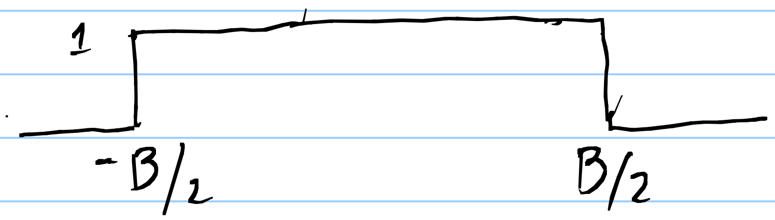
Example (Convolution)

$B \geq A$

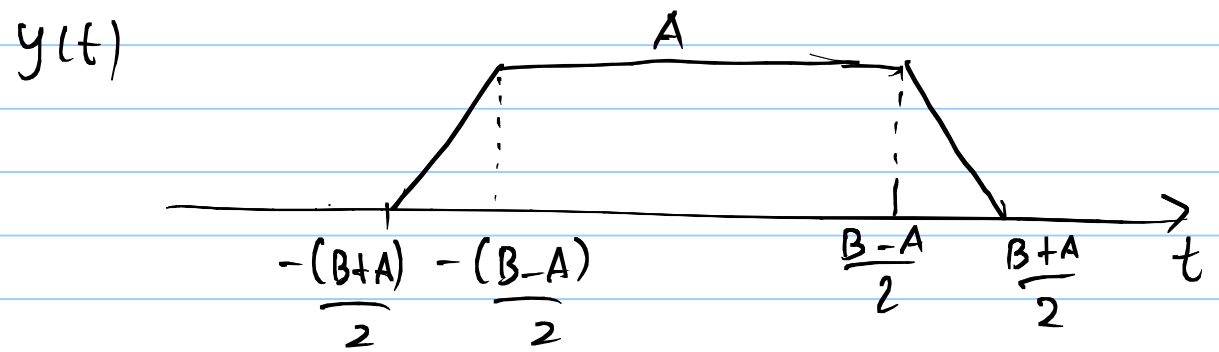
$x(t)$



$h(t)$



$$y(t) = x(t) * h(t)$$



Bandwidth of Signals

$$x(t) \xleftrightarrow{\mathcal{F}} X(f)$$

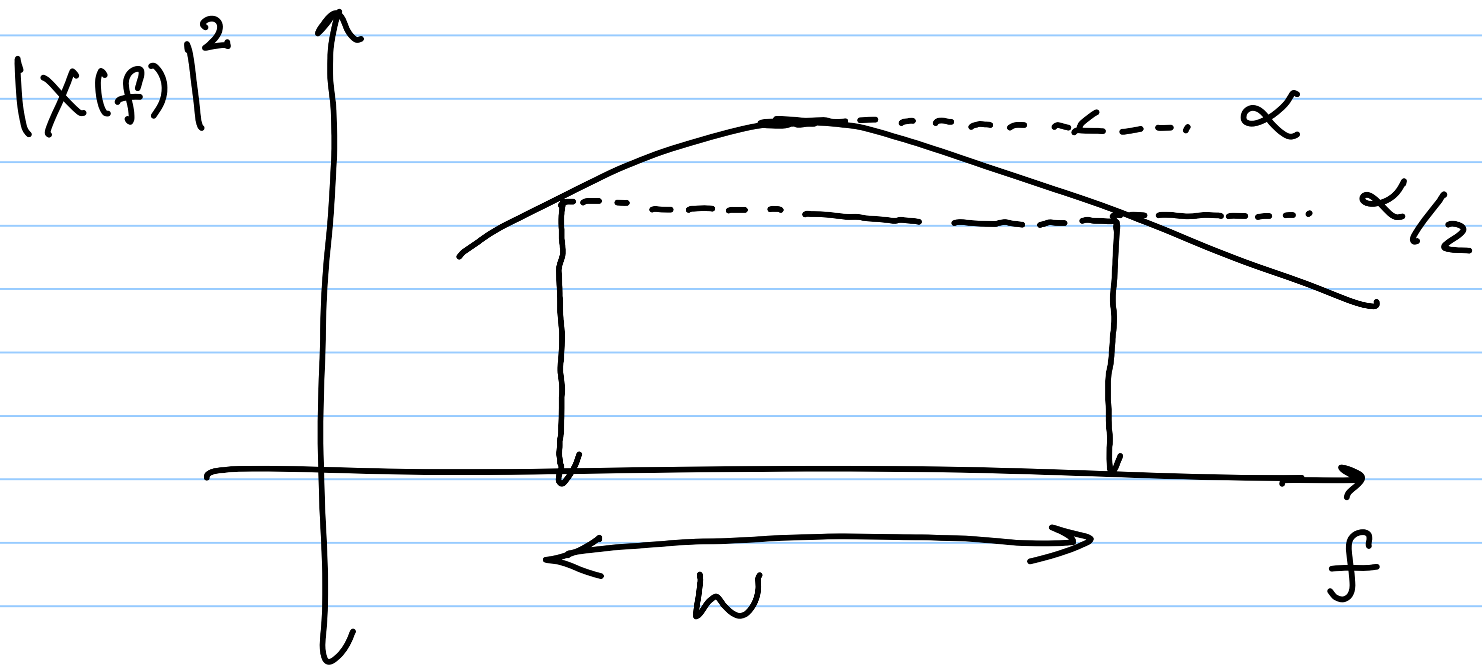
Spectrum $X(f)$ gives frequency components present in $x(t)$

Signals are rarely bandlimited

Spectrum $X(f)$ is non-zero from $-\infty$ to ∞

Two Notions of Bandwidth:

① 3 dB Bandwidth



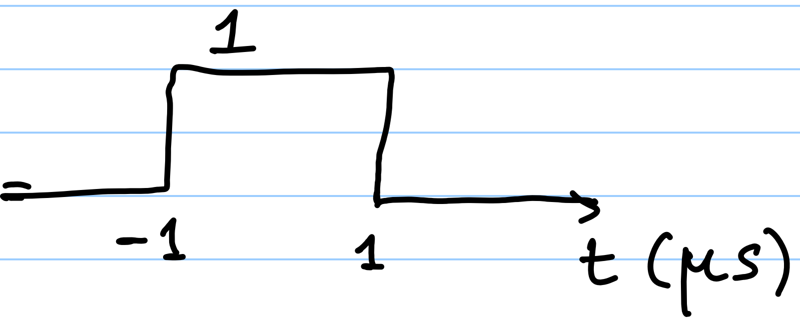
3 dB Bandwidth $= W$

② α- fraction bandwidth

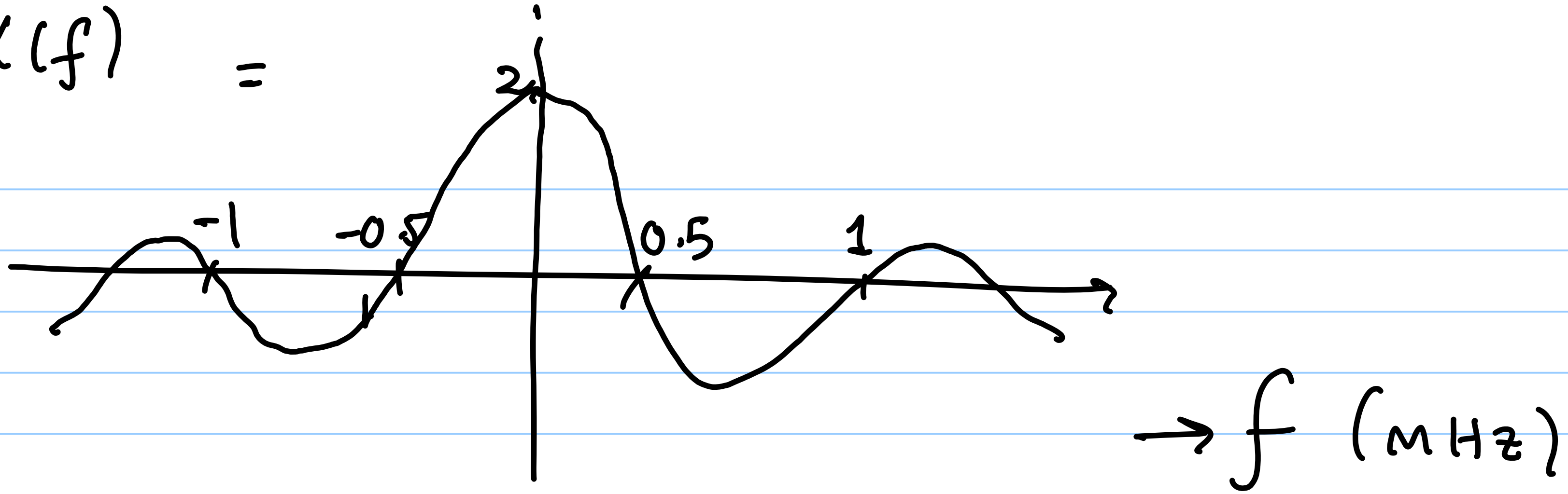
Width of Band of frequencies (Continuous) which contain α % of total energy of signal.

