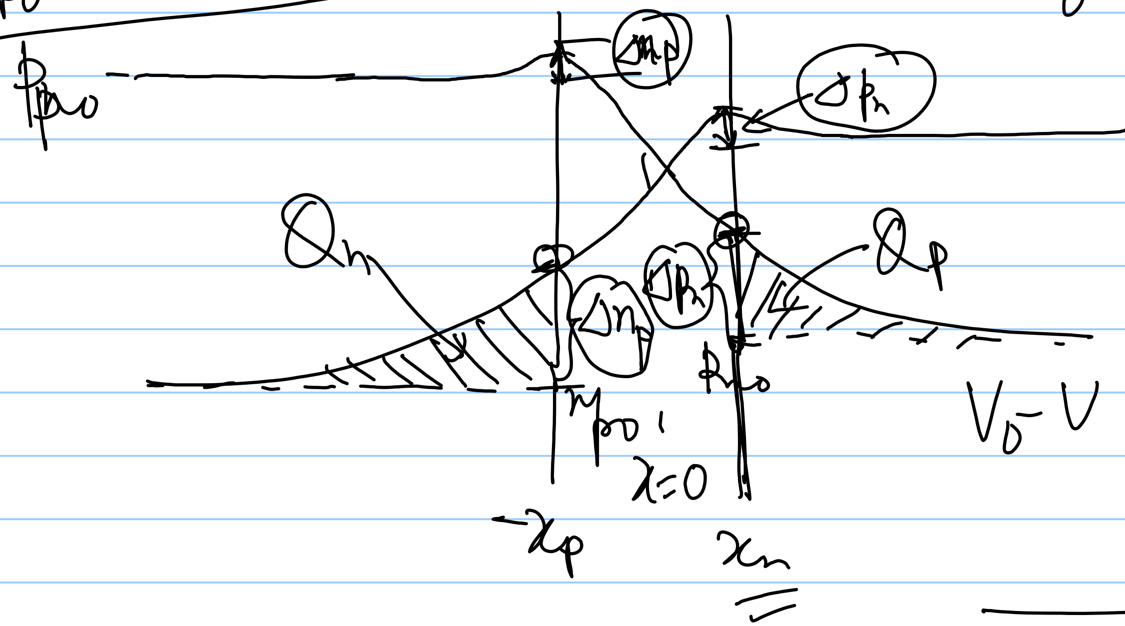
Derivation of Diode current

19/9/2014

$$\boxed{V_0 = V_T \ln\left(\frac{p_{p0}}{p_{n0}}\right) = V_T \ln\left(\frac{p(-x_{p0})}{p(x_{n0})}\right)}$$

$$p_{p0} = p_{n0}\, e^{V_0/V_T}$$



$$V_0 - V = V_T \ln\left(\frac{p_p}{p_n}\right)$$

$$= V_T \ln\left(\frac{p(-x_p)}{p(x_n)}\right)$$

$$V_0 - V = V_T \ln\left(\frac{p_{p0} + \Delta n_p}{p_{n0} + \Delta p_n}\right)$$

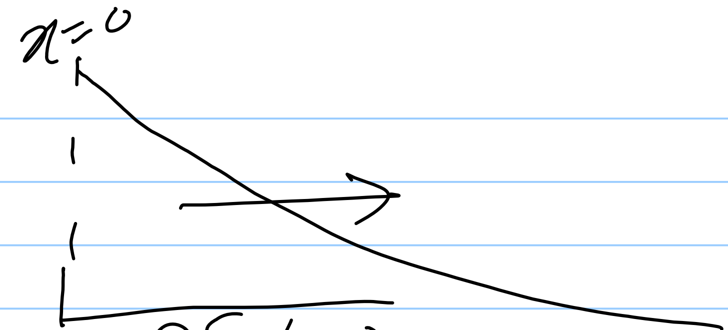
$$p_{p0} \simeq p_{p0} + \Delta n_p = (p_{n0} + \Delta p_n)\, e^{(V_0 - V)/V_T}$$

