EE 5320 Analog IC Design

Note Title 01-02-2021

2/2/21

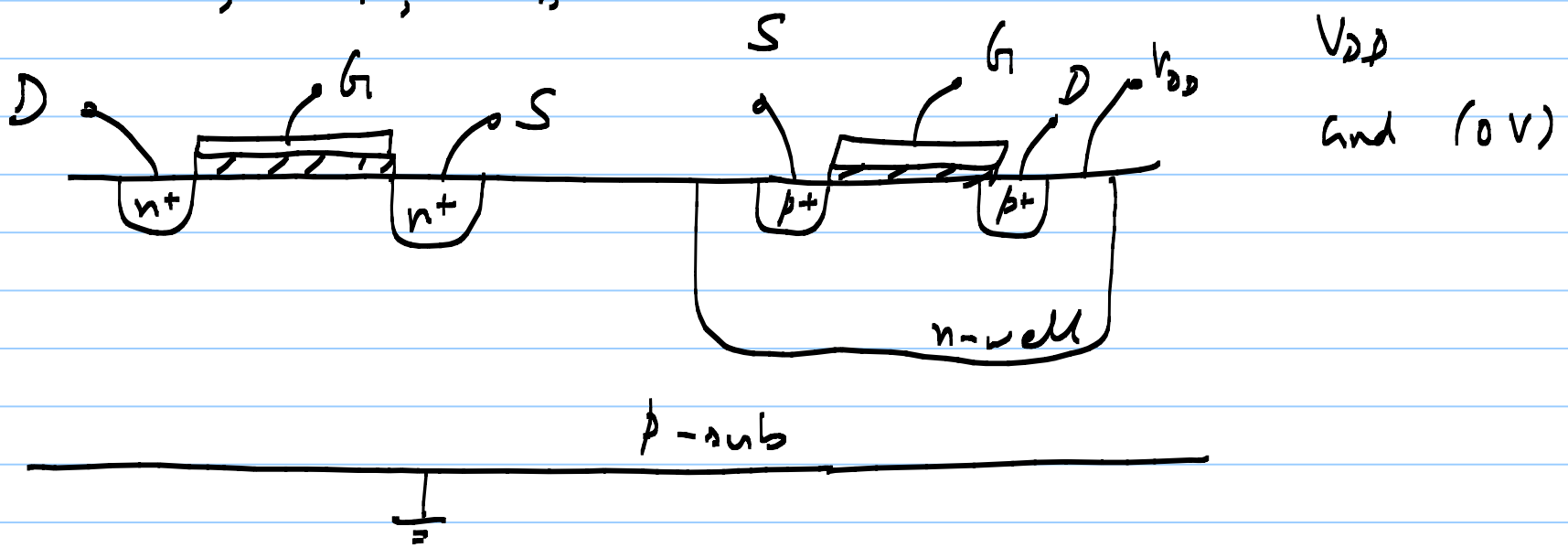
Lecture 1

IC – Any 2 similarly sized devices can be assumed to be nominally identical.

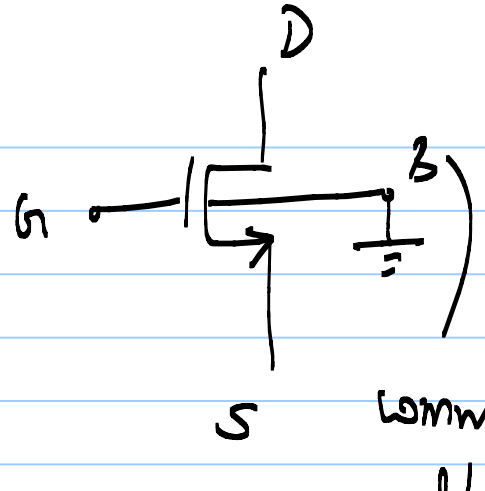
e.g. 2 MOSFETs with same W & L

MOSFET's, R's, C's

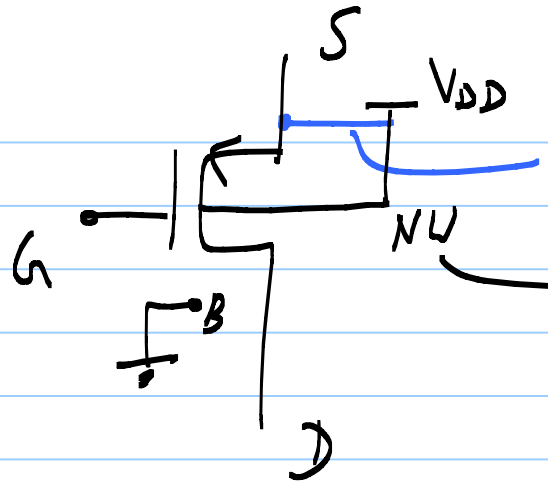
1) MOS



V_{DD} and (0 V)



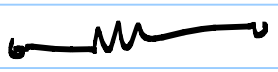
common for all NMOS's



als. possible

can be independent node for each PMOS.

2) R's



polysilicon

n^+ / p^+ diffusion

n-well

metal

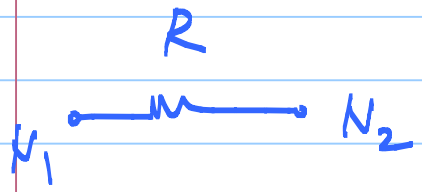
} mΩ to 100 kΩ

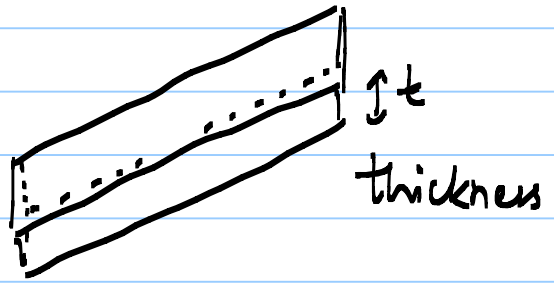
f, Temp. coefficient, linearity, tolerance, size

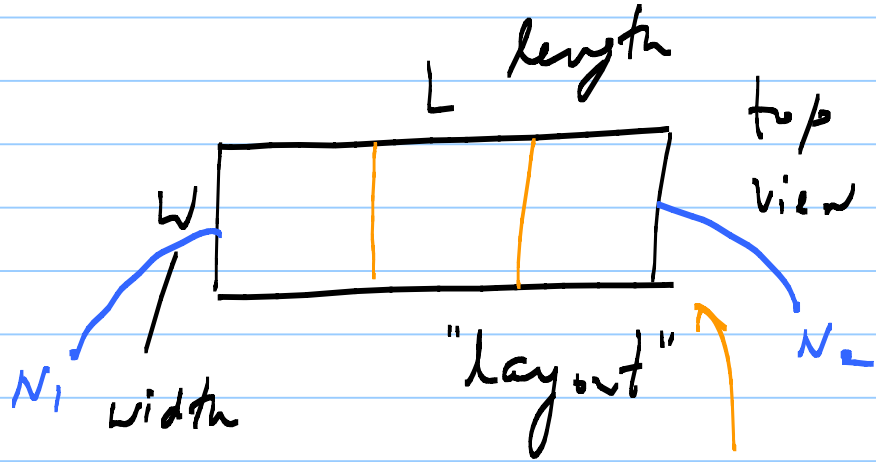
↓

fn. of material

ρ – function of material



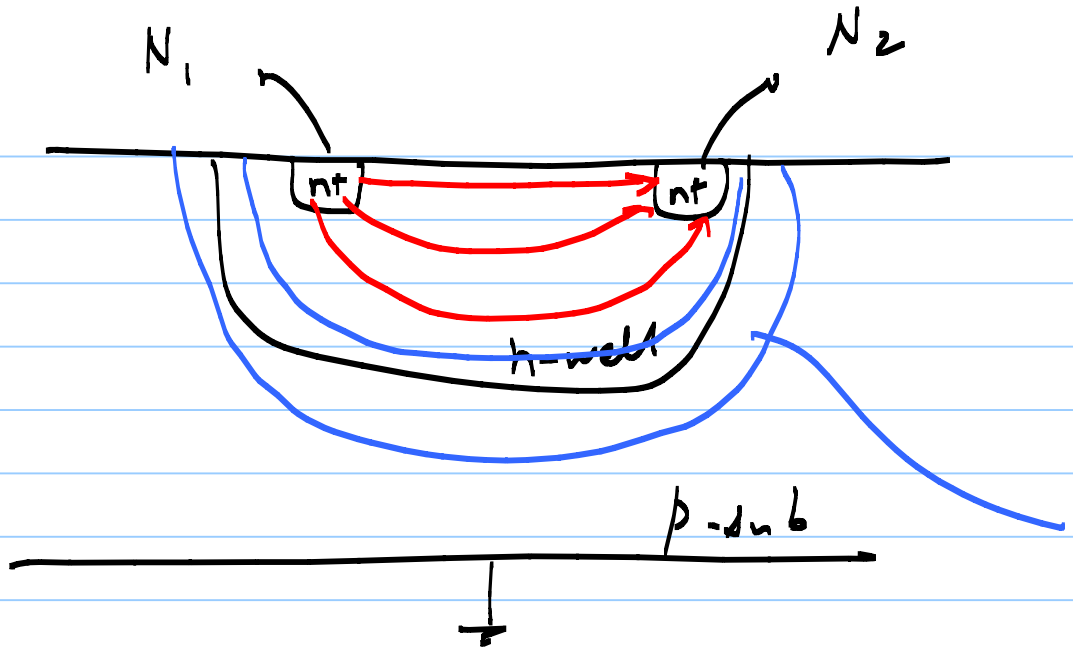


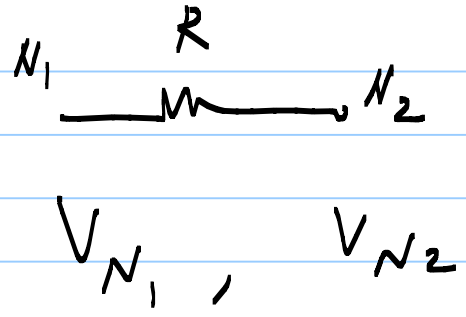


$$R = \frac{\rho \times \text{length}}{A} = \frac{\rho \times L}{W \times t} = \left(\frac{\rho}{t}\right) \times \left(\frac{L}{W}\right)$$

$\frac{\rho}{t}$: not under your control; "sheet resistance"; $\Omega/\square$

$\frac{L}{W}$: design & layout parameters; = "# of squares"

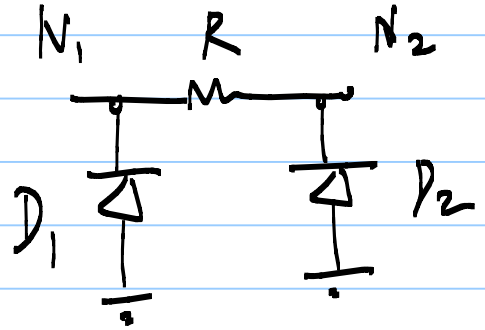




$V = IR$ or $I = V/R$

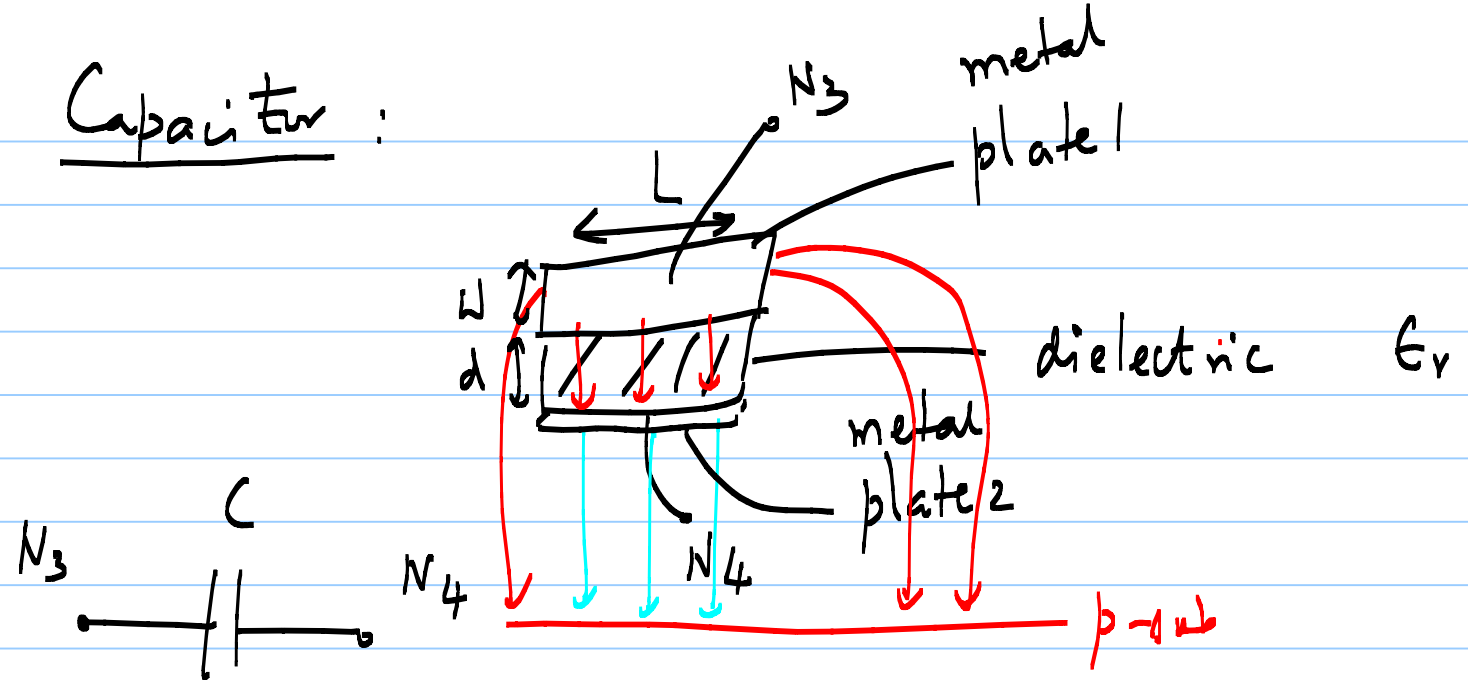
actual : $I = \frac{V}{R} + (\ )V^2 + (\ )V^3 + \ldots$

also



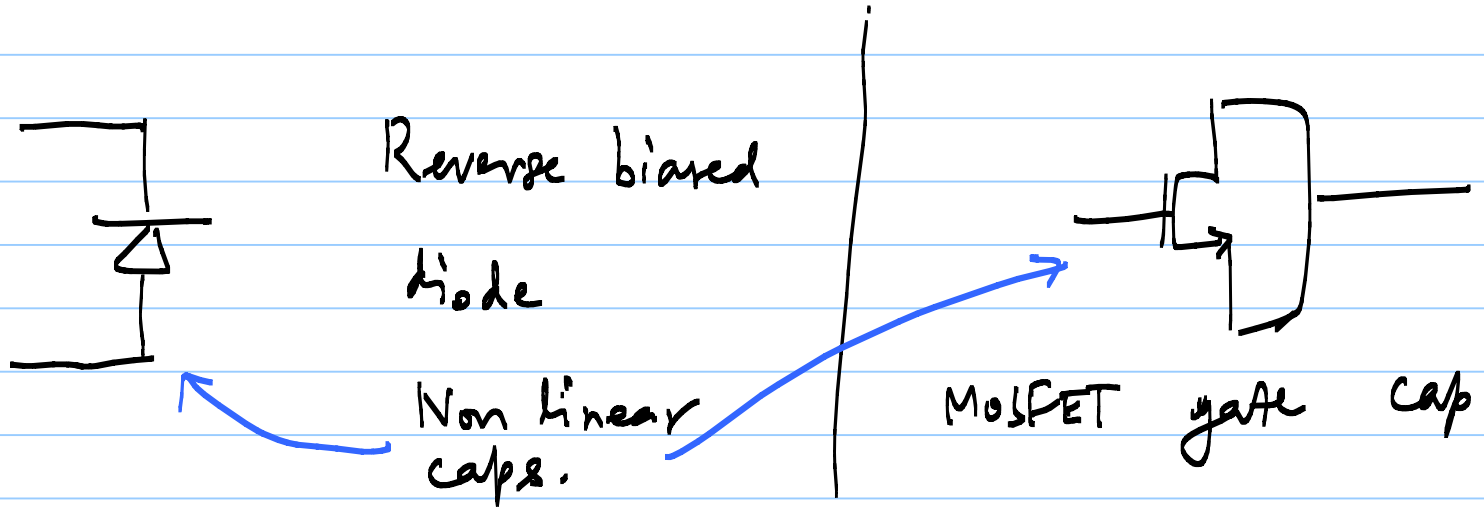
D_1 & D_2 have parasitic caps

3) Capacitor :

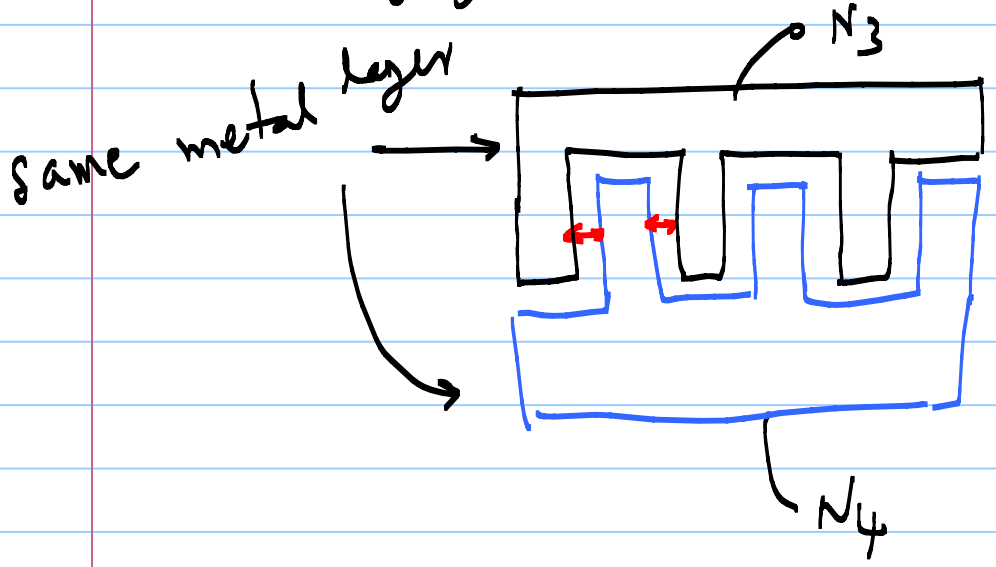


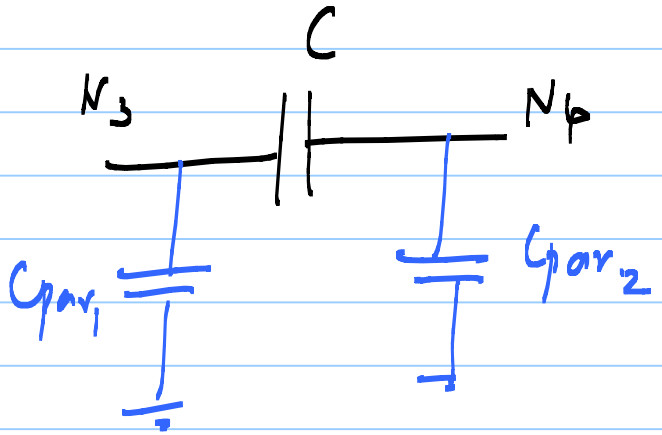
$$C = \frac{\epsilon_r \cdot \epsilon_0 \cdot A}{d} \cong \frac{\epsilon_r \epsilon_0 \cdot W \cdot L}{d}$$

under designer's control

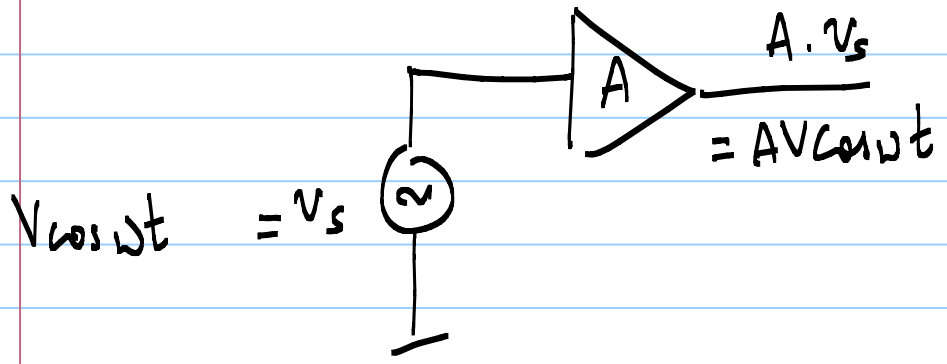


Fringing cap: (top view)





Noise

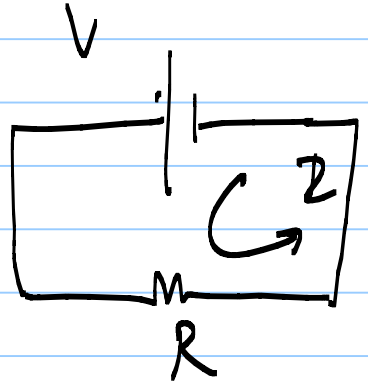


* As $V \uparrow$, sinusoid can. clip. (swing limits of Amp.)

* As $V \downarrow$: reach "noise" limit.

Noise: Random motion of charge becomes comparable to deterministic flow

→ happen at low V's and I's in circuit



$$I = \frac{V}{R}$$

Uncertainty is a function of temperature T in K

"Thermal Noise" of resistor

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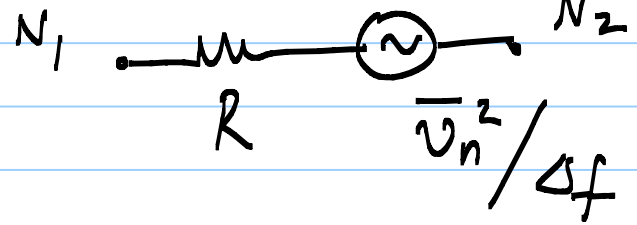
Lecture 2

1)

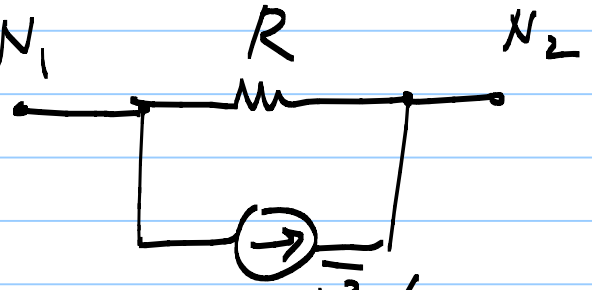


"Thermal noise"

power spectral density



$\equiv$



$$\frac{\overline{v_n^2}}{\Delta f} = 4kTR \quad V^2/Hz$$

$$\frac{\overline{v_n^2}/\Delta f}{R^2} = \frac{4kT}{R} \quad A^2/Hz$$

observation BW

$$\frac{\overline{v_n}}{\sqrt{\Delta f}} = \sqrt{4kTR} \quad V/\sqrt{Hz}$$

$$\frac{\overline{i_n}}{\sqrt{\Delta f}} = \sqrt{\frac{4kT}{R}} \quad A/\sqrt{Hz}$$

"White Noise" → No freq. dependence



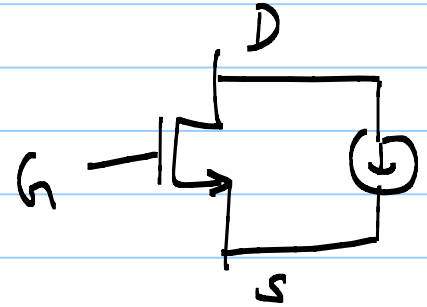
Use either

$\frac{\overline{v_n^2}}{\Delta f}$ or $\frac{\overline{i_n^2}}{\Delta f}$

2) pure C, L, M do not produce noise

3) MOSFETs in saturation

(a) flicker noise or 1/f noise



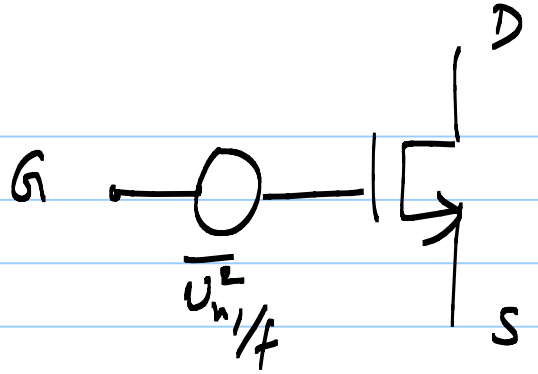
$$\frac{\overline{i_{n\,1/f}^2}}{\Delta f} = \frac{K_f \cdot g_m^2}{W \cdot L \cdot C_{ox}^2} \cdot \frac{1}{f}$$

transconductance

width

length

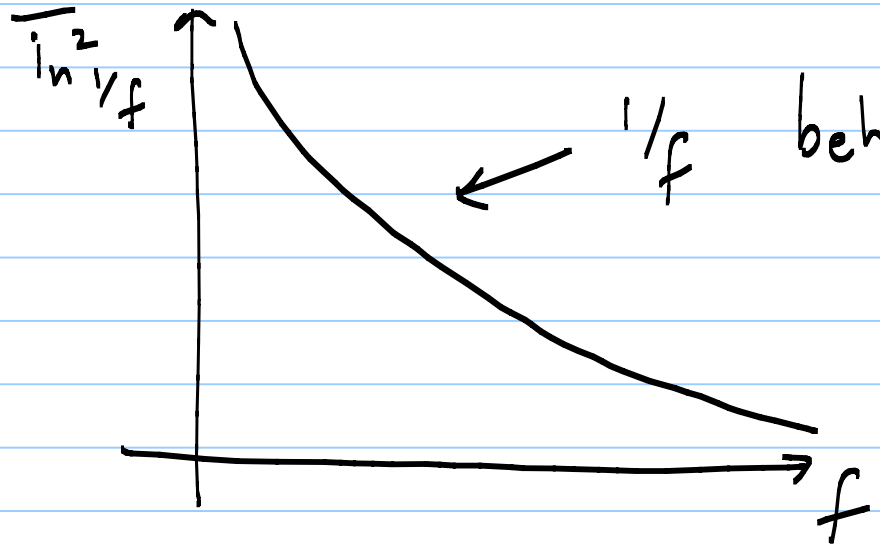
oxide cap



$$\frac{\overline{U_{n\,1/f}^2}}{\Delta f} = \frac{K_f}{W \cdot L \cdot C_{ox}^2} \cdot \frac{1}{f}$$

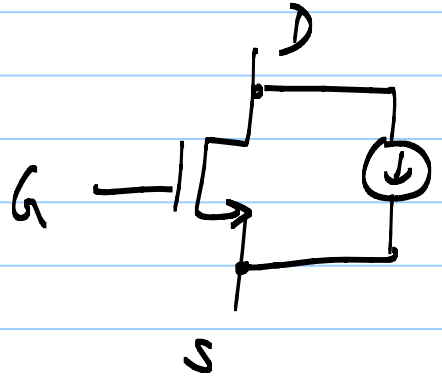
K_f → empirical constant

$W \cdot L$ → area of gate



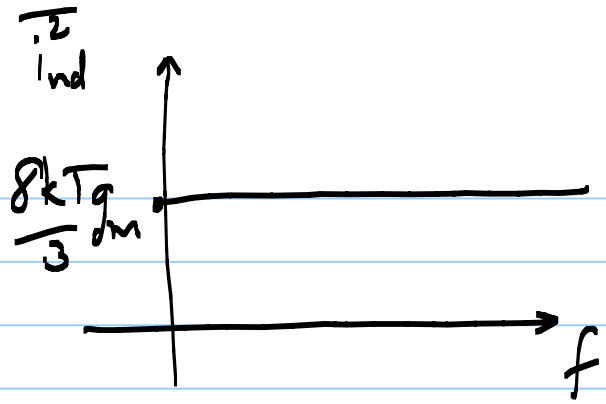
* 1/f noise is significant only at low frequencies

(b) Drain thermal noise

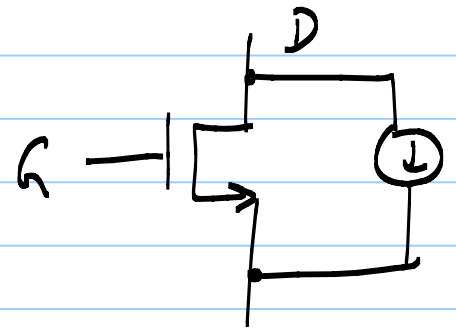


$$\frac{\overline{i_{nd}^2}}{\Delta f} = \frac{8kT}{3} g_m$$

→ White noise

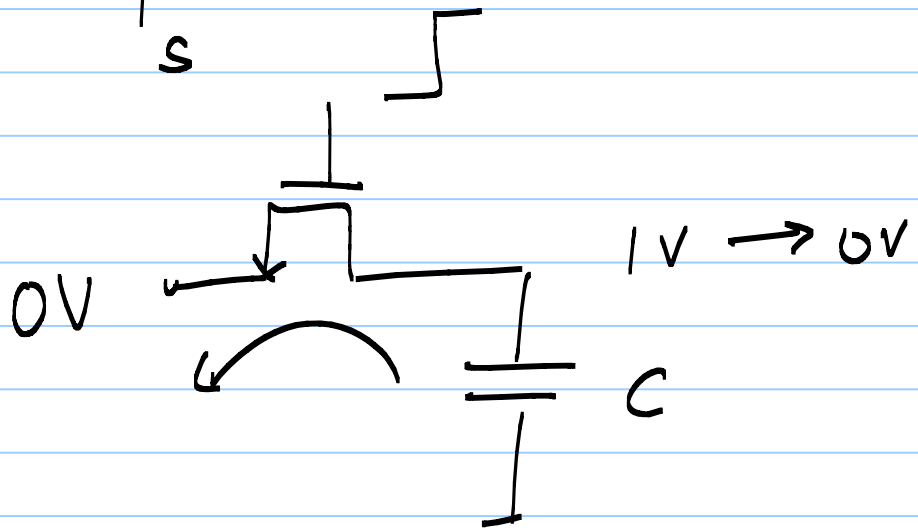


4) MOSFET thermal noise in triode



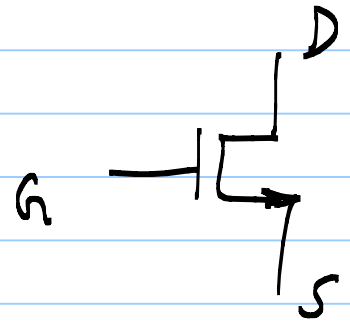
$$\frac{\overline{i_{nd}^2}}{\Delta f} = 4kT\, g_{ds}$$

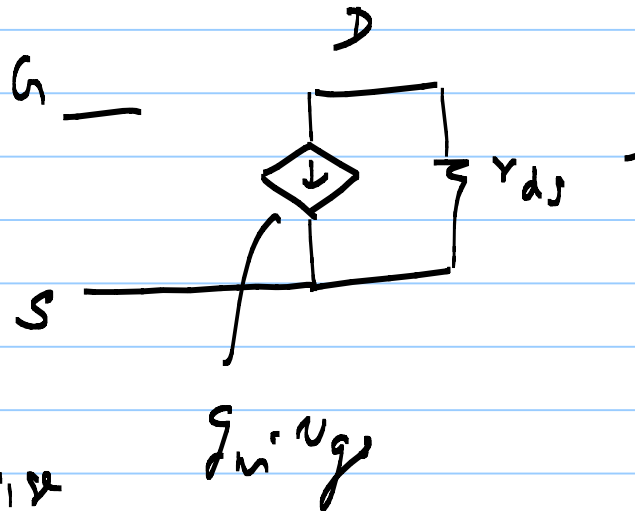
similar to resistor



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Lecture 3





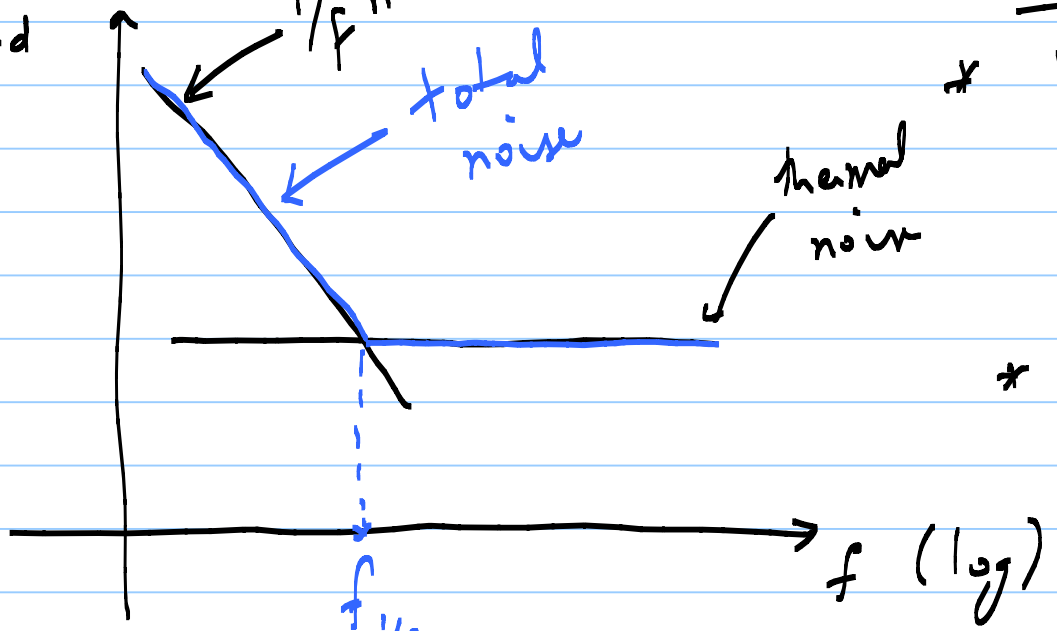
→ not a real resistor does not produce noise

$\overline{i_n^2} = \overline{i_{n,1/f}^2} + \overline{i_{nd}^2}$

(log)

w

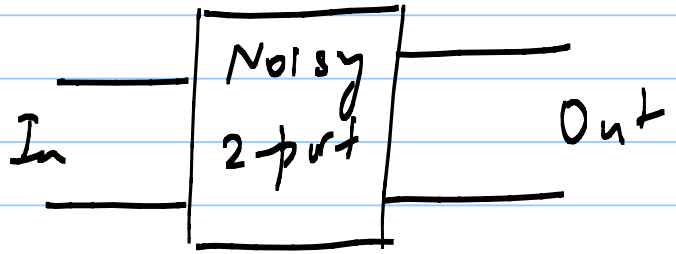
dB



* $\overline{i_{n,1/f}}$ and $\overline{i_{nd}}$ are uncorrelated

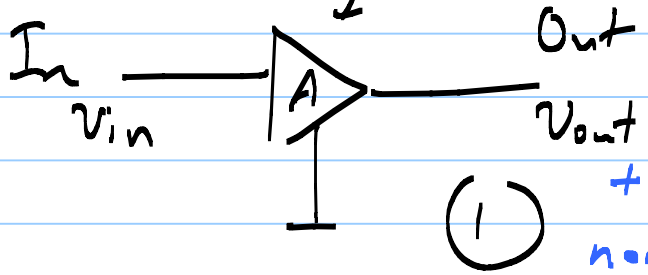
* Add as mean squared quantities

Noisy 2-port networks

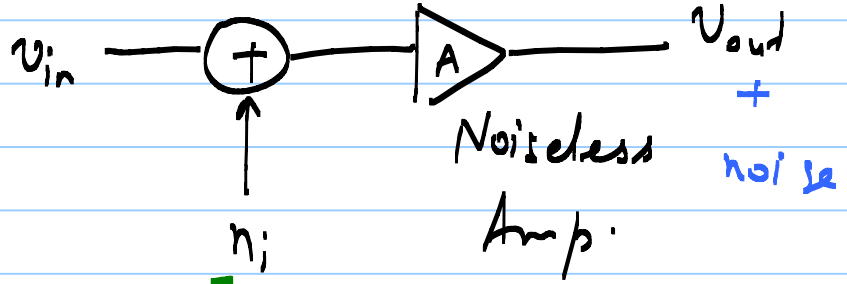


R, L, C, Mosfets

e.g. noisy amplifier

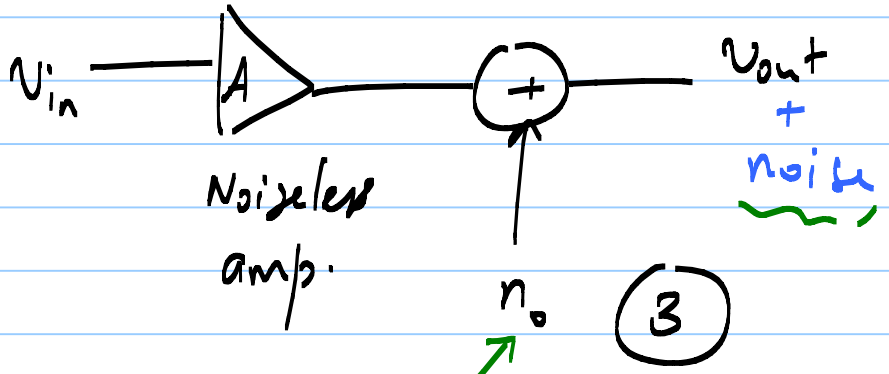


(1)



Input referred noise

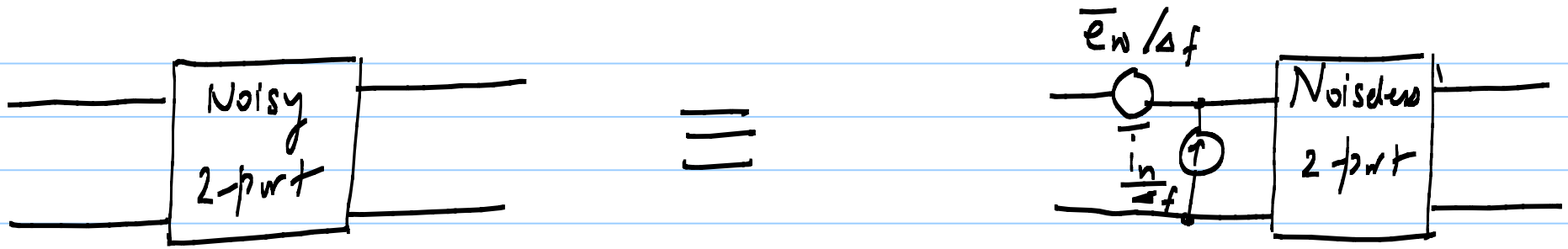
(2)



(3)

Output referred noise

↳ allows you to determine SNR degradation @ input



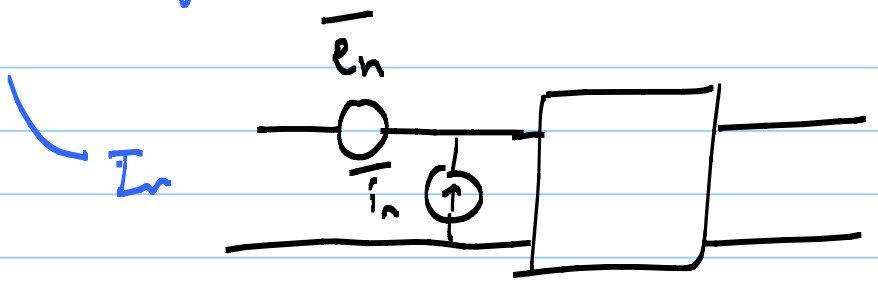
$\bar{e}_n/\sqrt{\Delta f}$ — $V/\sqrt{Hz}$ $\qquad$ $\bar{e}_n^2/\Delta f$ — V^2/Hz

$\bar{i}_n/\sqrt{\Delta f}$ — $A/\sqrt{Hz}$ $\qquad$ $\bar{i}_n^2/\Delta f$ — A^2/Hz

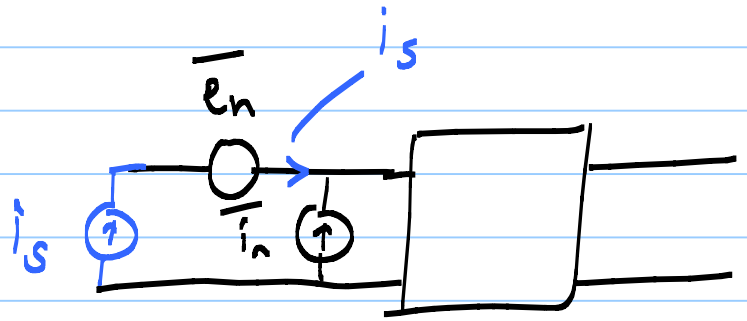
* Need both $\bar{e}_n$ & $\bar{i}_n$ to represent the input referred noise of a noisy 2-port

* Any polarity assumed for noise sources is only "nominal", unless noise sources are correlated

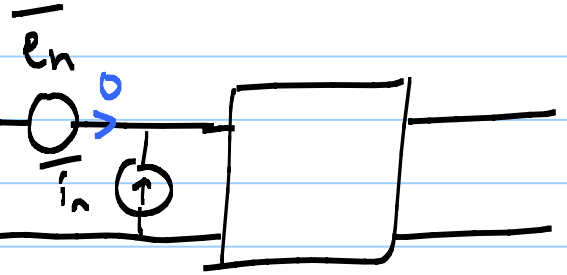
represent by Thevenin or Norton eq.



