

Derivation of n & p

$$\begin{aligned}
 n &= \int_{E_C}^{E_C'} D(E) f(E) dE \\
 &= \int_{E_C}^{E_C'} \frac{(2m_e^*)^{3/2}}{2\pi^2 \hbar^3} \sqrt{E - E_C} \cdot \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} dE \\
 &= \int_{E_C}^{E_C'} \frac{(2m_e^*)^{3/2}}{2\pi^2 \hbar^3} \sqrt{E - E_C} \exp\left[-\frac{E - E_F}{kT}\right] dE \quad \boxed{E - E_F \gg 3kT} \\
 &= \int_{E_C}^{E_C'} \frac{(2m_e^*)^{3/2}}{2\pi^2 \hbar^3} \sqrt{E - E_C} \exp\left[-\frac{E_C - E_F}{kT}\right] \cdot \exp\left[-\frac{E - E_C}{kT}\right] dE
 \end{aligned}$$

$f(E) \approx \frac{1}{B(E)}$
 $E \gg E_F$

$$\frac{E - E_c}{kT} = \epsilon$$

$$dE = kT d\epsilon$$

$$E_c' - E_c \approx 4 \text{ eV}$$

$$\frac{E_c' - E_c}{kT} \approx 160$$

$$n = \frac{(kT)^{3/2} (2m_e^*)^{3/2}}{2\pi^2 \hbar^3} \exp\left(-\frac{E_c - E_F}{kT}\right) \int_0^{\infty} \sqrt{\epsilon} \exp(-\epsilon) d\epsilon$$

$$n = \left(\frac{2m_e^* kT}{2\pi \hbar^2} \right)^{3/2} \exp\left(-\frac{E_c - E_F}{kT}\right)$$

\uparrow
 $\sqrt{\pi}/2$

$$n = N_c \exp\left\{-\frac{E_c - E_F}{kT}\right\}$$

(N_V) $N_c \rightarrow$ Effective density of states in the conduction band (Valence band)

$$p = \int_{-\infty}^{E_V} D_V(E) [1 - f(E)] dE$$

$$p = N_V \exp\left[-\frac{(E_F - E_V)}{kT}\right]$$

N_c replace m_e^* by m_h^*

$$N_V = 2 \left(\frac{m_h^* kT}{2\pi \hbar^2} \right)^{3/2}$$

$$n = N_C \exp \left\{ - \frac{(E_C - E_F)}{kT} \right\}$$

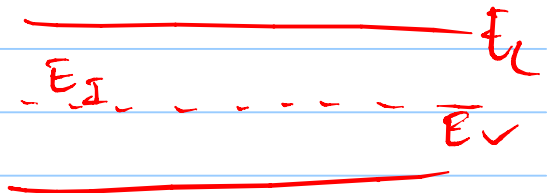
$$p = N_V \exp \left\{ - \frac{(E_F - E_V)}{kT} \right\}$$

$$\frac{n}{p} = \frac{N_C}{N_V} \exp \left[\frac{+2E_F - (E_C + E_V)}{kT} \right]$$

$$E_F = \frac{E_C + E_V}{2} + \frac{kT}{2} \ln \frac{N_V}{N_C} + \frac{kT}{2} \ln \left(\frac{n}{p} \right)$$

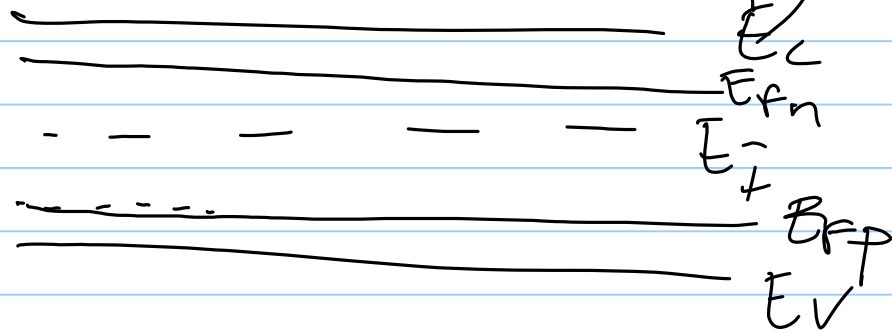
Fermi level of intrinsic² semi-cond

$$\underline{E_I} = \frac{E_C + E_V}{2} + \frac{kT}{2} \ln \left(\frac{N_V}{N_C} \right)$$



