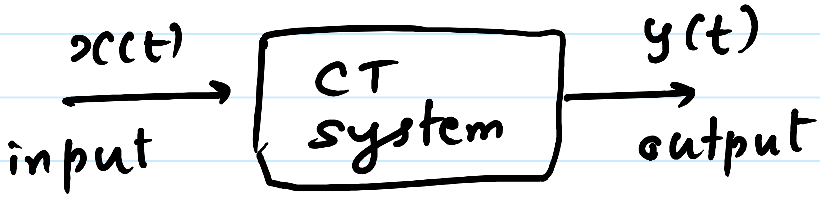
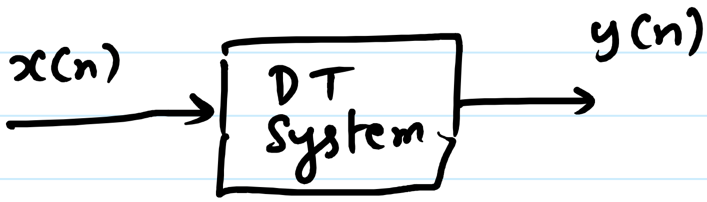
Sprop

22 January 2018 11:03

Properties of Systems





① Memory / Memoryless

A system is memoryless if output at any given time depends only on input at the same time, then system is memoryless.

Example: amplifier

① $y(t) = A\, x(t)$

A → gain

② $y(n) = 2x(n) - x^2(n)$

A system is said to have memory if output at a given time depends on input at other times

Example: delay

1. $y(t) = x(t-1)$

2. Accumulator

$$y(n) = \sum_{k=-\infty}^{n} x(k)$$

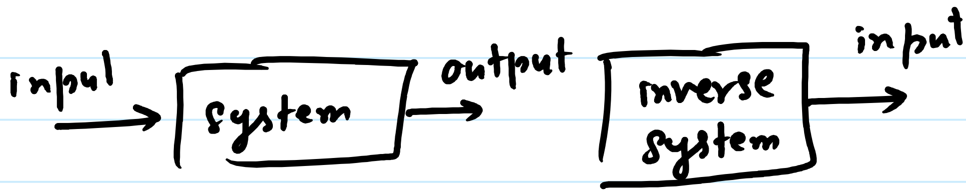
sum of input samples upto time n

3.

2. Invertible / Non-invertible systems

A system is invertible if **distinct** input signals produce **distinct** output signals.

For invertible systems, we have



inverse system gives back the original input.

Examples (invertible)

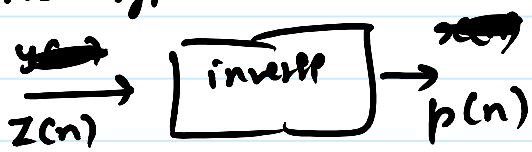
1. Amplifier

$$y(t) = 2\,x(t)$$

2. Delay

$$y(n) = x(n-1)$$

Inverse system



[redacted]

$$p(n) = z(n+1)$$

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③ Accumulator

$$y(n) = \sum_{k=-\infty}^{n} x(k)$$

Note.

$$y(n-1) = \sum_{k=-\infty}^{n-1} x(k)$$

$$y(n) = y(n-1) + x(n)$$

$$x(n) = y(n) - y(n-1)$$

↓
inverse
(difference)

Non-invertible system

① $y(t) = x^2(t)$

(Sign information is lost)

② $y(n) = 0 \cdot x(n)$

$= 0$
always

Causal / Non-Causal Systems

A system is causal if output at any given time depends only on input values upto that time.

ie) $y(t)$ depends on $\{x(\tau);\ \tau \leq t\}$

Examples (Causal)

① Accumulator

$$y(n) = \sum_{k=-\infty}^{n} x(k)$$

delay ② $y(t) = x(t-1)$

Non-causal systems

Time reversal ① $y(t) = x(-t)$

Timing advance ② $y(n) = x(n+2)$

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Stable / Unstable Systems

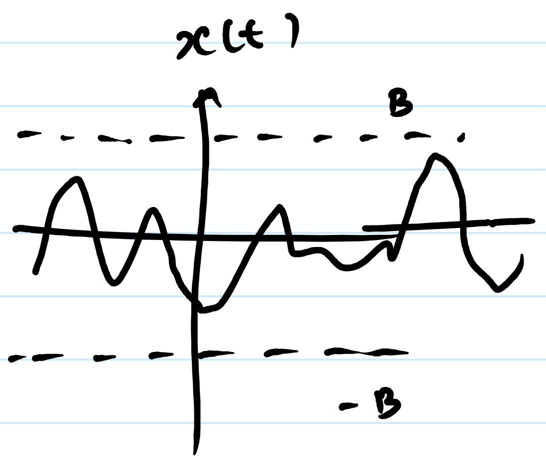
a Bounded input Bounded Output (BIBO)

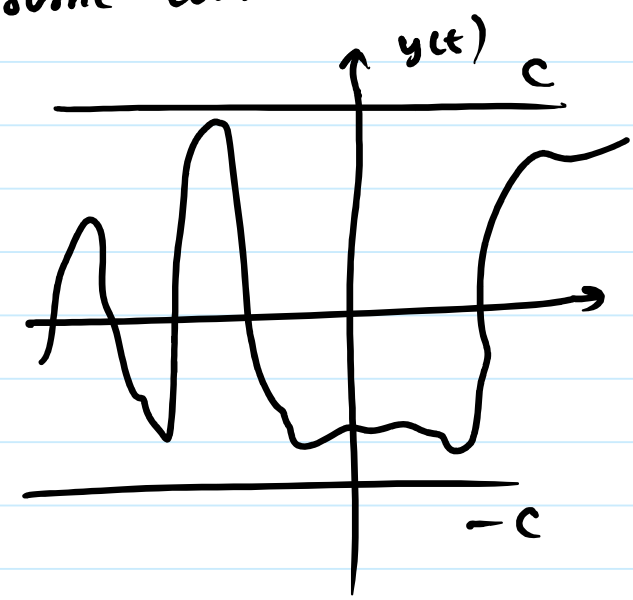
A system is called BIBO stable whenever the input remains bounded $|x(t)| \le B$