Example:

$x(t)$ =



$X(f) =$



$$X(f) = \frac{\sin 2\pi f}{\pi f}$$ (Unit is MHz of f)

Find Bandwidth which contains 99% of total energy

Find W such that

$$\int_{-W}^{W} |X(f)|^2 \, df = 0.99 \underbrace{\int_{-\infty}^{\infty} |x(t)|^2 \, dt}_{2}$$

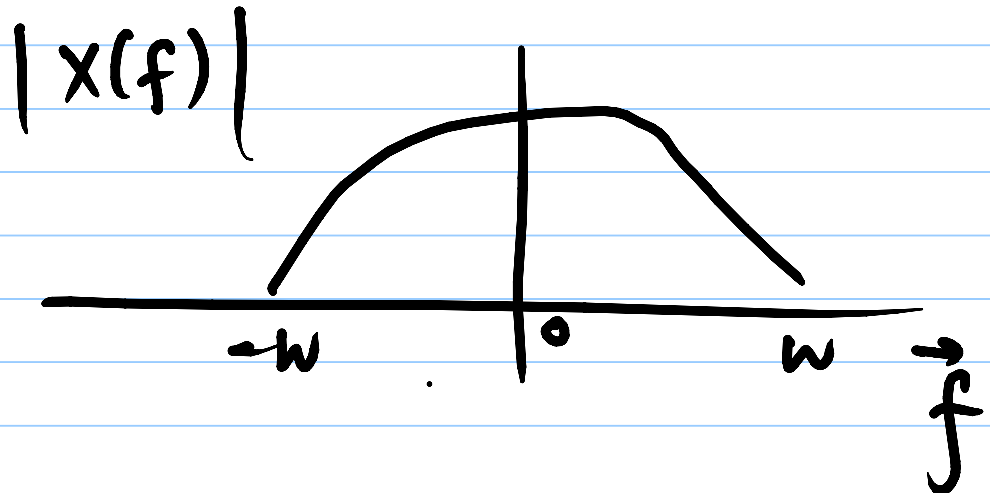
$$= 1.98$$

$$\int_{-W}^{W} \frac{\sin^2 2\pi f}{(\pi f)^2} \, df = 1.98$$

Numerically find $W \approx 5.1$ MHz

Baseband and Passband Signals

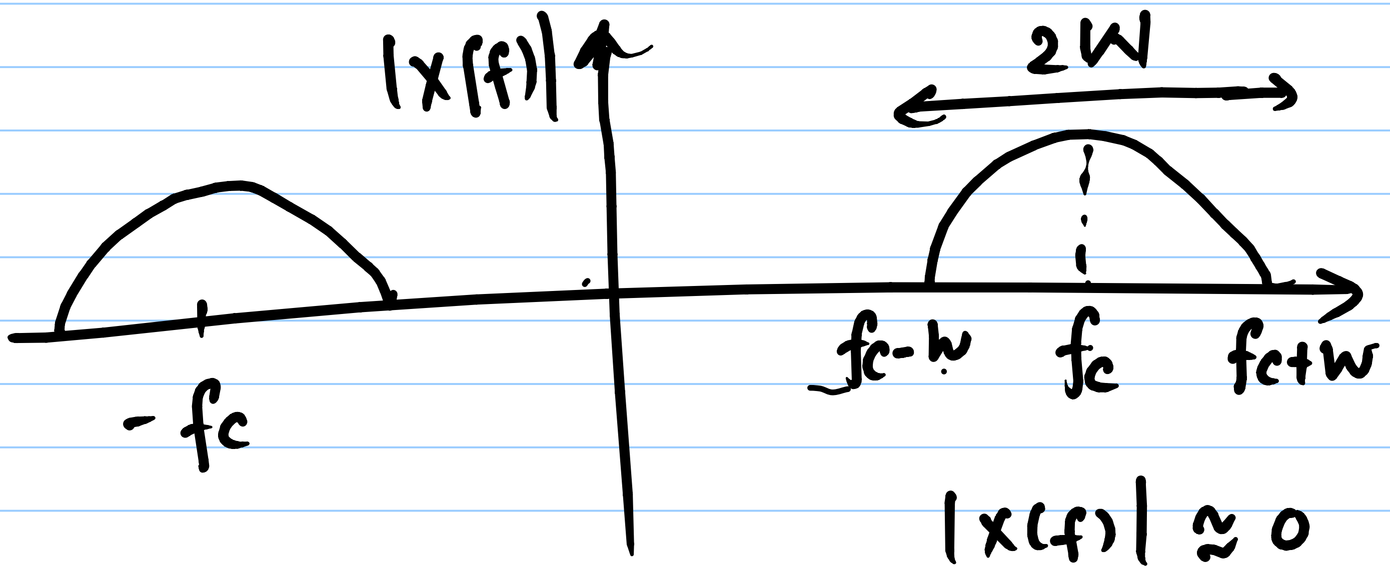
Signal $x(t)$ is called baseband Signal if its energy is concentrated **around DC (zero frequency)**



$|X(f)| \approx 0$ for $|f| \geq W$

<u>Passband Signal</u>

Energy is concentrated around a band <u>away</u> from zero



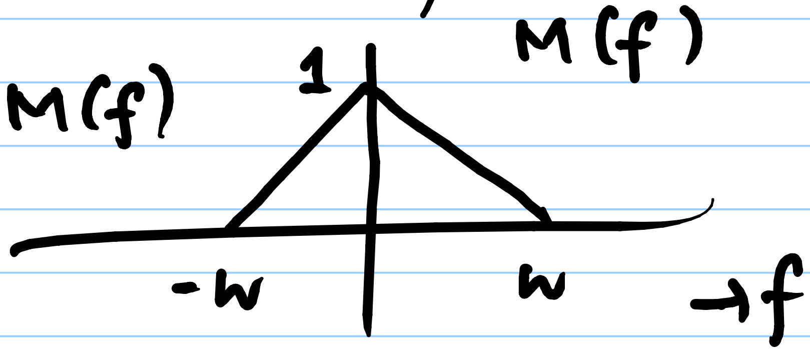
$W < f_c$

$|X(f)| \approx 0$ if $|f \pm f_c| > W$

Connection between baseband & Passband Signals

Say we have message signal $m(t)$
(real valued)
(base band)