$$p_{p0} = p_{n0}\, e^{V_0/V_T} \Longrightarrow n_{n0} = n_{p0}\, e^{V_0/V_T}$$

$$1 = \frac{p_{n0} + \Delta p_n}{p_{n0}}\, e^{-V/V_T}$$

$$\Rightarrow \boxed{\underbrace{p_{n0} + \Delta p_n}_{(x = x_n)} = p_{n0}\, e^{V/V_T}} \Longrightarrow \Delta p_n = p_{n0}\left(e^{V/V_T} - 1\right)$$

$$\boxed{\underbrace{n_{p0} + \Delta n_p}_{(x = -x_p)} = n_{p0}\, e^{V/V_T}} \Longrightarrow \Delta n_p = n_{p0}\left(e^{V/V_T} - 1\right)$$



$$0 = \frac{\partial \delta p(x,t)}{\partial t} = -\frac{1}{q}\frac{\partial}{\partial x} J_p(x,t) - \frac{\delta p(x,t)}{\tau_p}$$

$$\boxed{\frac{d^2 \delta p(x)}{dx^2} = \frac{1}{D_p \tau_p}\,\delta p(x) = \frac{\delta p(x)}{L_p^2}}$$

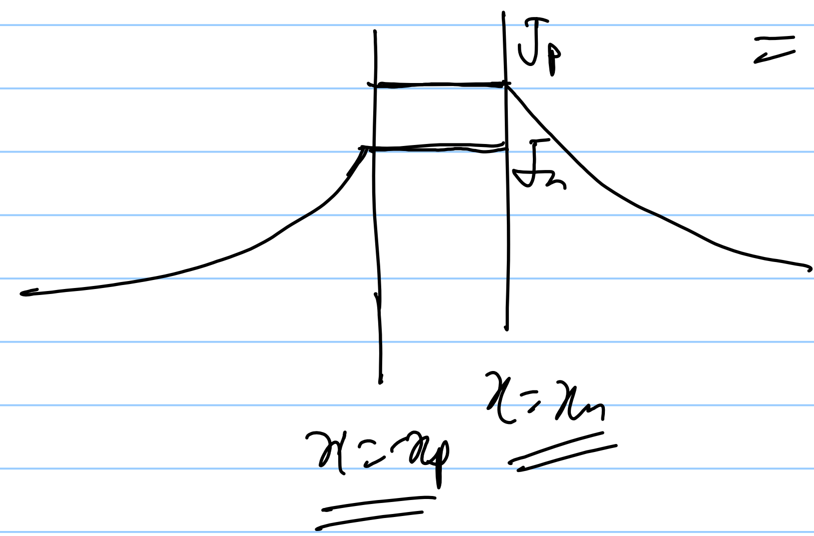
$$\Rightarrow \delta p(x) = \delta p(x=0)\, e^{-x/L_p}$$

$$\Delta p_n = p_{n0}\left(e^{V/V_T} - 1\right)$$

$$\delta p_n(x) = \Delta p_n\, e^{-(x-x_n)/L_p} = \delta p_n(x=x_n)\, e^{-\frac{(x-x_n)}{L_p}}$$

$$\delta p_n(x) = \underbrace{\delta p_n(x = x_n)}_{\Delta p_n}\, e^{-(x - x_n)/L_p} \qquad x \geq x_n.$$

$$\delta n_p(x) = \underbrace{\delta n_p(x = -x_p)}_{\Delta n_p}\, e^{(x + x_p)/L_n} \qquad x \leq -x_p.$$



$$J = J_p(x = x_n) + J_n(x = -x_p)$$

$$= -qD_p \frac{d\,\delta p_n(x)}{dx}\bigg|_{x = x_n} + qD_n \frac{d\,\delta n_p(x)}{dx}\bigg|_{x = -x_p}$$

$$J = -\frac{q D_p}{-L_p}\,\delta p_n(x = x_n) + \frac{q D_n}{L_n}\,\delta n_p(x = -x_p)$$