* In general, $\bar{e}_n$ & $\bar{i}_n$ can be uncorrelated, fully or partially correlated.



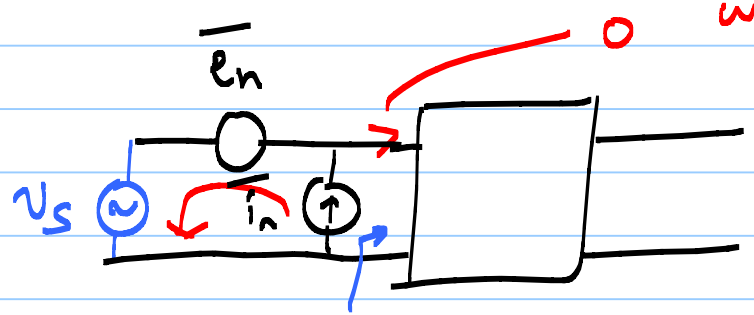
* Noise sources may be assumed to be uncorrelated with signal sources



No portion of $\bar{e}_n$ appears at output

i_s replaced with open circuit

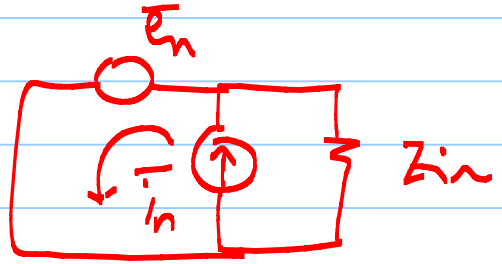
* $\bar{e}_n$ & $\bar{i}_n$ are properties of the 2 port



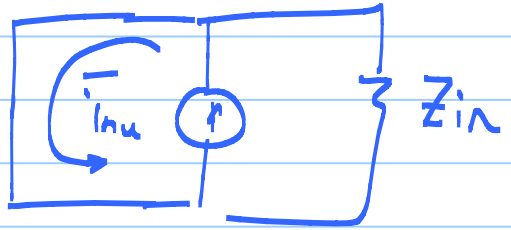
current into 2-port due to $\bar{i}_n$

← No part of $\bar{i}_n$ appears @ output

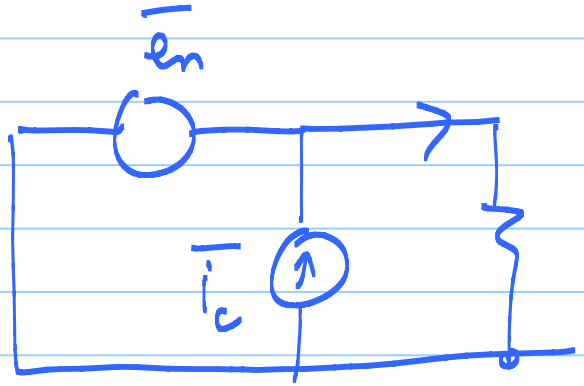
finite Z_{in}

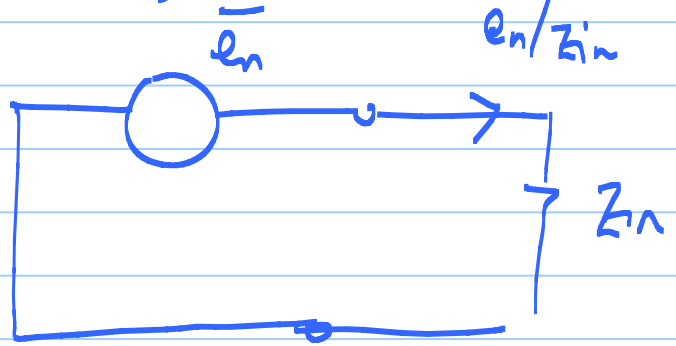


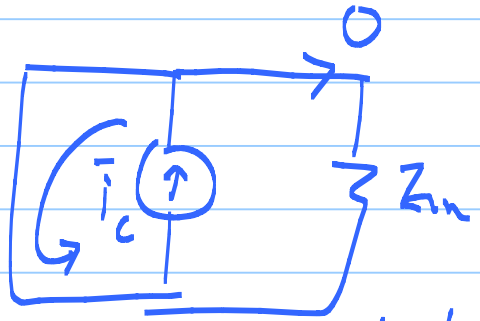
$$\bar{i}_n^2 = \bar{i}_{n_c}^2 + \bar{i}_{n_u}^2$$



→ add mean squared noise (uncorrelated)

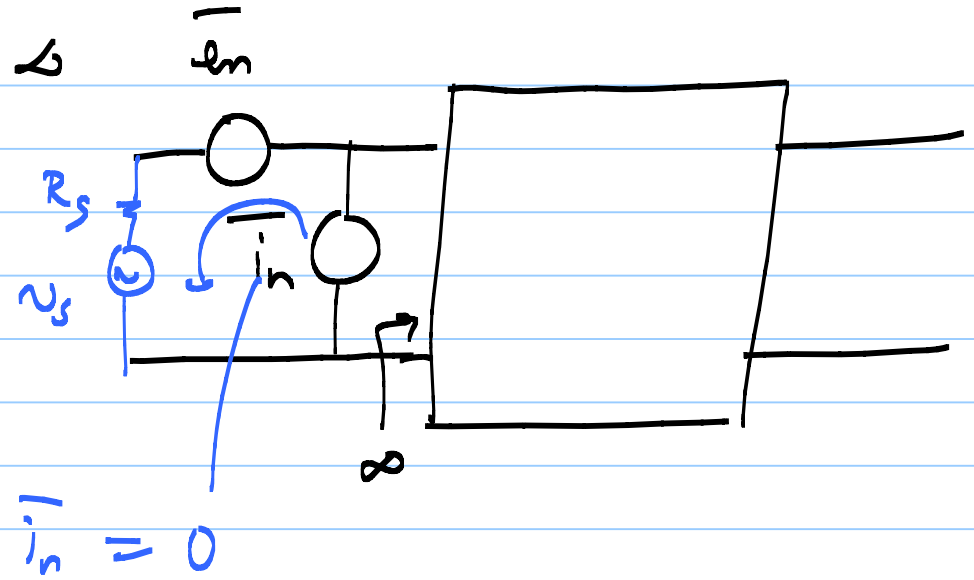


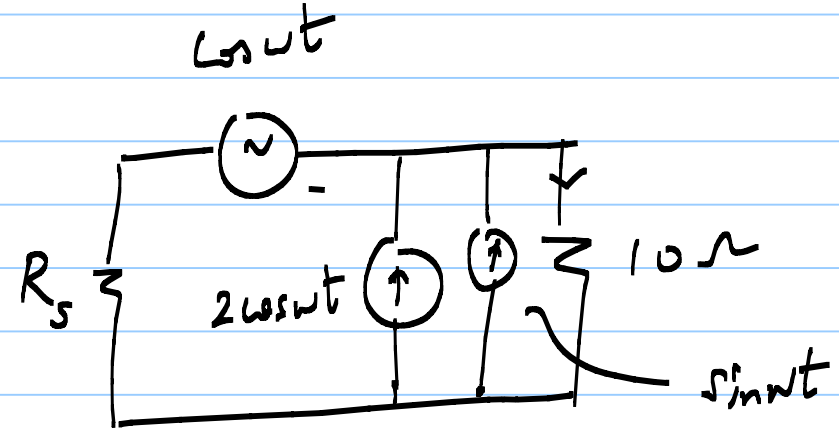


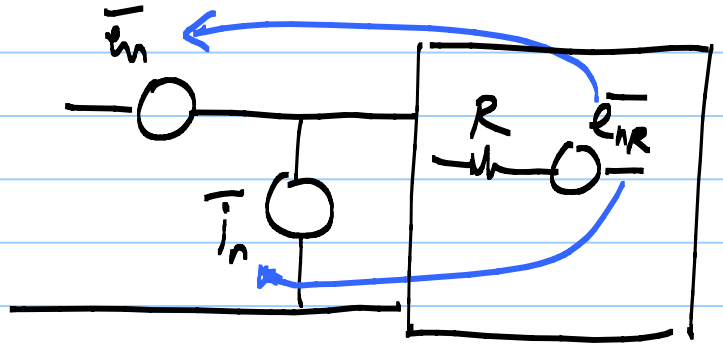


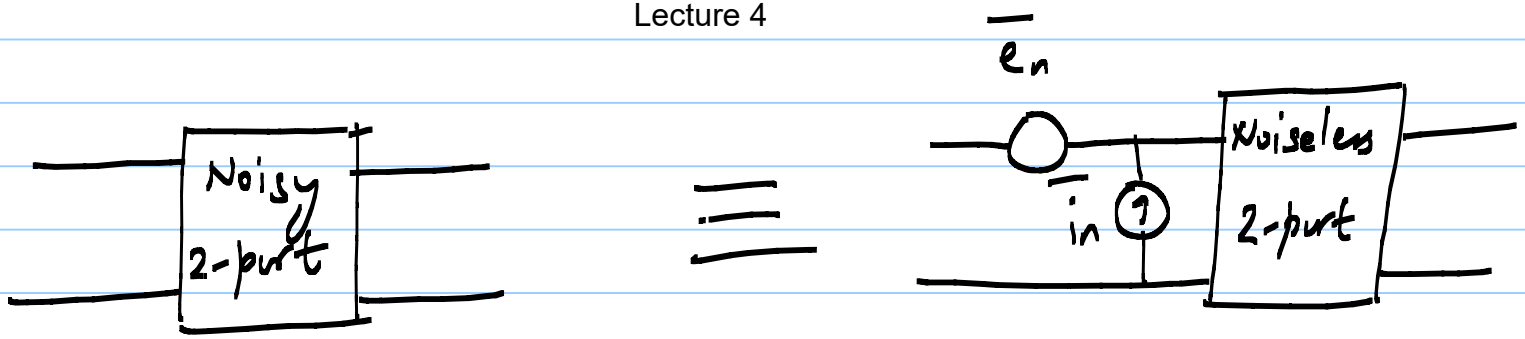
→ add noise amplitudes & then square

e.g. $Z_{th} = \infty$

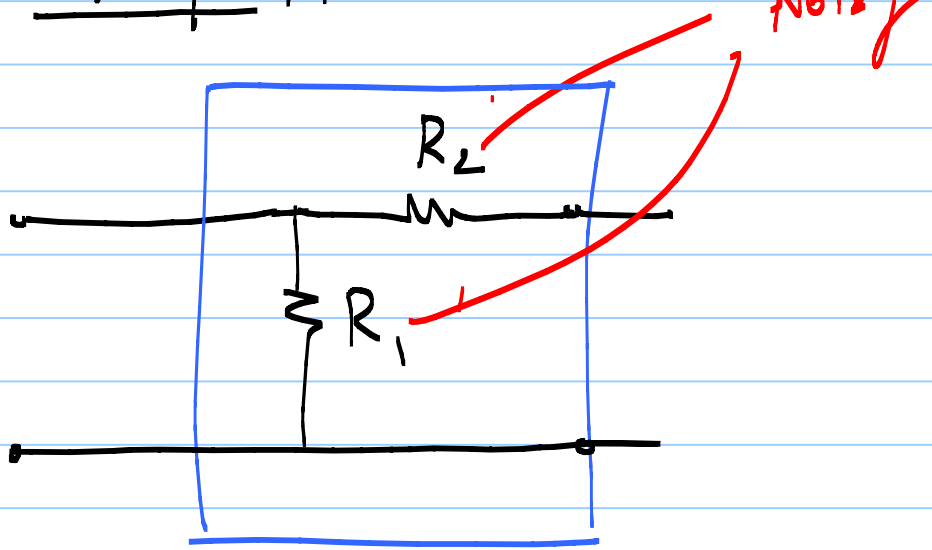

$\bar{e}_n$
R_s
v_s
$\bar{i}_n$
∞
$\bar{i}_n = 0$


cos ωt
R_s
2 cos ωt
10 Ω
sin ωt

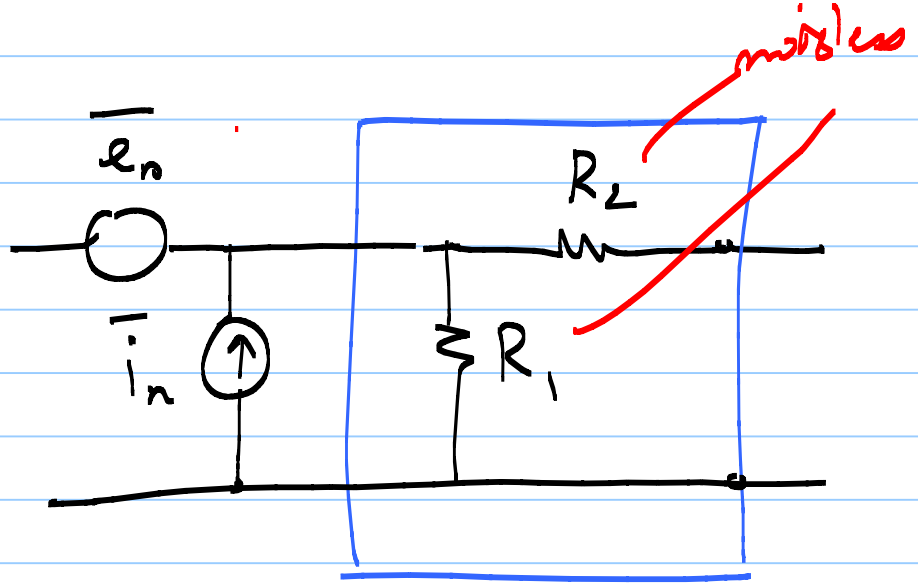

$\bar{e}_n$
$\bar{i}_n$
R
$\bar{e}_{nR}$



Example 1:

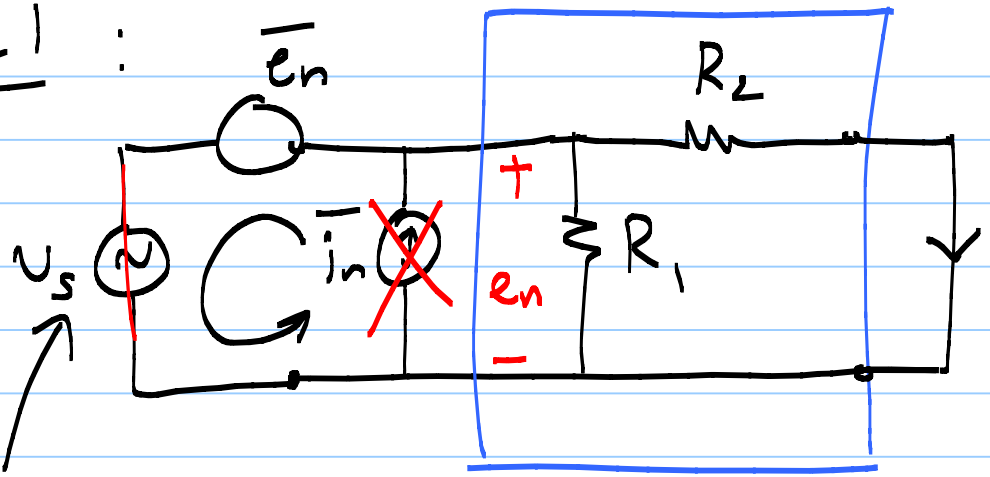


Noisy 2-port



Noiseless 2-port

case 1 :

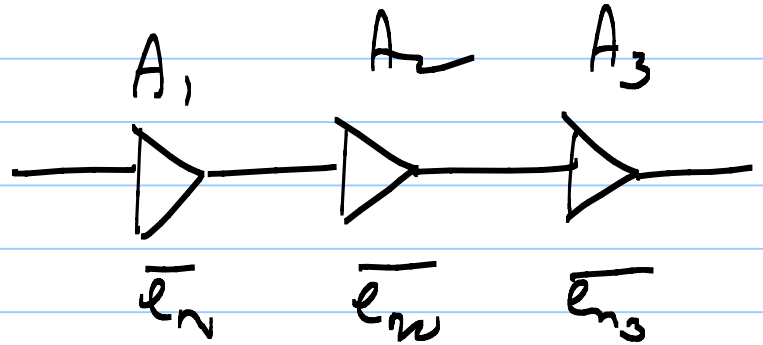


$\bar{v}_{on_1} = f(\bar{e}_n \text{ only})$

$= \dfrac{\bar{e}_n}{R_2}$

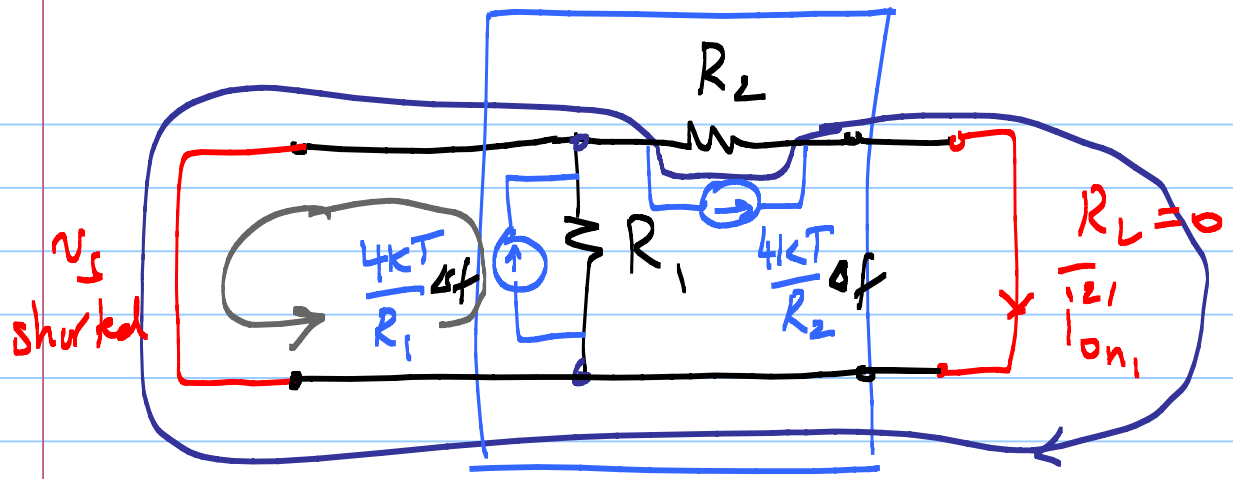
Short v_s for noise analysis

Noiseless 2-port



$v_{on} = f(\bar{e}_{n_1}, \bar{e}_{n_2}, \bar{e}_{n_3})$

refer to input



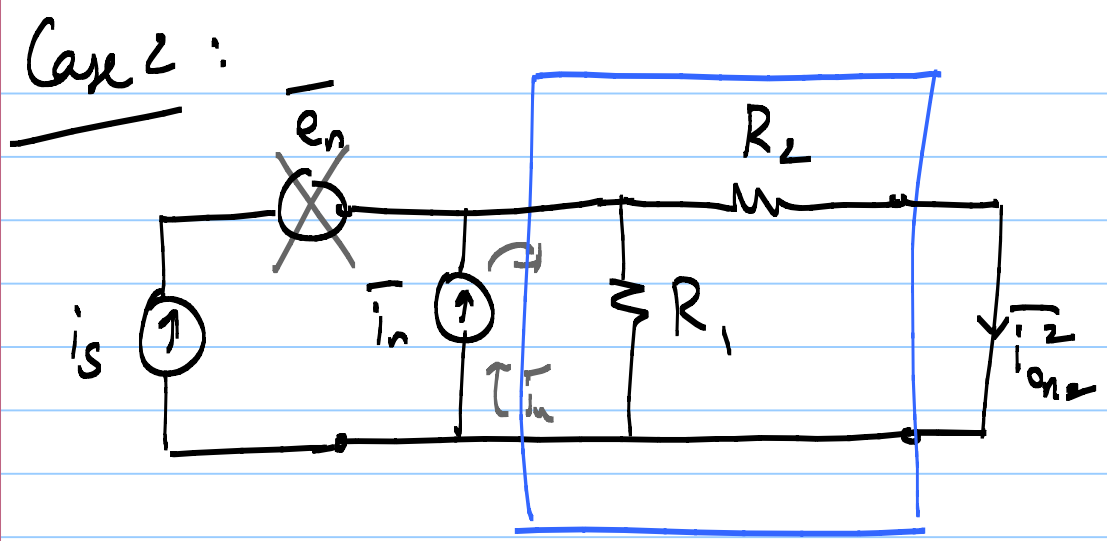
$$\overline{i_{on_1}^2} = \frac{4kT}{R_2}\Delta f$$

Noisy 2-port

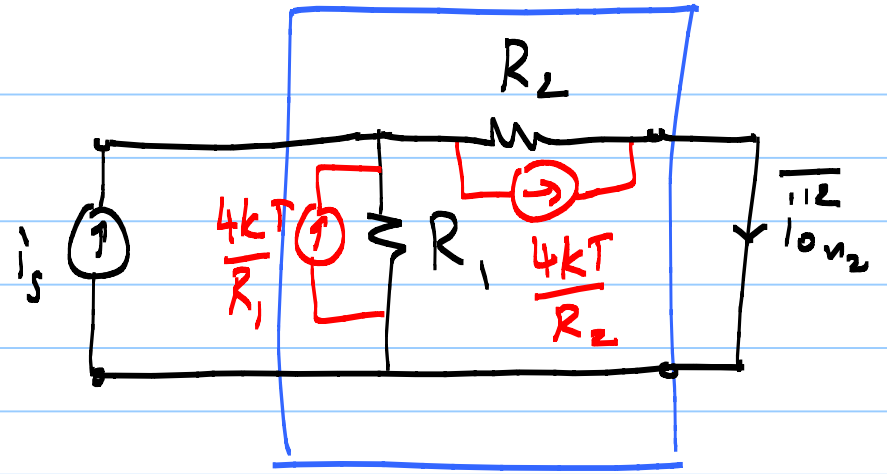
$$\overline{i'^2_{on_1}} = \overline{i^2_{on_1}} \quad \Bigg\} \quad \frac{\overline{e_n^2}}{R_2^2} = \frac{4kT}{R_2}\Delta f$$

$$\boxed{\frac{\overline{e_n^2}}{\Delta f} = 4kT R_2 \quad V^2/Hz}$$

Case 2 :



Noiseless 2-port



Noisy 2-port

$$\frac{\overline{i_{on_2}^2}}{\Delta f} = \frac{\overline{i_n^2} \cdot R_1^2}{(R_1+R_2)^2}$$

$$\frac{\overline{i_{on_2}^2}}{\Delta f} = \frac{4kT}{R_1} \cdot \left(\frac{R_1}{R_1+R_2}\right)^2 + \frac{4kT}{R_2} \cdot \left(\frac{R_2}{R_1+R_2}\right)^2$$

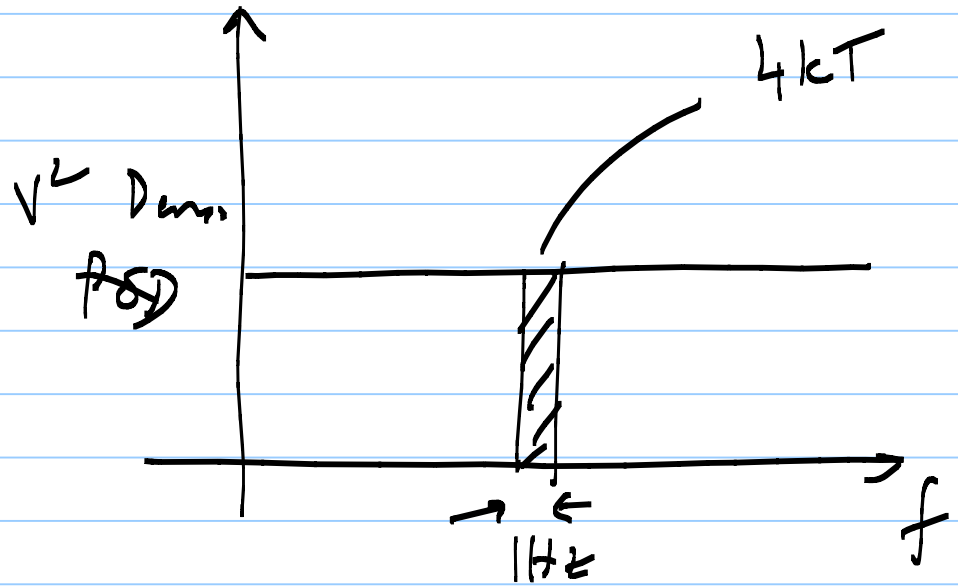
$$\overline{i_{on_2}^{\prime 2}} = \overline{i_{on_2}^2}$$

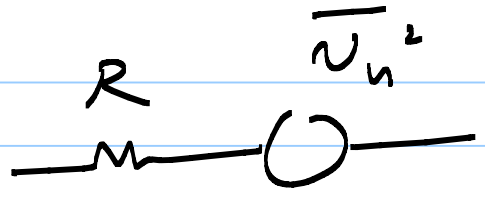
HW1

$$\frac{\overline{i_n^2}}{\Delta f} = (\quad) \quad A^2/Hz$$

10/2/2021

Lecture 5





$$\text{Noise power} = \int_{f_1}^{f_2} PSD\, df$$

$$M.S.V = \int_{f_1}^{f_2} (4kTR)\, df$$

$\bar{e}_n$, $\bar{i}_n$, $\bar{v}_n$ etc $\rightarrow$ RMS $\rightarrow |Z|$

$\overline{e_n^2}$, $\overline{i_n^2}$, $\overline{v_n^2}$ etc. $\rightarrow$ M.S. $\rightarrow |Z|^2$

$$\overline{i_{on_2}^2} = \overline{i_{on_2}'^2}$$

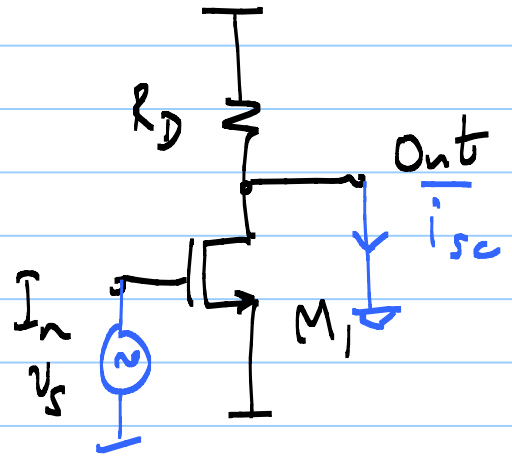
$$\frac{\overline{i_n^2}}{\Delta f}\left(\frac{R_1}{R_1+R_2}\right)^2 = \frac{4kT}{R_1}\left(\frac{R_1}{R_1+R_2}\right)^2 + \frac{4kT}{R_2}\left(\frac{R_2}{R_1+R_2}\right)^2$$

$$\frac{\overline{i_n^2}}{\Delta f} = \frac{4kT}{R_1} + \frac{4kT}{R_2}\left(\frac{R_2}{R_1}\right)^2$$

$$\frac{e_n^2}{\Delta f} = 4kTR_2$$

partially correlated

Example 2 — Common Source Amplifier @ low freq.



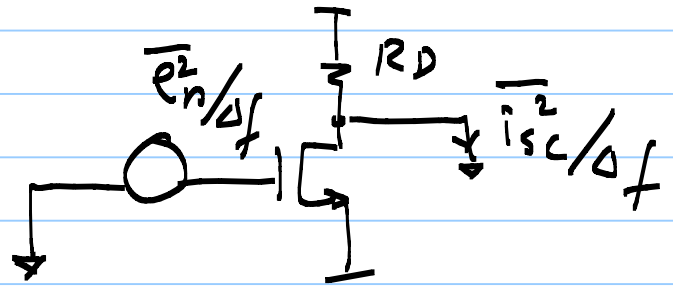
$$Z_{in} = \infty \quad \text{(at low freq.)}$$

$$\frac{\overline{i_n^2}}{\Delta f} = 0$$

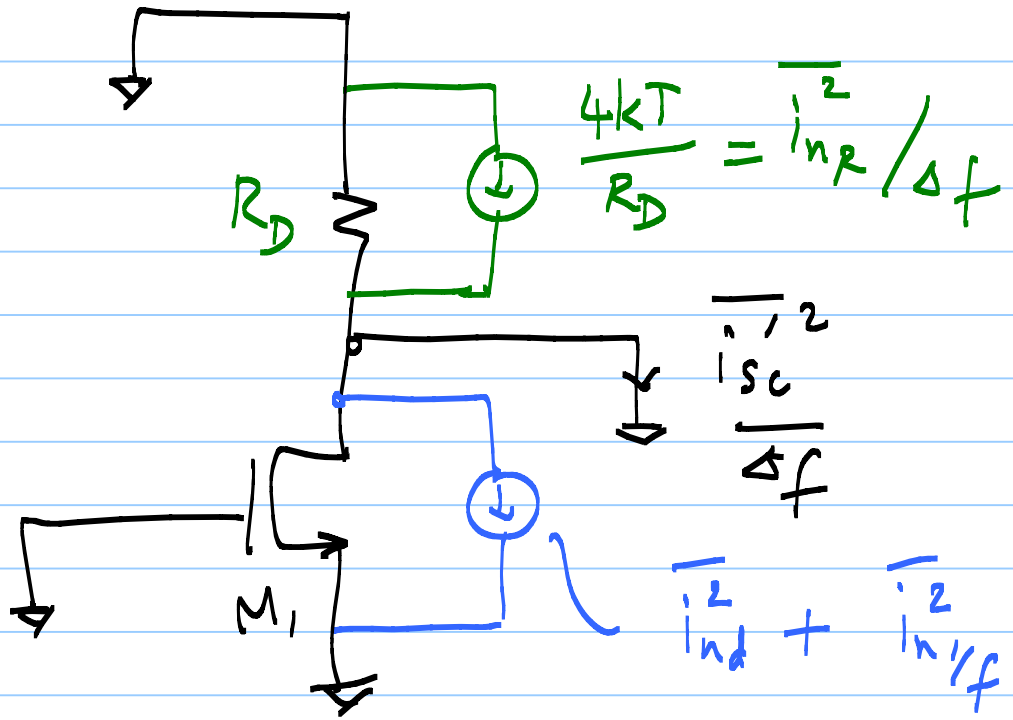
$$\frac{\overline{e_n^2}}{\Delta f} = ?$$

Input port: Drive with ideal v_s

Output port: choose $R_L = 0$



$$\frac{\overline{i_{sc}^2}}{\Delta f} = g_m^2 \frac{\overline{e_n^2}}{\Delta f}$$



$$\frac{\overline{i_{nd}^2}}{\Delta f} = \frac{8kT}{3} g_{m_1}$$

$$\frac{\overline{i_{n1/f}^2}}{\Delta f} = \frac{K_f g_{m_1}^2}{WL C_{ox}^2} \cdot \frac{1}{f}$$

$$\frac{\overline{i_{sc}'^2}}{\Delta f} = \frac{\overline{i_{nR}^2}}{\Delta f} + \frac{\overline{i_{nd}^2}}{\Delta f} + \frac{\overline{i_{n1/f}^2}}{\Delta f}$$

$$\overline{i_{sc}^2} = \overline{i_{sc}'^2}$$

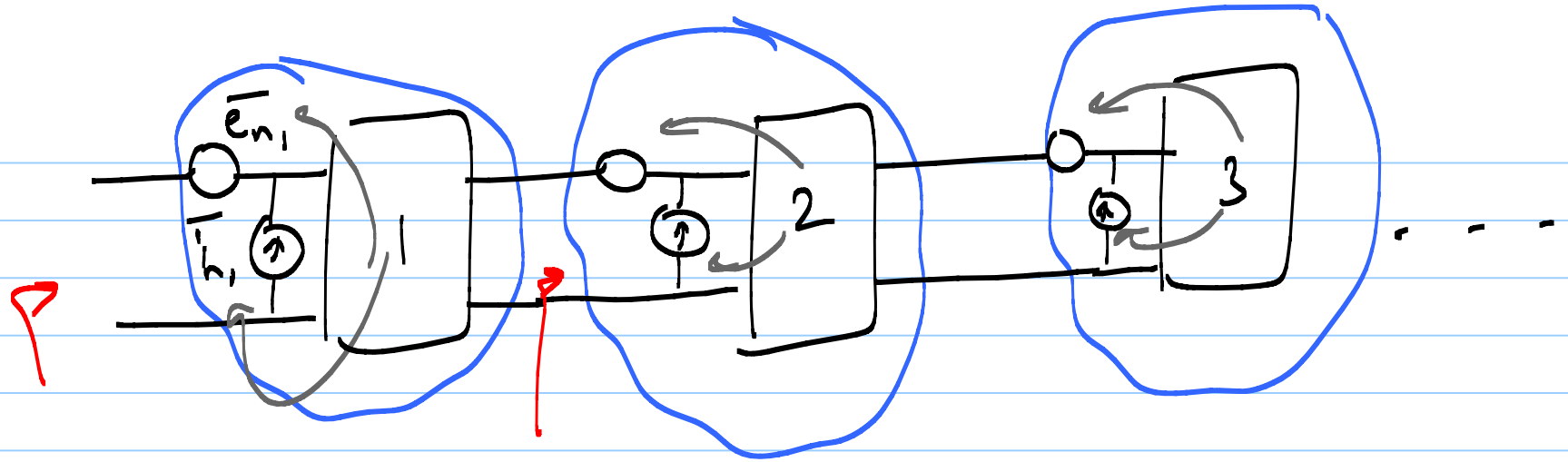
$$g_{m_1}^2 \overline{e_n^2} = \overline{i_{nR}^2} + \overline{i_{nd}^2} + \overline{i_{n1/f}^2}$$

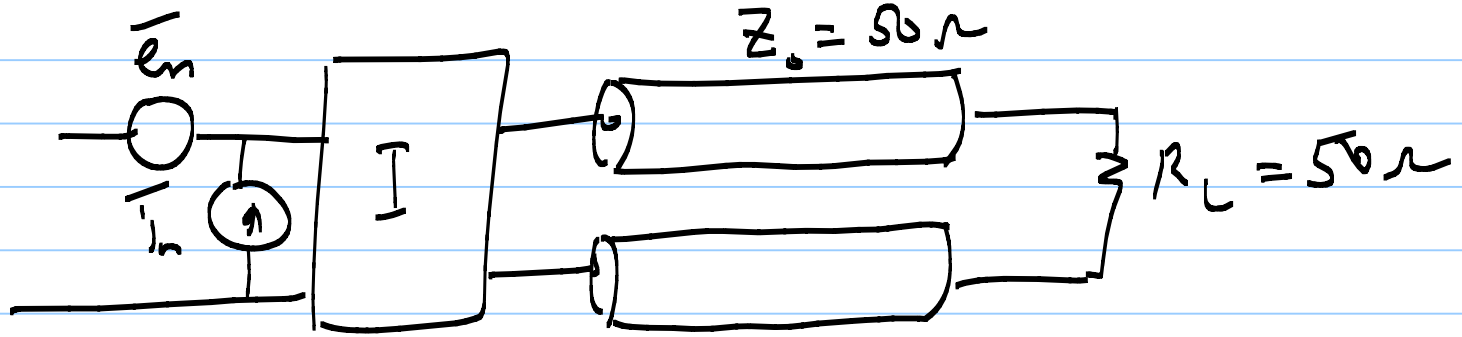
$$\frac{\overline{e_n^2}}{\Delta f} = \frac{\overline{i_{nR}^2}/\Delta f + \overline{i_{nd}^2}/\Delta f + \overline{i_{n1/f}^2}/\Delta f}{g_{m_1}^2}$$

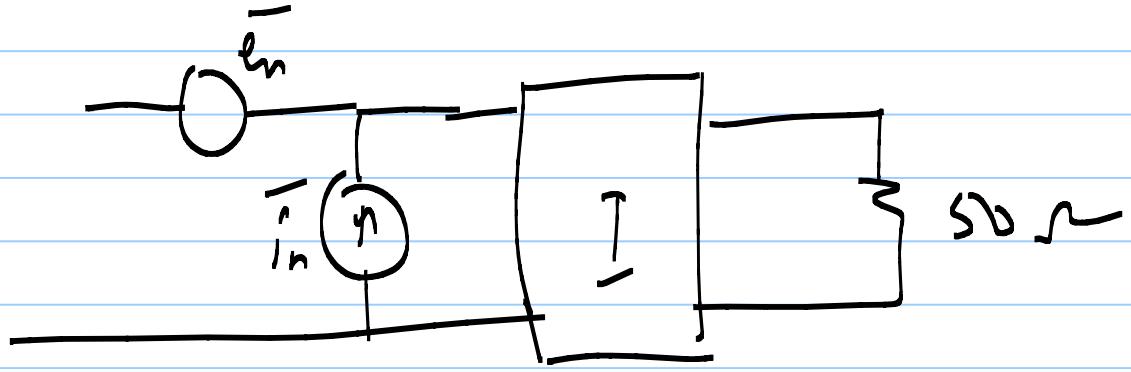
$$\frac{\overline{e_n^2}}{\Delta f} = \frac{4kT}{g_{m_1}^2 R_D} + \frac{8kT}{3 g_{m_1}} + \frac{K_f}{W_1 L_1 C_{ox}^2} \cdot \frac{1}{f}$$

HW2 — Open circuit output ($R_L = \infty$) and find out $\frac{\overline{e_n^2}}{\Delta f}$

{ you should get same answer }


$\bar{e}_{n_1}$
$\bar{i}_{n_1}$
1
2
3


$\bar{e}_n$
$\bar{i}_n$
I
$Z_0 = 50\,\Omega$
$R_L = 50\,\Omega$

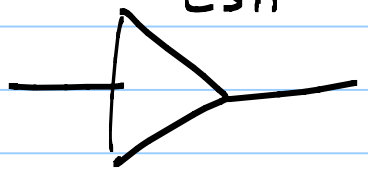

$\bar{e}_n$
$\bar{i}_n$
I
50 Ω

11/2/2021

Lecture 6

CSA :

$$\frac{\overline{e_n^2}}{\Delta f} = \frac{4kT}{g_{m_1}^2 R_D} + \frac{8kT}{3 g_{m_1}} + \frac{K_f}{W_1 L_1 C_{ox}^2} \cdot \frac{1}{f}$$

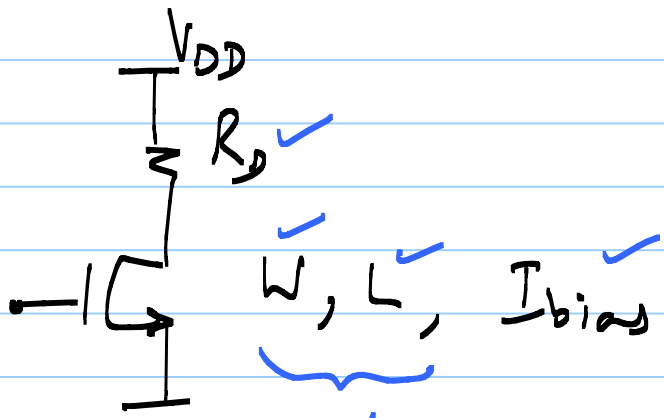


gain ✓, BW ✓, swing ✓ limits,

noise → PSD (w)