$$E_{Fn} = E_I + kT \ln \left(\frac{N_D}{n_i} \right)$$

$$E_{Fp} = E_I - kT \ln \left(\frac{N_A}{n_i} \right)$$

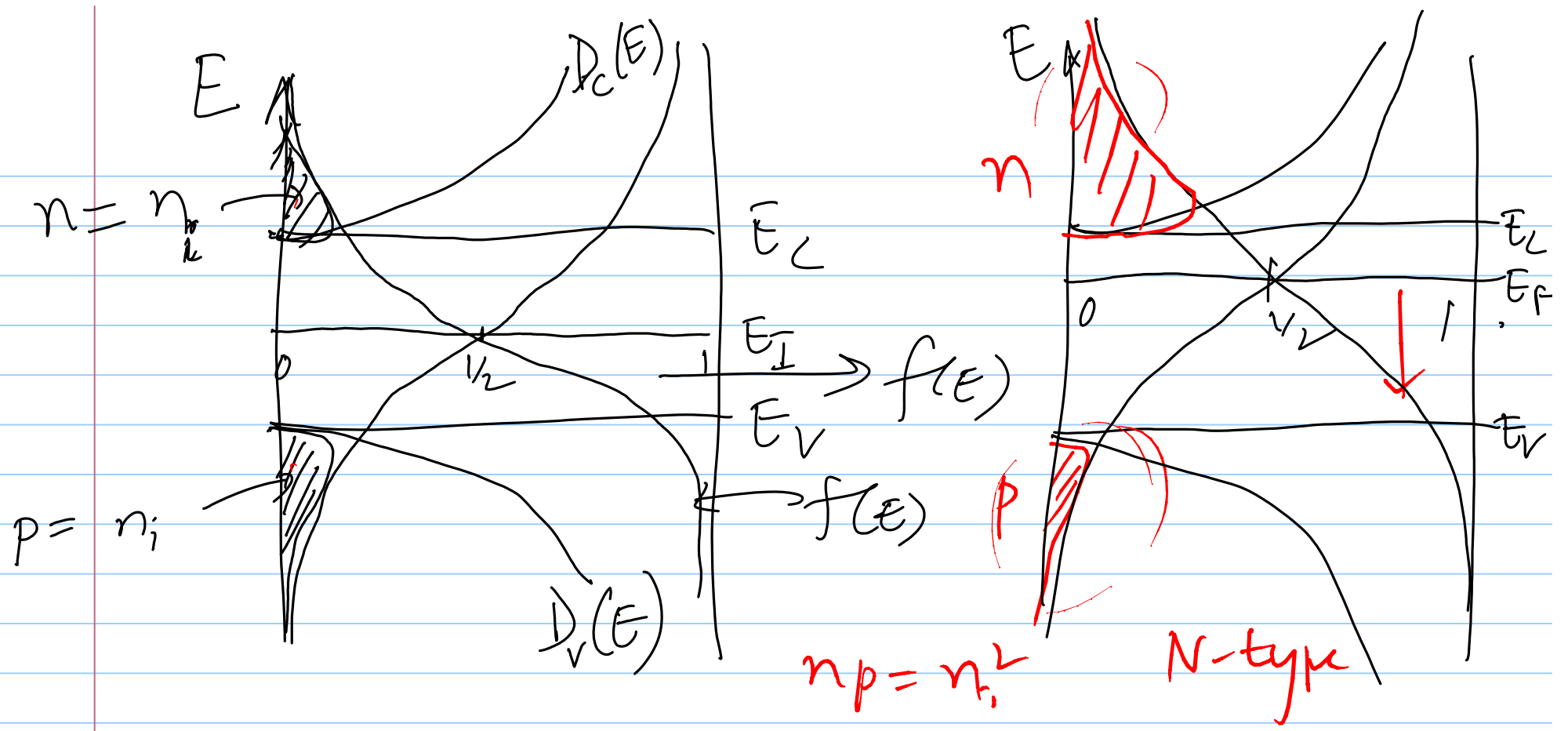


$$n \approx N_D$$

$$p = \frac{n_i^2}{N_D}$$

$$p = N_A$$

$$n = \frac{n_i^2}{N_A}$$



$$n = N_D^+ \neq N_D$$



$$N_D^0 + N_D^+ = N_D$$



$$\eta_D = \frac{N_D^+}{N_D}$$

$$f(E_D)$$

$$= \frac{1}{1 + 2 \exp\left(\frac{E_D - E_F}{kT}\right)}$$

$$\frac{N_D^0}{N_D} = f(E_D) = \frac{1}{1 + \frac{1}{2} \exp\left(\frac{E_D - E_F}{kT}\right)}$$

$$\eta_D = \frac{N_D^+}{N_D} = 1 - \frac{N_D^0}{N_D} = \frac{1}{1 + 2 \exp\left(\frac{E_F - E_D}{kT}\right)}$$

$$\rightarrow \eta_D = \frac{1}{1 + 2 \exp\left(\frac{E_F - E_D}{kT}\right)}$$

$$n = N_D \eta_D = N_C \exp\left[-\frac{(E_C - E_F)}{kT}\right]$$

$$\eta_D = \frac{1}{1 + 2 \exp\left[-\frac{(E_C - E_F)}{kT}\right] \exp\left(\frac{E_C - E_D}{kT}\right)}$$

$$\eta_D = \frac{1}{1 + 2 \cdot \frac{N_D N_D}{N_C} \exp\left(\frac{E_{ion}}{kT}\right)}$$

$$\frac{2N_D}{N_C} \exp\left(\frac{E_{ion}}{kT}\right) \eta_D^2 + \eta_D - 1 = 0$$

$$\eta_D = \frac{-1 + \sqrt{1 + \frac{8N_D}{N_C} \exp\left(\frac{E_{ion}}{kT}\right)}}{\frac{4N_D}{N_C} \exp\left(\frac{E_{ion}}{kT}\right)}$$

↑
ionization
coeffⁿ

$$\eta_A \approx \eta_D \quad (N_D \leftarrow N_A)$$