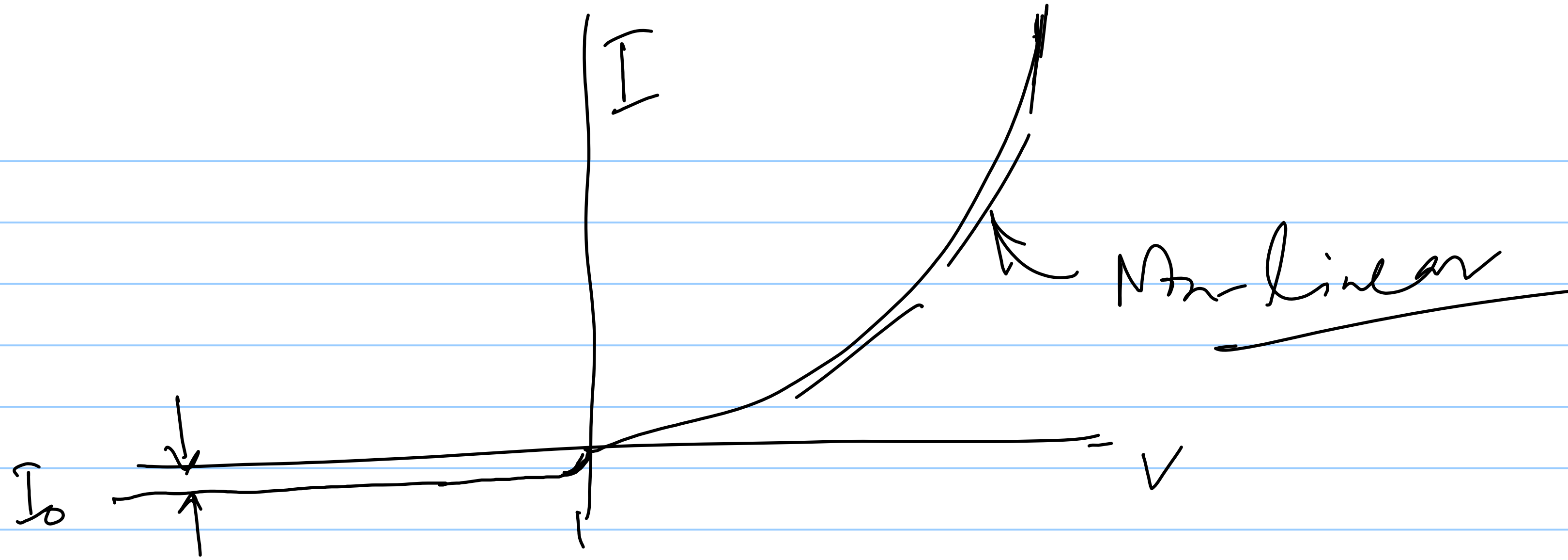
$$= q\left(\frac{D_p}{L_p}\Delta p_n + \frac{D_n}{L_n}\Delta n_p\right)$$

$$= \underbrace{q\left(\frac{D_p}{L_p}p_{n0} + \frac{D_n}{L_n}n_{p0}\right)}_{J_0}\left(e^{V/V_T} - 1\right)$$

$$J = J_0\left(e^{V/V_T} - 1\right)$$

$$I = A J_0\left(e^{V/V_T} - 1\right) = I_0\left(e^{V/V_T} - 1\right)$$

$I_0 \rightarrow$ reverse saturation current



$I_0 = 1pA$

$V = 0.6V$

$V_T = 0.0259V$

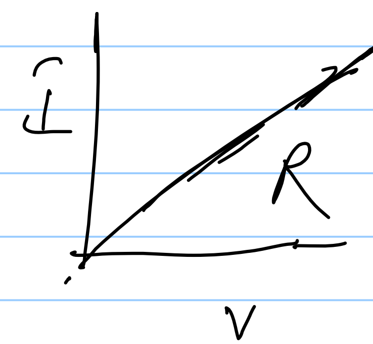
$I = ? = 11.5mA$

Diode resistance?