$m(t) \overset{F}{\longleftrightarrow} M(f)$



$f_c > W$

Suppose we want to send this message via carrier frequency f_c

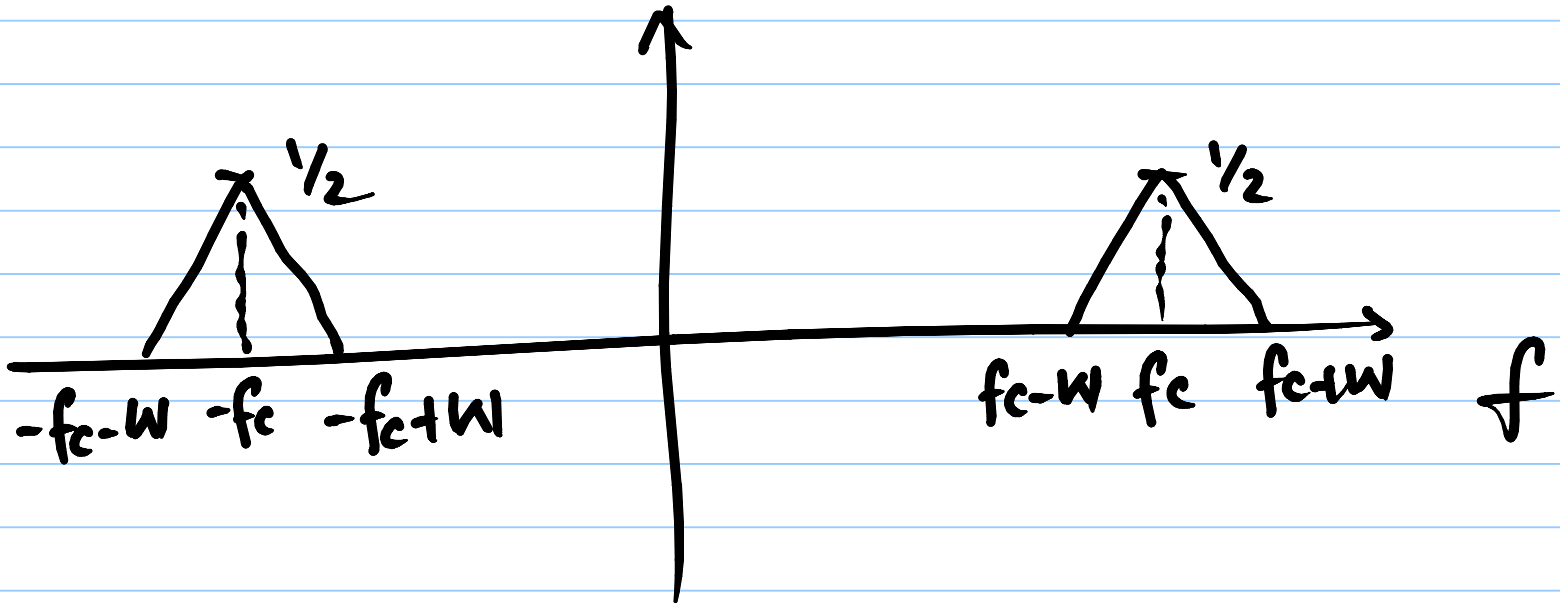
Let $m_p(t) = m(t)\cos(2\pi f_c t)$

$$\Big\downarrow \mathcal{F}$$

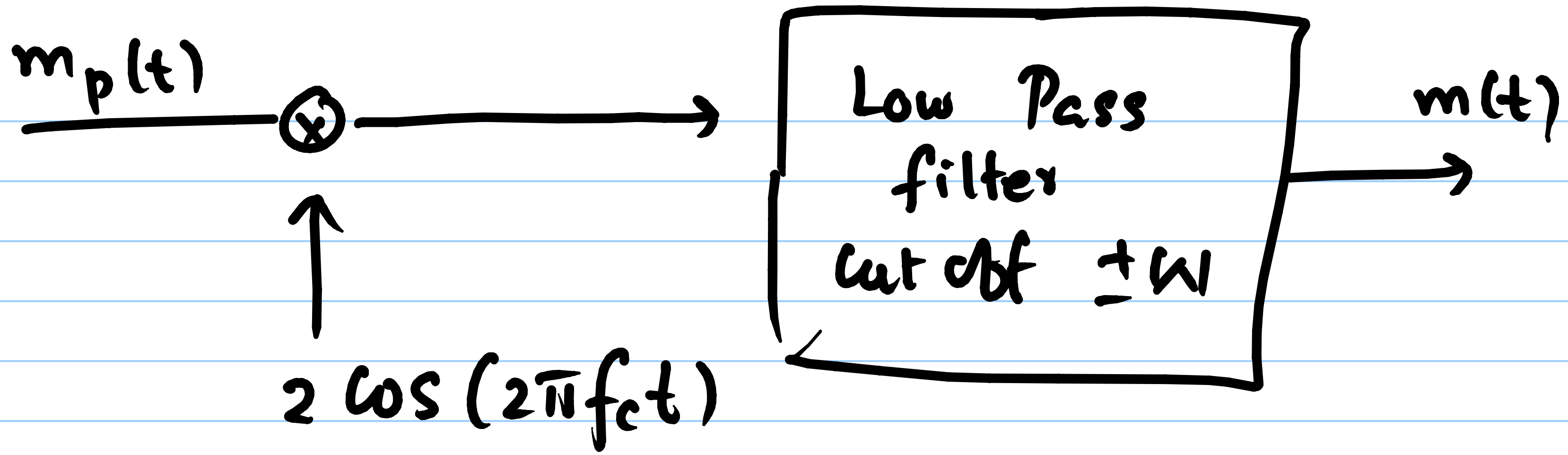
Spectrum $M_p(f) = M(f) * \left(\frac{1}{2}\delta(f-f_c) + \frac{1}{2}\delta(f+f_c)\right]$