→ Integrated noise

⋮



Once designed:

$\frac{\overline{e_n}}{\Delta f}$ is too large

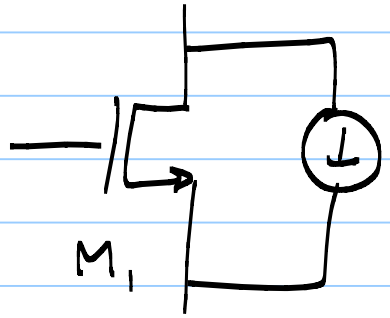
What to do?

$$\frac{\overline{e_n^2}}{\Delta f} = \frac{4kT}{g_{m_1}(g_{m_1} R_D)} + \frac{8kT}{3 g_{m_1}} + \frac{k_f}{W_1 L_1 C_{ox}^2} \cdot \frac{1}{f}$$

gain

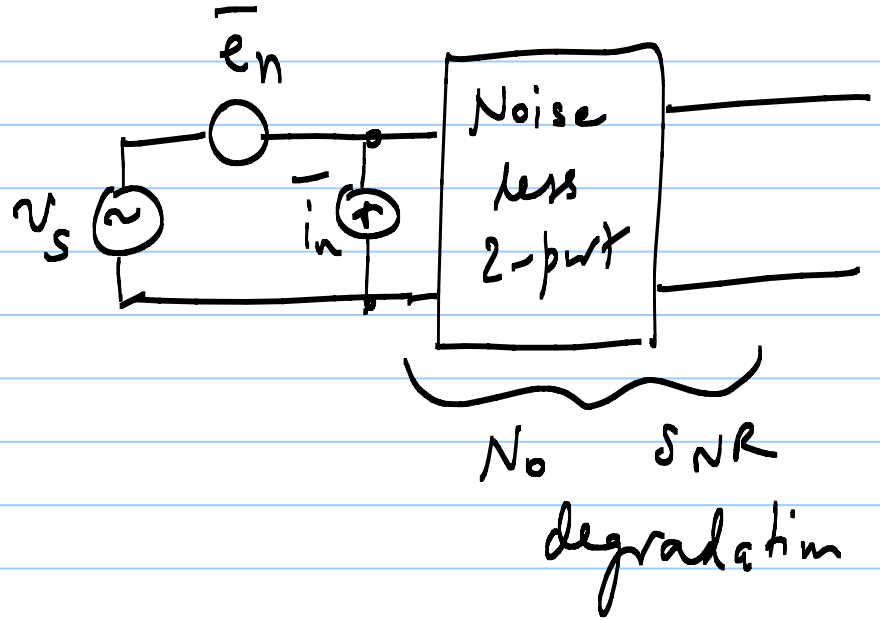
↓ 1/f noise by ↑ gate area (keep g_m constant)

thermal noise flow is inversely related to g_{m_1}

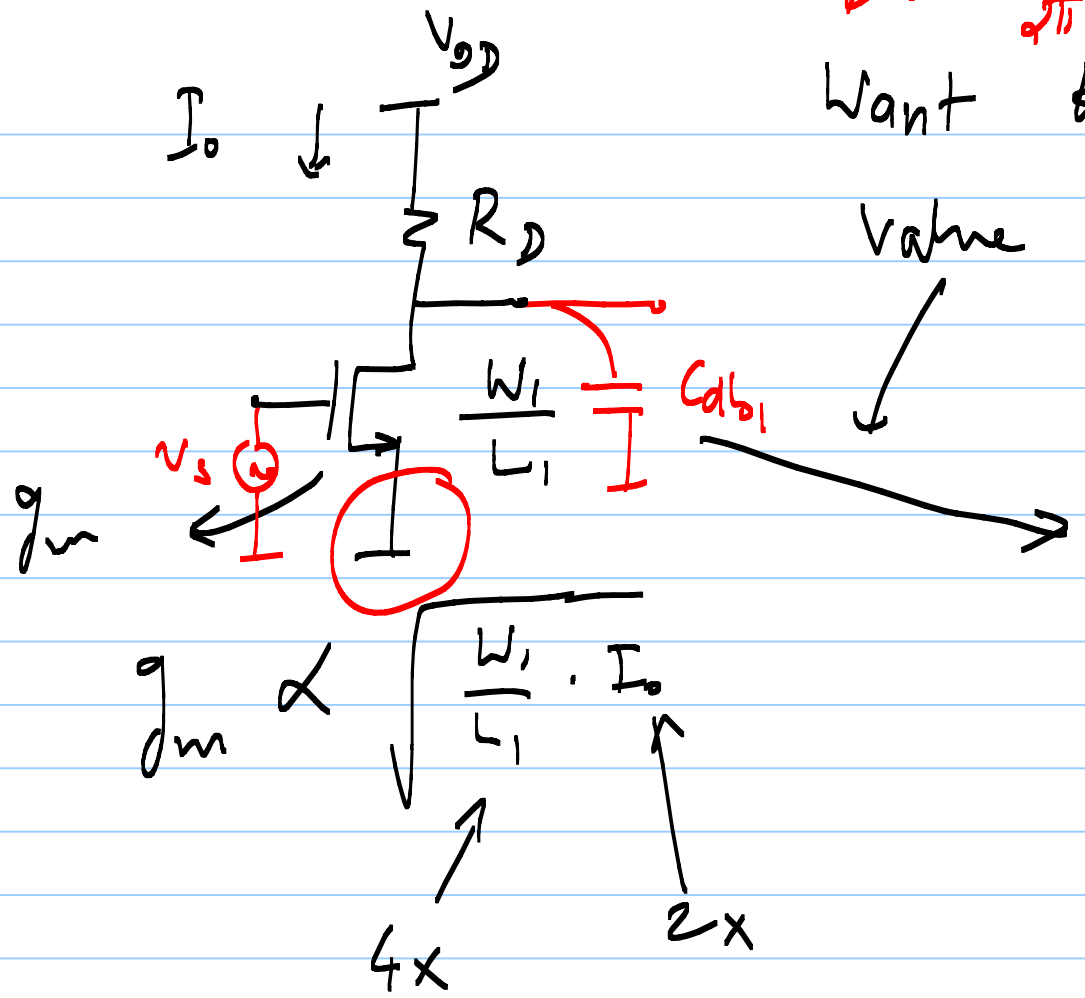


$$\frac{\overline{i_{nd}^2}}{\Delta f} = \frac{8kT}{3} g_m$$

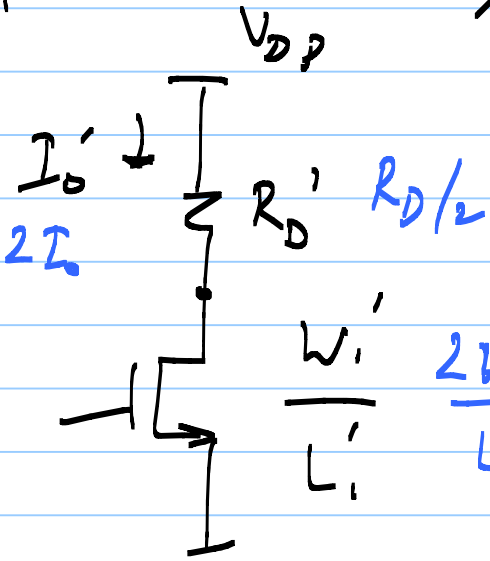
output signal power ↑ as g_m^2


$\bar{e}_n$
v_s
$\bar{i}_n$
Noise
less
2-port
N_o
SNR
degradation

$BW = \frac{1}{2\pi R_D C_{db_1}}$; $BW' = \frac{1}{2\pi \frac{R_D}{2} \cdot 2 C_{db_1}}$



Want to reduce $\overline{e_n^2}$ to half its value (thermal noise), keep other parameters the same



(gain, swing limits, etc.) BW

V_{GS} & V_{DS} remain the same

$g_m' = 2\sqrt{2} \cdot g_m$! ✗

$$g_m' = 2 g_m \Rightarrow \overline{e_n'^2} = \frac{\overline{e_n^2}}{2}$$

$4 \times \frac{W}{L}$ $\quad 4 \times I_D$ $\quad 2 \times \frac{W}{L}, \; 2 \times I_D$

$$g_m \propto \sqrt{I_D \cdot \frac{W}{L}}$$

$$\cdots \; g_m \propto \frac{2 I_D}{V_{D,sat}}$$

$$R_D' = \frac{R_D}{2} \quad \{ \text{keep gain same} \}$$

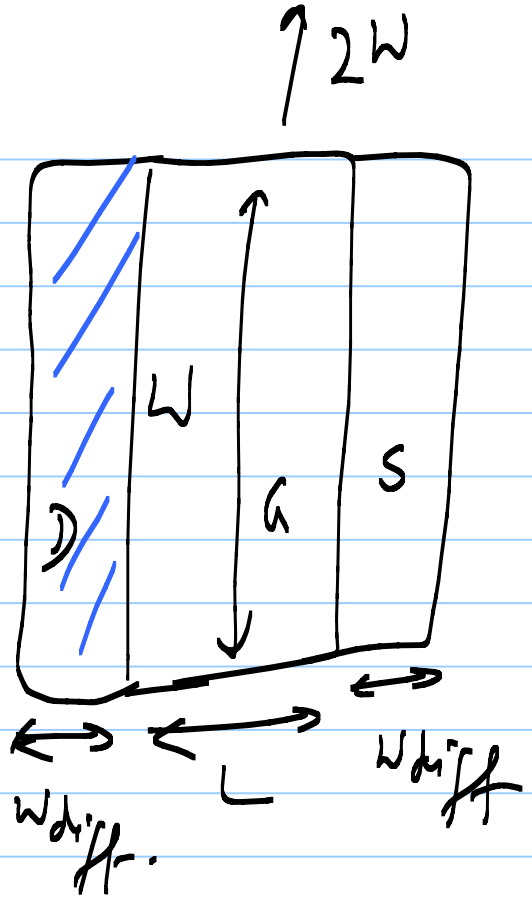
$\leftarrow V_{GS} - V_T$

$V_{D,sat}$ = same $\{$ keep swing limits same $\}$

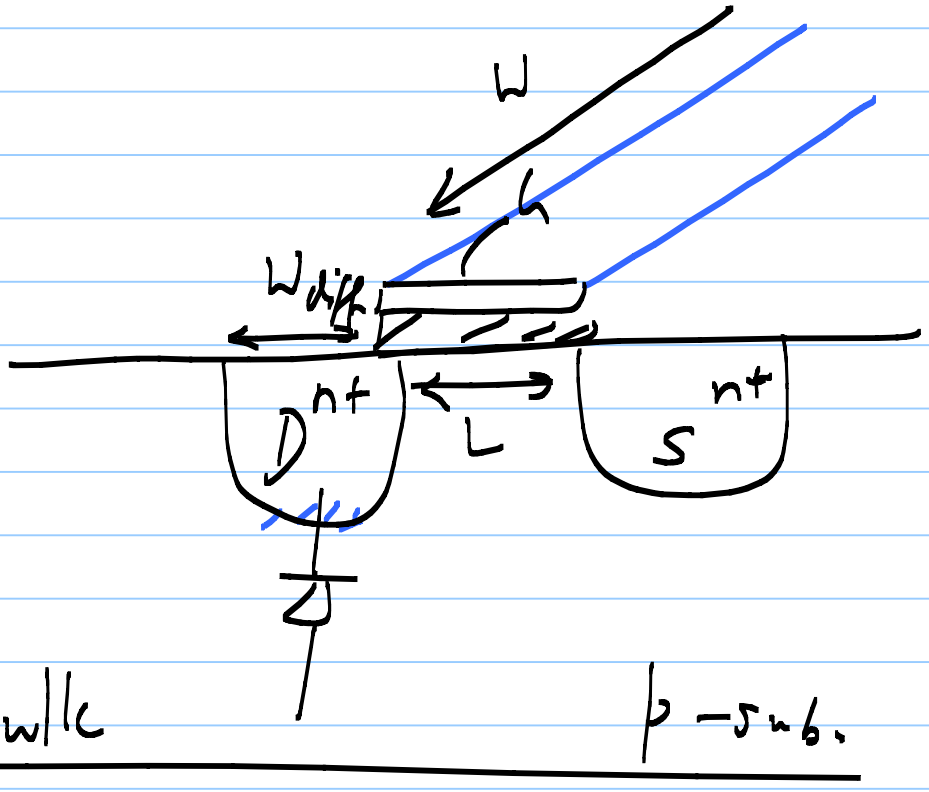
$\hookrightarrow \propto \sqrt{\frac{I_D}{(W/L)}} \rightarrow$ keep current density the same

"Noise Scaling"

Top view of MOSFET



Side view

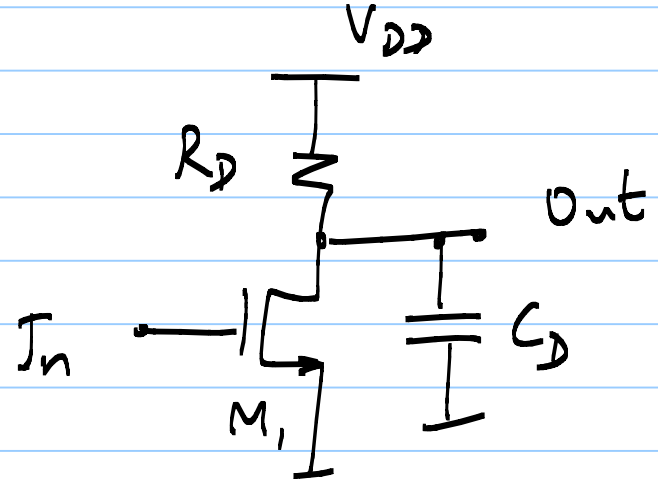


$C_{db} \propto W$

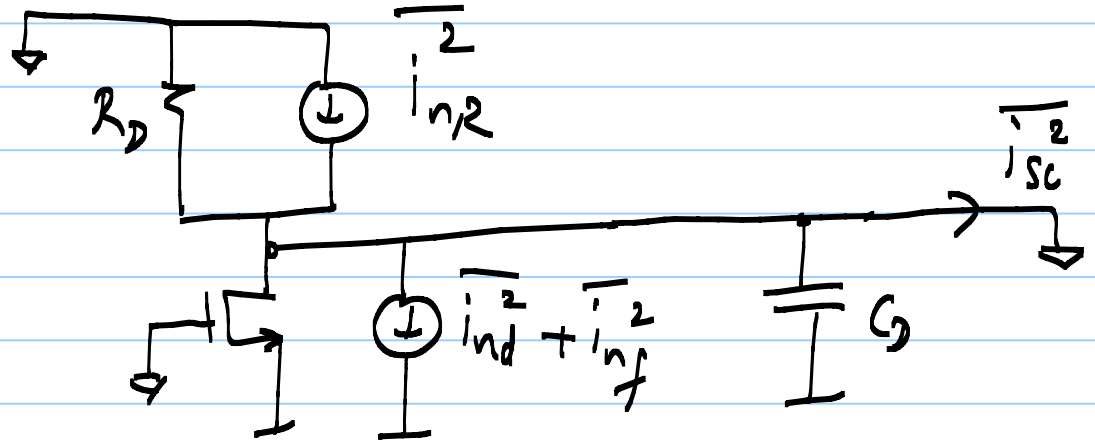
15|2|2021

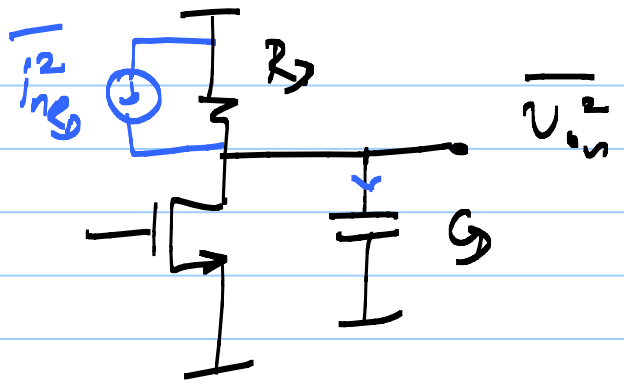
Lecture 7

Example 3



$\overline{e_n^2}$ = Same as without C_D

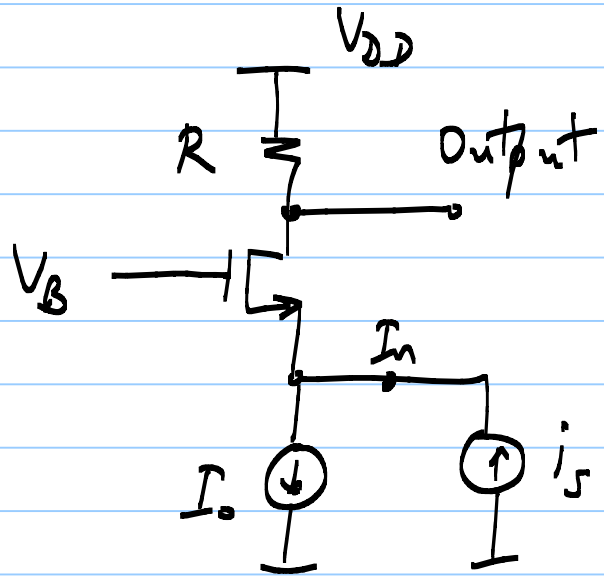


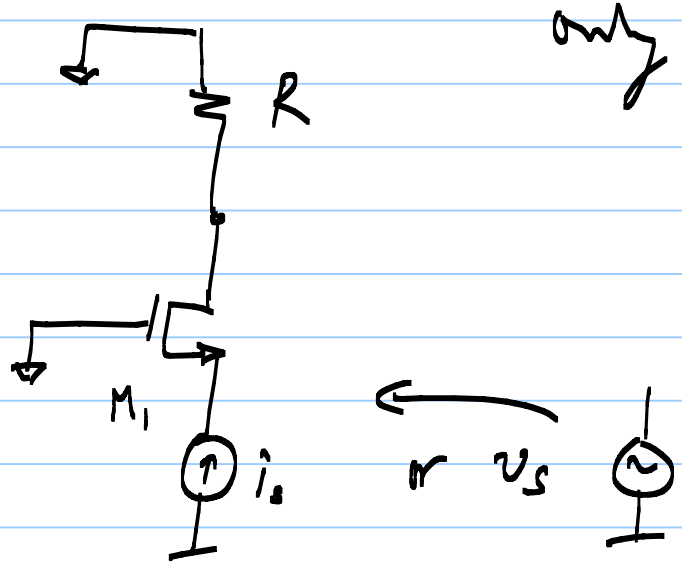


Use o.c. voltage to determine $\overline{e_n^2}$

HW3 : Do noise analysis & find $\overline{e_n}$ for CSA with C_D load

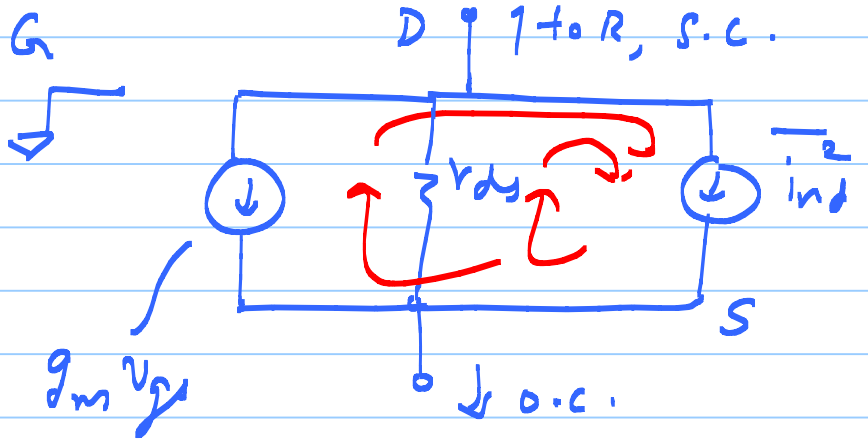
Example 4 : Common Gate Amplifier (CGA), thermal noise only





$\overline{e_n^2}$ = non-zero, finite

$\overline{i_n^2}$ = "



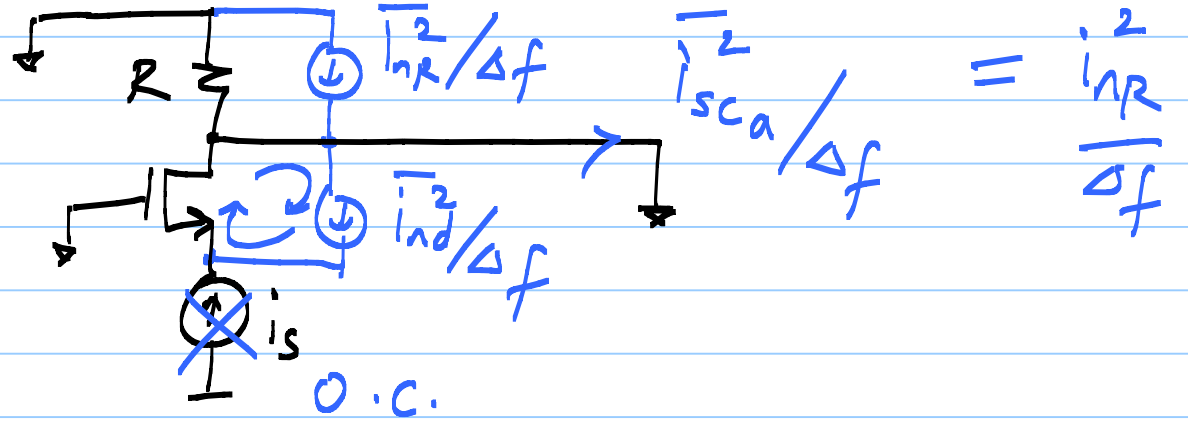
Case 1 – $\overline{i_n^2}$

S.C. output, note $\overline{i_{sc}^2}$

(a) – noisy circuit

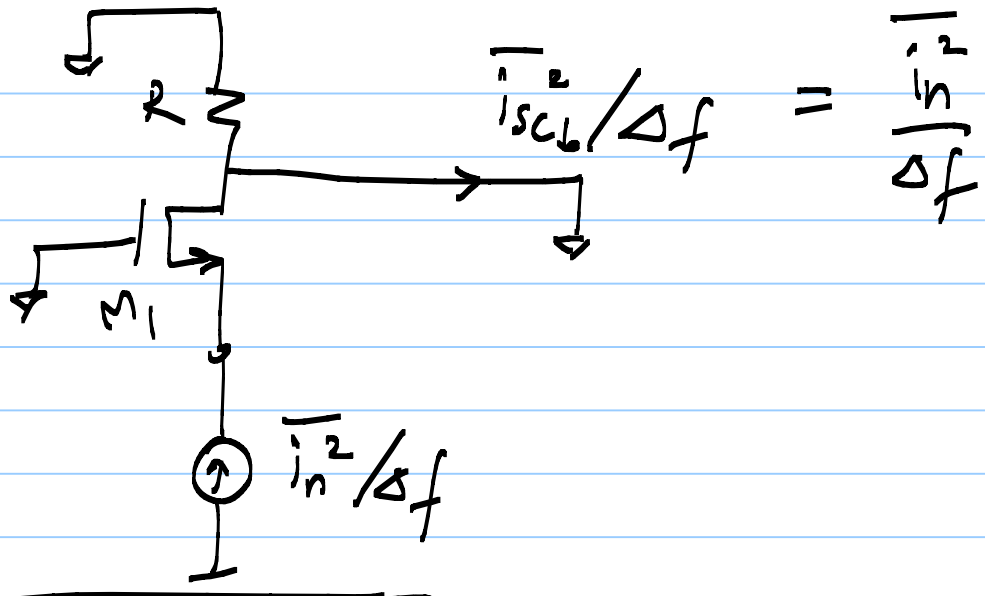
(b) – noiseless cir. with $\overline{i_n^2}$

(a)



$\overline{i_{sca}^2}/\Delta f = \dfrac{\overline{i_{nR}^2}}{\Delta f}$

(b)



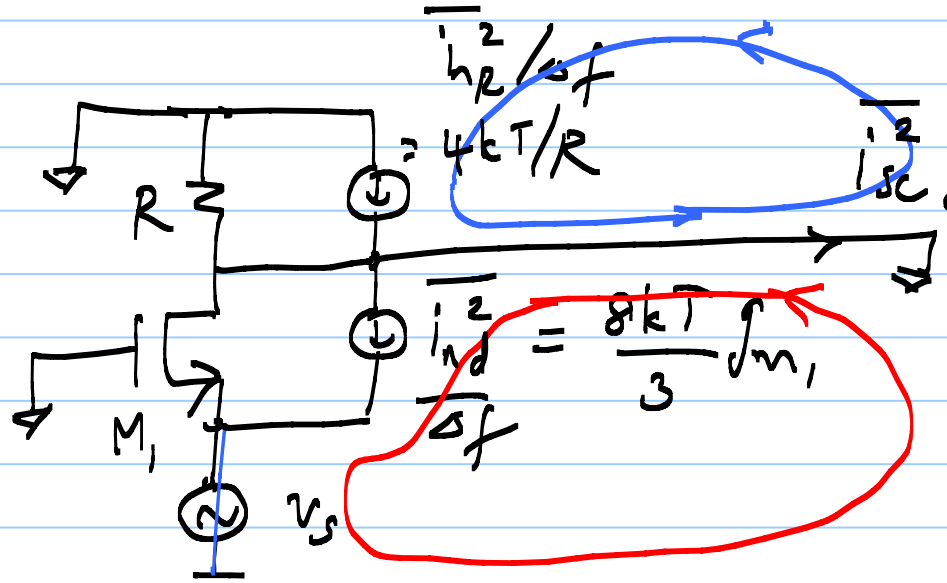
$$\frac{\overline{i_n^2}}{\Delta f} = \frac{4kT}{R}$$

Case 2 : $\overline{e_n^2}$, excite the circuit with ideal v_s

(c) noisy element

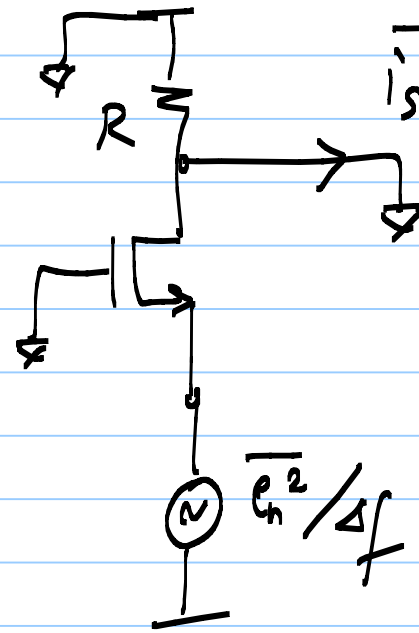
(d) $\overline{e_n^2}$ with noiseless circuit

(c)



$$\overline{i_{sc}^2}/\Delta f = \frac{4kT}{R} + \frac{8kT}{3} g_{m_1}$$

(d)



$$\frac{\overline{i_{sc_d}^2}}{\Delta f} = g_{m_1}^2 \frac{\overline{e_n^2}}{\Delta f}$$

$$\frac{\overline{i_{sc_c}^2}}{\Delta f} = \frac{\overline{i_{sc_d}^2}}{\Delta f}$$

$$g_{m_1}^2 \frac{\overline{e_n^2}}{\Delta f} = \frac{\overline{i_{n_R}^2} + \overline{i_{n_d}^2} + \overline{i_{n_f}^2}}{\Delta f}$$

$$g_{m_1}^2 \frac{\overline{e_n^2}}{\Delta f} = \frac{4kT}{R} + \frac{8kT}{3} g_{m_1}$$

$$\boxed{\frac{\overline{e_n^2}}{\Delta f} = \frac{4kT}{g_{m_1}^2 R} + \frac{8kT}{3 g_{m_1}}}$$

$\overline{e_n}$ partially correlated with $\overline{i_n}$

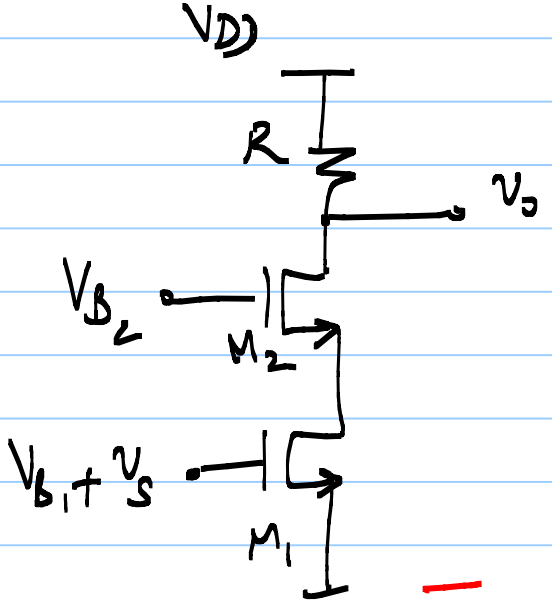
$+ \dfrac{K_f}{WL C_{ox}^2} \cdot \dfrac{1}{f}$ ← if you considered flicker noise too.

Example 5 — Cascode Amplifier (thermal noise only)

(HW - with 1/f noise)

HW - without approximations

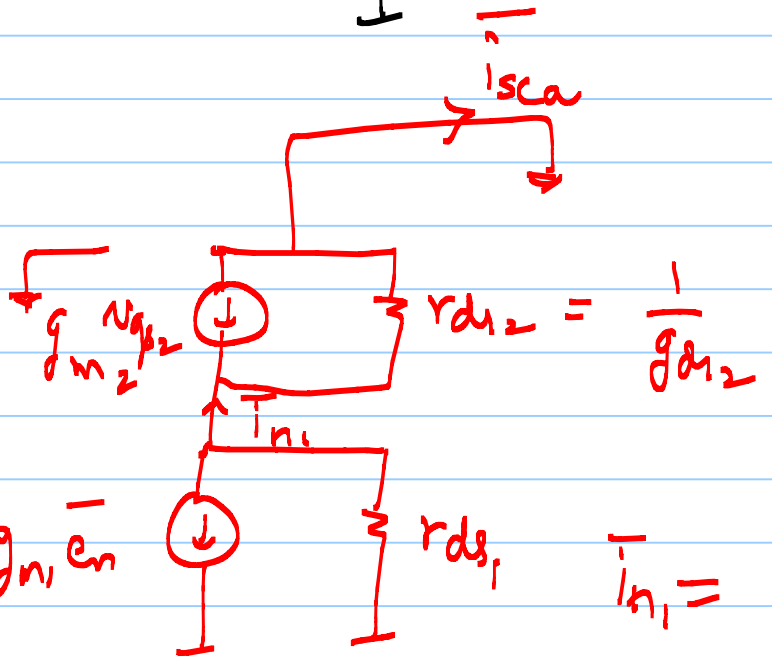
e.g. $g_m r_{ds} \gg 1$

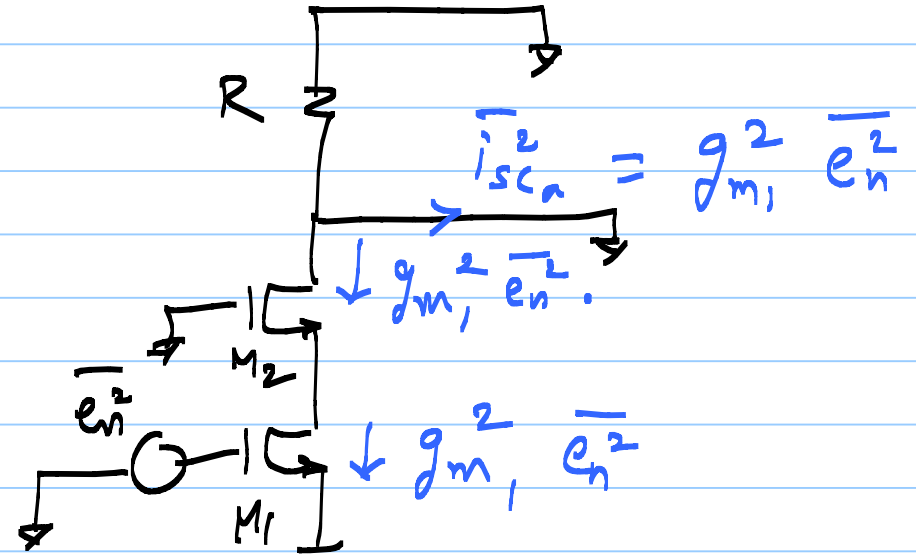


$\overline{i_n^2} = 0$

$\overline{e_n^2} = ?$

Case (a)



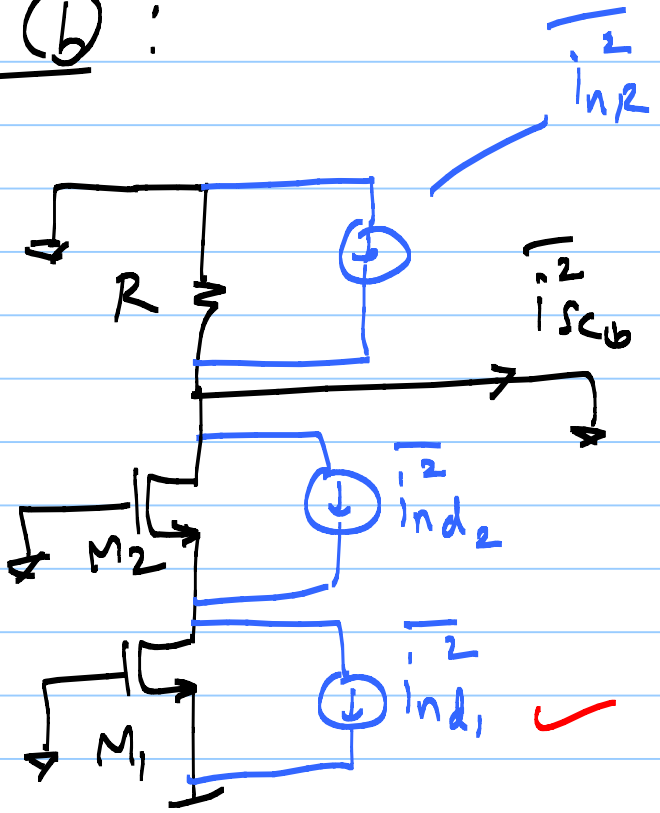


$$\overline{i_{n_1}} = \frac{r_{ds_1} \cdot g_{m_1} \overline{e_n}}{r_{ds_1} + \frac{1}{g_{m_2} + g_{ds_2}}} = \overline{i_{sca}}$$

$$\overline{i_{sC_a}^2} = \frac{g_{m_1}^2 \, r_{ds_1}^2}{\left(r_{ds_1} + \dfrac{1}{g_{m_2} + g_{ds_2}}\right)^2} \cdot \overline{e_n^2} \simeq g_{m_1}^2 \cdot \overline{e_n^2}$$

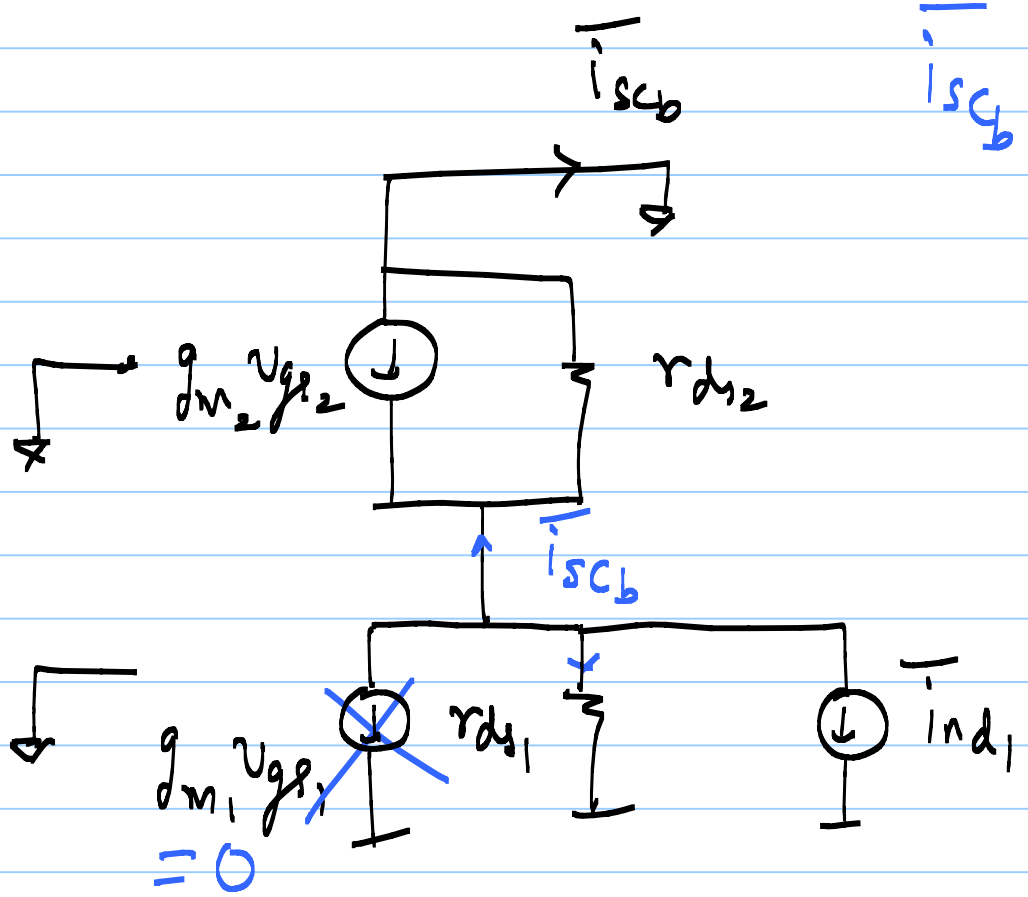
if $r_{ds_1} \gg \dfrac{1}{g_{m_2} + g_{ds_2}}$

case (b):



$$\overline{i_{sC_b}^2} = \underbrace{\overline{i_{nR}^2}}_{\text{I}\ (R)} + \underbrace{\overline{i_{nd_1}^2}}_{\text{II}\ (M_1)} + \underbrace{0}_{\text{III}\ (M_2)}$$

II only M_1 noise:

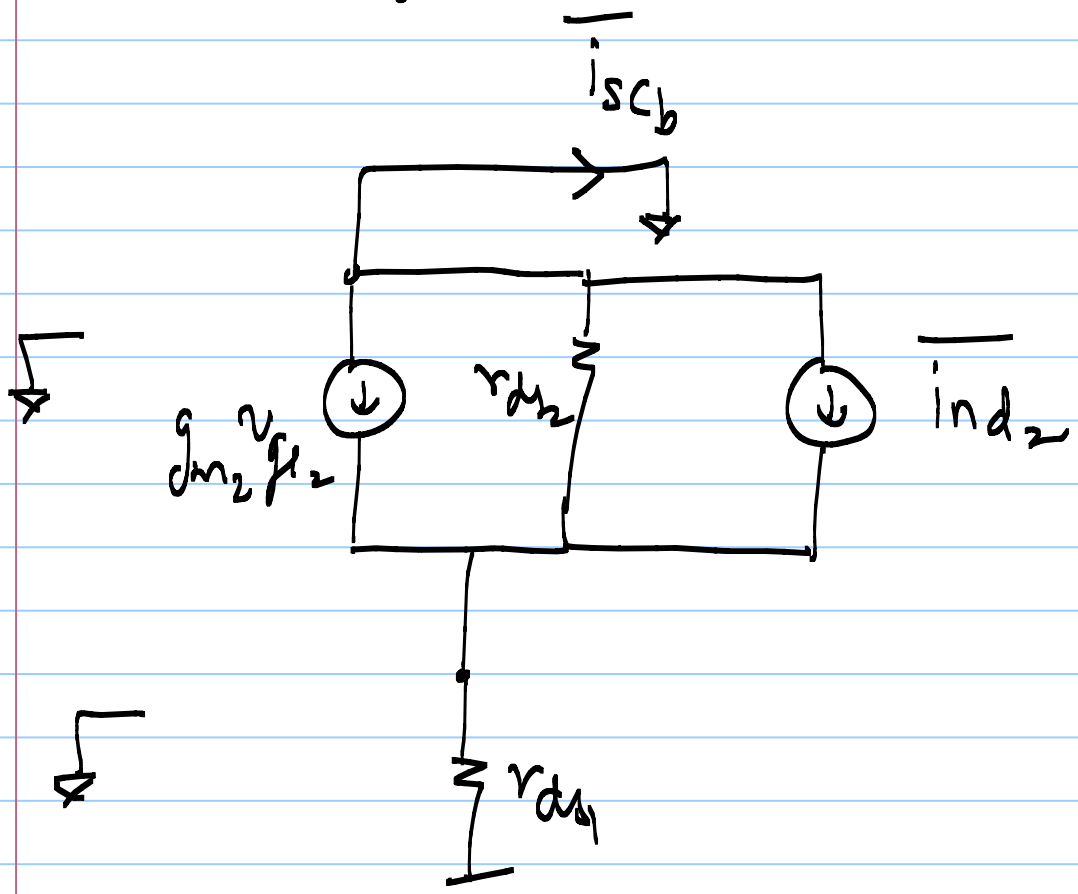


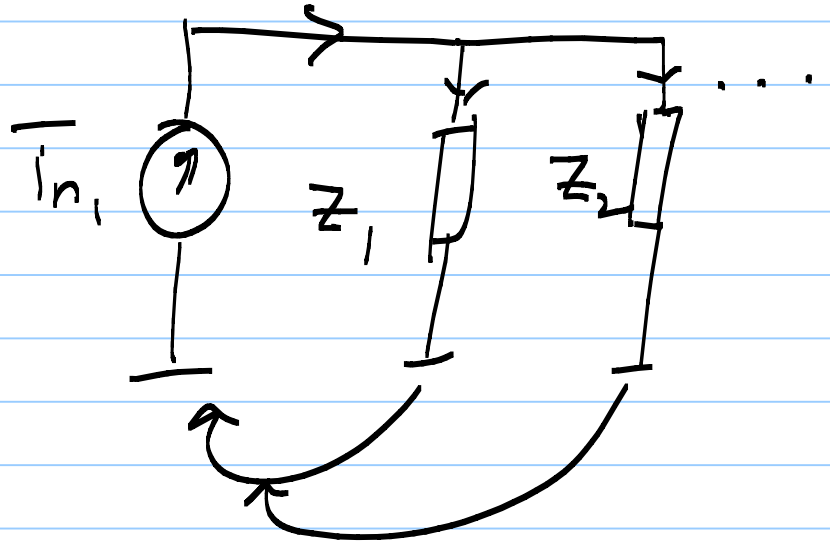
$$\overline{i_{sc_b}} = \overline{i_{nd_1}} \cdot \frac{r_{ds_1}}{r_{ds_1} + \dfrac{1}{g_{m_2} + g_{ds_2}}}$$

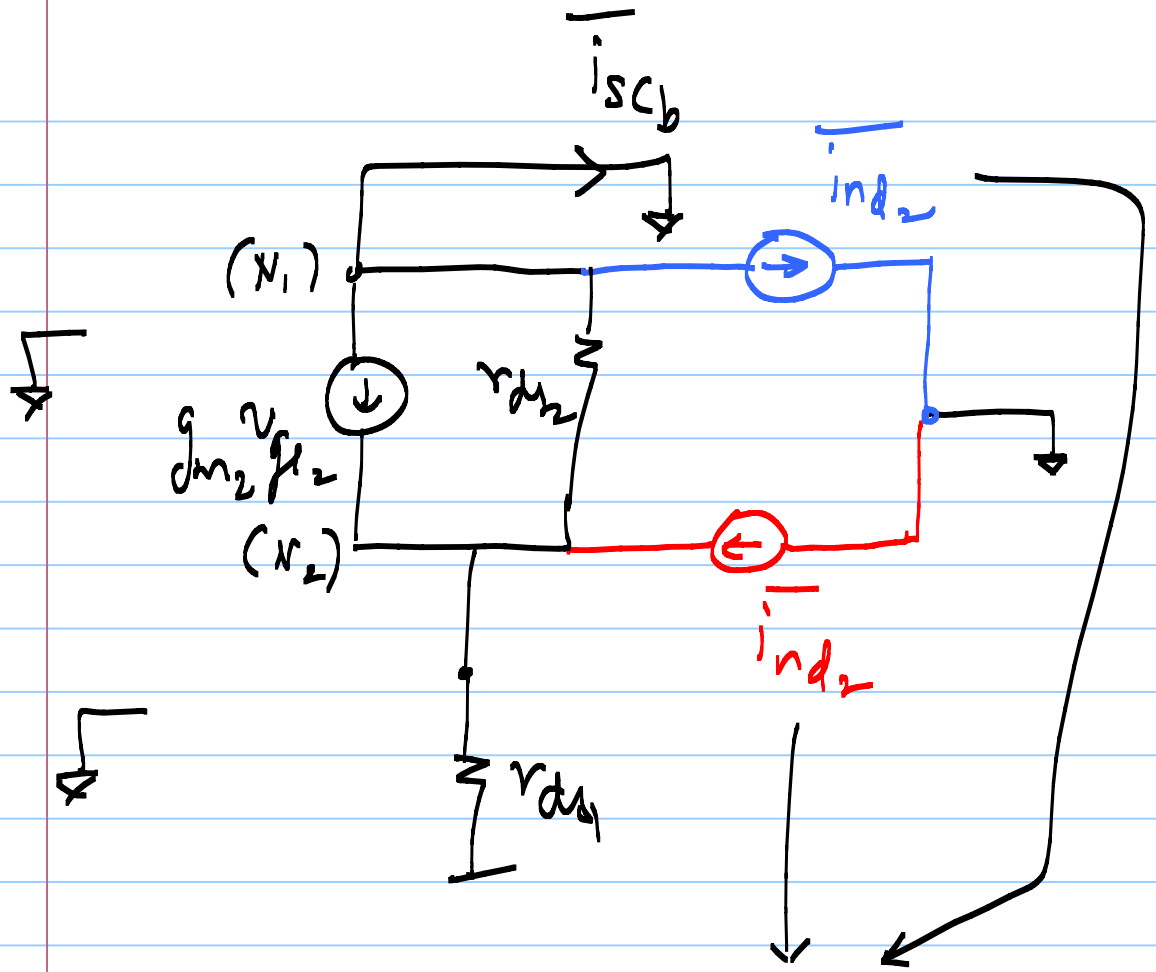
$$\approx \overline{i_{nd_1}} \quad \text{if} \quad r_{ds_1} \gg \frac{1}{g_{m_2} + g_{ds_2}}$$

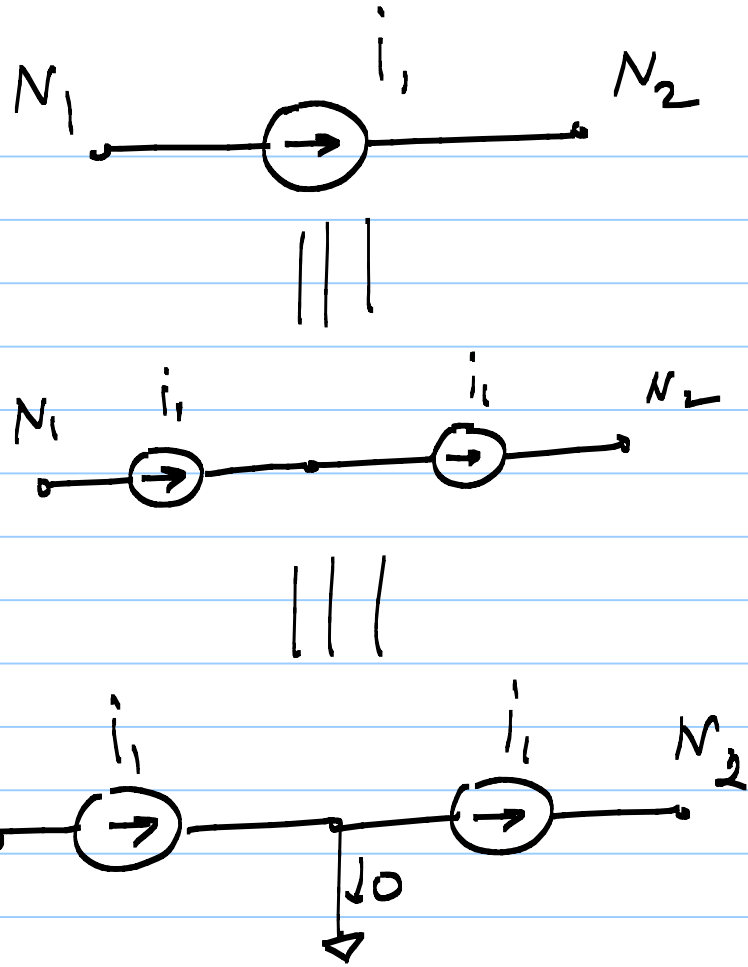
III

only M_2 noise

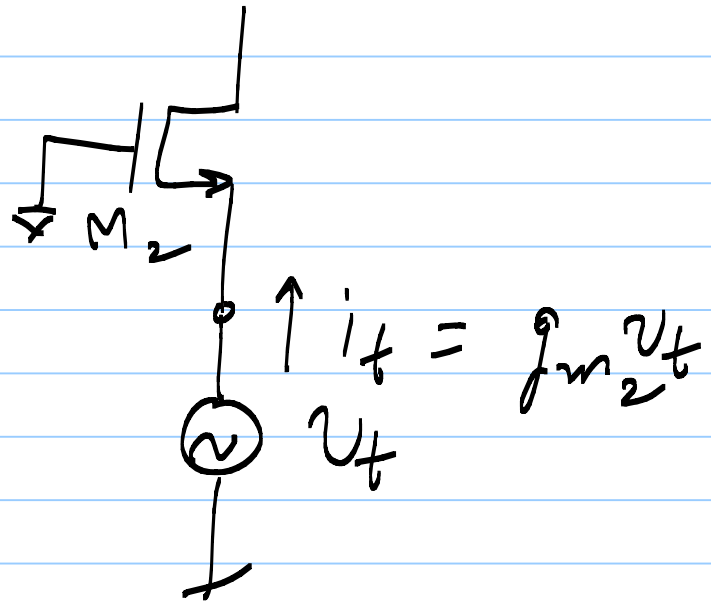


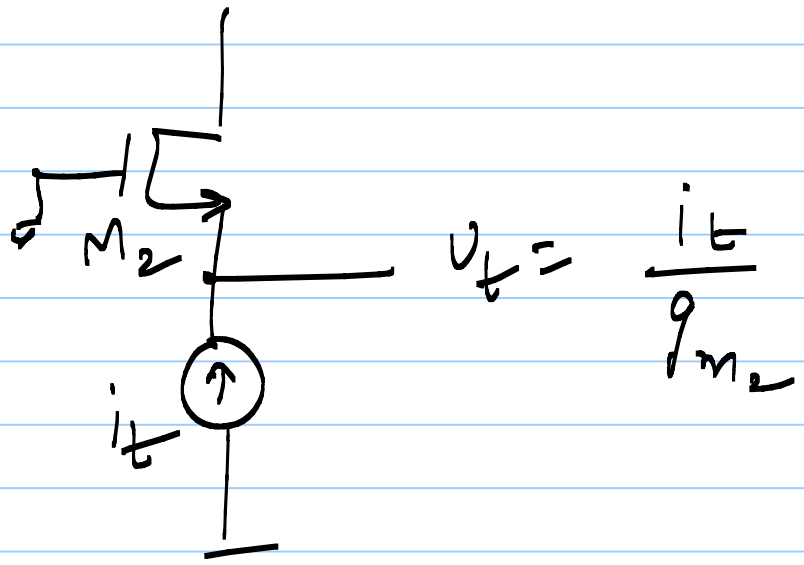


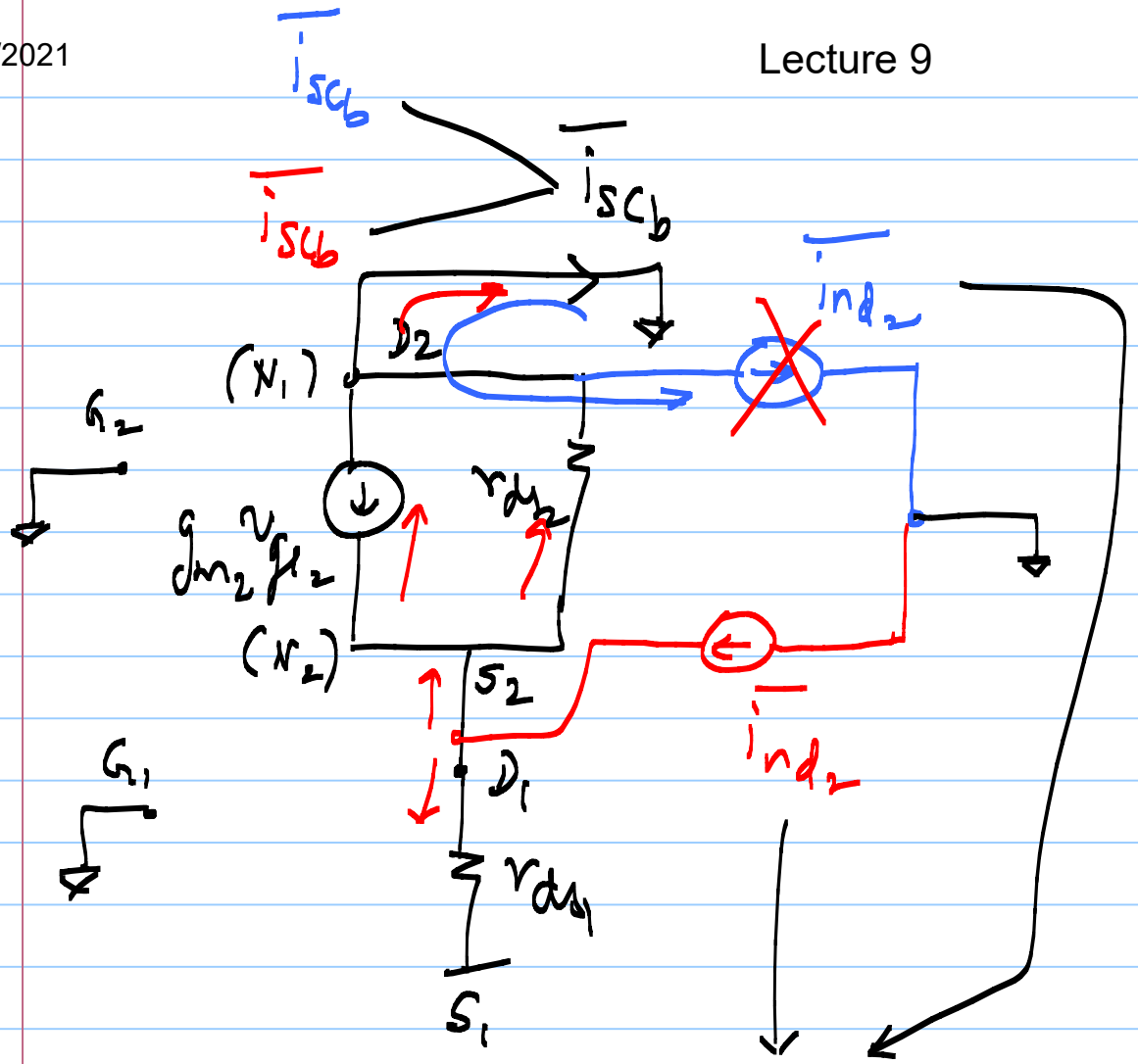




these two current sources are fully correlated


M_2
$i_t = g_{m_2} v_t$
v_t

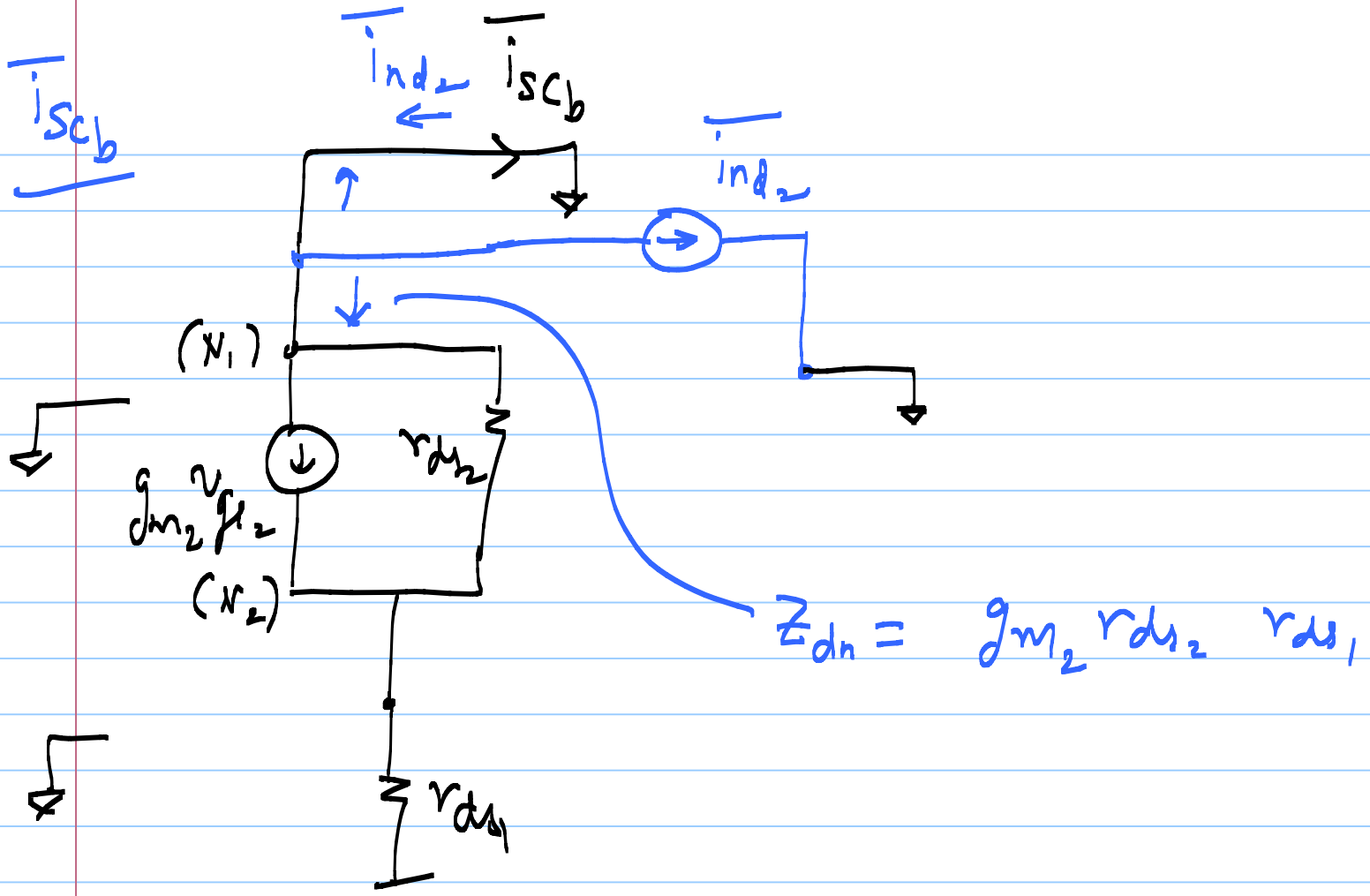

M_2
$v_t = \frac{i_t}{g_{m_2}}$
i_t



$\overline{i_{nd_2}}$ and $\overline{i_{nd_2}}$ are fully correlated.

$$\overline{i_{sc_b}} = \overline{i_{nd_2}}$$

$$\overline{i_{x_b}} = \frac{g_{m_2} + g_{ds_2}}{g_{m_2} + g_{ds_2} + g_{ds_1}} \cdot \overline{i_{nd_2}}$$

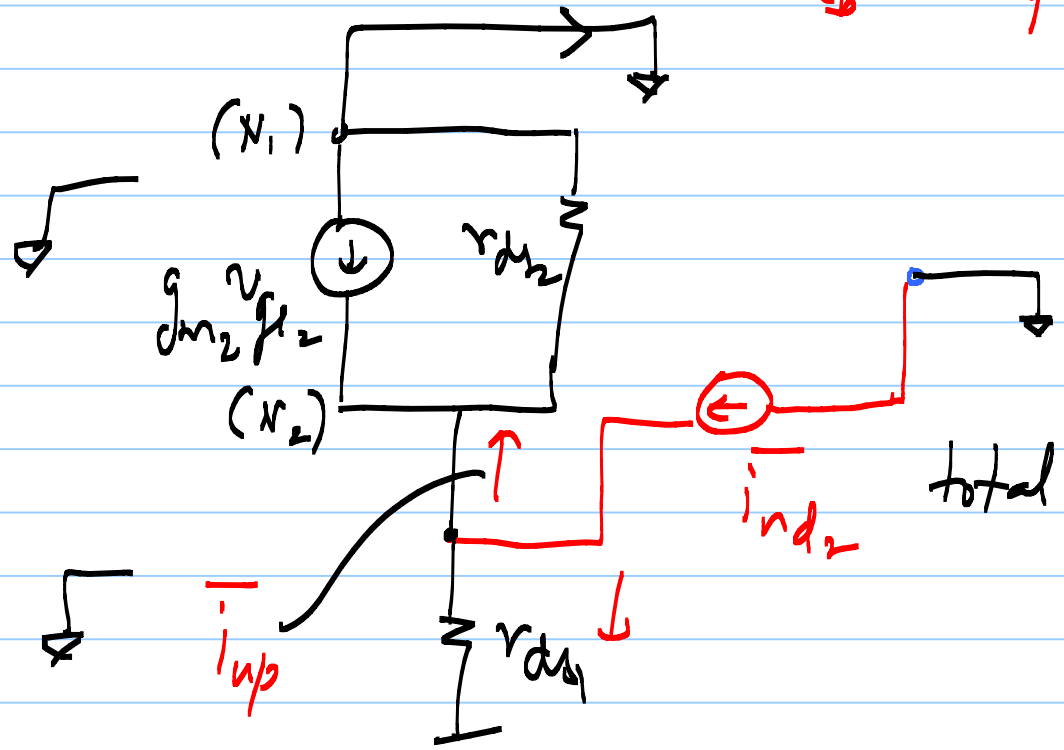

i_{sc_b}
i_{nd_2}
i_{sc_b}
i_{nd_2}
(V_1)
$g_{m_2} v_{gs_2}$
r_{ds_2}
(V_2)
r_{ds_1}
$Z_{dn} = g_{m_2} r_{ds_2} r_{ds_1}$

$\overline{i_{sc_b}}$

$\overline{i_{up}} = \overline{i_{sc_b}}$

$\overline{i_{sc_b}}$

$$\overline{i_{sc_b}} = \overline{i_{up}} = \frac{g_{m_2} + g_{ds_2}}{g_{m_2} + g_{ds_2} + g_{ds_1}} \cdot \overline{i_{nd_2}}$$



total $\overline{i_{sc_b}} = \overline{i_{sc_b}} - \overline{i_{sc_b}}$

$$= \overline{i_{nd_2}} - \frac{g_{m_2} + g_{ds_2}}{g_{m_2} + g_{ds_2} + g_{ds_1}} \overline{i_{nd_2}}$$

$$\overline{i_{sc_b}} = \frac{g_{ds_1}}{g_{m_2} + g_{ds_2} + g_{ds_1}} \cdot \overline{i_{nd_2}} \approx 0 \quad \text{if } g_{ds's} << g_{m's}$$

Noise of cascoded CSA $\approx$ noise of CSA

$$\frac{\overline{i_{sc_b}^2}}{\Delta f} = \frac{\overline{i_{n_R}^2}}{\Delta f} + \frac{\overline{i_{n_{d_1}}^2}}{\Delta f}$$

$$= \frac{4kT}{R} + \frac{8kT}{3} g_{m_1}$$

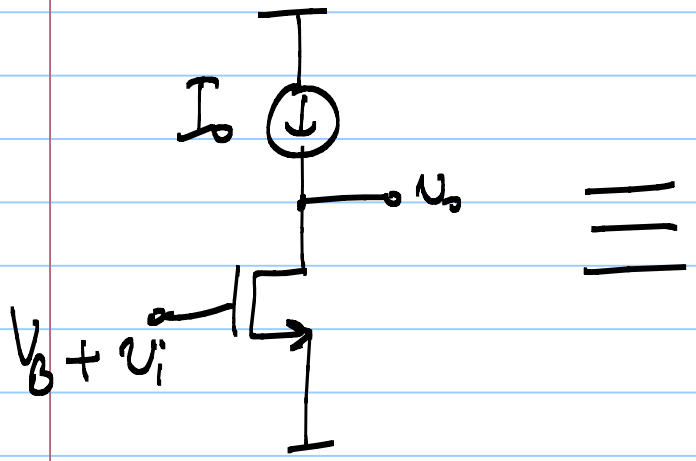
} case b with noisy circuit

$$\overline{i_{sc_a}^2}/\Delta f = g_{m_1}^2 \cdot \overline{e_n^2}/\Delta f$$

} case a with $\overline{e_n}$

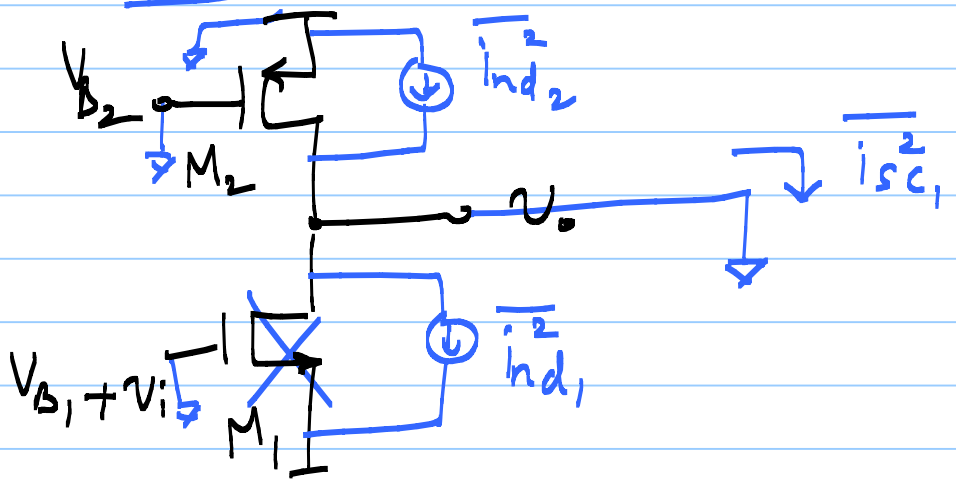
$$\frac{\overline{e_n^2}}{\Delta f} = \frac{4kT}{g_{m_1}^2 R} + \frac{8kT}{3 g_{m_1}}$$ = same as that for CSA

Example 6: CSA with active load



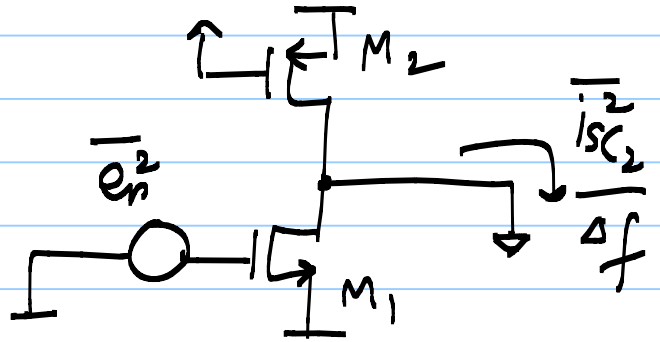
$\equiv$

case 1 – noisy 2-port



$$\frac{\overline{i_{sc_1}^2}}{\Delta f} = \frac{\overline{i_{nd_1}^2}}{\Delta f} + \frac{\overline{i_{nd_2}^2}}{\Delta f}$$

Case 2:



$$= g_{m_1}^2 \frac{\overline{e_n^2}}{\Delta f}$$

$$\frac{\overline{i_{sc_1}^2}}{\Delta f} = \frac{\overline{i_{sc_2}^2}}{\Delta f}$$

$$g_{m_1}^2 \frac{\overline{e_n^2}}{\Delta f} = \frac{\overline{i_{nd_1}^2}}{\Delta f} + \frac{\overline{i_{nd_2}^2}}{\Delta f}$$

$$\boxed{\frac{\overline{e_n^2}}{\Delta f} = \frac{8kT}{3 g_{m_1}} + \frac{8kT}{3} \frac{g_{m_2}}{g_{m_1}^2}}$$

For low noise: g_{m_1} as large as possible ✓

g_{m_2} as small as possible ?

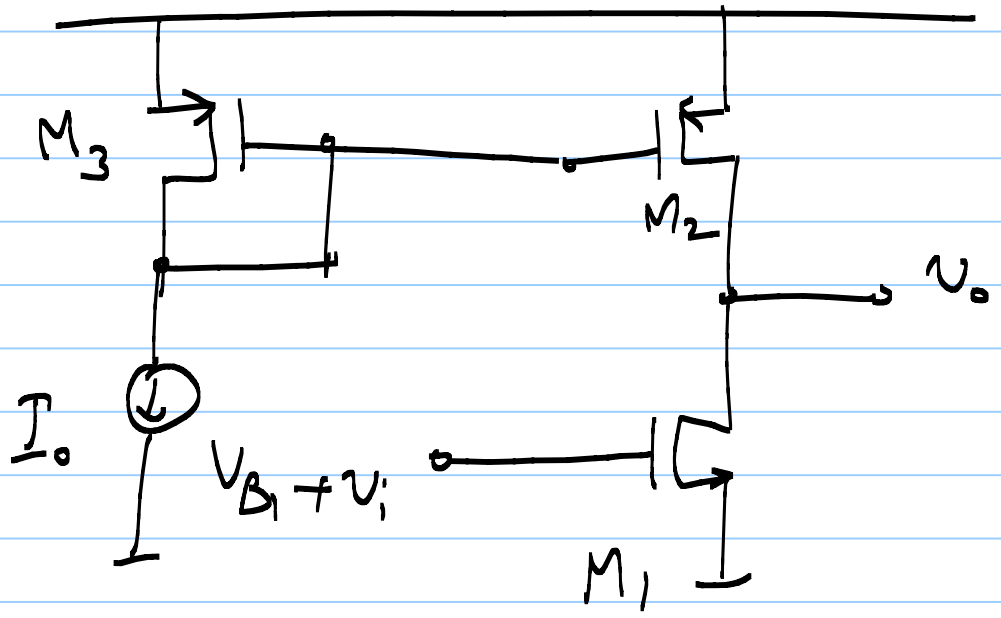
Same I_D flows through M_1 & M_2

$\Rightarrow$ Make $\left(\frac{W}{L}\right)_2$ – as small as possible

$\rightarrow$ V_{as} & V_{Dsat} will increase

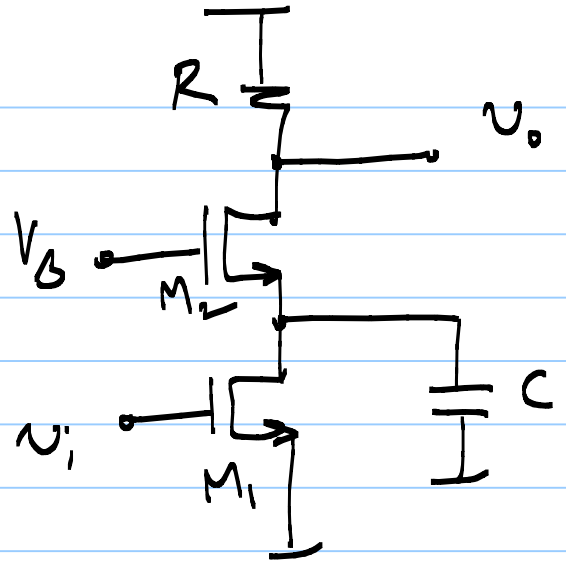
$\rightarrow$ will impact swing limits

HW:

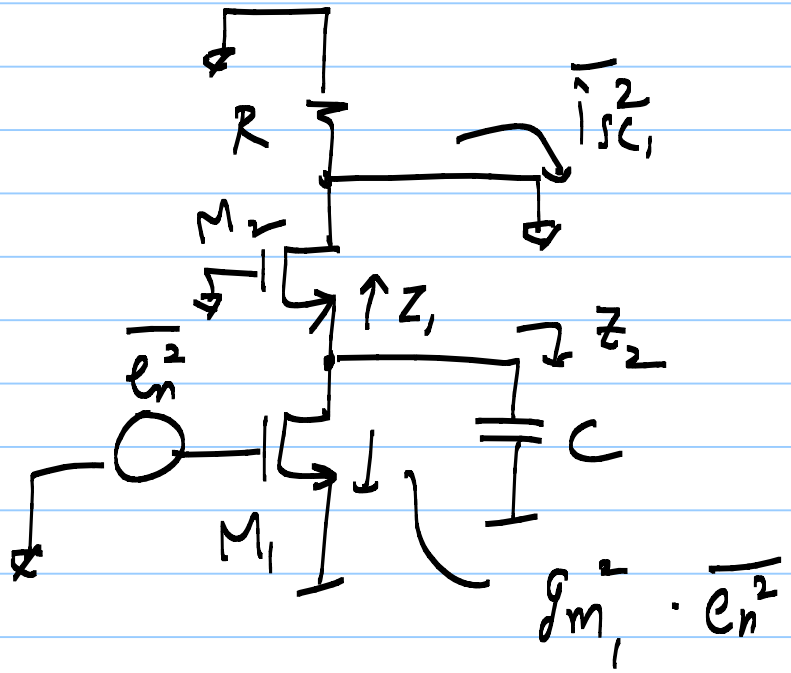


Noise of M_3?

Example 7



Case I with $\overline{e_n}$



$$Z_1 = \frac{1}{g_{m_2}}$$

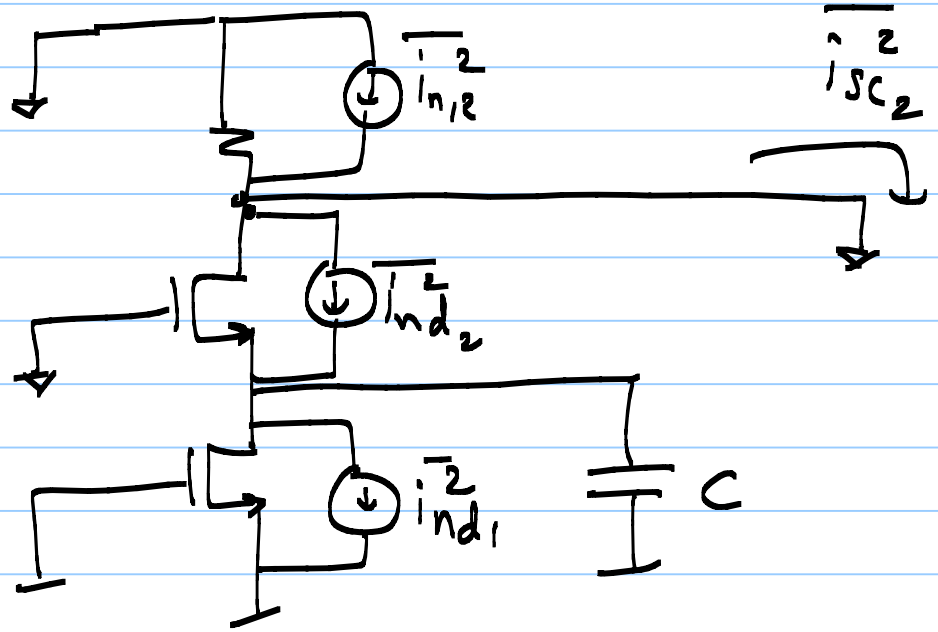
$$Z_2 = \frac{1}{j\omega C}$$

$$\overline{i_{sc_1}^2} = g_{m_1}^2 \cdot \overline{e_n^2} \cdot \left| \frac{Z_2}{Z_1 + Z_2} \right|^2$$

$$\frac{Z_2}{Z_1+Z_2} = \frac{1/j\omega C}{\frac{1}{g_{m_2}} + \frac{1}{j\omega C}} = \frac{g_{m_2}}{g_{m_2} + j\omega C}$$

$$\overline{i_{sc_1}^2} = g_{m_1}^2 \cdot \overline{e_n^2} \cdot \frac{g_{m_2}^2}{g_{m_2}^2 + \omega^2 C^2}$$

Case 2 :



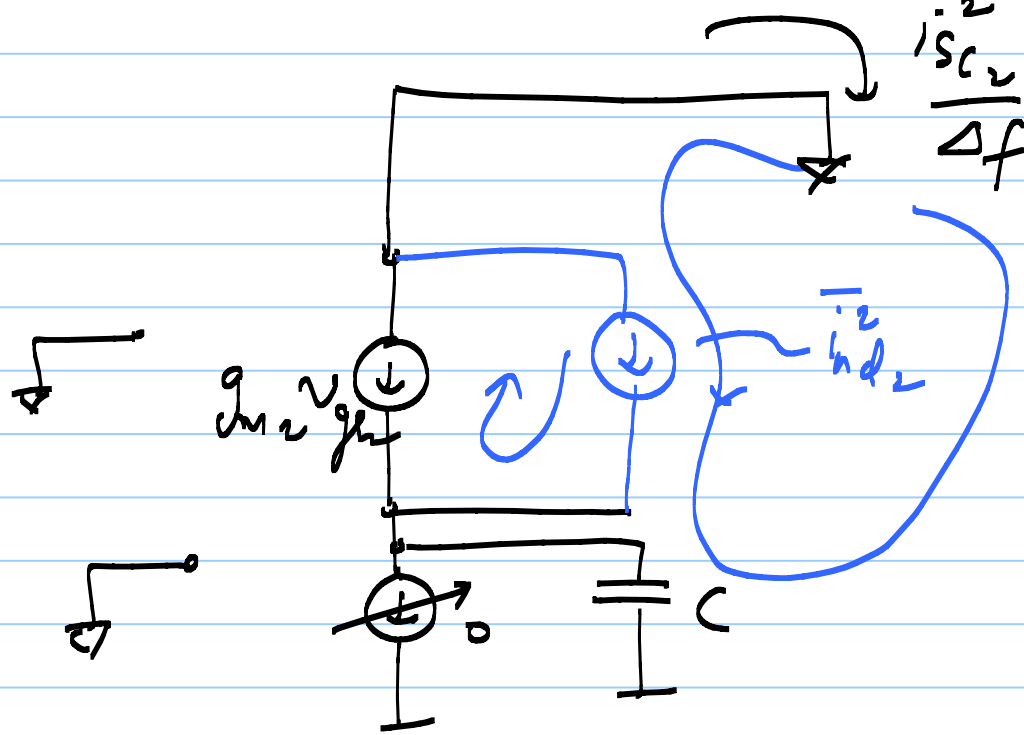
$$\overline{i_{sc_2}^2} = f_1\left(\overline{i_{n,R}^2}\right) + f_2\left(\overline{i_{nd_1}^2}\right) + f_3\left(\overline{i_{nd_2}^2}\right)$$

(I) (II) (III)

(I) : $\dfrac{\overline{i_{sc_2}^2}}{\delta f} = \dfrac{4kT}{R}$

(II) : $\dfrac{\overline{i_{sC_2}^2}}{\Delta f} = \dfrac{8kT}{3} g_{m_1} \cdot \dfrac{g_{m_2}^2}{g_{m_2}^2 + \omega^2 C^2}$

(III) : $\overline{i_{sC_2}^2}$



$\dfrac{\overline{i_{sC_2}^2}}{\Delta f} = \dfrac{8kT}{3} g_{m_2} \cdot \dfrac{\omega^2 C^2}{g_{m_2}^2 + \omega^2 C^2}$