$r_d = ? = V/I$

$R = \frac{dV}{dI}$

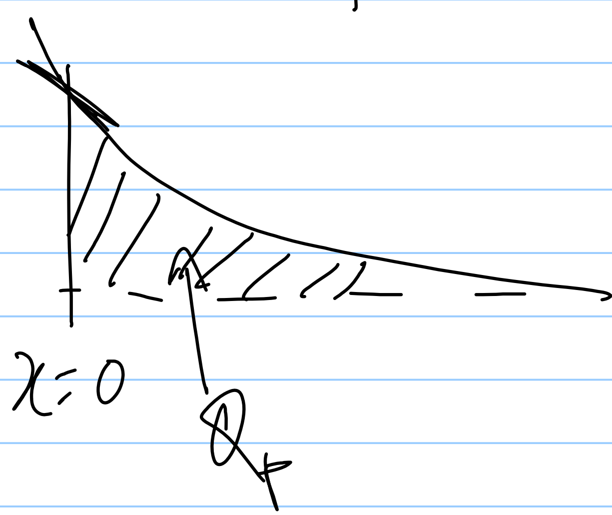
$r_d = \left(\frac{dI}{dV}\right)^{-1}$



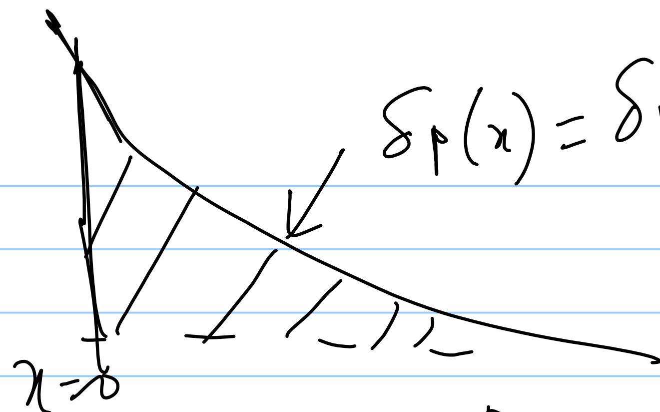
$$r_d = \left(\frac{dI}{dV}\right)^{-1} = ? = \left(\frac{I + I_0}{V_T}\right)^{-1} \cong \frac{V_T}{I} \approx 2.25\,\Omega$$

$$I = I_0\left(e^{V/V_T} - 1\right)$$

$$\frac{dI}{dV} = \frac{I_0}{V_T} e^{V/V_T} = \frac{I + I_0}{V_T} \approx I/V_T$$



$$I_p(x=0) = \frac{Q_p}{\tau_p}.$$



$$\delta_p(x) = \delta_p(x=0)\, e^{-x/L_p}$$

$$Q_p = qA \int_0^{\infty} \delta_p(x)\, dx = qA \cdot L_p\, \delta_p(x=0)$$

$$\frac{Q_p}{\tau_p} = qA \frac{L_p}{\tau_p} \delta_p(x=0)$$

$$I_p(x=0) = -qA D_p \left.\frac{d\delta_p(x)}{dx}\right|_{x=0} = \frac{qA D_p}{L_p} \delta_p(x=0)$$

$$\frac{L_p}{\tau_p} = \frac{D_p}{L_p} \Rightarrow D_p \tau_p = L_p^2$$

$$I = I_p(x = x_n) + I_n(x = -x_p)$$

$$I = \frac{Q_p}{\tau_p} + \frac{Q_n}{\tau_n} \simeq \frac{Q_{stored}}{\tau} = I_o\left(e^{V/V_T} - 1\right)$$

$$\underline{\underline{Q_{stored} = I\tau}}$$

Diffused minority charge.