$$= \frac{1}{2}M(f-f_c) + \frac{1}{2}M(f+f_c)$$

$M_p(f)$



From $m_p(t)$ we can get original message $m(t)$ as follows



$$2\, m_p(t) \cos(2\pi f_c t) = m(t)\, 2\cos^2(2\pi f_c t)$$

$$= m(t)\,[1 + \cos 4\pi f_c t]$$

$$= m(t) + m(t)\cos 4\pi f_c t$$

$$\Downarrow$$

Passband signal

centered around

$2f_c$

$$\Downarrow$$

eliminated by

LPF

base band

signal

frequencies

from $-W$ to W

IN-Phase & Quadrature Modulation

Let $x_c(t)$ & $x_s(t)$ be two

real different baseband signals

$$|X_c(f)| \approx 0 \quad \text{for} \quad |f| > W$$

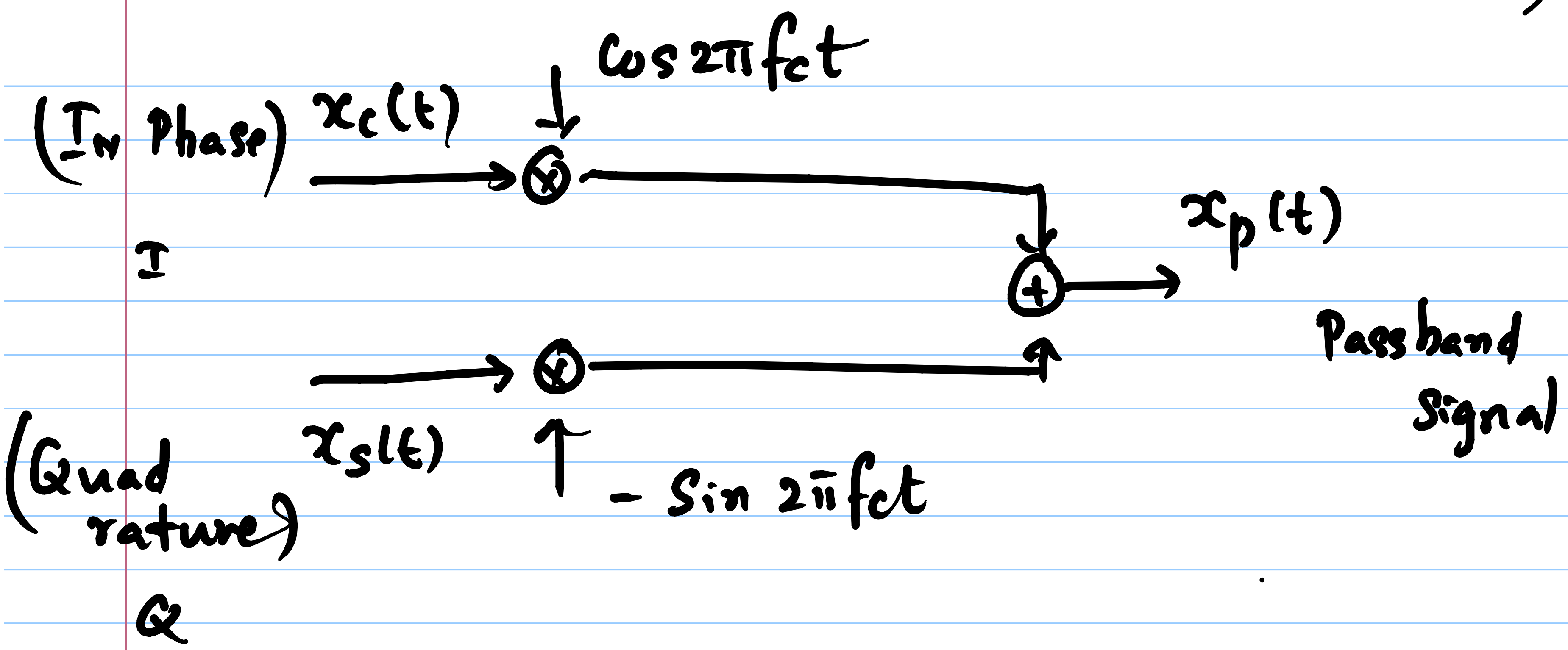
$$|X_s(f)| \approx 0 \quad \text{for} \quad |f| > W$$

$$x_p(t) = x_c(t)\cos 2\pi f_c t - x_s(t)\sin 2\pi f_c t$$

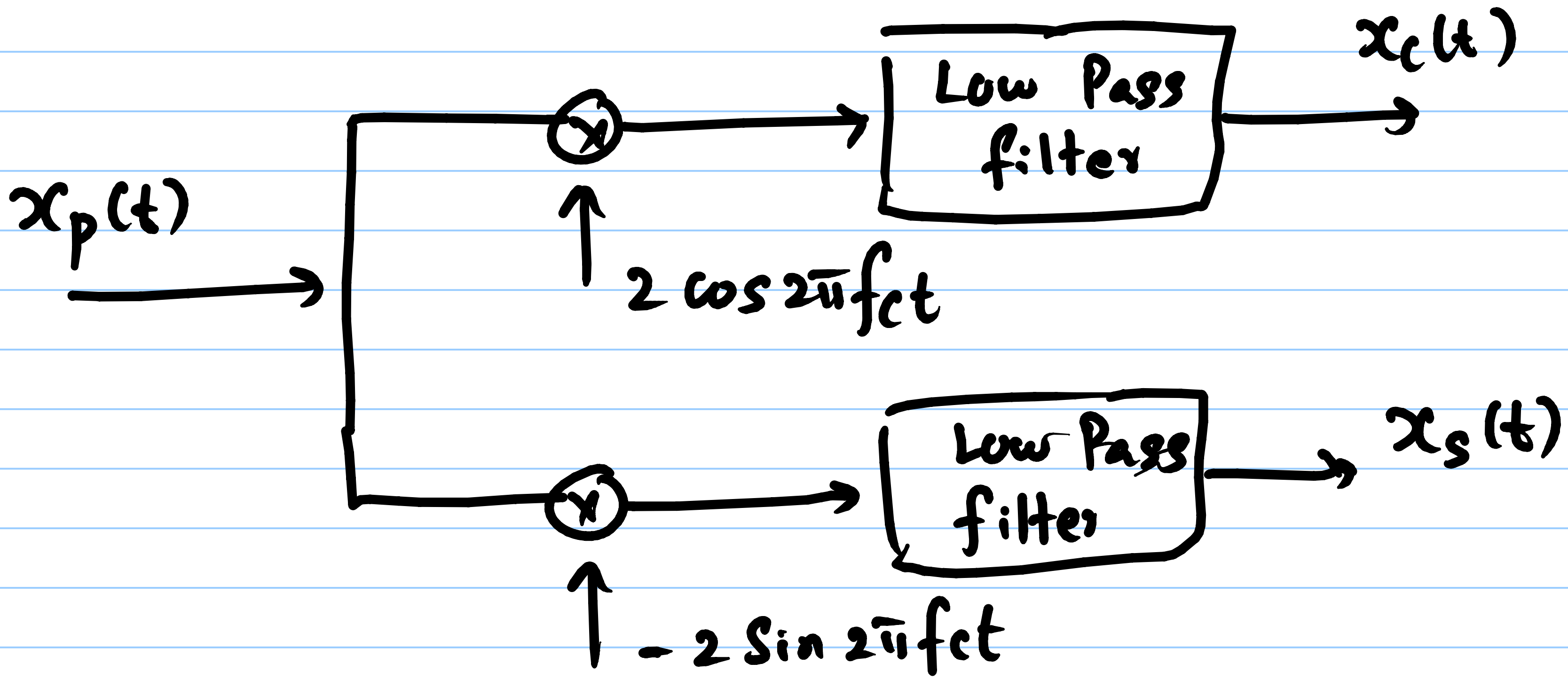
Passband Signal

Spectrum $|X_p(f)| \approx 0$ if $|f \pm f_c| > W$

Upconversion (baseband to Passband)



Down conversim (Passband to baseband)



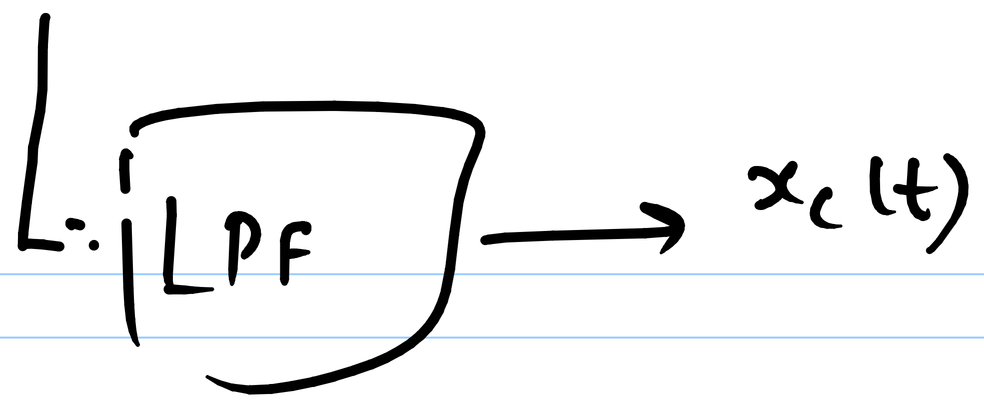
$$x_p(t) = x_c(t)\cos 2\pi f_c t - x_s(t)\sin 2\pi f_c t$$

<u>Top Branch</u> of Down Conversion

$$x_p(t)\, 2\cos(2\pi f_c t)$$

$$= 2\, x_c(t) \cos^2 2\pi f_c t - 2\, x_s(t) \sin(2\pi f_c t).\cos(2\pi f_c t)$$

$$= x_c(t)\, \underset{\text{Centered } 2f_c}{\big[1 + \cos 4\pi f_c t\big]} - \underbrace{x_s(t)\, \sin(4\pi f_c t)}_{\text{Pass band at } 2f_c}$$



Verify yourself for quadrature component

Main Result: Two different baseband Signals (In phase & Quadrature component) are sent via cos & Sin carriers