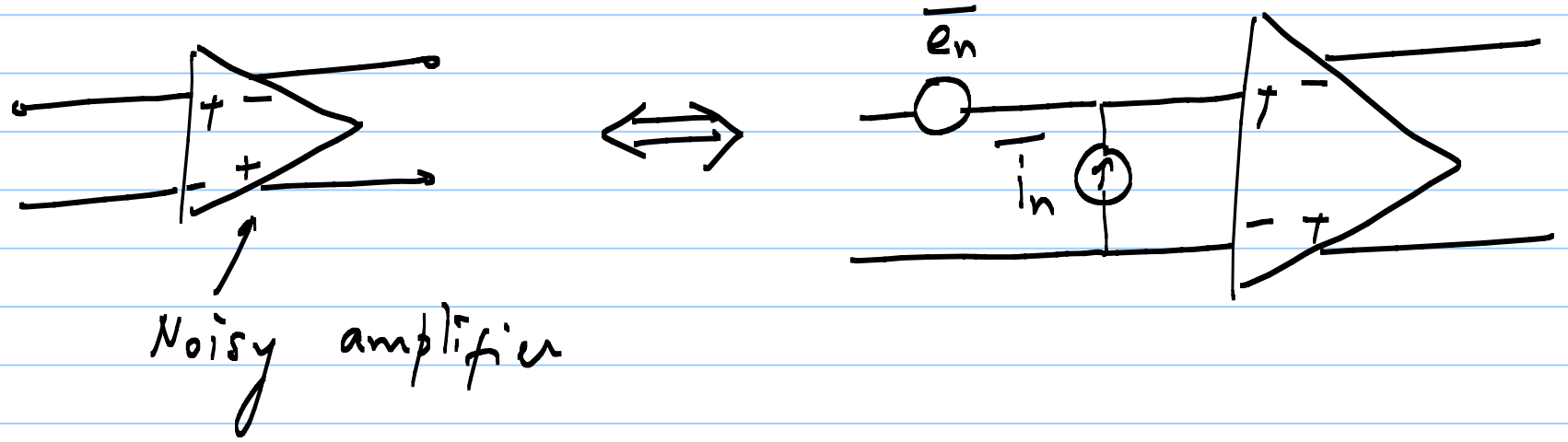
$$\frac{\overline{i_{sC_2}^2}}{\Delta f} = \frac{4kT}{R} + \frac{8kT}{3} g_{m_1} \cdot \frac{g_{m_2}^2}{g_{m_2}^2 + \omega^2 C^2}$$

$$+ \frac{8kT}{3} g_{m_2} \cdot \frac{\omega^2 C^2}{g_{m_2}^2 + \omega^2 C^2}$$

$$= \frac{\overline{i_{sC_1}^2}}{\Delta f} = g_{m_1}^2 \cdot \frac{\overline{e_n^2}}{\Delta f} \cdot \frac{g_{m_2}^2}{g_{m_2}^2 + \omega^2 C^2}$$

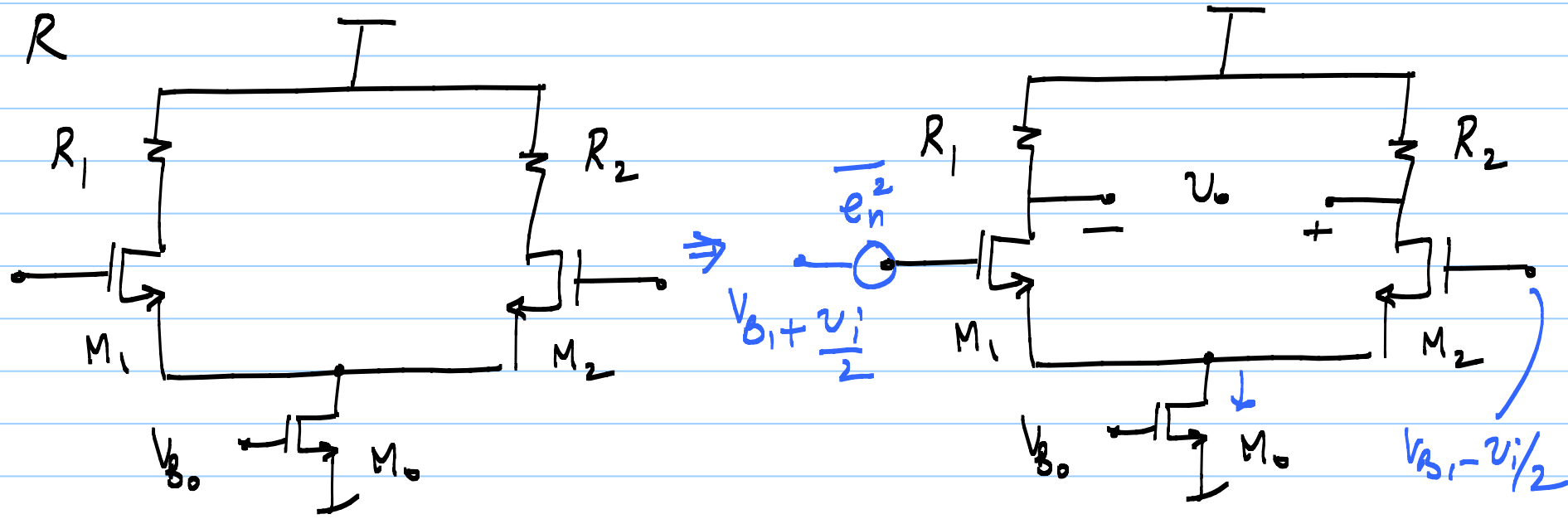
$$\frac{\overline{e_n^2}}{\Delta f} = \frac{4kT}{R} \cdot \frac{(g_{m_2}^2 + \omega^2 C^2)}{g_{m_1}^2 g_{m_2}^2} + \frac{8kT}{3 g_{m_1}} + \frac{8kT}{3 g_{m_2}} \cdot \frac{\omega^2 C^2}{g_{m_1}^2}$$

Noise in differential amplifiers

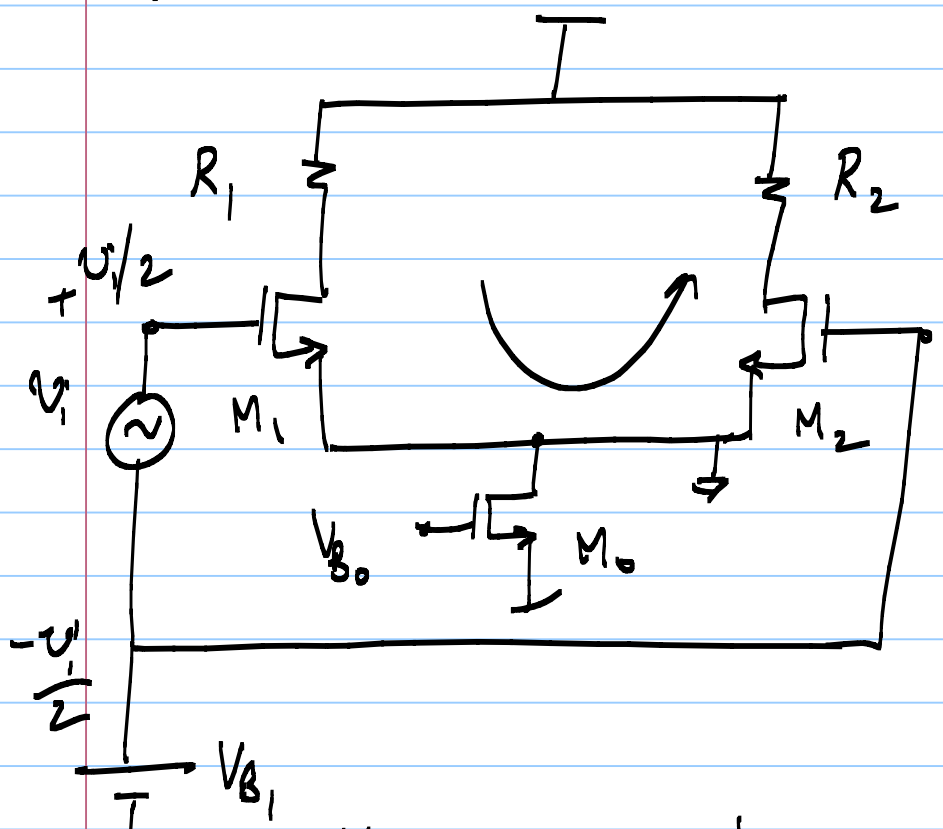


* Z_{in} of amp is $\infty \Rightarrow \overline{i_n^2} = 0$

$R_1 = R_2 = R$



Signal analysis

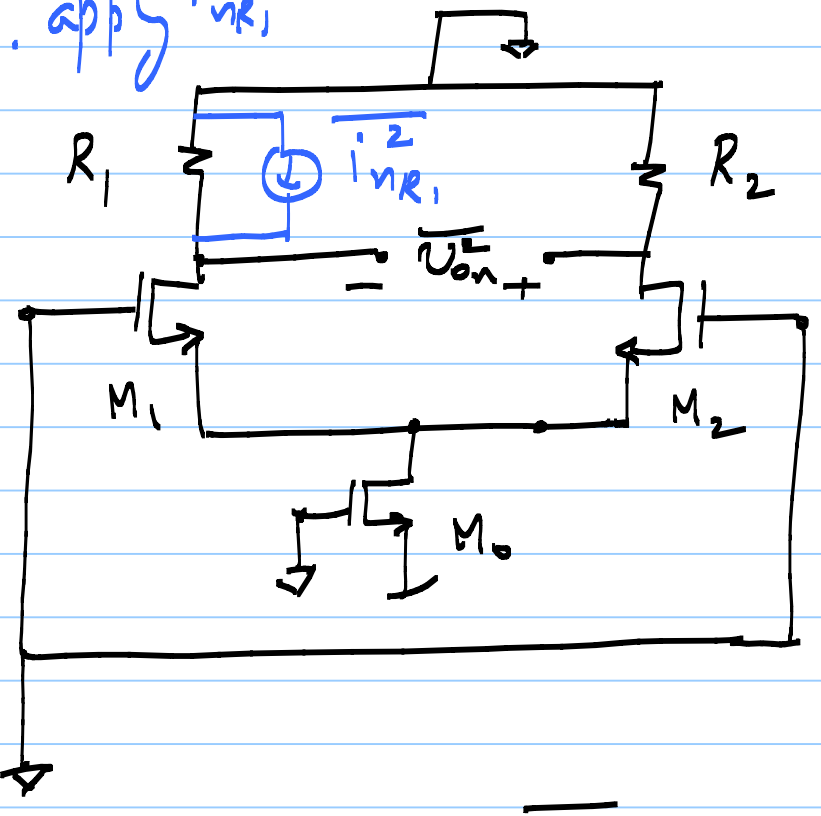


* Half circuit analysis

* Symmetric

Noise analysis

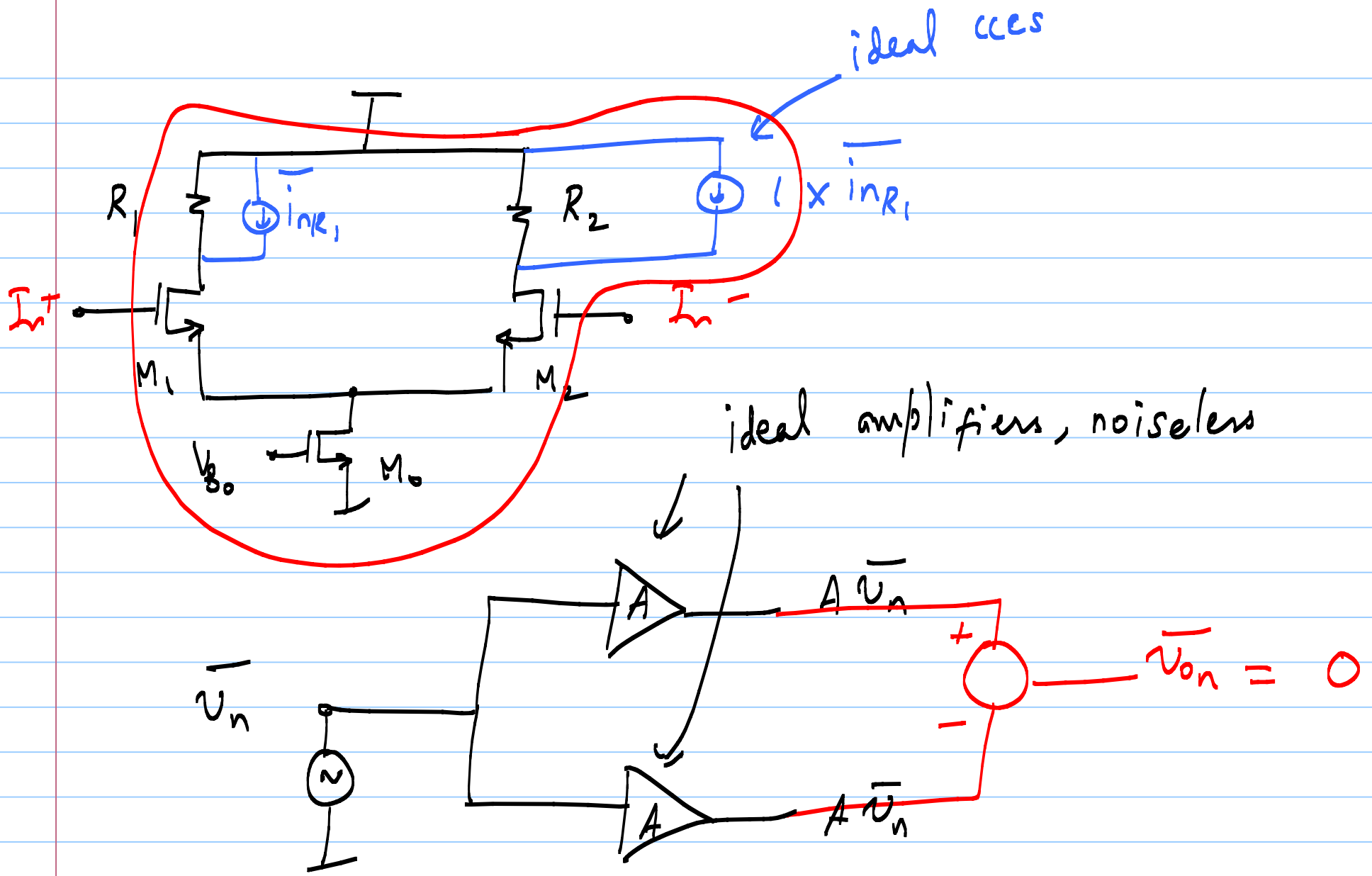
e.g. apply $\overline{i_{nR_1}^2}$

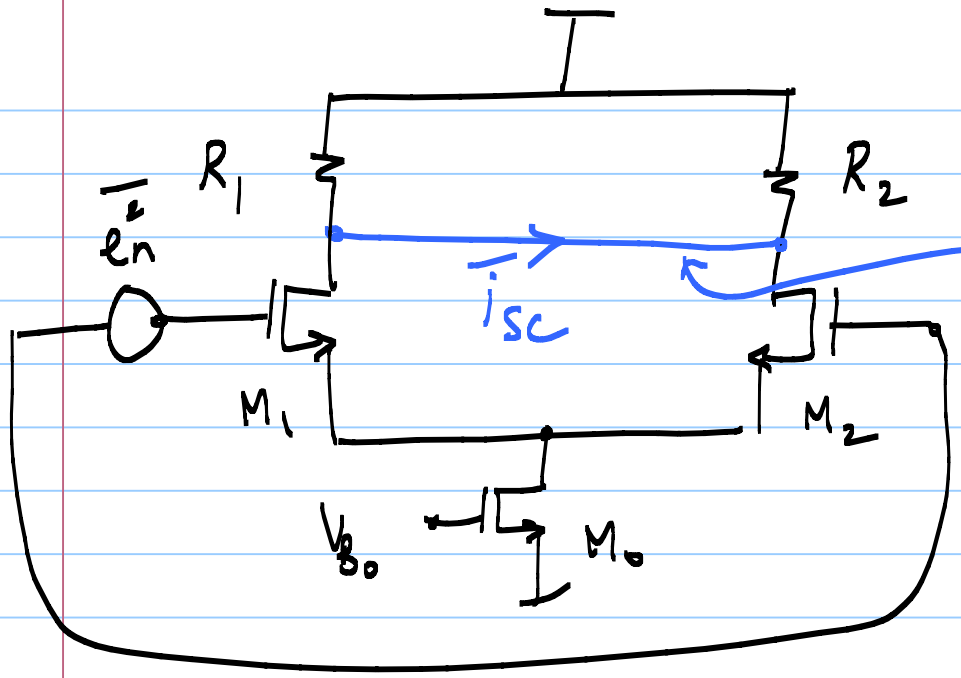


* Input source is $\overline{i_{nR_1}^2}$, $\overline{i_{nR_2}^2}$ etc.

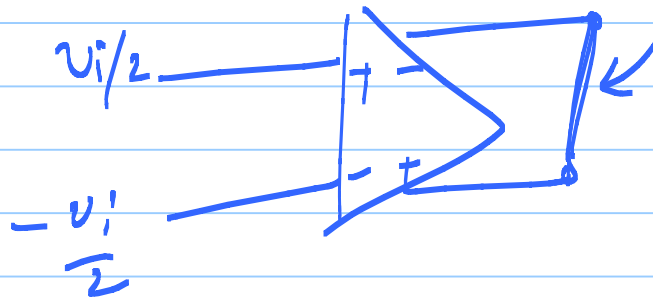
→ Not symmetric w.r.t two halves

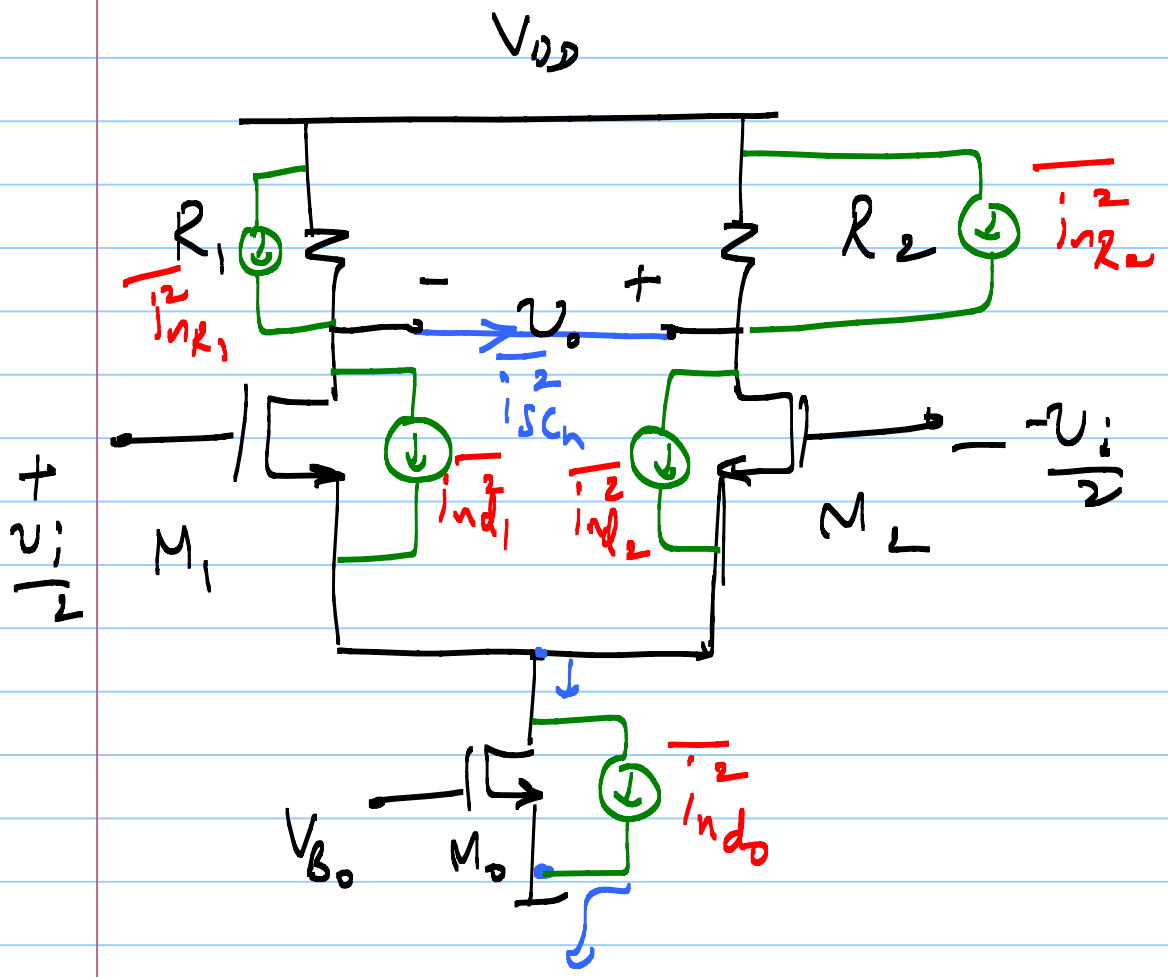
→ No half circuit analysis


ideal ccs
R_1
$\overline{i_{nR_1}}$
R_2
$1 \times \overline{i_{nR_1}}$
I_n^+
I_n^-
M_1
M_2
V_{B_0}
M_0
ideal amplifiers, noiseless
$\overline{v_n}$
A
$A\,\overline{v_n}$
+
−
$\overline{v_{on}} = 0$



short circuit load:





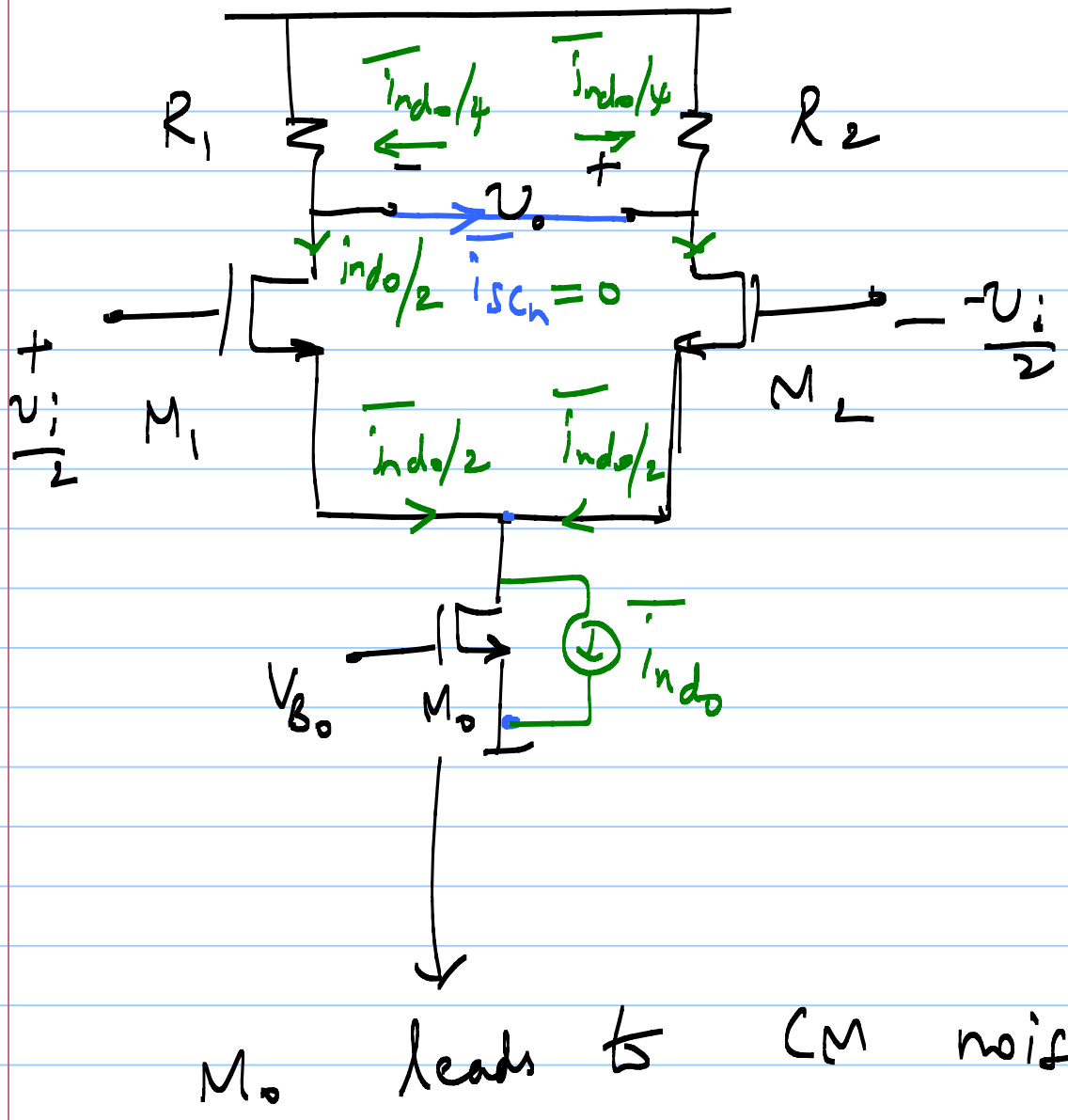
Case 1 :

Noise sources:

$\overline{i^2_{nR_1}}$, $\overline{i^2_{nR_2}}$, $\overline{i^2_{nd_1}}$, $\overline{i^2_{nd_2}}$

and $\overline{i^2_{nd_0}}$

1) Noise of M_0 :



$R_1 = R_2 = R$

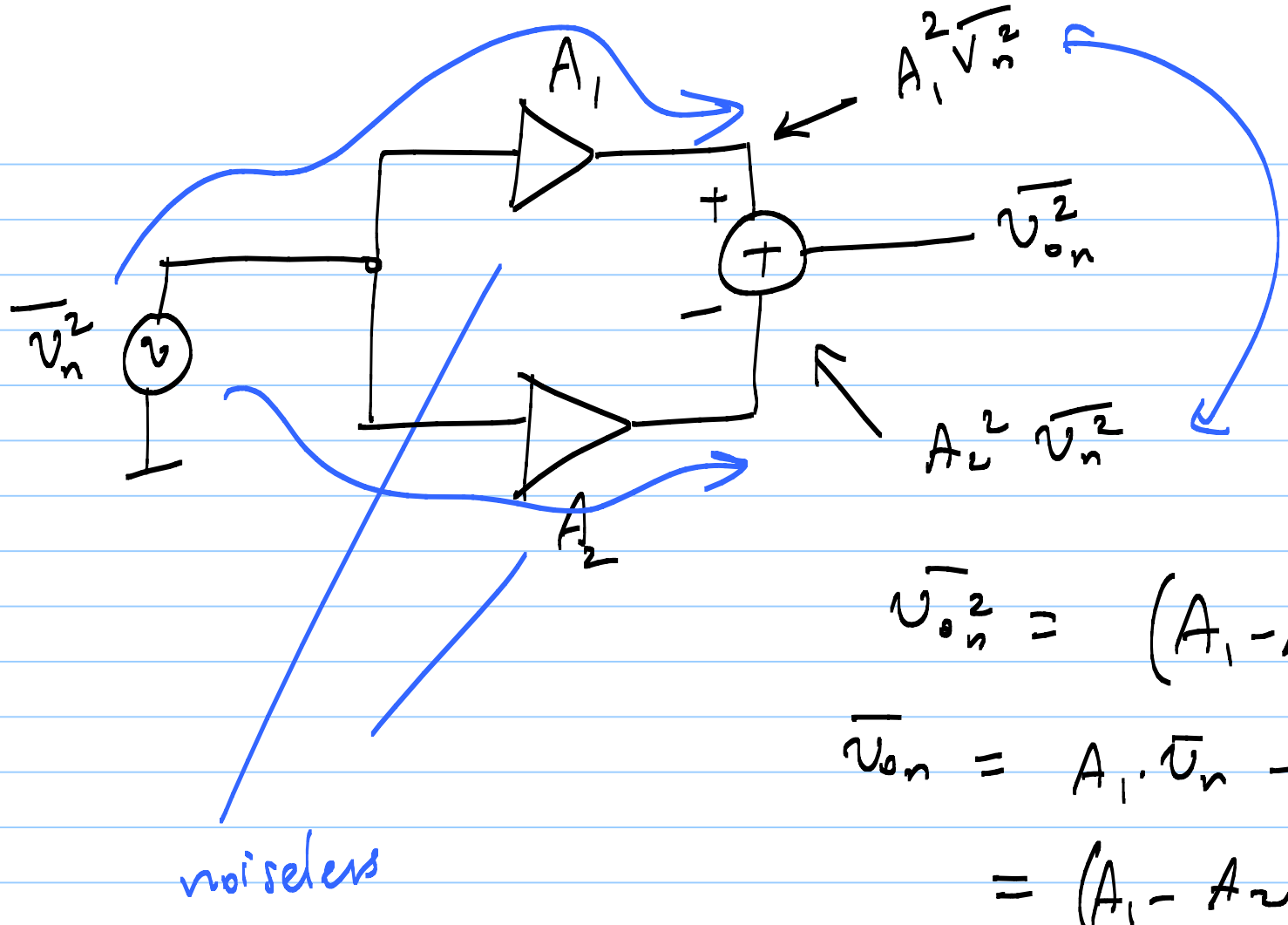
$\overline{i_{sc_n}} = 0$

$\overline{i_{sc_n}^2} = 0$

M_0 does not contribute to $\overline{e_n^2}$

(Differential mode noise)

M_0 leads to CM noise



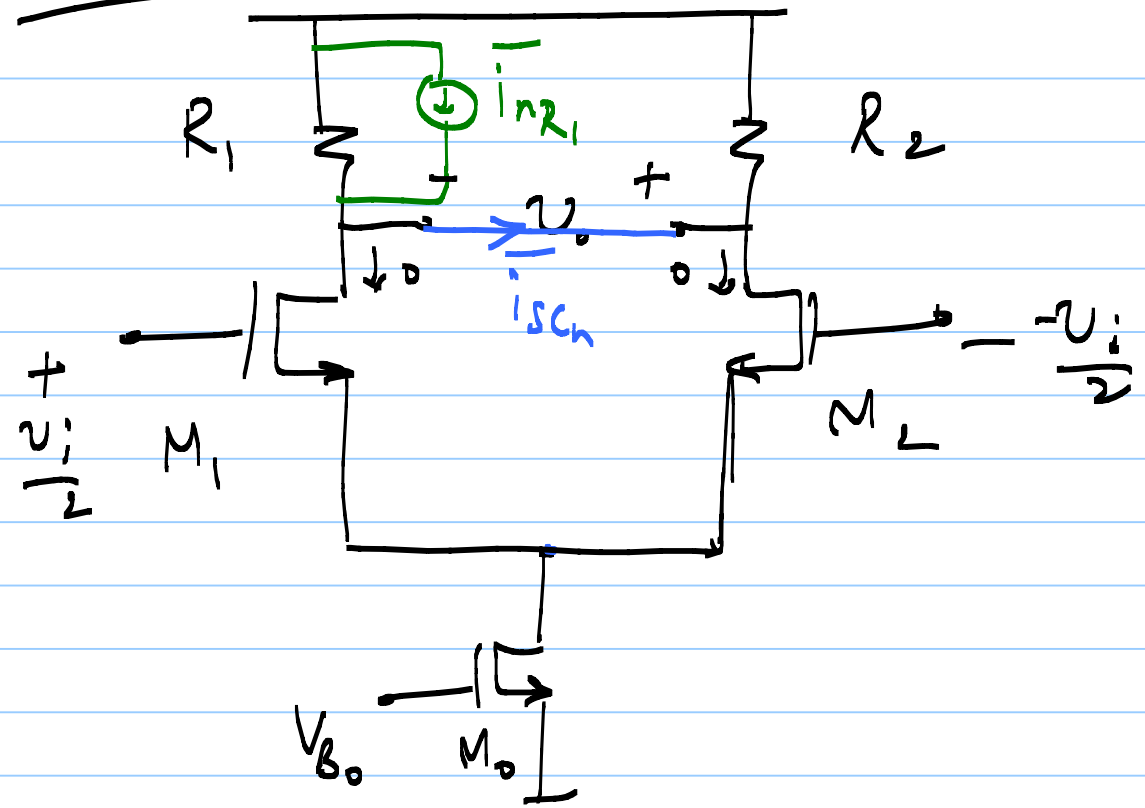
fully correlated

$$\overline{v_{on}^2} = (A_1 - A_2)^2 \overline{v_n^2}$$

$$\overline{v_{on}} = A_1 \cdot \overline{v_n} - A_2 \cdot \overline{v_n}$$

$$= (A_1 - A_2) \overline{v_n}$$

2) Noise from R_1



r_{ds} is large

$$\overline{i_{SC_n}} = \frac{\overline{i_{nR_1}}}{2}$$

$$\frac{\overline{i_{SC_n}^2}}{\Delta f} = \frac{1}{4}\frac{\overline{i_{nR_1}^2}}{\Delta f}$$

$$= \frac{kT}{R_1} = \frac{kT}{R}$$

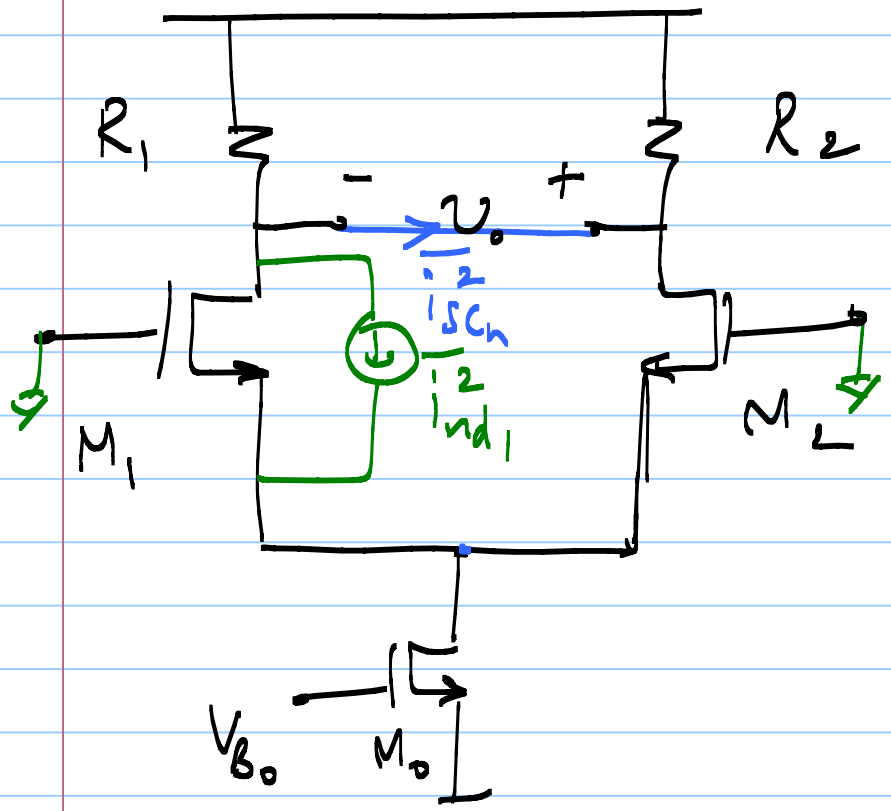
3) Noise from R_2: $\frac{\overline{i_{SC_n}^2}}{\Delta f} = \frac{kT}{R_2} = \frac{kT}{R}$ → uncorrelated

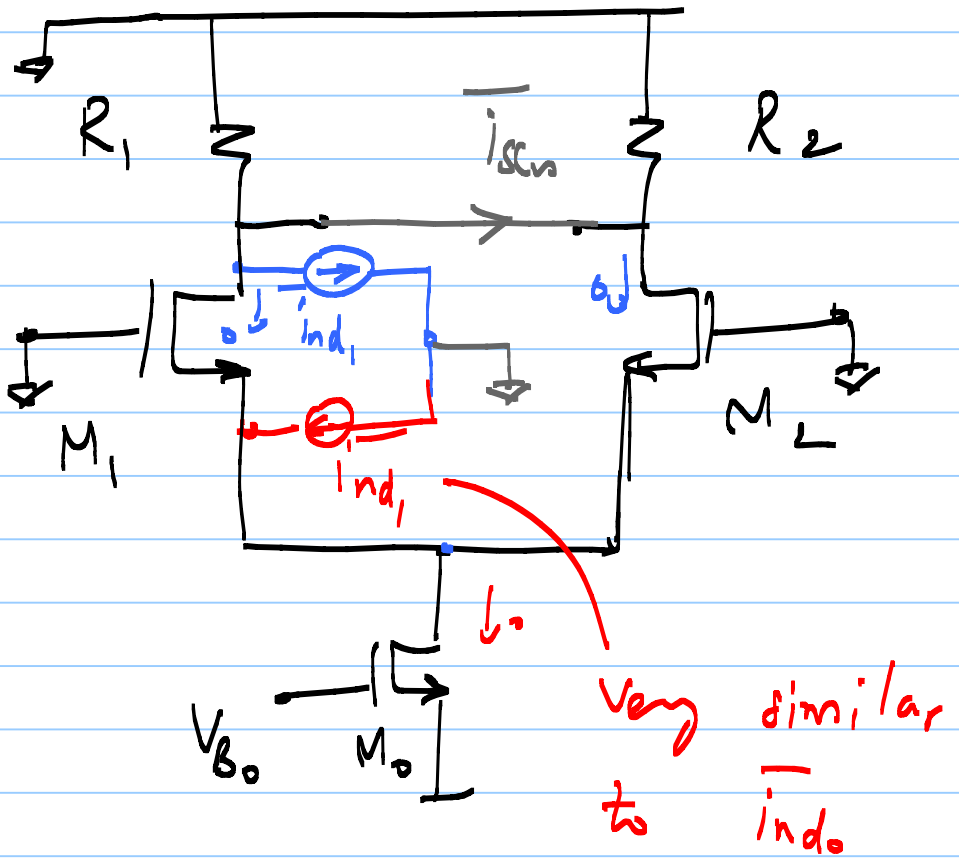
S.C.