$\rightarrow W \leftarrow$ depletion width

$$W = \sqrt{\frac{2\epsilon_{Si}(V_b - V)}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)}$$

$$C_{junction} = C_j = C_{dep} = \frac{\epsilon_{si} A}{W}$$

$$C_j = \frac{\epsilon_{si} A}{\sqrt{\frac{2 \epsilon_{si} V_0 \left(1 - V/V_0\right)}{q} \frac{N_A + N_D}{N_A N_D}}} = \frac{C_j(V=0)}{\sqrt{1 - V/V_0}} = \frac{C_{j0}}{\sqrt{1 - V/V_0}}$$

$$= \frac{\epsilon_{si} A}{\sqrt{\frac{2 \epsilon_{si} V_0}{q} \left(\frac{N_A + N_D}{N_A N_D}\right)} \sqrt{1 - V/V_0}} = \frac{\epsilon A}{W_0} \frac{1}{\sqrt{1 - V/V_0}}$$

$$I = I_0\left(e^{V/V_T} - 1\right)$$

$$Q_{stored} = I\tau$$

$$\frac{dQ_{stored}}{dV} = C_{diff} = \tau g_d.$$

Large Signal Variables

I, Q

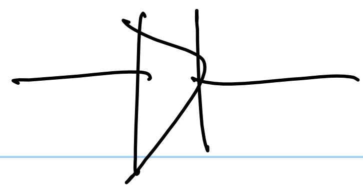
g_d, r_d

$$C_j(V) = \frac{C_{j0}}{\sqrt{1 - V/V_0}}$$

$$Q_j(V) = \int^{V} C_j(V)\,dV$$

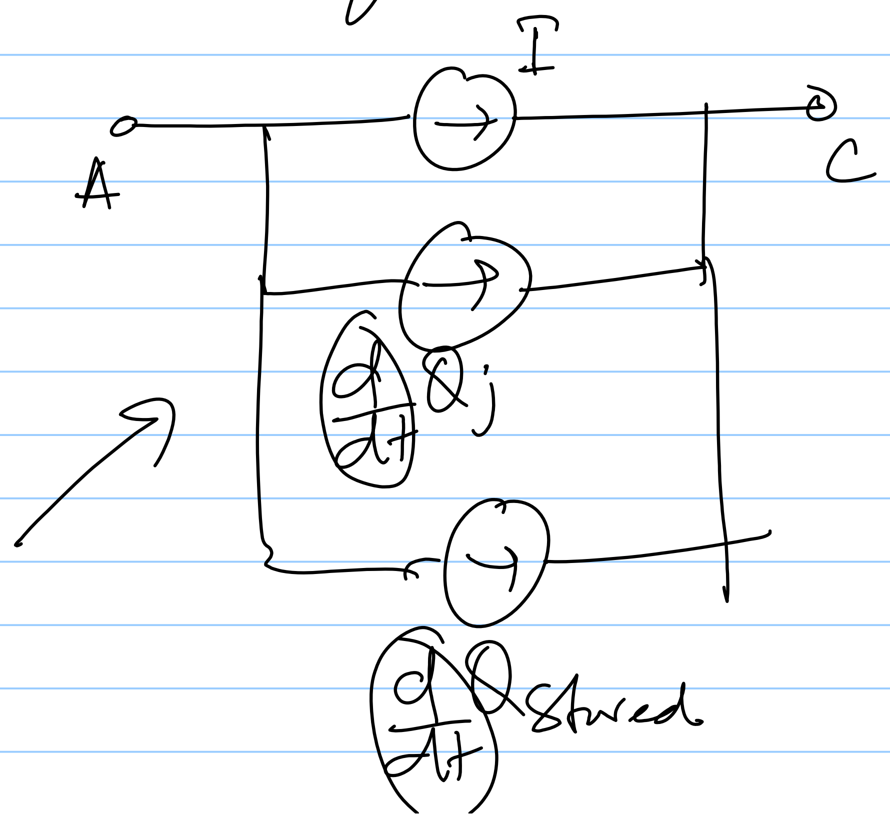
Small-Signal Variables

C, g

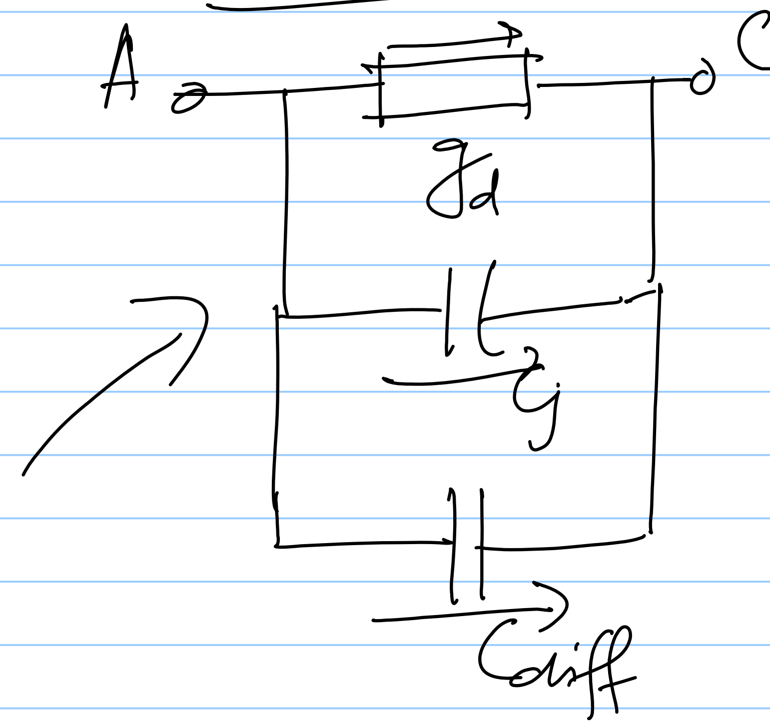


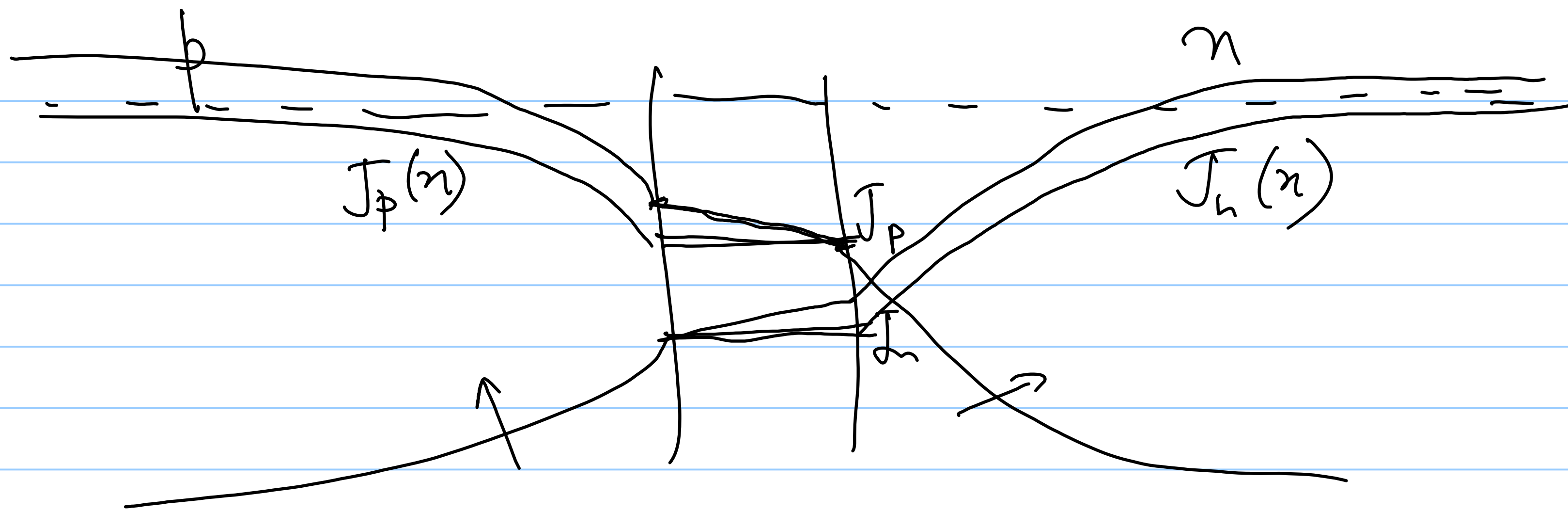
Symbol

L-S Equivalent Circuit



S-S EC





$$I(x) = \text{Constant}$$