Orthogonality of I & Q components

$$x_p(t) = \underbrace{x_c(t)\cos 2\pi f_c t}_{I} - \underbrace{x_s(t)\sin 2\pi f_c t}_{Q}$$

$$a(t) = x_c(t)\cos 2\pi f_c t$$

$$b(t) = x_s(t)\sin 2\pi f_c t$$

<u>Claim:</u> $a(t)$ & $b(t)$ are orthogonal

$$\int_{-\infty}^{\infty} a(t)\, b(t)\, dt = 0$$

Proof:

Let $c(t) = x_c(t)\, x_s(t)$

Spectrum $C(f) = X_c(f) * X_s(f)$

limited to $-W$ to W

limited to $-2W$ to $+2W$

$c(t)$ is base band signal (Spectrum $-2W$ to $+2W$)

$$a(t)\,b(t) = x_c(t)\,x_s(t)\cos 2\pi f_c t \sin 2\pi f_c t$$

$$= c(t)\,\underbrace{\frac{\sin 4\pi f_c t}{2}}$$

$\downarrow$

Passband Signal centered $2f_c$

$\downarrow$

Spectrum exits $2f_c - 2W$ to $2f_c + 2W$

(on positive side)

(Conjugate Symmetry on negative side)

Since $f_c > W$, we have

$$2f_c - 2W > 0$$

DC component in spectrum of

$a(t) \cdot b(t)$ is zero

So $$\int_{-\infty}^{\infty} a(t)\, b(t)\, dt = 0$$

Complex Envelope

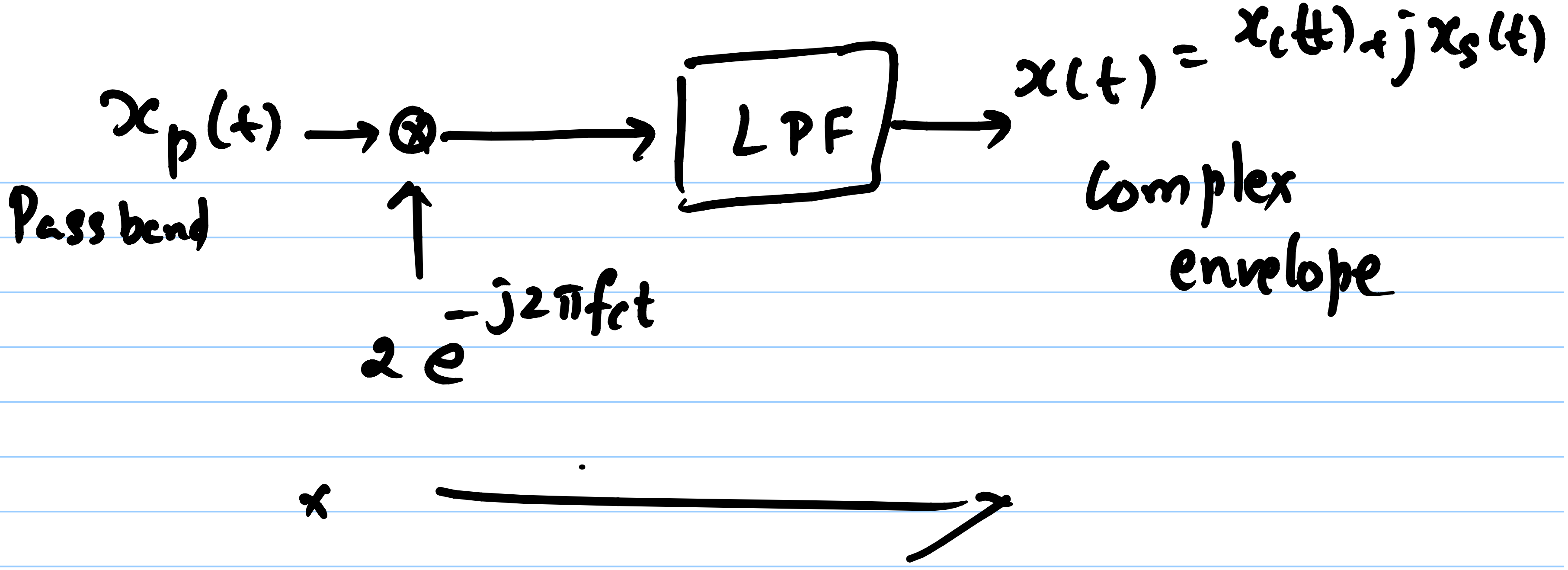
Let $x(t) = \underbrace{x_c(t)}_{}^{(I)} + j\, x_s(t)$ (I) (Q)

$x_c(t)$, $x_s(t)$ are real baseband signals

Complex envelope

Passband Signal $x_p(t) = x_c(t)\cos 2\pi f_c t - x_s(t)\sin 2\pi f_c t$

$$x_p(t) = \mathrm{Re}\left\{ x(t)\, e^{j2\pi f_c t} \right\}$$



$x_p(t)$
Passband
$2e^{-j2\pi f_c t}$
LPF
$x(t) = x_c(t) + j x_s(t)$
complex envelope

FREQUENCY DOMAIN RELATIONSHIP between baseband & Passband Signals

Let $x_c(t)$ & $x_s(t)$ be real baseband signal

freq. components from $-W$ to $+W$

Complex envelope $x(t) = x_c(t) + j\,x_s(t)$

$\downarrow$ freq. components $-W$ to W

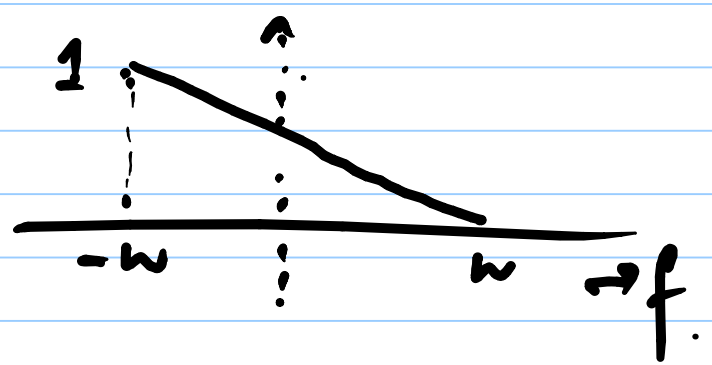
$$x_p(t) = \mathrm{Re}\{x(t)\, e^{j2\pi f_c t}\}$$

$x(t)$ has all the information about $x_c(t)$, $x_s(t)$

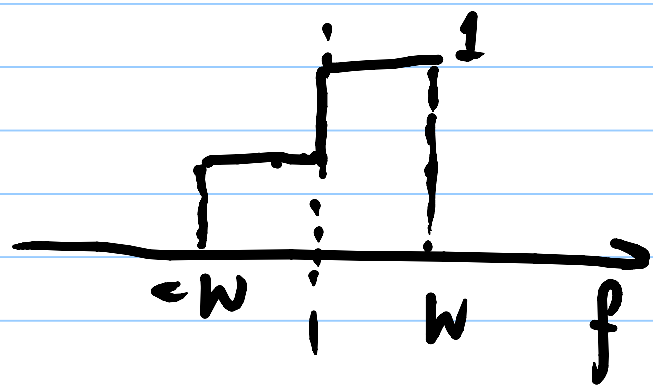
Spectrum $\quad x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(f)$

