EE 5140: Tutorial

Noncoherent Demodulation, MLSE

October 26, 2017

1. Textbook Problem: 3.21

2. Consider an ON/OFF keying scheme (equal priors) where the receiver makes its decision based on a real observation Y with observation model

$$\begin{aligned} Y &= H + W \qquad 1 \text{ sent} \\ Y &= W \qquad\qquad 0 \text{ sent} \end{aligned}$$

   where H is a real zero-mean Gaussian random variable with variance 1. Value of H is unknown but its distribution is known. W is zero-mean Gaussian random variable with variance $\frac{N_0}{2}$. H and W are independent.

   (a) Find the ML rule for the above binary hypothesis testing problem

   (b) Find the average probability of error in terms of $\frac{E_b}{N_0}$.

3. Let $u(t)$ and $v(t)$ be the rectangular and triangular pulses shown below:

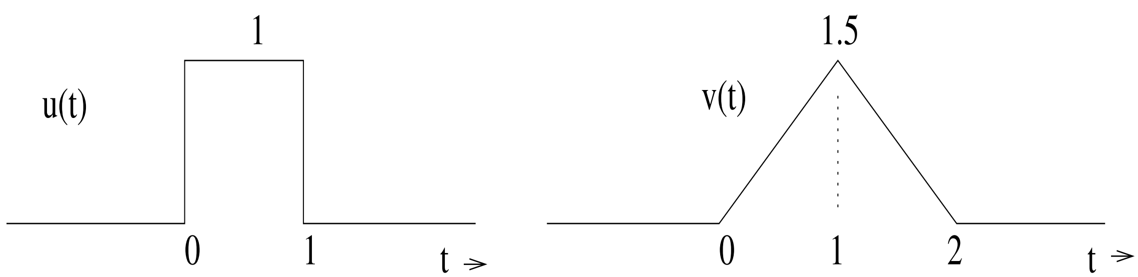

Figure 1:

   Consider the following three signal sets:

$$\begin{aligned} &\text{Signal set } \mathcal{A}: \quad s_0(t) = v(t) \text{ and } s_1(t) = -v(t) \\ &\text{Signal set } \mathcal{B}: \quad s_0(t) = u(t) \text{ and } s_1(t) = u(2(t-0.5)) - u(2t) \\ &\text{Signal set } \mathcal{C}: \quad s_0(t) = u(t) \text{ and } s_1(t) = -v(t) \end{aligned}$$

(a) For *coherent* binary signalling under AWGN, which of the above signal sets gives the *worst* error performance? Justify your answer.

(b) For *noncoherent* binary signalling under AWGN, which of the above signal sets gives the *best* error performance? Justify your answer. (*Hint:* What does the asymptotic error performance of noncoherent signalling depend upon?)

4. Consider a sequence of binary symbols $\{b(n); n = 0, ..., N\}$ corresponding to a ON/OFF signalling scheme, i.e., $b(n) = 0$ or 1. The discrete time output of the ISI channel is given by

$$y(n) \quad = \quad 2b(n) + b(n-1) + w(n); \quad n = 0, ..., N$$

where $w(n)$ is real i.i.d. white Gaussian noise with unit variance. Define the observation vector $\underline{\boldsymbol{y}} = [y(0), \cdots, y(n)]$ and symbol vector $\underline{\boldsymbol{b}} = [b(0), \cdots, b(N)]$.

(a) Given the transmit symbols $\underline{\boldsymbol{b}}$, are the observations $\{y(n)\}$ independent?

(b) Find the conditional pdf of $\underline{\boldsymbol{y}}$ given transmit symbols $\underline{\boldsymbol{b}}$, that is $f(\underline{\boldsymbol{y}}|\underline{\boldsymbol{b}})$

(c) Can you find an equivalent to the ML metric $f(\underline{\boldsymbol{y}}|\underline{\boldsymbol{b}})$, which can be written as additive form with n.

(d) Give the expression for MLSE rule to find $\{b(n)\}$ using $\{y(n)\}$.

(e) Suppose $N = 4$, $b(0) = 0$, $b(4) = 0$ and $y(0) = 1$, $y(1) = 2$, $y(2) = 1$, $y(3) = -4$, $y(4) = -3$. Using trellis diagrams and viterbi algorithm, find the maximum likelihood sequence for $\{b(0), b(1), b(2), b(3), b(4)\}$.