Total noise, from Resistors = $\frac{2kT}{R}$

PSD

4) Noise from M_1

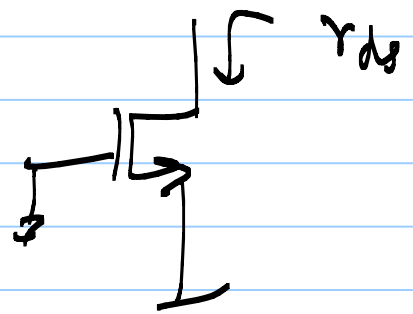


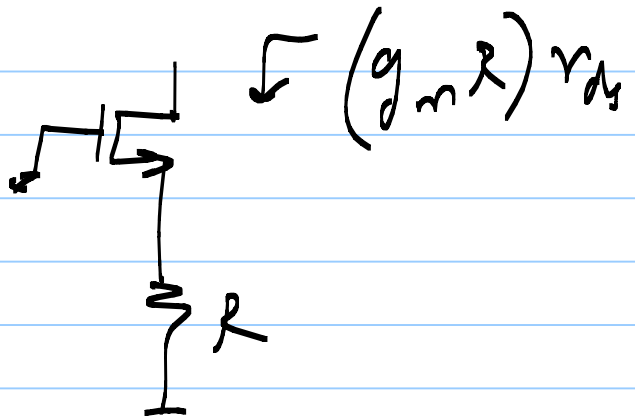


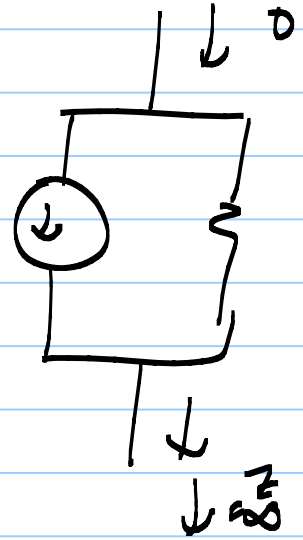
$$\overline{i_{SC_n}} = \frac{-\overline{i_{ind_1}}}{2} \quad ; \quad \overline{i_{SC_n}} = 0$$

Due to M_1

$$\overline{i_{SC_n}} = \frac{-\overline{i_{ind_1}}}{2} \Rightarrow \frac{\overline{i^2_{SC_n}}}{\Delta} = \frac{\overline{i^2_{ind_1}}/\Delta f}{4} = \frac{2kT}{3} \cdot g_{m_1}$$





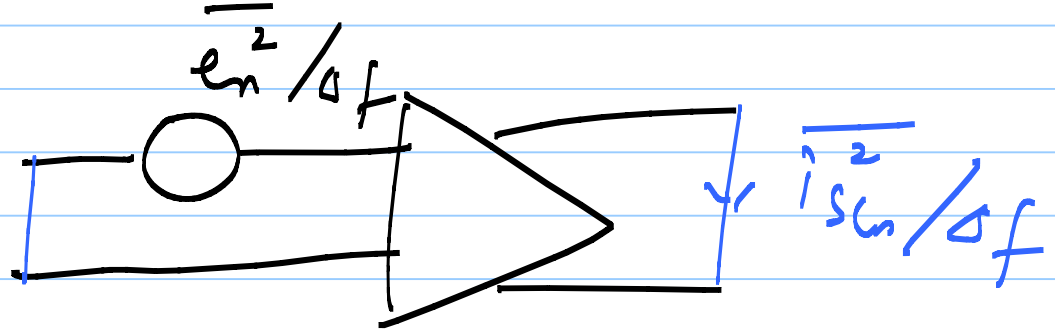


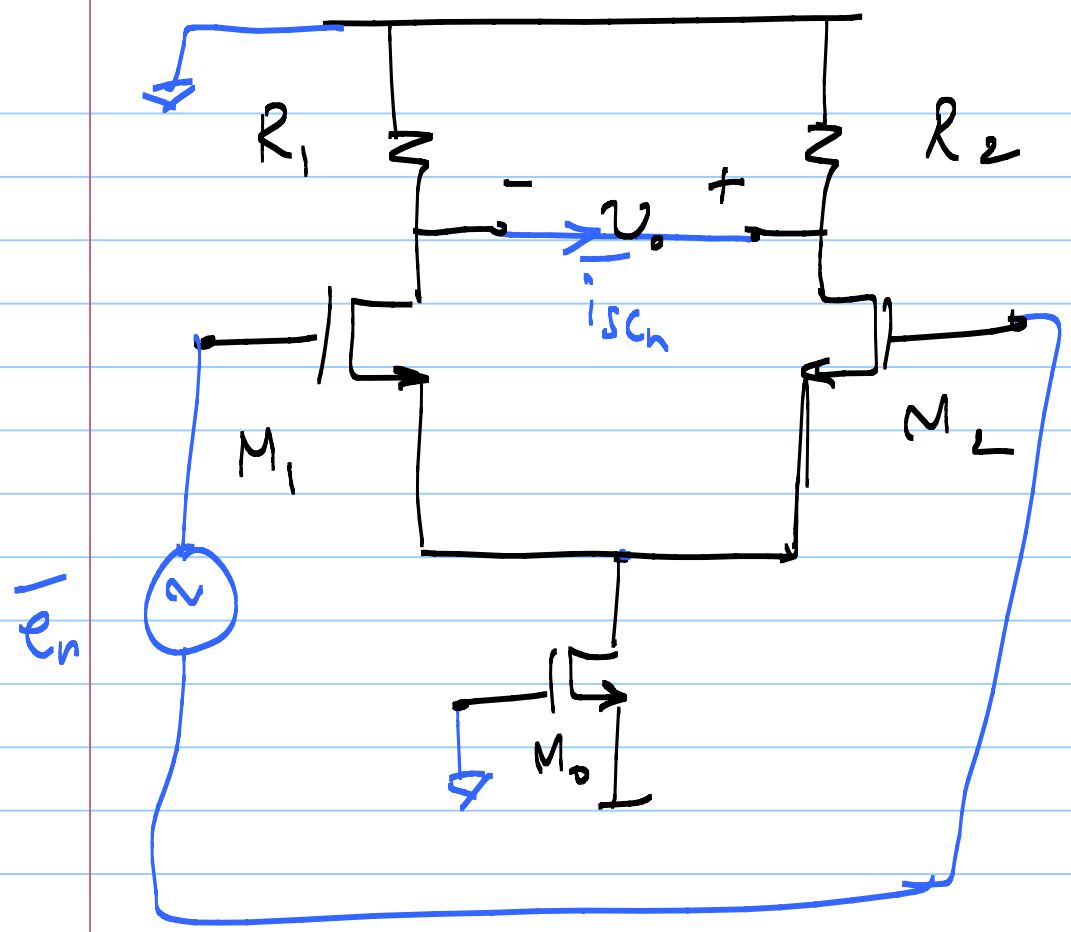
4) Noise from M2

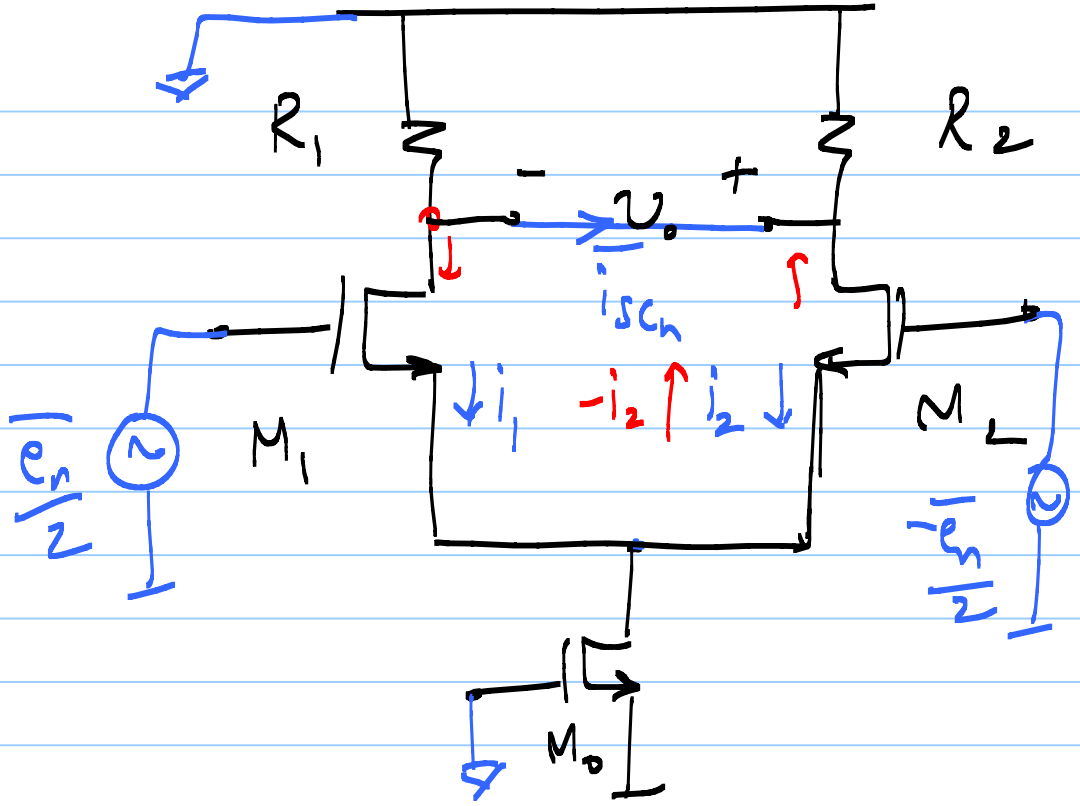
$$\overline{i_{sC_n}^2} = \frac{2kT}{3} g_{m_2} = \frac{2kT}{3} g_{m_1}$$

$$\frac{\overline{i_{sC_n\,total}^2}}{\Delta f} = \frac{2kT}{R} + \frac{4kT}{3} g_{m_1}$$

Case 2 :







$$\overline{i_1} = g_{m_1}\,\frac{\overline{e_n}}{2} \quad ; \quad \overline{i_2} = -g_{m_2}\,\frac{\overline{e_n}}{2}$$

$$-\overline{i_2} = g_{m_2}\,\overline{e_n}/2$$

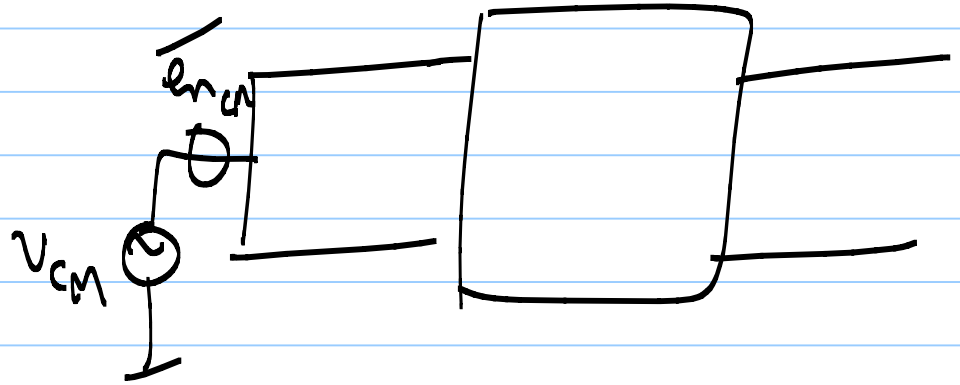
$$\overline{i_{sc_n}} = -g_{m_1}\,\overline{e_n}/2 \Rightarrow \frac{\overline{i_{sc_n}^2}}{\Delta f} = \left(\frac{g_{m_1}^2}{4}\right)\cdot \overline{e_n^2}/\Delta f$$

Equate $\frac{\overline{i_{sn}^2}}{\Delta f}$ in the two cases:

$$\frac{g_{m_1}^2}{4}\,\frac{\overline{e_n^2}}{\Delta f} = \frac{2kT}{R} + \frac{4kT}{3}\,g_{m_1}$$

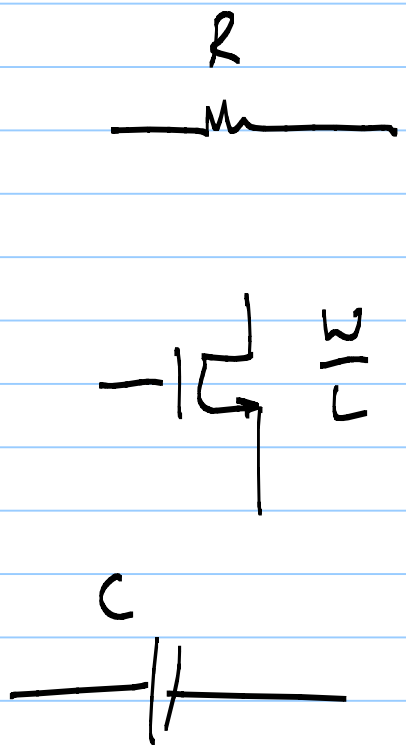
$$\boxed{\frac{\overline{e_n^2}}{\Delta f} = \frac{8kT}{g_{m_1}^2 R} + \frac{16kT}{3 g_{m_1}}}$$

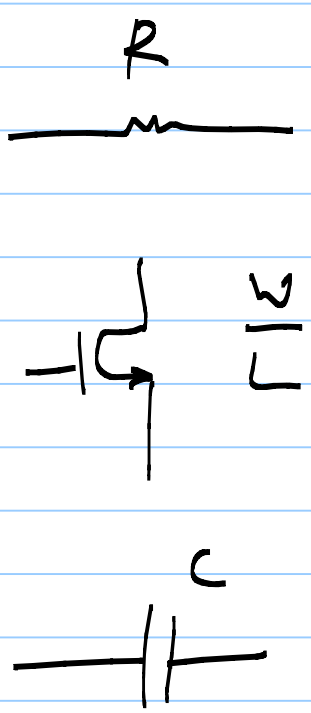
HW: Calculate $\overline{v_{on}^2}$ o.c. noise voltage @ output


$e_{n_{CM}}$
v_{CM}

Lecture 12

Mismatch





Only nominally identical

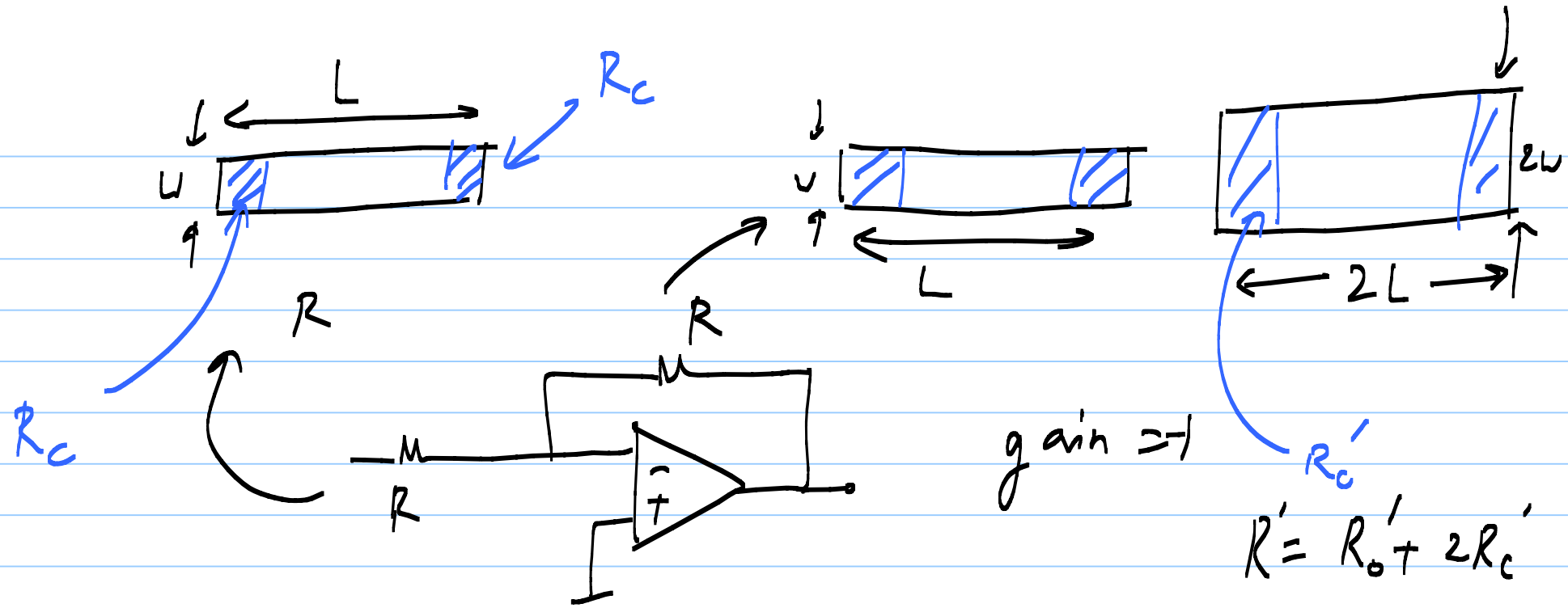
Small differences

Mismatch

Design ← Systematic → Random

Systematic → layout, package ...

Ex 1

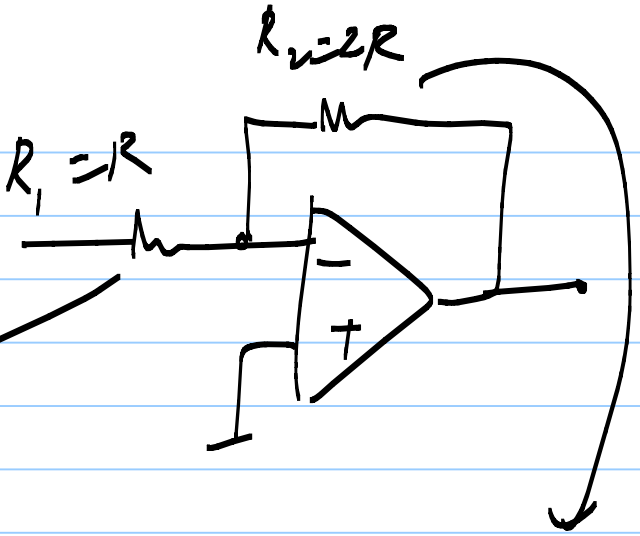


$R = R_0 + 2R_c$

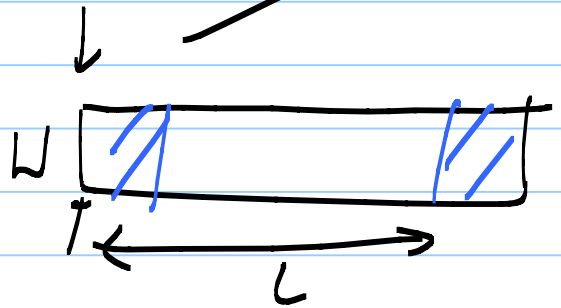
↑ contact resistance

$R' = R_0' + 2R_c'$

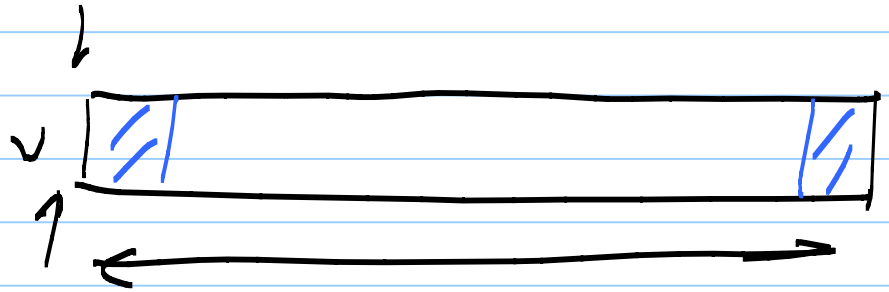
Ex 2



gain = −2

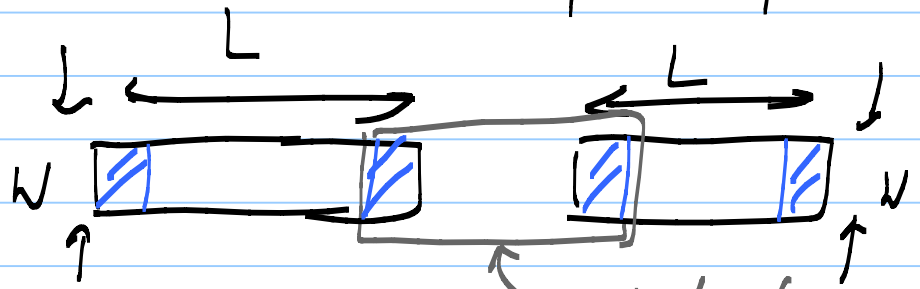


$R_1 = R_0 + 2R_c$

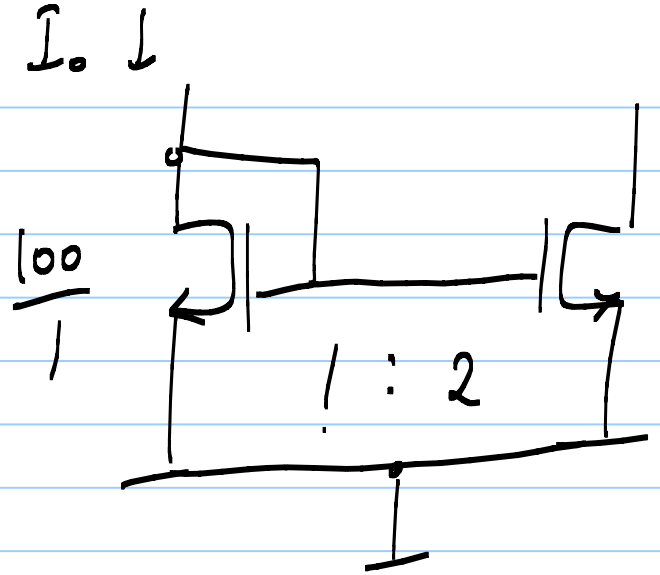


$R_2 = 2R_0 + 2R_c$

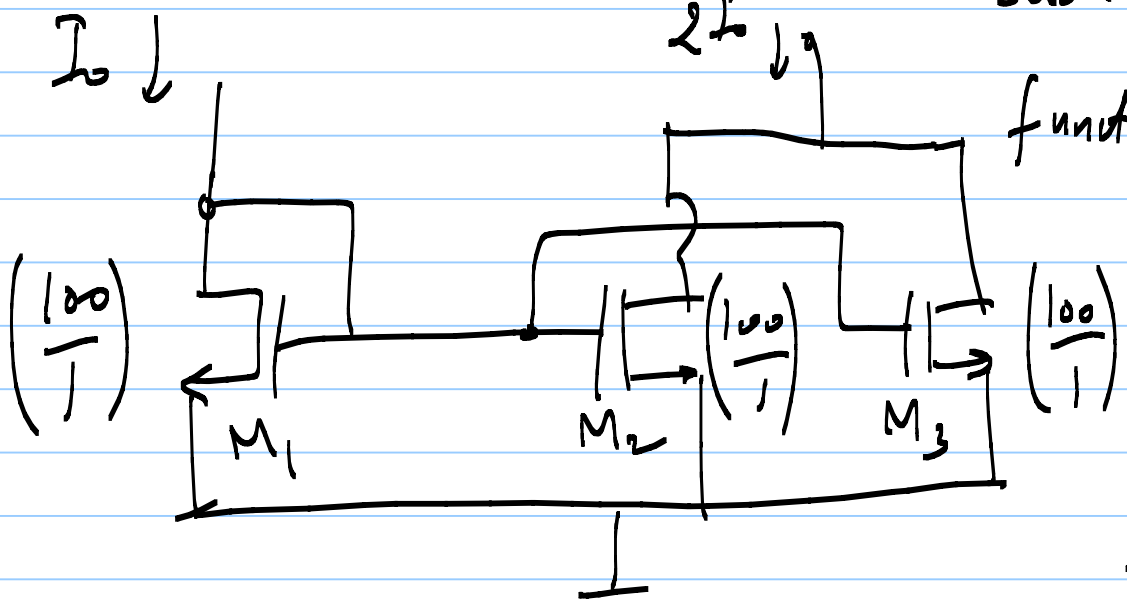
$\neq 2R_1$



$R_2 = 2R_0 + 4R_c + R_m$



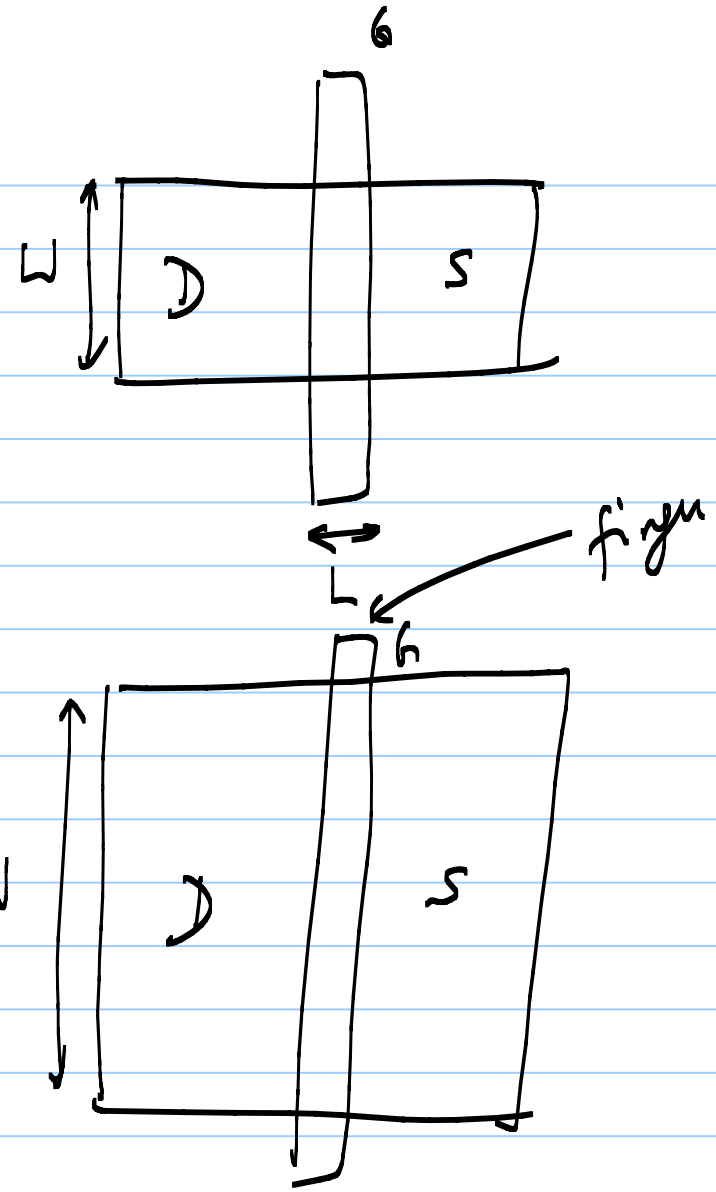
parameters of MOSFETs such as V_T are weak functions of W, L ✓

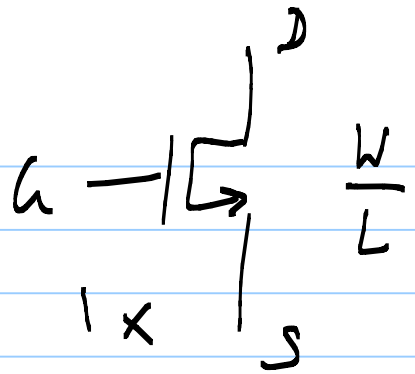


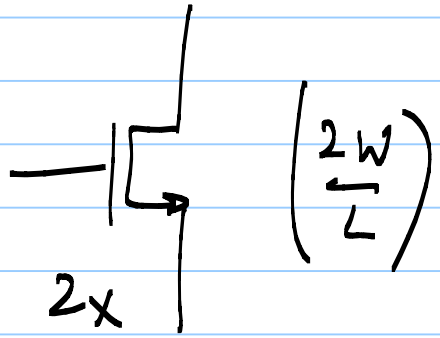
$M_1 = M_2 = M_3$

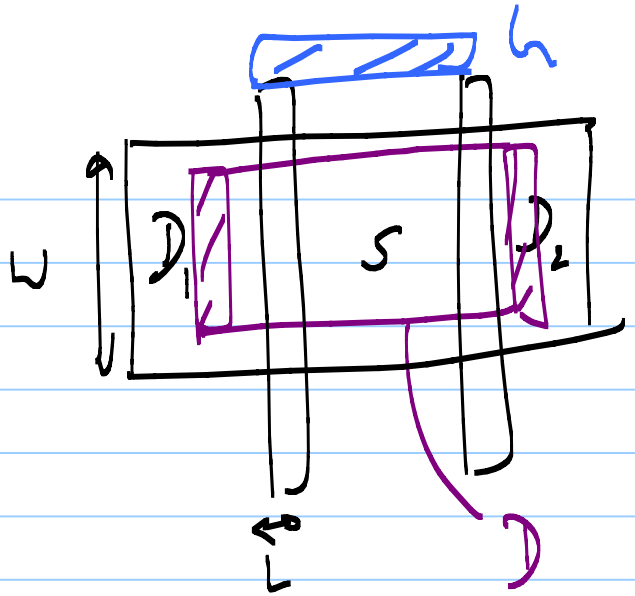
Separate device, not just 2x # of fingers

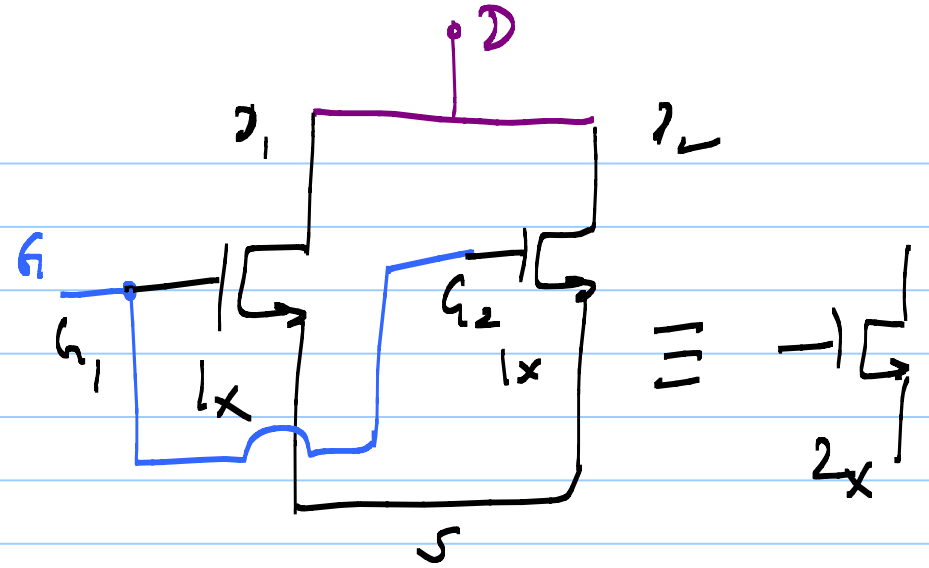
Top view (layout)

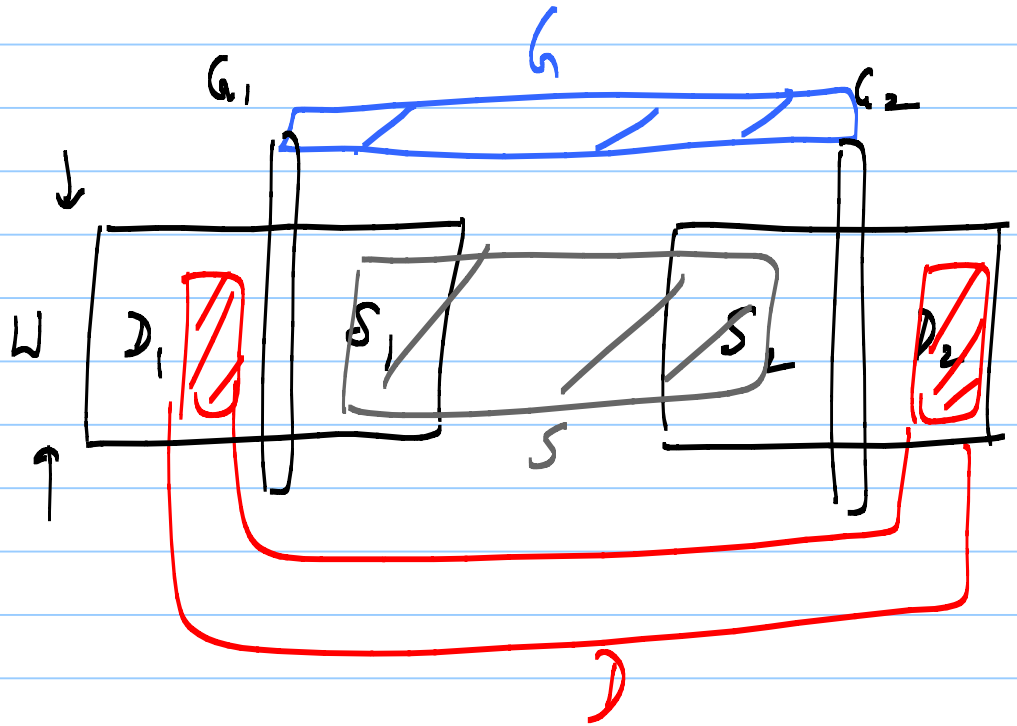


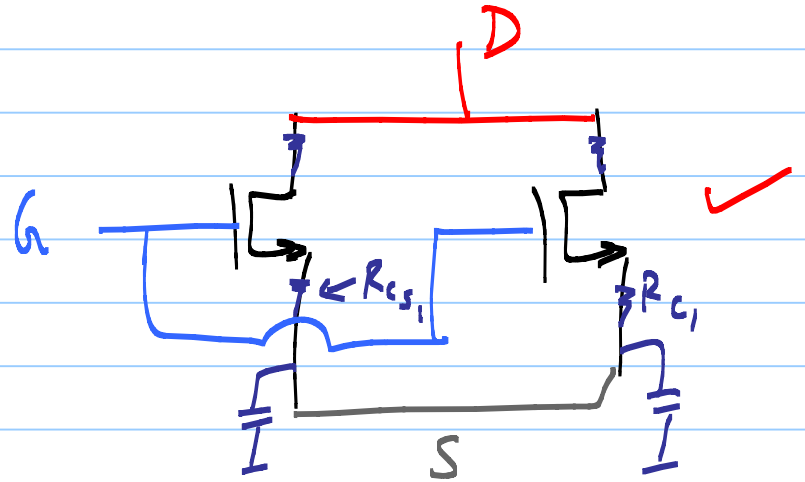




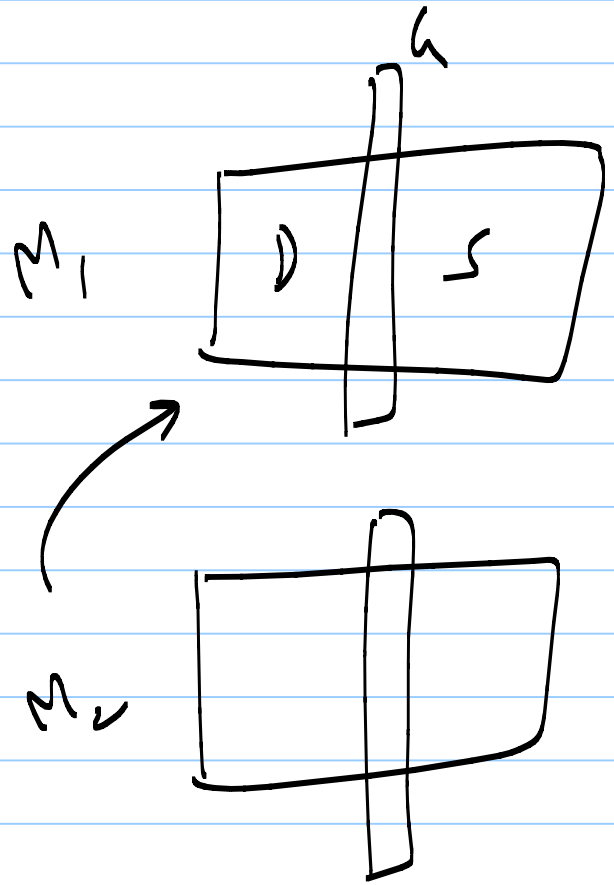

G
W
D1
S
D2
D
L

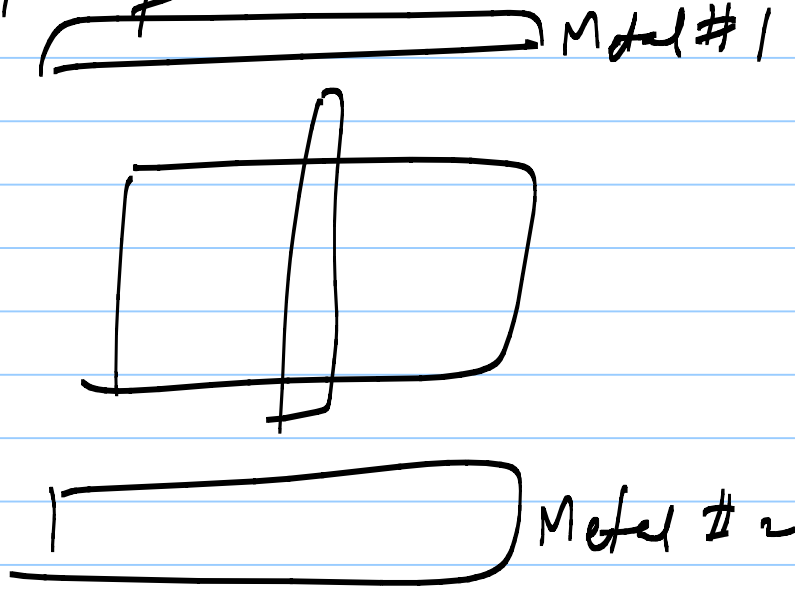

D
D1
D2
G
G1
G2
Lx
Lx
S
≡
2x


G
G1
G2
W
D1
S1
S2
D2
S
D


D
G
RCS1
RC1
S

Ex 3 : Device parameter weak fn. of environment

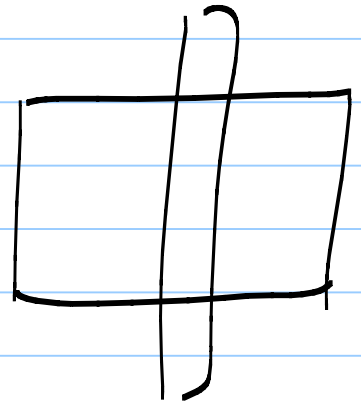




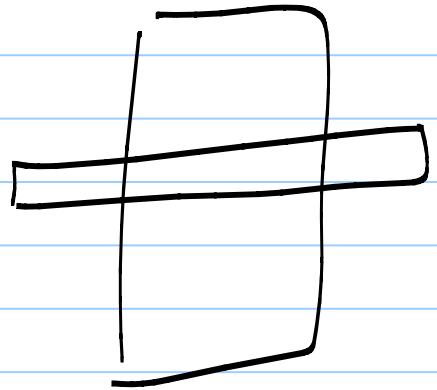
e.g. Replicate Metal routing also

Ex 4 $M_1 \equiv M_2$ is needed $\Rightarrow$ Same orientation

(a)