Suppose

$\mathrm{Re}\{X(f)\}$



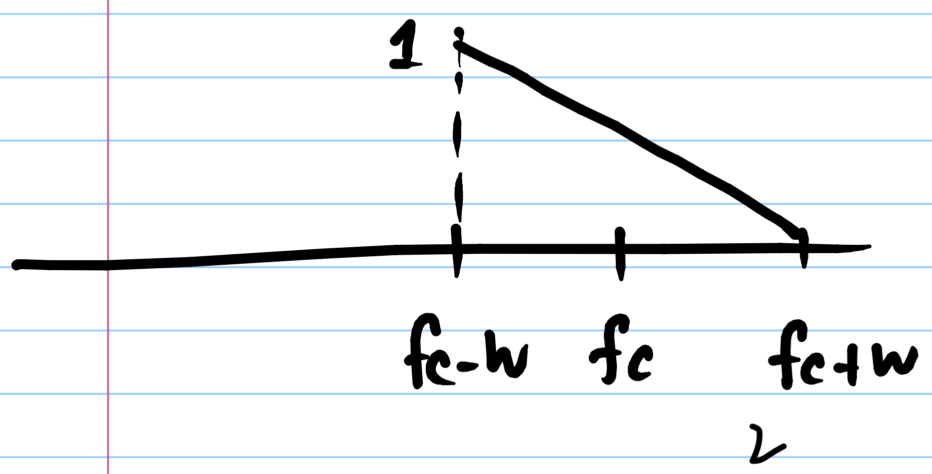
$\mathrm{Im}\{X(f)\}$



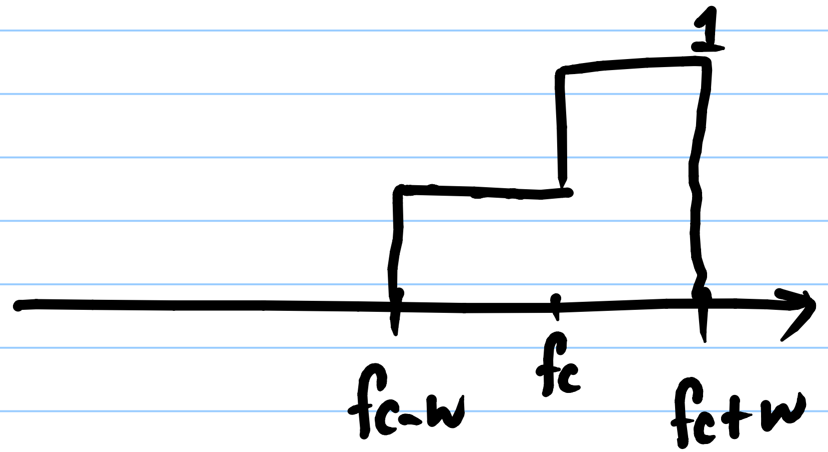
Define $c(t) = x(t)\, e^{j2\pi f_c t}$

We have $C(f) = X(f - f_c)$

$\text{Re}\{C(f)\}$



$\text{Im}\{C(f)\}$



$$x_p(t) = \mathrm{Re}\{c(t)\}$$

$$= \frac{c(t) + c^*(t)}{2}$$

Conjugation

$$c(t) \overset{\mathcal{F}}{\longleftrightarrow} C(f)$$

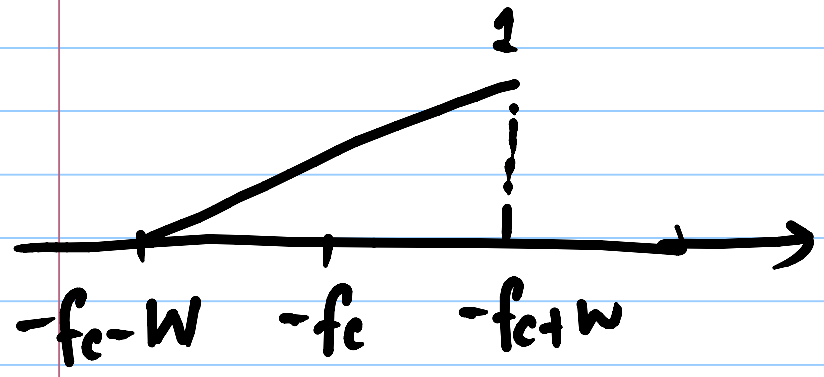
$$c^*(t) \longleftrightarrow C^*(-f)$$

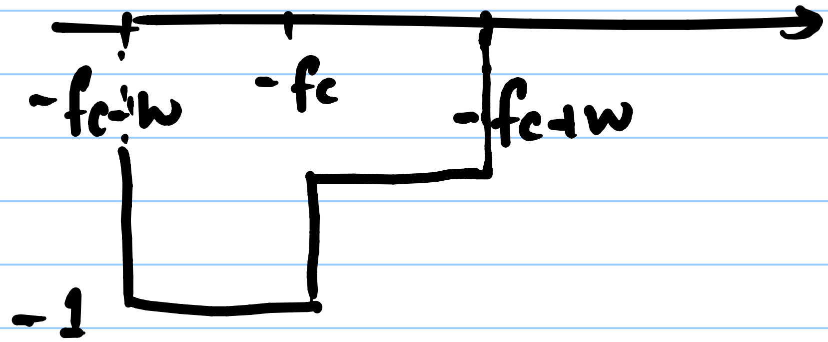
flipping and Conjugation

$$\mathrm{Re}\{C^*(-f)\} = \mathrm{Re}\{C(+f)\}$$

$$\mathrm{Im}\{C^*(-f)\} = -\mathrm{Im}\{C(+f)\}$$

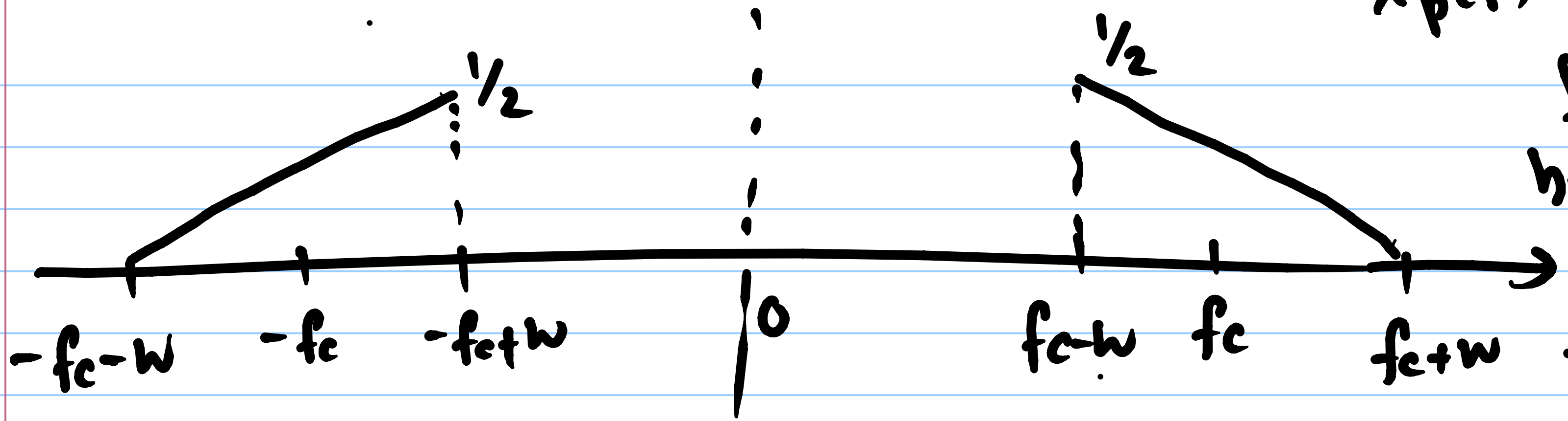
$\mathrm{Re}\{C^*(-f)\}$ $\mathrm{Im}\{C^*(-f)\}$