fixed constant

the output $y(t)$ also remains bounded $|y(t)| \le C$

for some constant C





Linear / Non linear Systems

A system is linear if it satisfies the following input-output relationships

(a) Additivity / Superposition

Suppose

$$x_1(n) \xrightarrow{\text{System}} y_1(n)$$

$$x_2(n) \longrightarrow y_2(n)$$

Then

$$x_1(n) + x_2(n) \longrightarrow y_1(n) + y_2(n)$$

(b) Homogeneity

Suppose: $x_1(n) \longrightarrow y_1(n)$

Then: $\alpha\, x_1(n) \longrightarrow \alpha\, y_1(n)$

where α is any complex number

$x_1(n)$, $x_2(n)$ are arbitrary input signals

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Additivity & homogeneity are two different conditions

One does not imply other.

example:

① $x(n)$ is input

$$y(n) = \begin{cases} \dfrac{x^2(n)}{x(n-1)}, & \text{if } x(n-1) \neq 0 \\ 0, & \text{if } x(n-1) = 0 \end{cases}$$

Satisfies homogeneity but not additivity

(2) $x(n)$ is input

$$y(n) = \underbrace{Re}_{\text{real part}}\{x(n)\}$$

satisfies additivity.

Homogeneity: Take $\alpha = j$ Not satisfied

$$x_1(n) = j\,x(n)$$

$$y_1(n) = Re\{x_1(n)\} = Re\{j\,x(n)\}$$

$$= -Im(x(n)) \neq y(n)$$

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Examples : Linear Systems

(1) Delay

$$y(n) = x(n-1)$$

(2) Accumulator

$$y(n) = \sum_{k=-\infty}^{n} x(k)$$

Non-linear system

(1) $y(n) = x^2(n)$

(2) $y(n) = 2x(n) + 3$

Addition of this constant violates linearity condition

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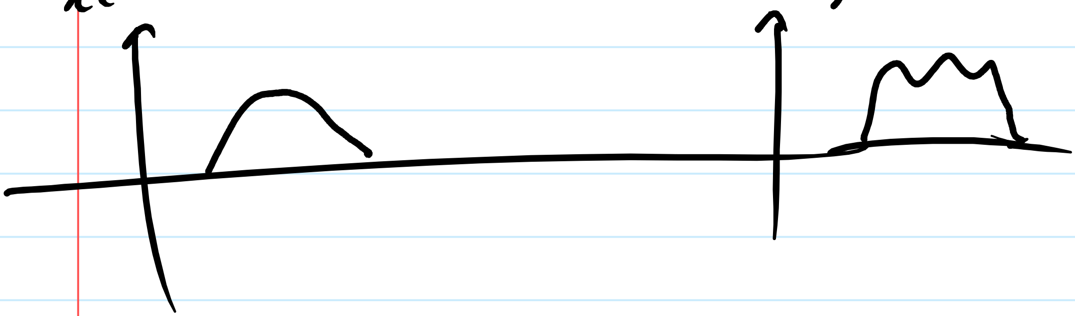
Time Invariant System

A system is time invariant if time shifting of input results in identical time shift in the output

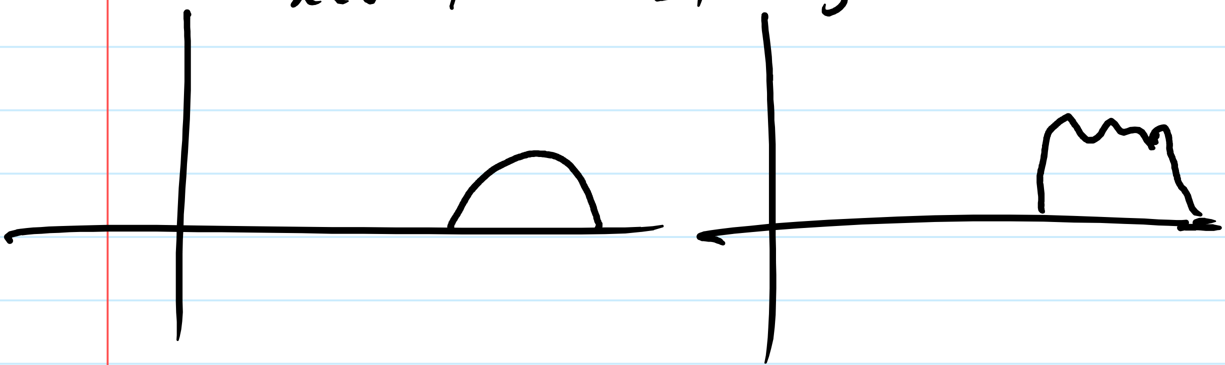
Suppose $x(t) \longrightarrow y(t)$

Then $x(t-T) \longrightarrow y(t-T)$

$x(t)$ $y(t)$



$x(t-T) \Rightarrow y(t-T)$



$x(t) \rightarrow$ arbitrary input

$T \rightarrow$ arbitrary delay.

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Examples: Time invariant systems

verify

① Delay $y(n) = x(n-1)$

② Accumulator

$$y(n) = \sum_{k=-\infty}^{n} x(k)$$

Time Varying systems

verify → ① $y(n) = n\, x(n)$

gain depends on time index.

② Time Scaling

$y(t) = x(2t)$