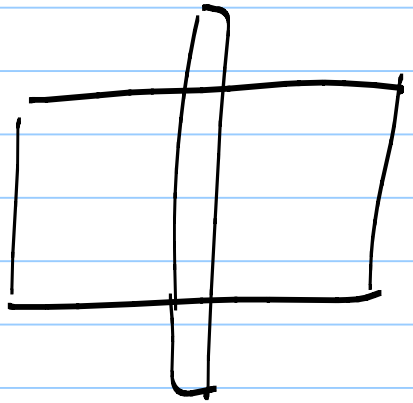


M_1

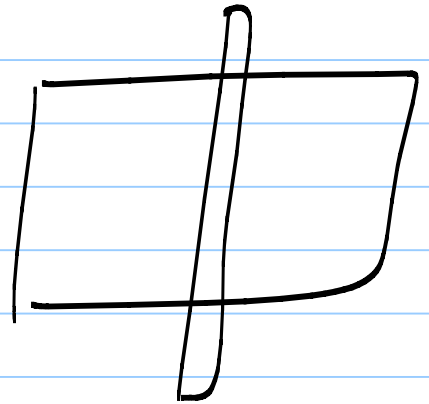


M_2 X

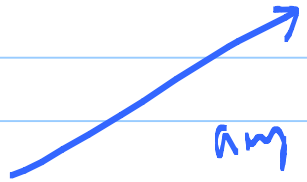
(b) M_1



Temp. gradient



M_2



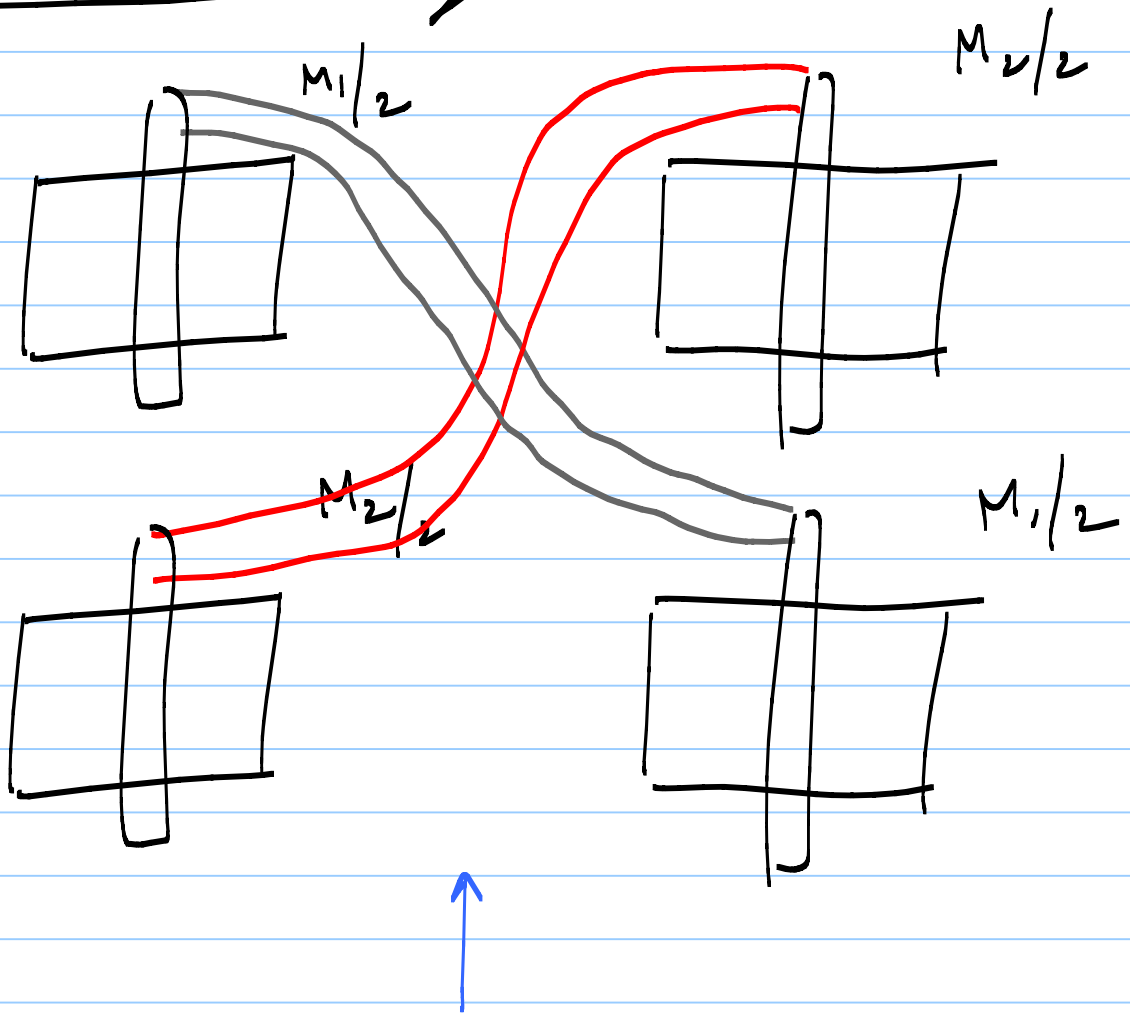
any random direction

Common-centroid layout

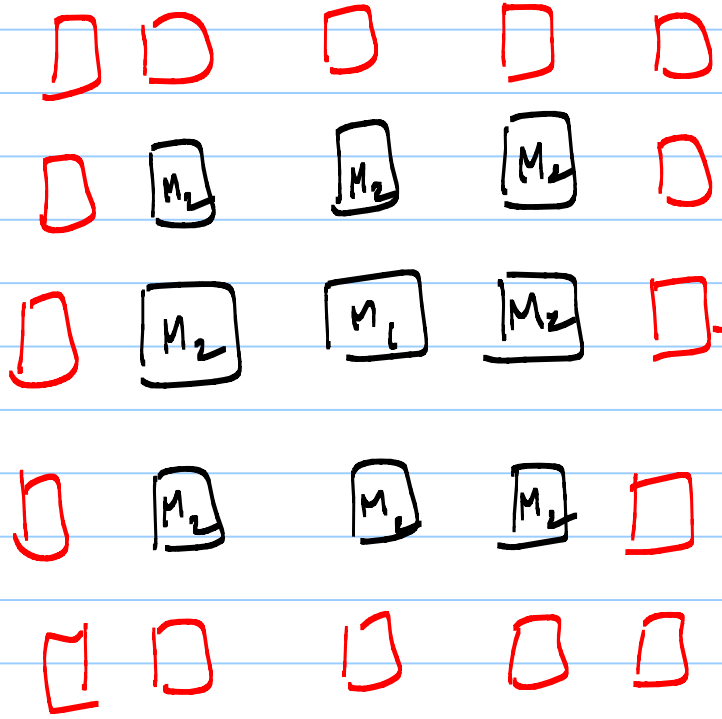
Split M_1 & M_2 into equal halves

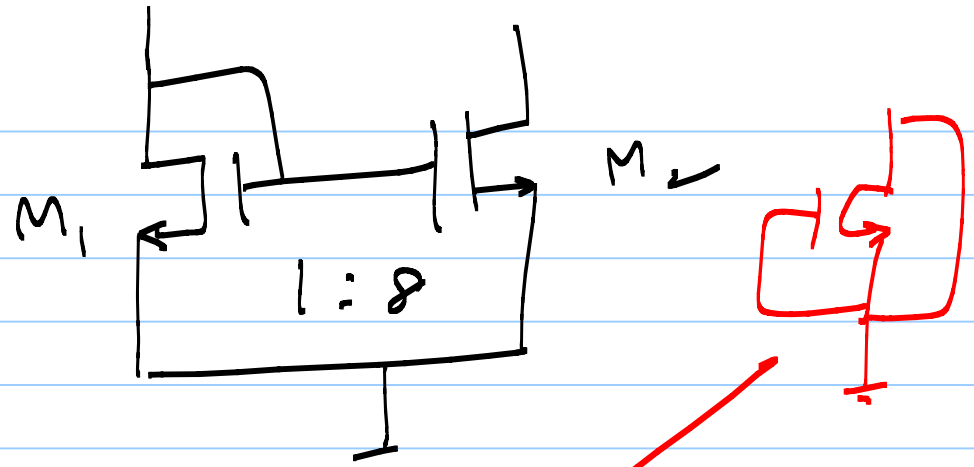
$W \rightarrow \frac{W}{2}$

L same

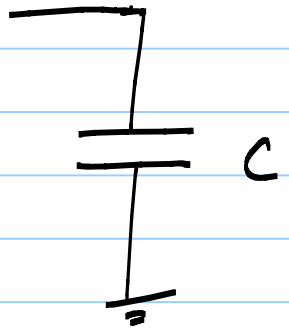


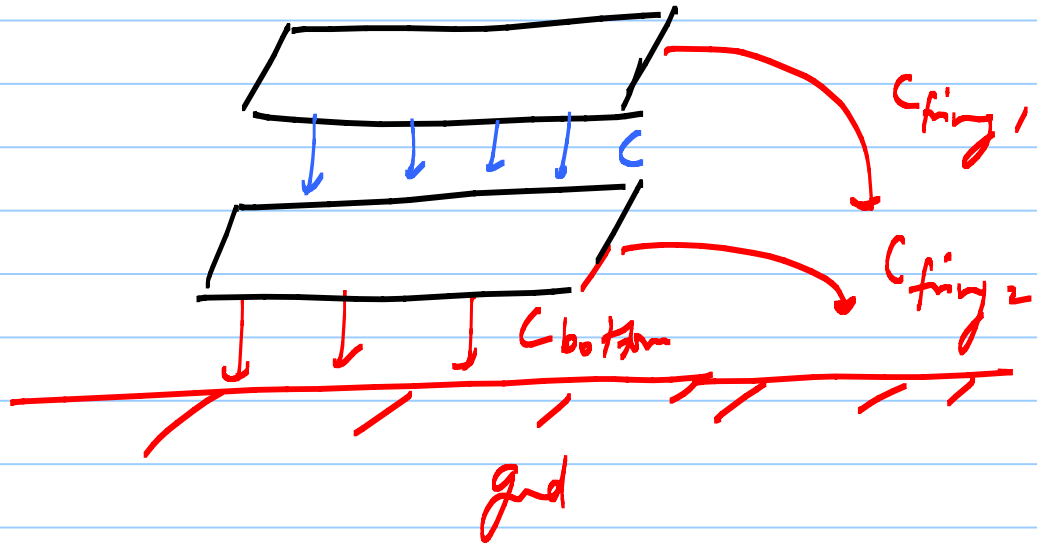
Array layout

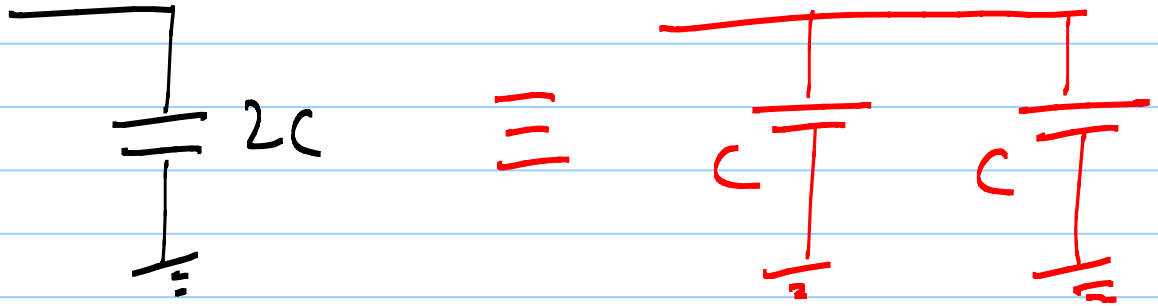




Capacitor

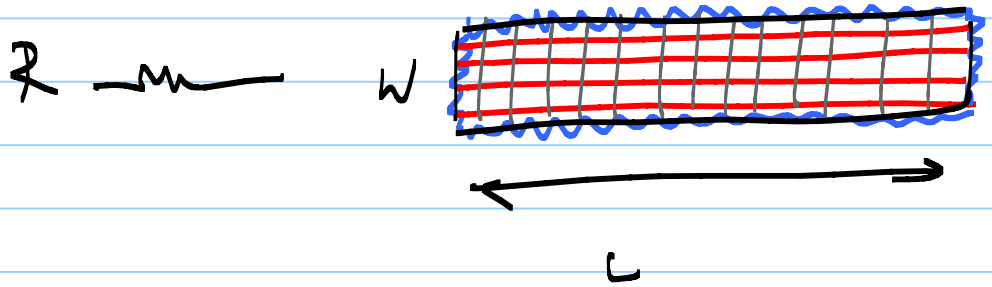






Random Mismatch

Mean = 0



represented by σ

slice is vertical & horizontal direction and determine dependence

Similar effects for

C, V_T, $\beta = \mu C_{ox}$, ...

$$\sigma \propto \frac{1}{\sqrt{Area}}$$

empirical constant of proportionality that depends on the process

$$\sigma_{\frac{\Delta C}{C}} = \frac{A_C}{\sqrt{W_C \cdot L_C}}$$

$$\sigma_{\frac{\Delta R}{R}} = \frac{A_R}{\sqrt{W_R \cdot L_R}}$$

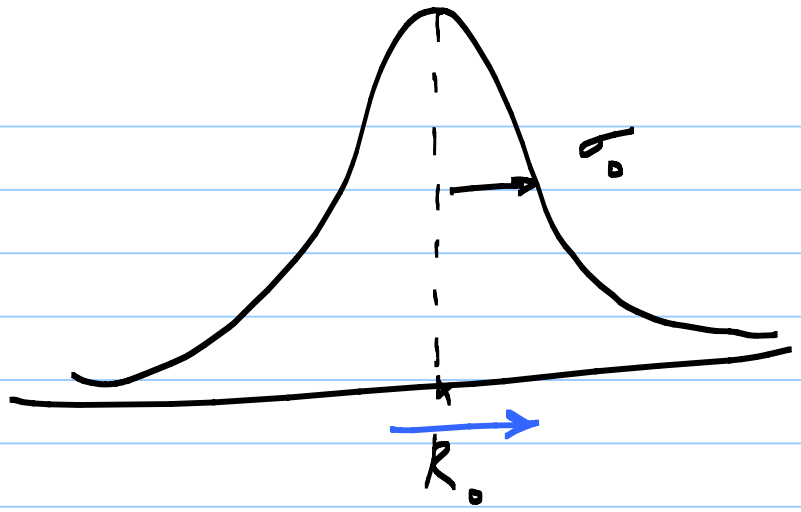
$$\sigma_{V_T} = \frac{A_{V_T}}{\sqrt{W \cdot L}}$$

$$\sigma_{\beta} = \frac{A_{\beta}}{\sqrt{W \cdot L}}$$

Low mismath

↕

Area of device

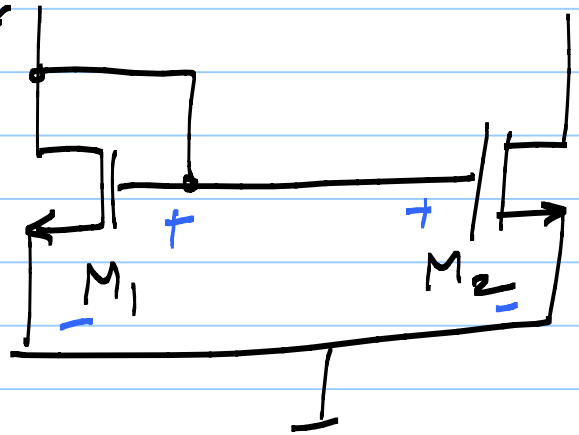


$R = R_0 \Rightarrow$ No systematic mismath

For random mismath → only σ matters

Choose W & L for acceptable σ

Example: $I_1 \downarrow$



$\downarrow I_2 = I_1 + \Delta I$

$M_1 \rightarrow V_T$

$M_2 \rightarrow V_T + \Delta V_T$

$V_{GS_1} = V_{GS_2}$

Only V_T mismath exists

$\frac{\Delta I}{I} = ?$ $\Rightarrow$ You need to find out $\sigma^2_{\frac{\Delta I}{I}}$ in terms of $\sigma^2_{V_T}$

$$I_D = \frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_T)^2$$

ΔV_T is small

$$\frac{\partial I_D}{\partial V_T} = -\frac{\partial I_D}{\partial V_{GS}} = -g_m$$

$$\Delta I_D = \frac{\partial I_D}{\partial V_T} \cdot \Delta V_T = -g_m \Delta V_T$$

$$\frac{\Delta I}{I} = -\frac{g_m}{I} \cdot \Delta V_T$$

$$\sigma^2_{\frac{\Delta I}{I}} = \frac{g_{m_2}^2}{I_2^2} \cdot \sigma^2_{V_T}$$

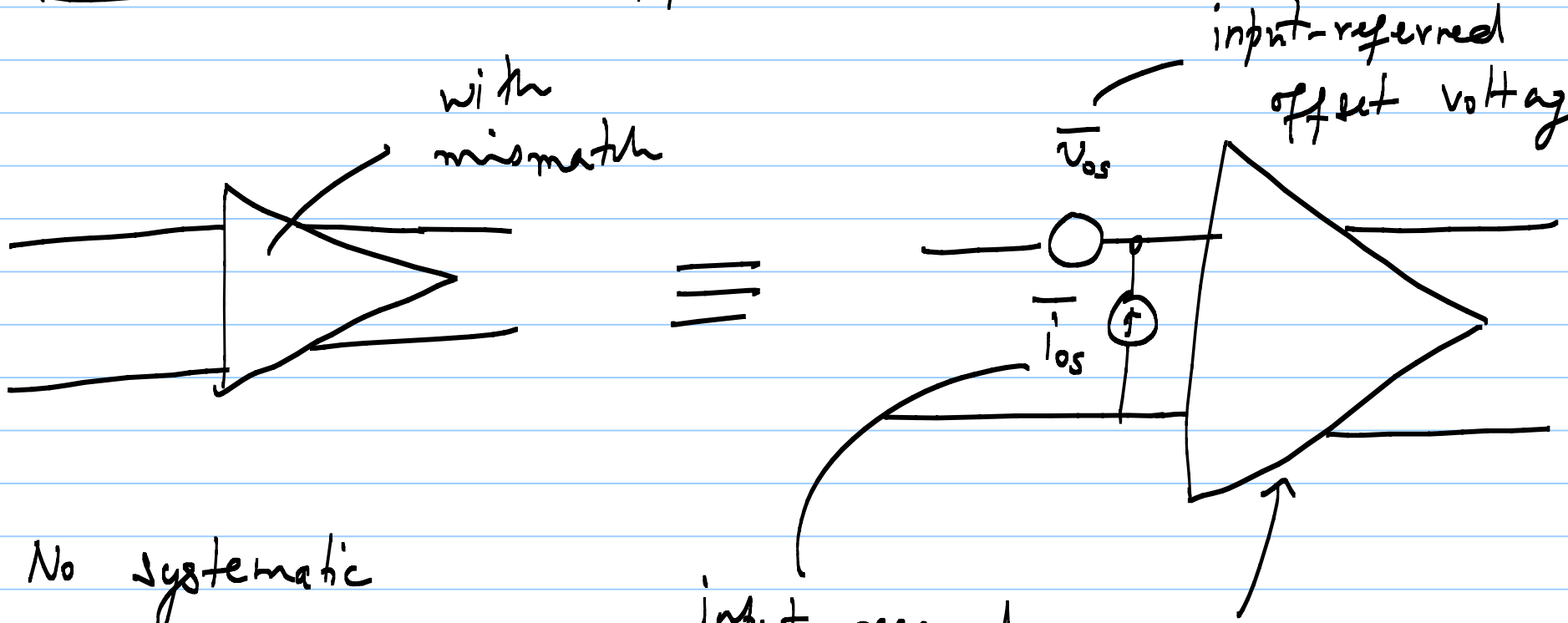
$$g_{m_2} = \frac{2 I_2}{V_{GS_2} - V_{T_2}} = \frac{2 I_2}{V_{DSat_2}}$$

$$\sigma^2_{\frac{\Delta I}{I}} = \frac{4}{V_{DSat}^2} \cdot \sigma^2_{V_T}$$

larger V_{DSat} $\longrightarrow$ smaller $\frac{\Delta I}{I}$

$V_{DSat} \Rightarrow I / \frac{W}{L}$; $\sigma^2_{V_T} \rightarrow 1/W \cdot L$

Mismatch in Differential Circuits

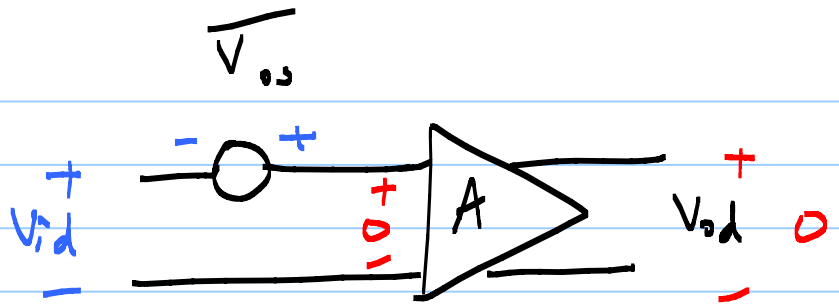


* No systematic mismatch
  ⇓
  $\overline{V_{os}}$ & $\overline{I_{os}}$ have zero mean

input referred offset current

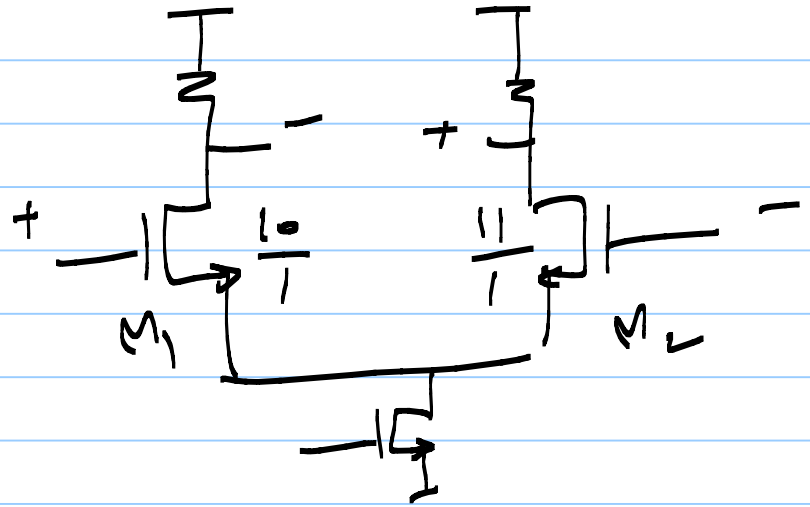
* $\overline{V_{os}}$ & $\overline{I_{os}}$ are r.m.s. values
* $\overline{I_{os}} = 0$ for MOSFET amplifier when input is applied at gate

no mismatch



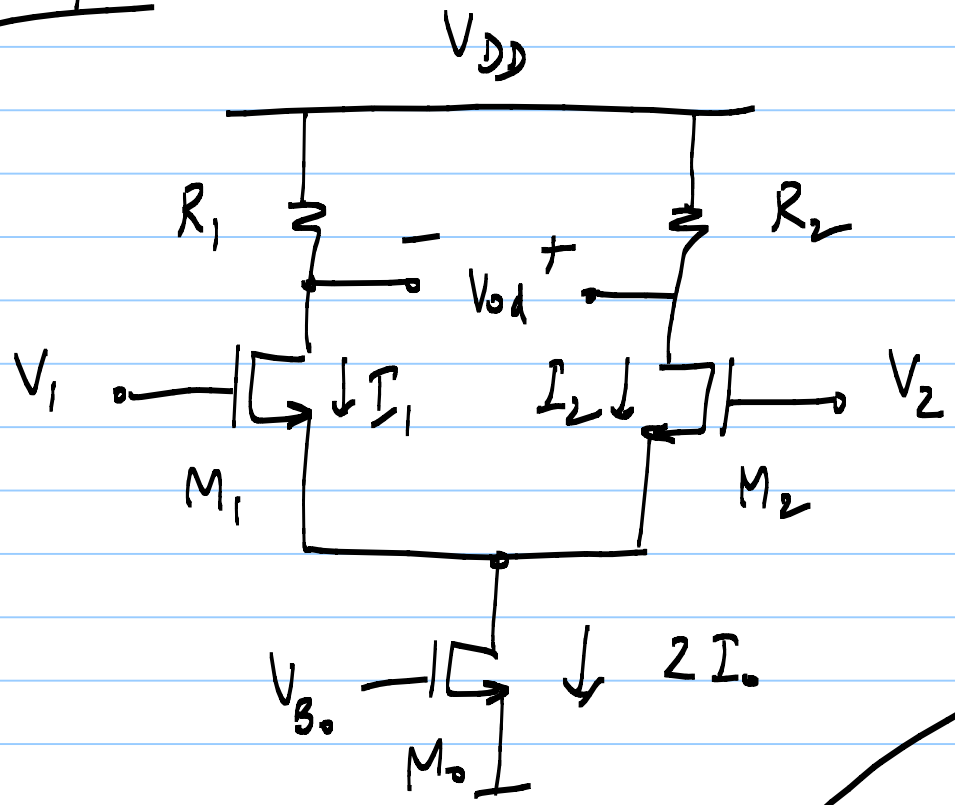
$V_{id} = V_{os}$

Input referred offset voltage V_{os} = voltage that needs to be applied at input so that output offset voltage = 0



$M_1 \neq M_2$

Example 2



$R_1 \neq R_2$, $V_{T_1} \neq V_{T_2}$,

$\left(\frac{W}{L}\right)_1 \neq \left(\frac{W}{L}\right)_2$

$\overline{V_{os}} = ?$

$V_1 - V_2 = V_{os}$ } conditions as per our definition

$V_{od} = 0$

$I_1 \neq I_2$; $V_{GS_1} \neq V_{GS_2}$

$V_1 - V_2 = V_{GS_1} - V_{GS_2}$

$I_1 R_1 = I_2 R_2$

(A) $R = \frac{R_1 + R_2}{2}$; $\Delta R = \frac{R_1 - R_2}{2}$

$R_1 = R + \Delta R$; $R_2 = R - \Delta R$

(B) $\left(\frac{W}{L}\right) = \frac{\left(\frac{W}{L}\right)_1 + \left(\frac{W}{L}\right)_2}{2}$; $\Delta\left(\frac{W}{L}\right) = \frac{\left(\frac{W}{L}\right)_1 - \left(\frac{W}{L}\right)_2}{2}$

(C) $I = \frac{I_1 + I_2}{2}$; $\Delta I = \frac{I_1 - I_2}{2}$

$I_1 = I + \Delta I$; $I_2 = I - \Delta I$

(D) $V_{T_1} - V_{T_2} = \Delta V_T$

$$I_1 R_1 = I_2 R_2$$

$$(I + \Delta I)(R + \Delta R) = (I - \Delta I)(R - \Delta R)$$

$$\left(1 + \frac{\Delta I}{I}\right)\left(1 + \frac{\Delta R}{R}\right) = \left(1 - \frac{\Delta I}{I}\right)\left(1 - \frac{\Delta R}{R}\right)$$

$$1 + \frac{\Delta I}{I} + \frac{\Delta R}{R} + \frac{\Delta I \Delta R}{IR} = 1 - \frac{\Delta I}{I} - \frac{\Delta R}{R} + \frac{\Delta I \Delta R}{IR}$$

$$\boxed{\frac{\Delta I}{I} = -\frac{\Delta R}{R}} \qquad \text{—— (1)}$$

$$V_1 - V_2 = V_{GS_1} - V_{GS_2}$$

$$= \left[V_{T_1} + \sqrt{\frac{2I_1}{\mu C_{ox}\left(\frac{W}{L}\right)_1}} \right] - \left[V_{T_2} + \sqrt{\frac{2I_2}{\mu C_{ox}\left(\frac{W}{L}\right)_2}} \right]$$

$$= \Delta V_T + \sqrt{\frac{2}{\mu C_{ox}}} \left[\sqrt{\frac{I + \Delta I}{\left(\frac{W}{L}\right) + \frac{\Delta W}{L}}} - \sqrt{\frac{I - \Delta I}{\frac{W}{L} - \frac{\Delta W}{L}}} \right]$$

$$= \Delta V_T + \underbrace{\sqrt{\frac{2I}{\mu C_{ox}\left(\frac{W}{L}\right)}}}_{\text{nominal } V_{Dsat}} \left[\sqrt{\frac{1 + \Delta I/I}{1 + \frac{\Delta W/L}{W/L}}} - \sqrt{\frac{1 - \Delta I/I}{1 - \frac{\Delta W/L}{W/L}}} \right]$$

$$= \Delta V_T + V_{Dsat}\left[\left(1+\frac{\Delta I}{2I}\right)\left(1-\frac{\Delta W/L}{2\,W/L}\right)-\left(1-\frac{\Delta I}{2I}\right)\left(1+\frac{\Delta W/L}{2\,W/L}\right)\right]$$

$$V_1-V_2 = \Delta V_T + V_{Dsat}\left[\left(\frac{\Delta I}{I}\right)-\left(\frac{\Delta W/L}{W/L}\right)\right]$$

Apply (1):

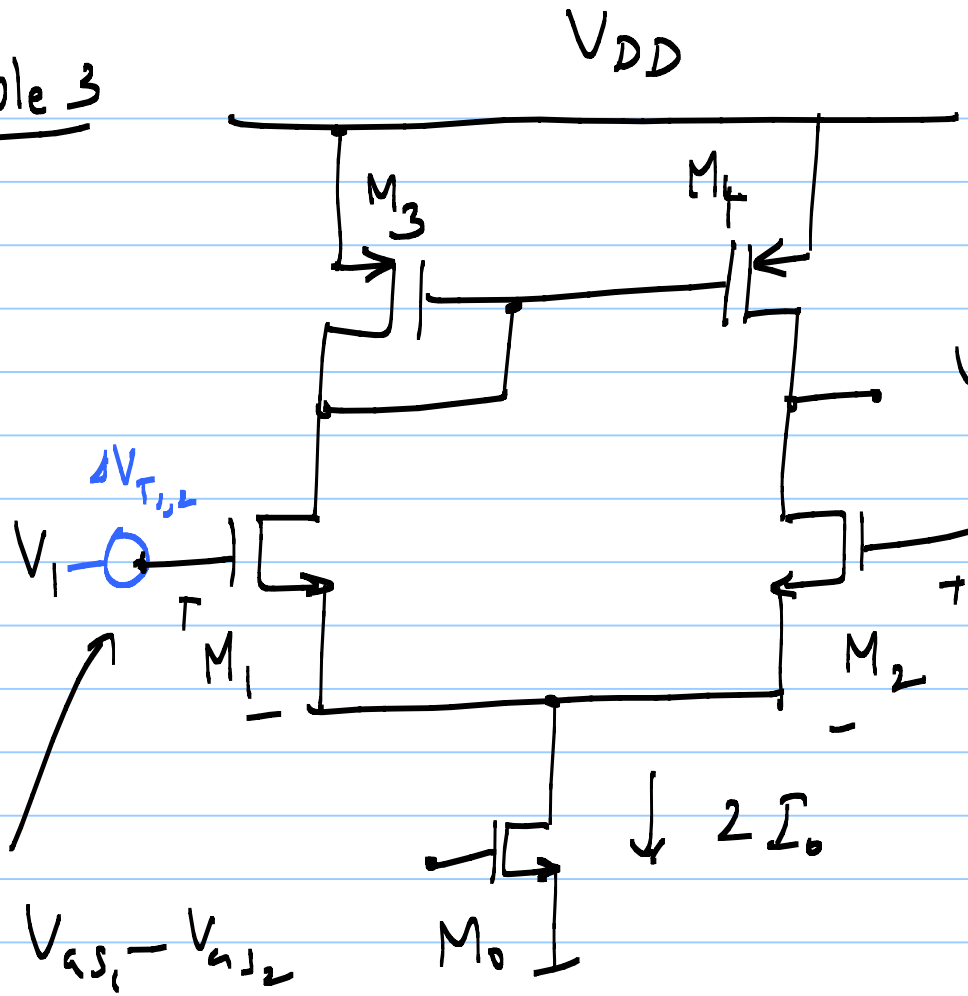
$$V_1-V_2 = V_{os} = \Delta V_T + V_{Dsat}\left[\frac{-\Delta R}{R}-\frac{\Delta W/L}{W/L}\right]$$

Input referred offset voltage

$$V_{os} = \Delta V_T - \frac{V_{Dsat}}{2}\left[2\frac{\Delta R}{R}+2\frac{\Delta W/L}{W/L}\right]$$

$$\sigma_{os}^2 = \underbrace{\sigma_{V_T}^2}_{A_{V_T}^2/WL} + \frac{V_{Dsat}^2}{4}\left[\underbrace{\sigma_{\Delta R/R}^2}_{A_R^2/WL} + \underbrace{\sigma_{\frac{\Delta W/L}{W/L}}^2}_{\frac{A_{\Delta W/L}^2}{WL}}\right]$$

Example 3



$V_{T_1} - V_{T_2} = \Delta V_{T,12}$

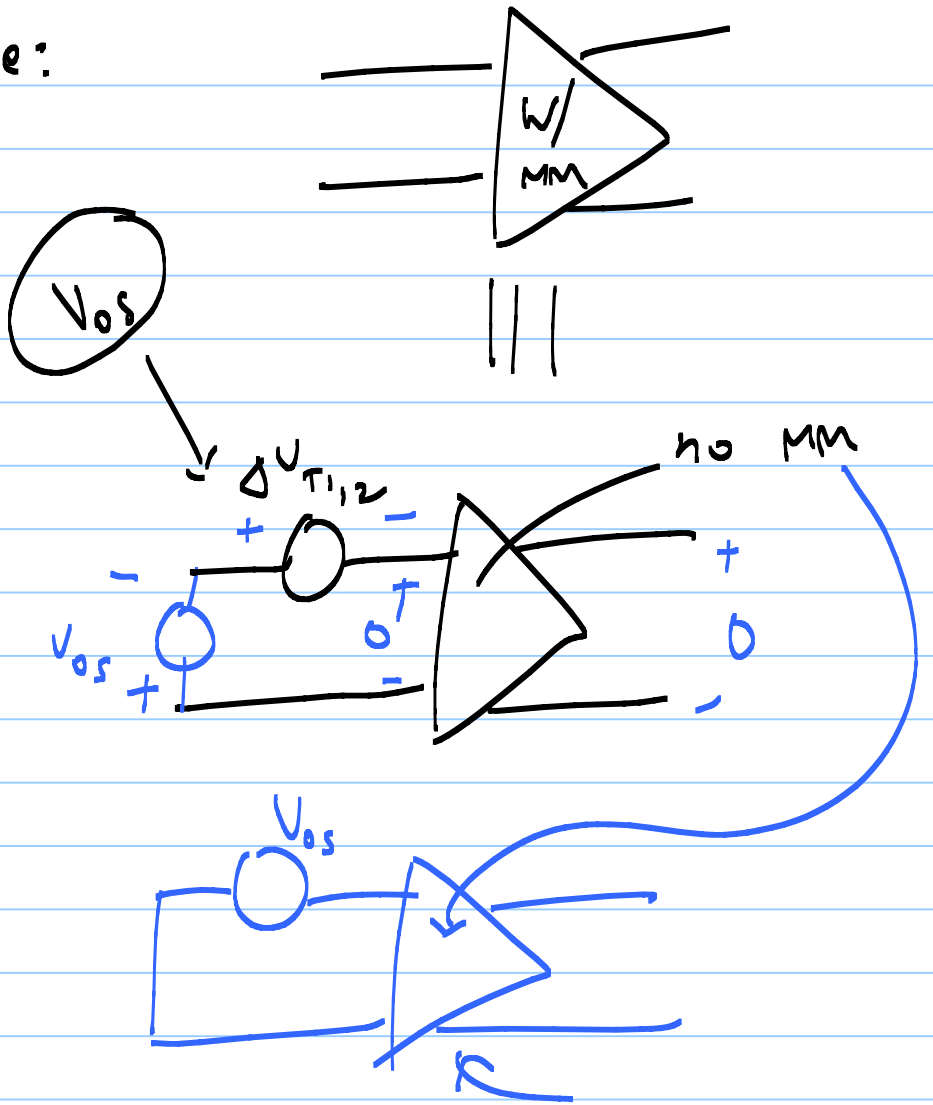
No other mismatches

$V_{DC_{nom.}} = V_{DD} - V_{SG_{34}}$ | I_0

$V_{OS} = \Delta V_{T,12}$

$V_1 - V_2 = V_{GS_1} - V_{GS_2}$

Previous example:



$$V_{OS} = V_1 - V_2 = V_{GS_1} - V_{GS_2}$$

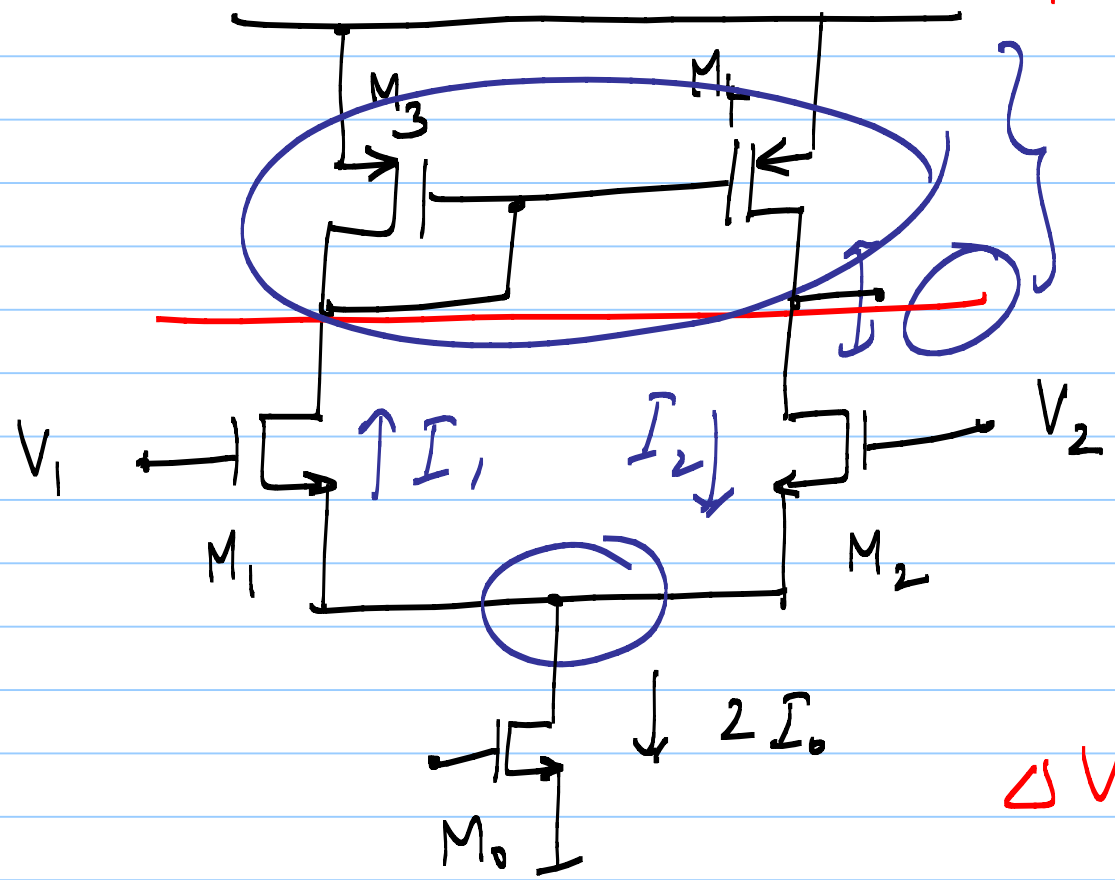
$$= \left(V_{T_1} - V_{T_2}\right) + \underbrace{\left(V_{DSAT_1} - V_{DSAT_2}\right)}$$