$$X_p(f) = \frac{C(f) + C^*(-f)}{2}$$

$Re\{X_p(f)\}$

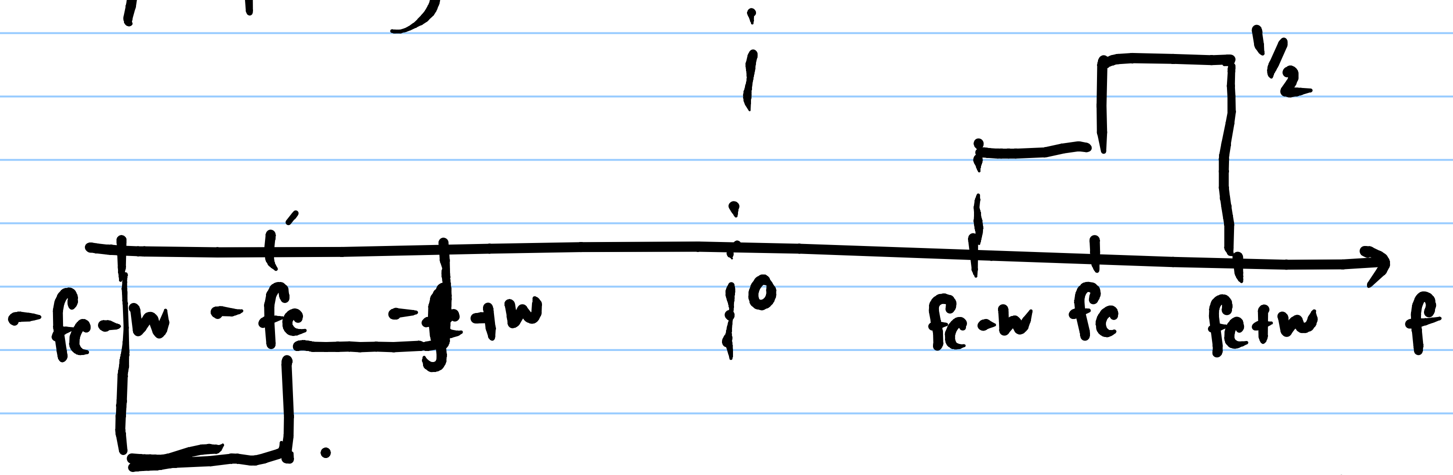


Note

$X_p(f) = X_p^*(-f)$

because $x_p(t)$ is real

$Im\{X_p(f)\}$



Notes:

1. Shape of Spectrum $X(f)$ & Shape of $X_p(f)$ are Same (positive Side).

2. Complex base band Signals, the Spectrum is not symmetric

   Bandwidth $= 2W$ (include both positive and negative freq)

3. Real passband signal $x_p(t)$

has symmetric spectrum

Bandwidth (look at only positive freq)

$\doteq 2W$

4. $$X_p(f) = \frac{X(f-f_c)}{2} + \frac{X^*(-f-f_c)}{2}$$

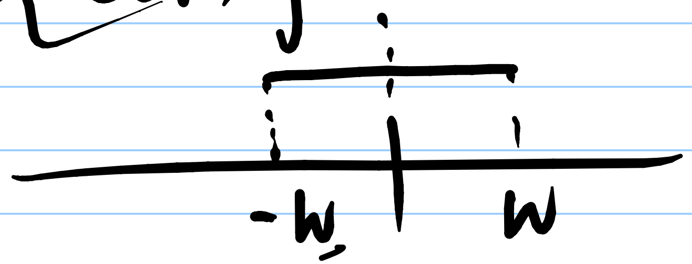
Find $X_c(f)$ & $X_s(f)$ from $X(f)$

$$x(t) = x_c(t) + j\,x_s(t)$$

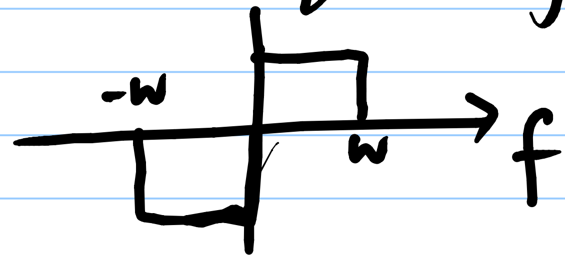
$$x_c(t) = \mathrm{Re}\{x(t)\} = \frac{x(t) + x^*(t)}{2}$$

$$X_c(f) = \frac{1}{2}\left[X(f) + X^*(-f)\right]$$

$\mathrm{Re}\{X_c(f)\}$

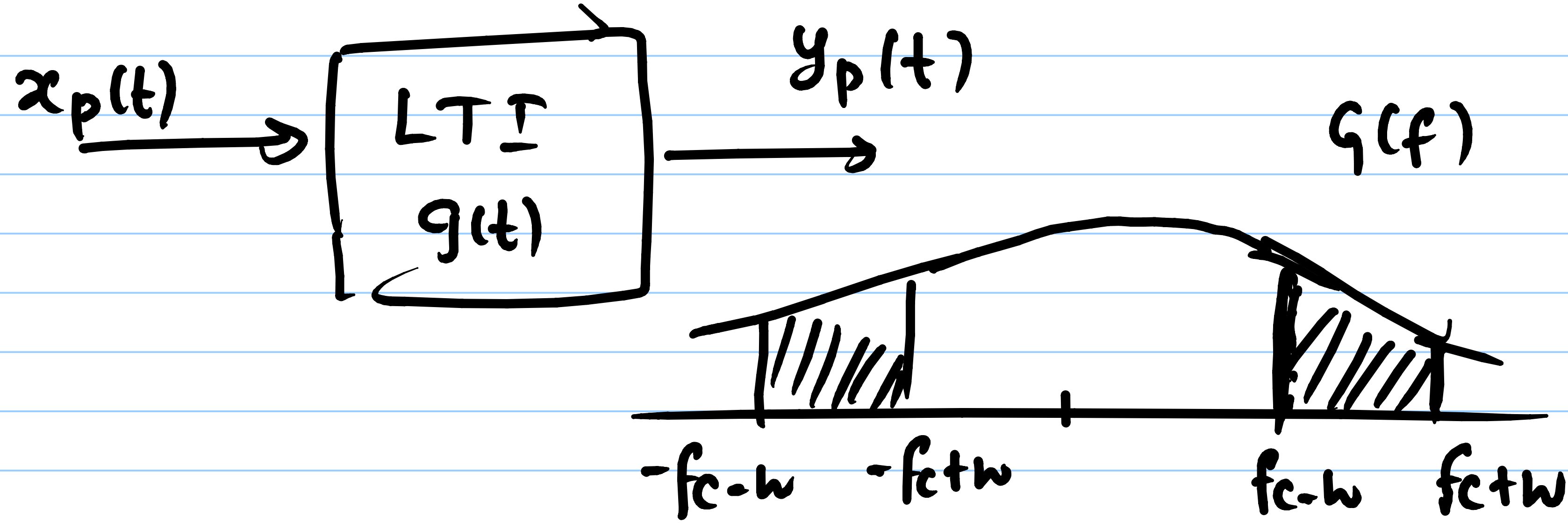


$\mathrm{Im}\{X_c(f)\}$



Similarly

$$X_s(f) = \frac{1}{2j}\left[X(f) - X^*(-f)\right]$$



Restricting $G(f)$ to bands $f_c \pm w$, $-f_c \pm w$

We get $H_p(f)$ (and impulse response $h_p(t)$).

Other Equivalences between baseband & Passband

Let $x(t) = x_c(t) + j\, x_s(t)$

$\downarrow$

baseband complex envelope

$$x_p(t) = \mathrm{Re}\left\{ x(t)\, e^{j2\pi f_c t} \right\}$$

Energy of $x_p(t) = \int_{-\infty}^{\infty} (x_p(t))^2 \, dt$

$$= \frac{1}{2} \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

Let $y_p(t) = \text{Re}\left\{ y(t)\, e^{j2\pi f_c t} \right\}$

$y(t) \rightarrow$ complex envelope in baseband

Inner product between $x_p(t)$ & $y_p(t)$

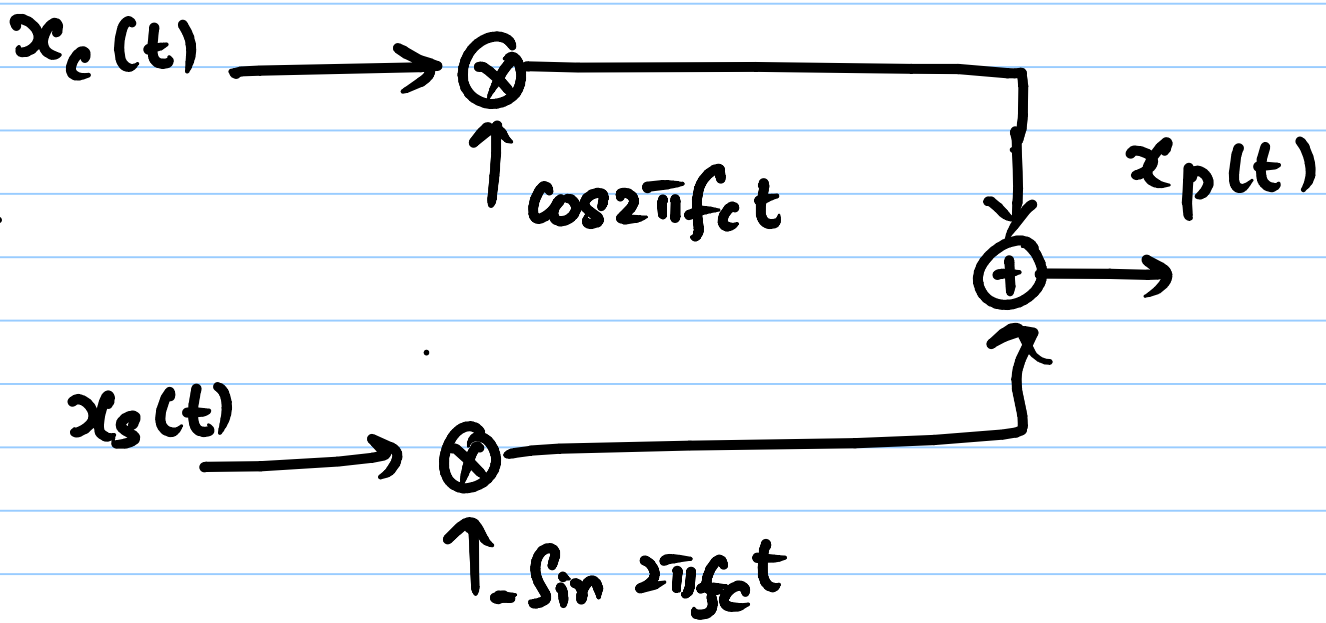
$$\langle x_p(t), y_p(t) \rangle = \int_{-\infty}^{\infty} x_p(t)\, y_p(t)\, dt$$

Verify $\Rightarrow$

$$= \frac{1}{2} \text{Re}\left\{ \int_{-\infty}^{\infty} x(t)\, y^*(t)\, dt \right\}$$

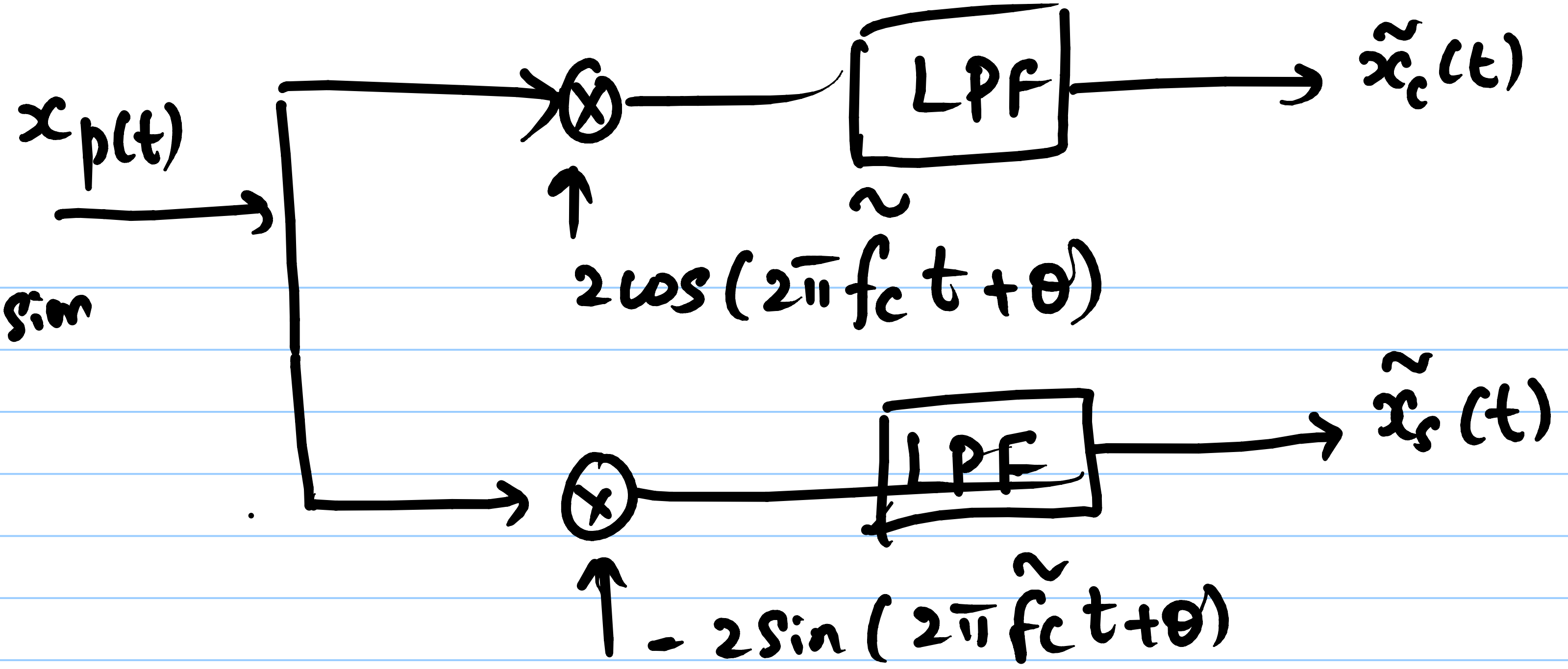
MisMatched Down Conversion

Up conversion



Down Convert with mismatched carrier freq $\tilde{f_c}$ & phase θ

Down Conversion



How is $\tilde{x}_c$ & $\tilde{x}_s$ related to x_c & x_s?

$$x(t) = x_c(t) + j\, x_s(t)$$

$$\tilde{x}(t) = \tilde{x}_c(t) + j\, \tilde{x}_s(t)$$

We know

$$x_p(t) = \mathrm{Re}\left\{ x(t)\, e^{j2\pi f_c t} \right\}$$

We also have

$$x_p(t) = \mathrm{Re}\left\{ \tilde{x}(t)\, e^{j(2\pi \tilde{f}_c t + \theta)} \right\}$$

It follows that

$$x(t)\, e^{j2\pi f_c t} = \tilde{x}(t)\, e^{j(2\pi \tilde{f}_c t + \theta)}$$

$$\tilde{x}(t) = x(t)\, e^{j(2\pi(f_c - \tilde{f}_c)t - \theta)}$$

$$\phi(t) = 2\pi(f_c - \tilde{f}_c)t - \theta$$

If $f_c = \tilde{f}_c$ then $\phi(t)$ is constant over time

$$\tilde{x}(t) = x(t)\, e^{j\phi(t)}$$

$$\tilde{x}_c(t) = x_c(t)\cos\phi(t) - x_s(t)\sin\phi(t)$$

$$\tilde{x}_s(t) = x_c(t)\sin\phi(t) + x_s(t)\cos\phi(t)$$

$\tilde{x}_c(t)$ has "interference" from $x_s(t)$

I & Q components interfere with each other due to carrier mismatch.

Our down conversion process
requires strict alignment of
(transmit & receive) carrier frequencies
and phase

This is called <u>coherent</u> down conversion