$= 0$ if $I_1 = I_2 = I$

$= \Delta V_{T_{1,2}}$

$\left(\frac{W}{L}\right)$, μC_{ox} are same

Example 4 — HW



$$I_{D3} = \frac{\beta}{2}(V_{SG3} - V_{T3})^2(1 + \lambda V_{DS3})$$

$$I_{D4} = \frac{\beta}{2}(V_{SG4} - V_{T4})^2(1 + \lambda V_{DS4})$$

Only mismatch in $V_{T3,4}$

$$= \frac{\beta}{2}(V_{SG3} - V_{T4})^2$$

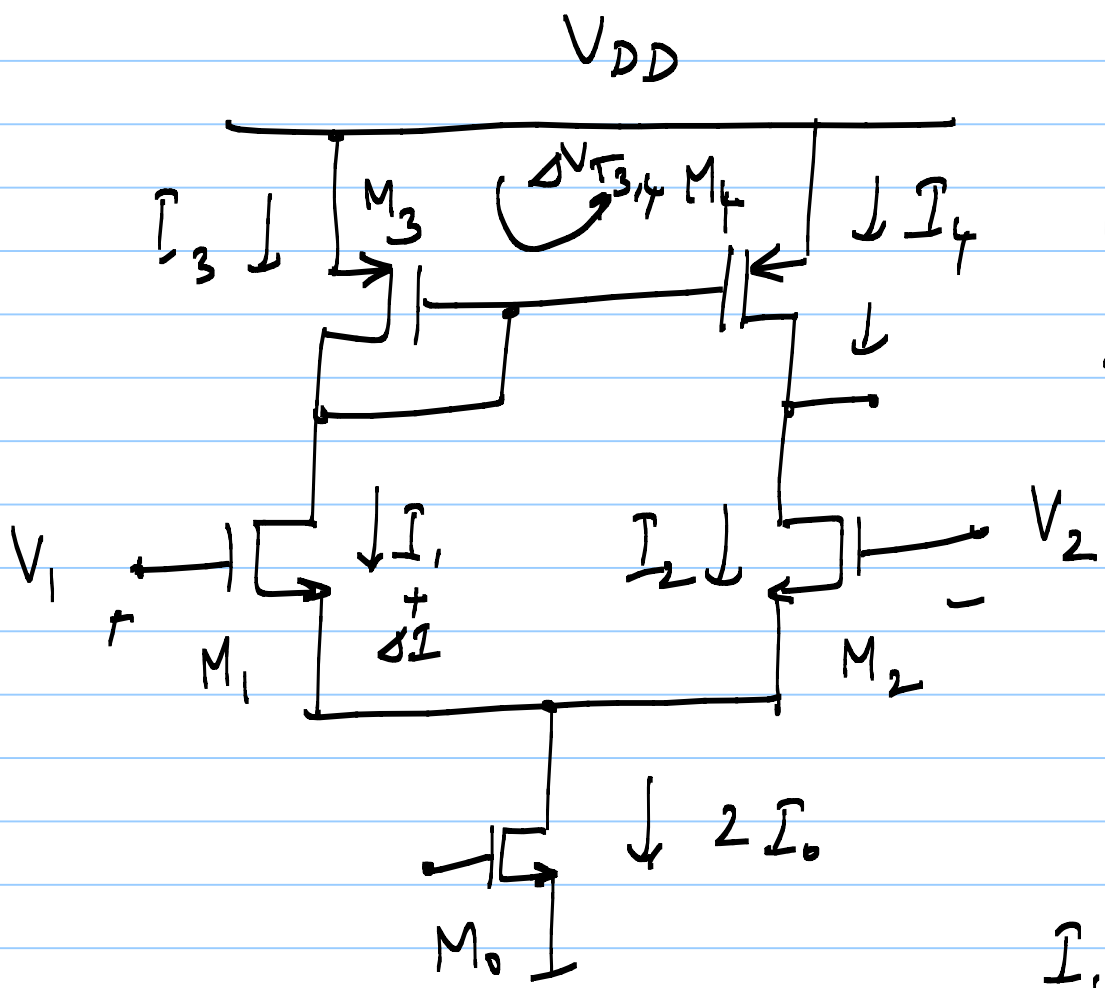
$$V_{T3} - V_{T4} = \Delta V_{T3,4}$$

$V_{OS} = ?$

$$\Delta V_{in} = \frac{g_{m3}}{g_{m1}} \Delta V_{T3,4}$$

$$I_1 + I_2 = 2I_0$$

Lecture 15



only $\Delta V_{T3,4}$ exists

$V_{T3} = V_{T4} + \Delta V_{T3,4}$

$\Delta I_{3,4} = \Delta V_{T3,4} \cdot g_{m3,4}$

$I_3 < I_4$

$I_4 = I_3 + \Delta I_{3,4}$

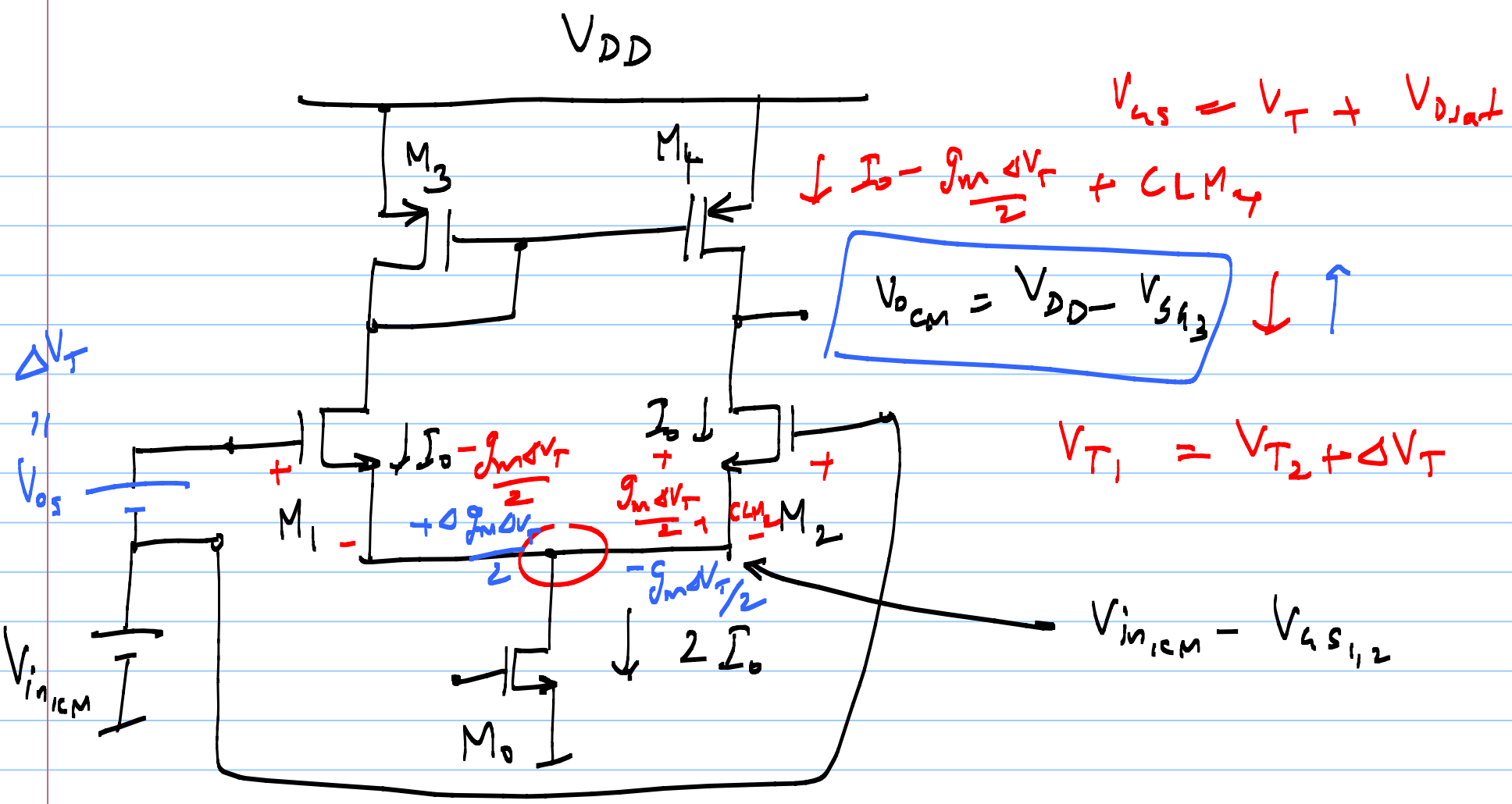
$V_{SG_3} = V_{SG_4}$

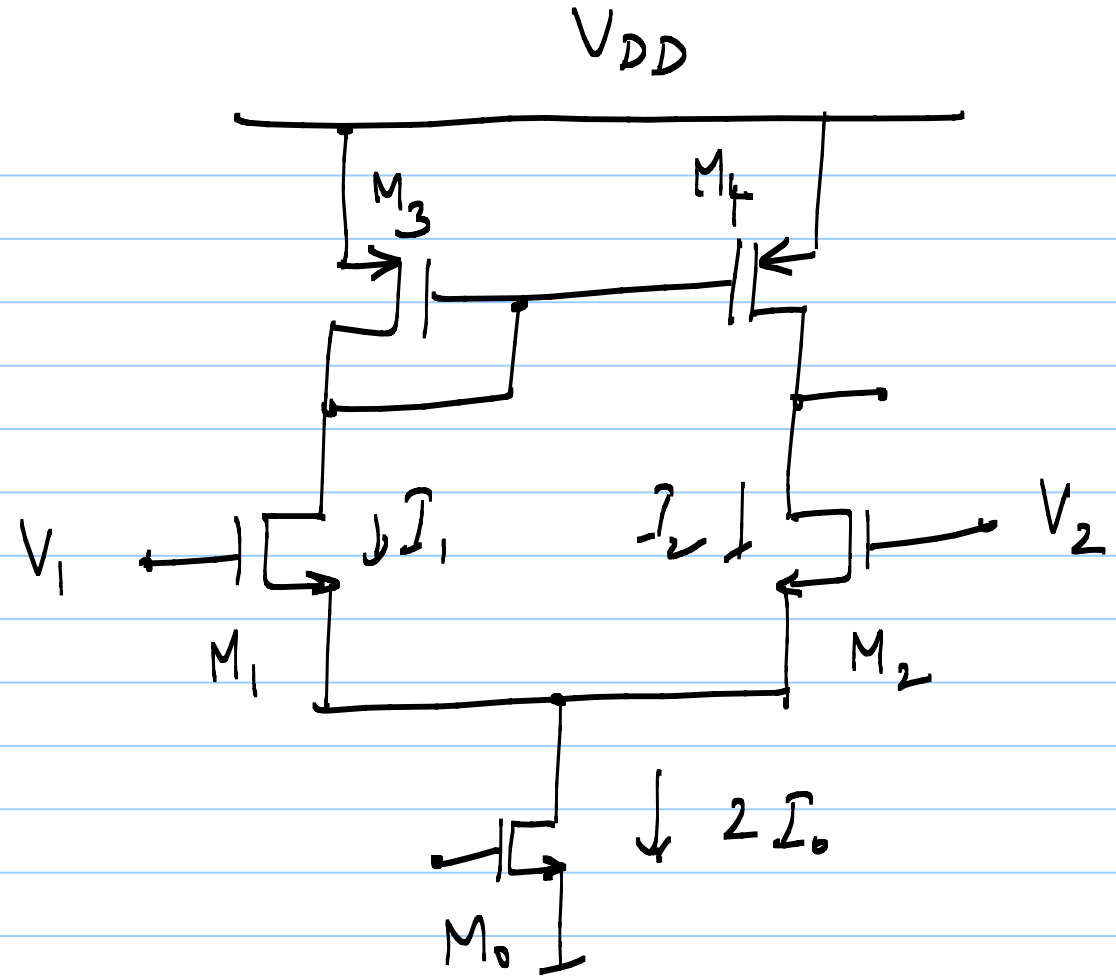
$I_1 = I_3, \quad I_2 = I_4$

$I_1 + I_2 = 2I_0$

$V_{in,CM} = \frac{V_1 + V_2}{2}$

$V_1 - V_2 = V_{GS_1} - V_{GS_2}$


V_{DD}
M_3
M_4
M_1
M_2
M_0
$V_{in_{ICM}}$
V_{OS}
ΔV_T
$I_0 - \frac{g_m \Delta V_T}{2}$
I_0
$+ \frac{g_m \Delta V_T}{2}$
$+ \Delta g_m \frac{\Delta V_T}{2}$
$- g_m \Delta V_T / 2$
$2 I_0$
$V_{in,CM} - V_{GS_{1,2}}$
$V_{GS} = V_T + V_{Dsat}$
$\downarrow I_D - g_m \frac{\Delta V_T}{2} + CLM_4$
$V_{O_{CM}} = V_{DD} - V_{SG_3}$
$V_{T_1} = V_{T_2} + \Delta V_T$



Apply $V_1 - V_2 = \Delta V$

$$I_1 = I_0 + g_{m_1}\frac{\Delta V}{2}$$

$$I_2 = I_0 - g_{m_1}\frac{\Delta V}{2}$$

$$\Delta I_{3,4} = g_{m_{3,4}} \cdot \Delta V_{T_{3,4}}$$

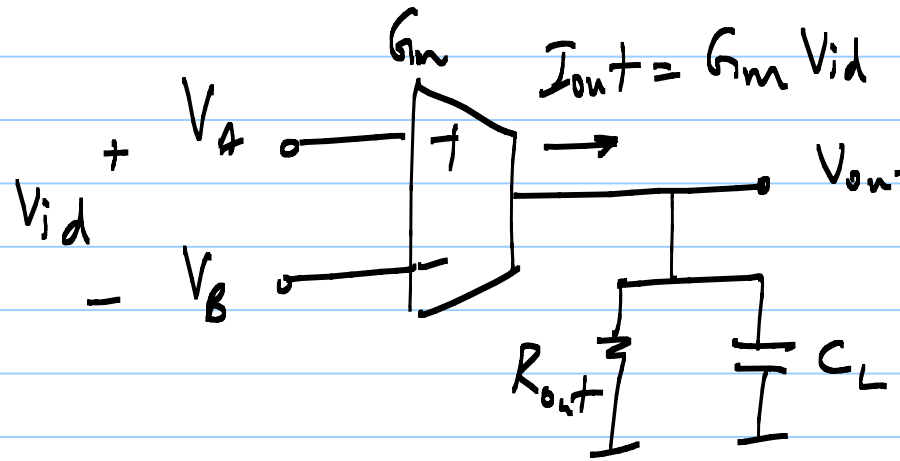
$$V_1 - V_2 = V_{OS} = \pm \frac{g_{m_3}}{g_{m_1}} \cdot \Delta V_{T_{3,4}}$$

$$\sigma_{OS}^2 = \left(g_{m_3}/g_{m_1}\right)^2 \sigma_{V_{T_{3,4}}}^2$$

Overall

$$\sigma_{OS}^2 = \sigma_{V_{T_{1,2}}}^2 + \left(\frac{g_{m_3}}{g_{m_1}}\right)^2 \sigma_{V_{T_{3,4}}}^2$$

One-stage opamp Datasheet



$$V_{out} = G_m R_{out} \cdot \frac{1}{1 + s C_L R_{out}} V_{id}$$

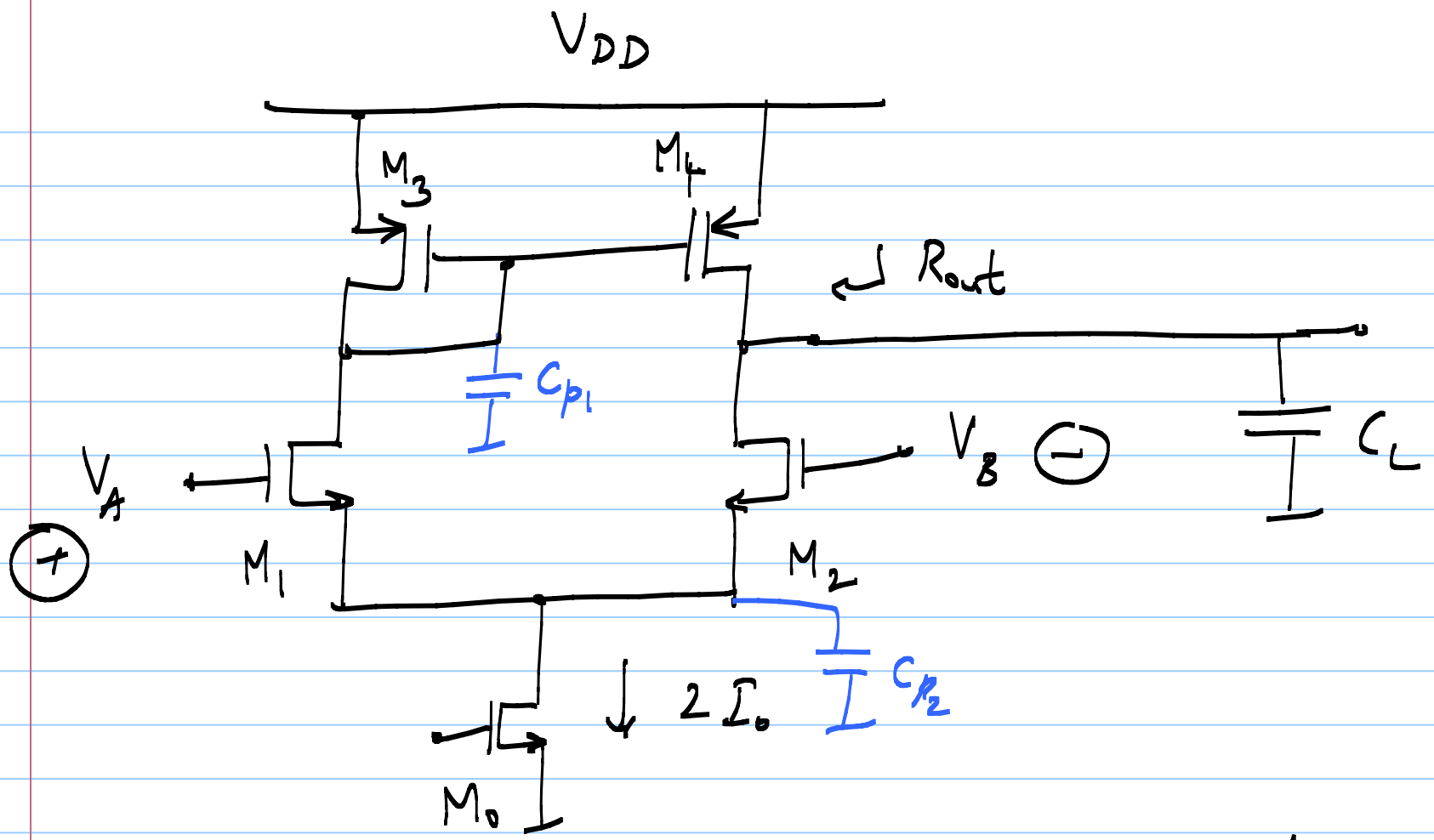
ideal response

$$\frac{V_{out}}{V_{id}}(s) = \frac{G_m}{s C_L} = \frac{\omega_u}{s}$$

$$\omega_u = \frac{G_m}{C_L}$$

1) DC gain = $g_{m_1} \underbrace{(r_{ds_2} \parallel r_{ds_4})}_{R_{out}}$

2) $G_m = g_{m_1}$



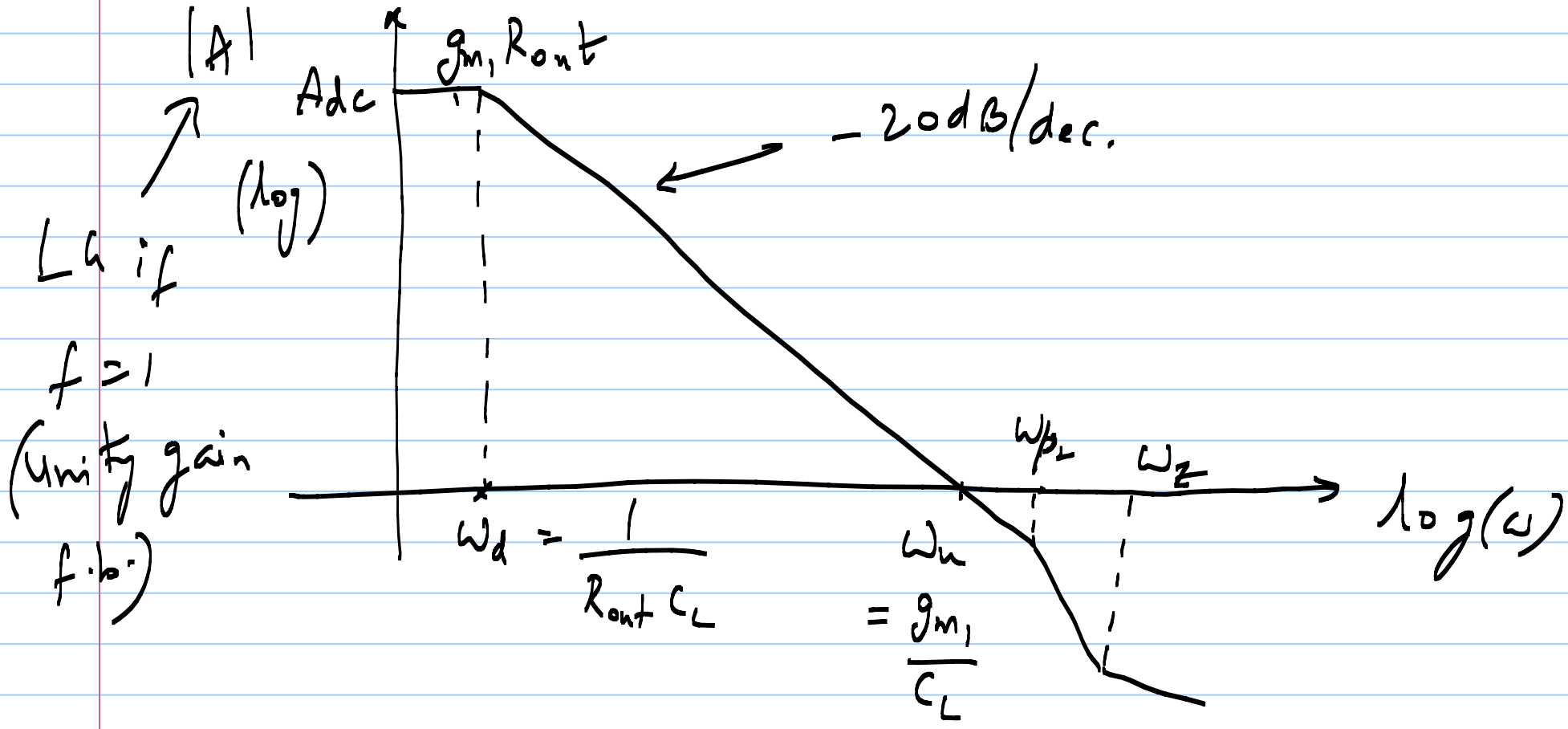
3) $\omega_u = \frac{g_{m1}}{C_L}$; $\omega_d = \frac{-1}{C_L R_{out}}$

UGF

Dominant pole

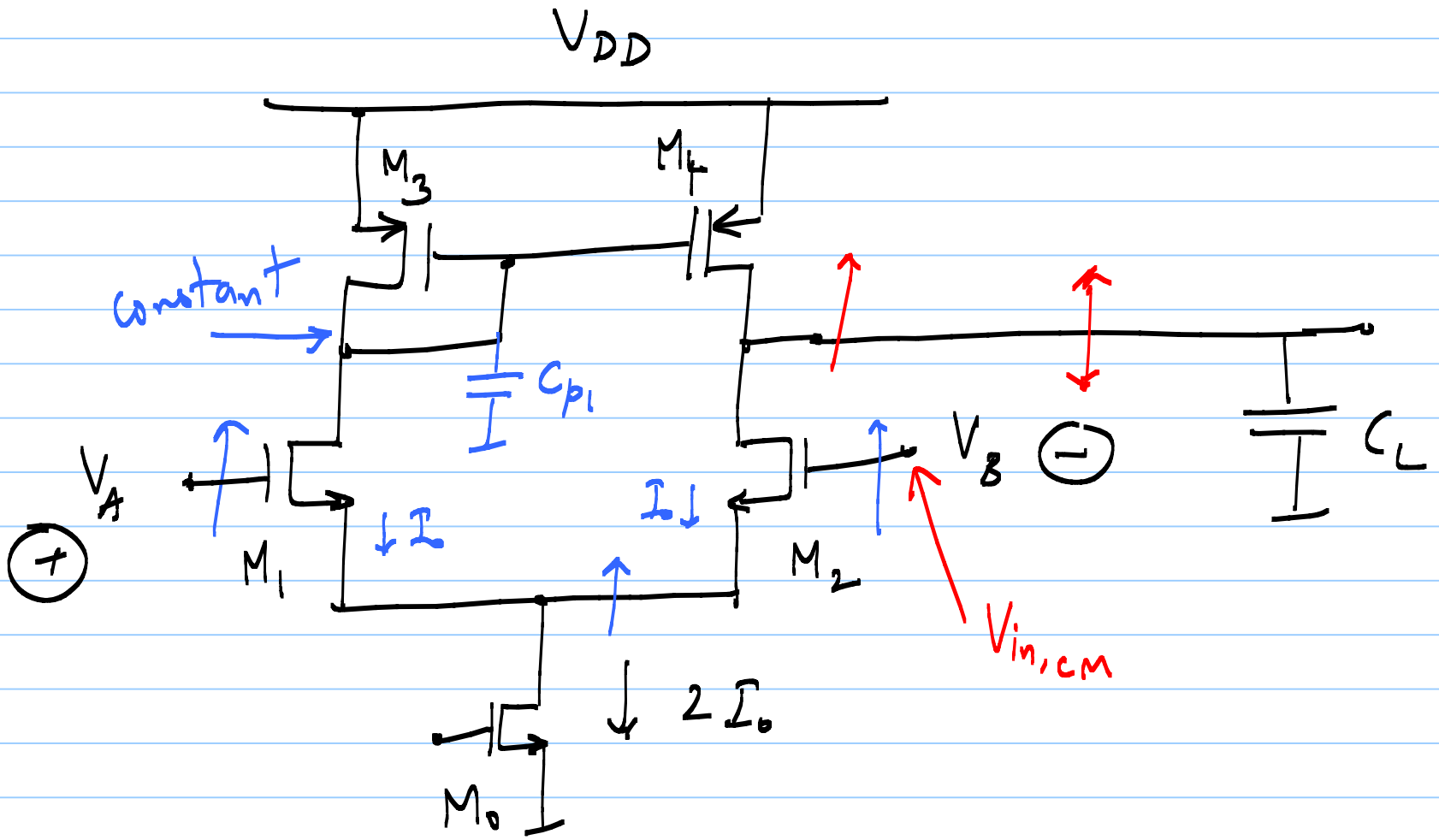
4) Non-dominant poles & zeroes : ω_{p_2}, ω_z

$$\omega_{p_2} = \frac{-g_{m_3}}{C_{p_1}} \; ; \quad \omega_z = \frac{-2g_{m_3}}{C_{p_1}} = 2\omega_{p_2}$$



* C_{p_2} affects only CMRR

One-stage opamp Datasheet



5) Input CM range :

$$V_{in,CM} = \left\{ V_{in,CM_{min}},\ V_{in,CM_{max}} \right\}$$

$$V_{in,CM_{min}} = V_{GS_1}\Big|_{I_0} + V_{DSat_0}\Big|_{2I_0}$$

$$V_{in,CM_{max}} = V_{DD} - V_{SG_3}\Big|_{I_0} + V_{T_1}$$

6) Output CM range

$$V_{out,CM_{min}} = V_{in,CM} - V_{T_2}$$

$$V_{out,CM_{max}} = V_{DD} - V_{SDsat_4}\Big|_{I_0}$$

$V_{SG_4} - V_{T_4}$

7) Input referred noise $\overline{e_n}$ (thermal only)

$\overline{i_n} = 0$

$$\overline{e_n^2} = \frac{16}{3} kT \cdot \frac{\left(g_{m_1} + g_{m_3}\right)}{g_{m_1}^2}$$

8) Input-referred offset voltage σ_{OS}^2 or $\overline{V_{OS}}$ (V_T mismatch only)

$\overline{I_{OS}} = 0$

$$V_{OS} = \Delta V_{T_{1,2}} + \left(\frac{g_{m_3}}{g_{m_1}}\right) \Delta V_{T_{3,4}}$$

$$\sigma_{OS}^2 = \sigma_{V_{T_{1,2}}}^2 + \left(\frac{g_{m_3}}{g_{m_1}}\right)^2 \sigma_{V_{T_{3,4}}}^2$$

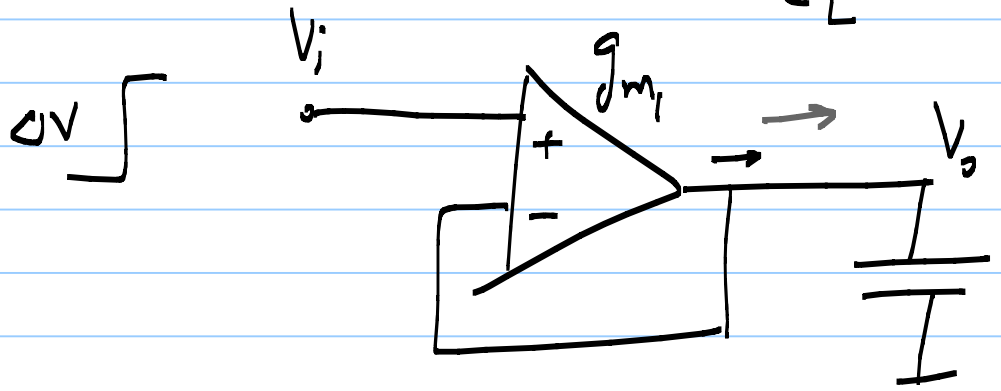
Low noise & offset: low g_{m_3}

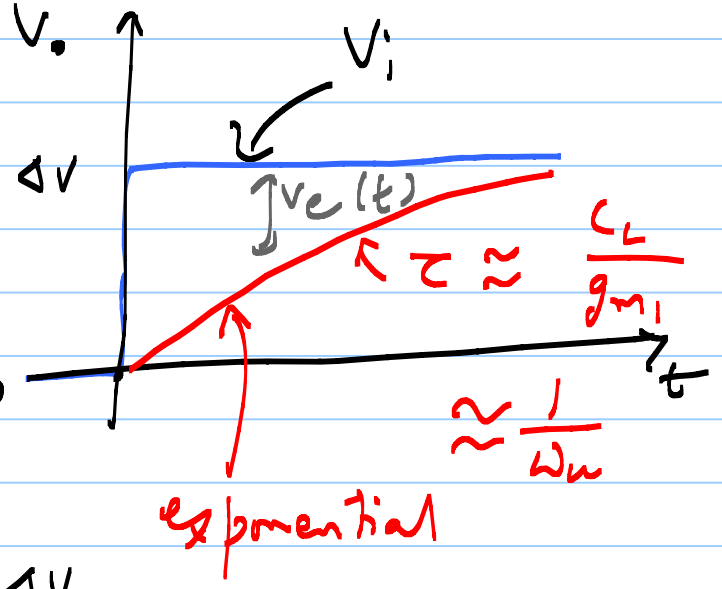
high g_{m_1}

large W_1, L_1 } reduces 1/f noise

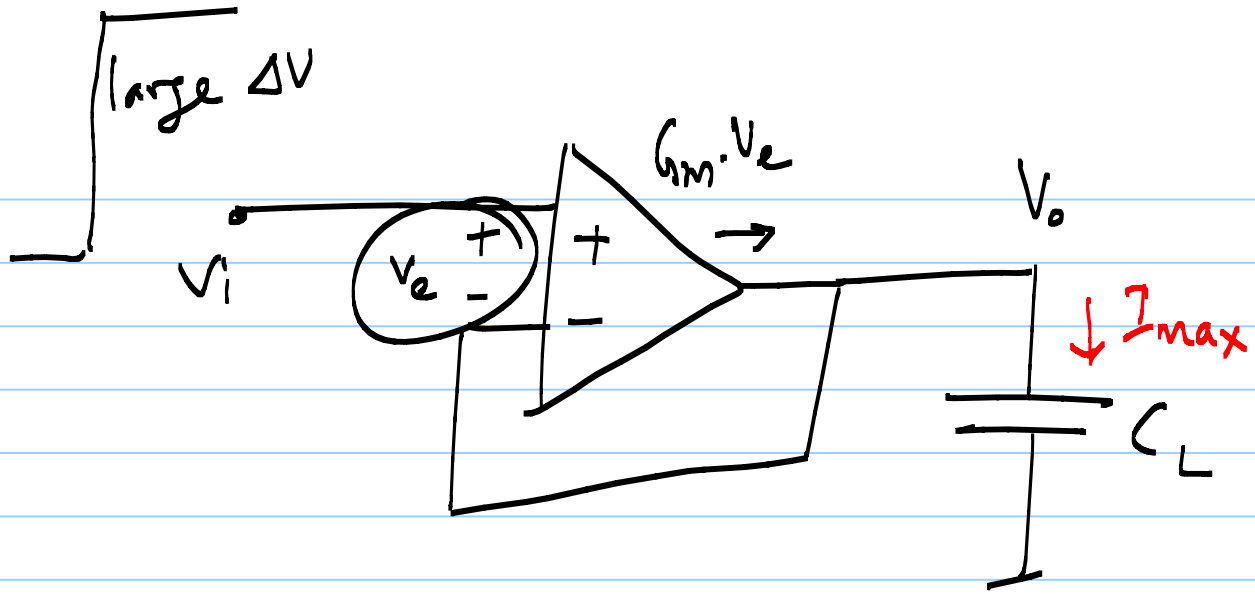
maybe large W_3, L_3 } also

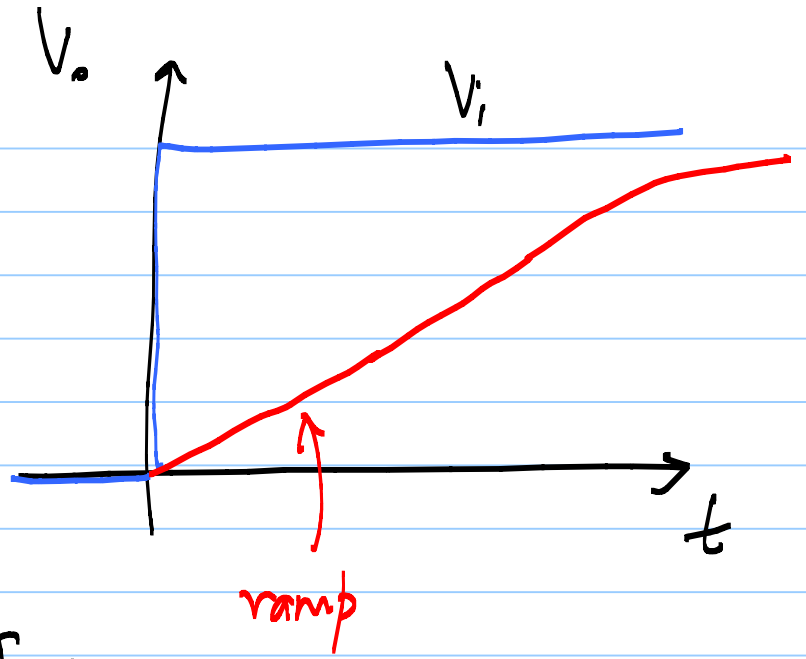
9) Slew rate $= \pm \frac{\alpha I_0}{C_L}$





$v_e(t=0^+) = \Delta V \Rightarrow I_{out}(t=0^+) = g_{m_1} \Delta V$

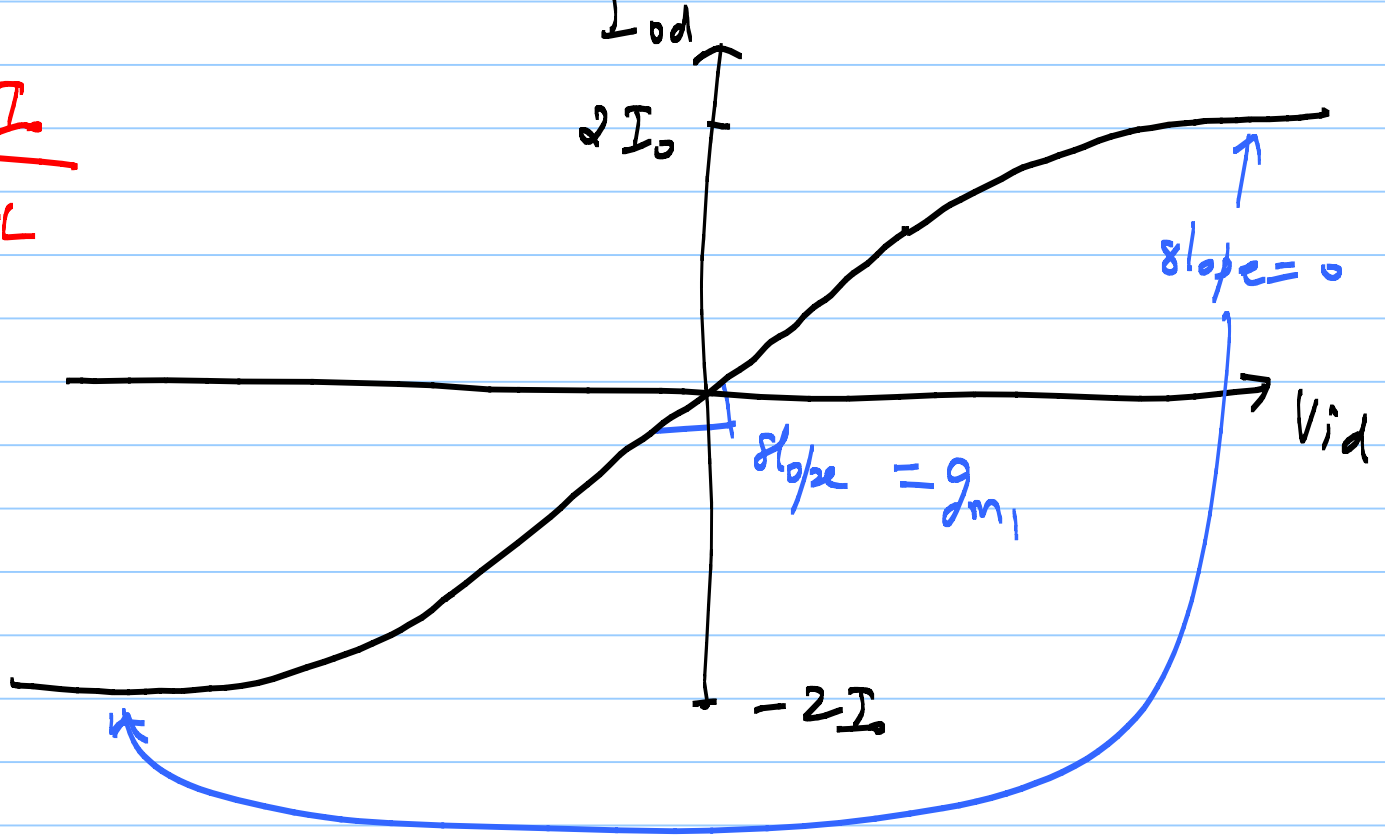


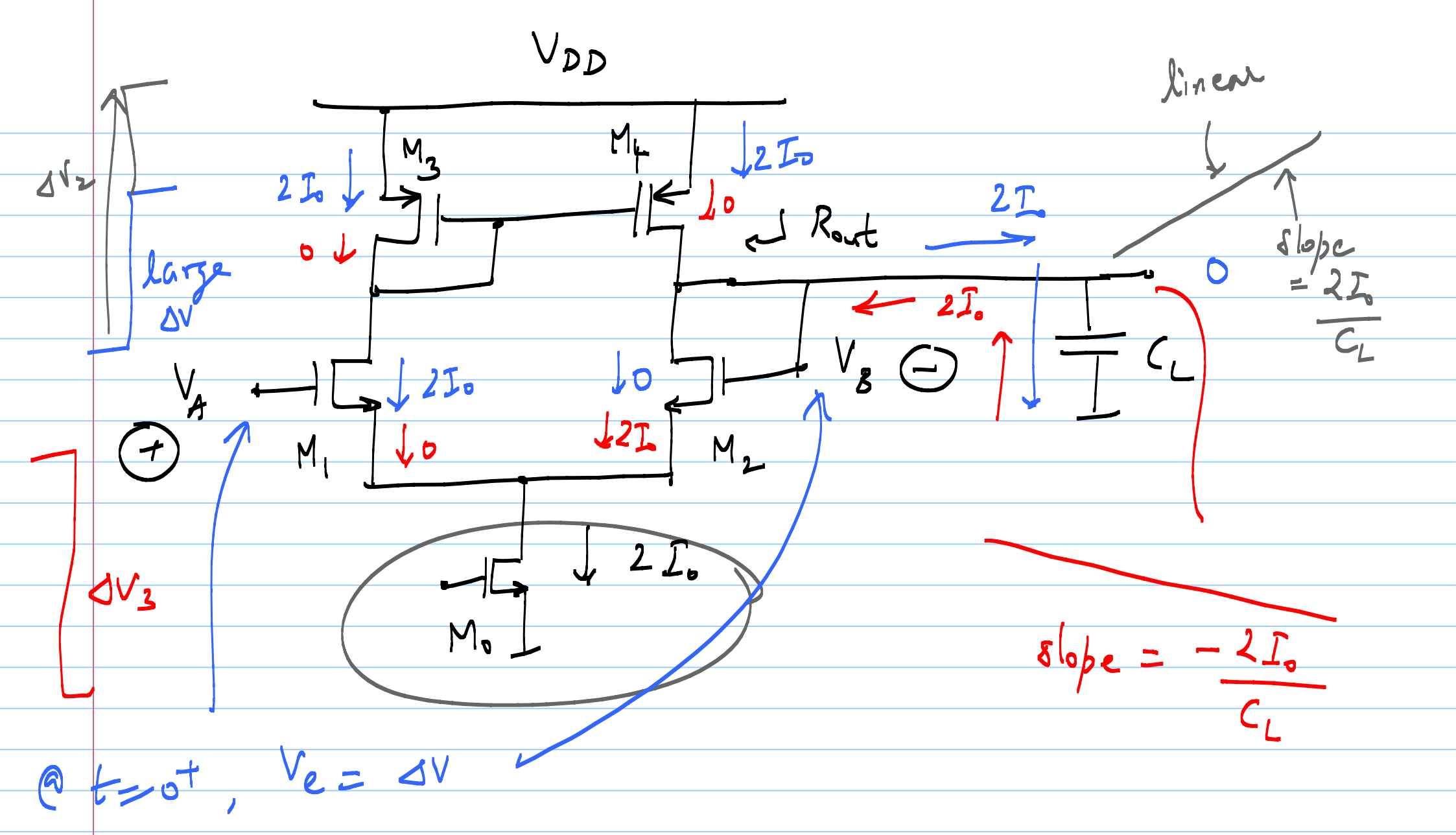


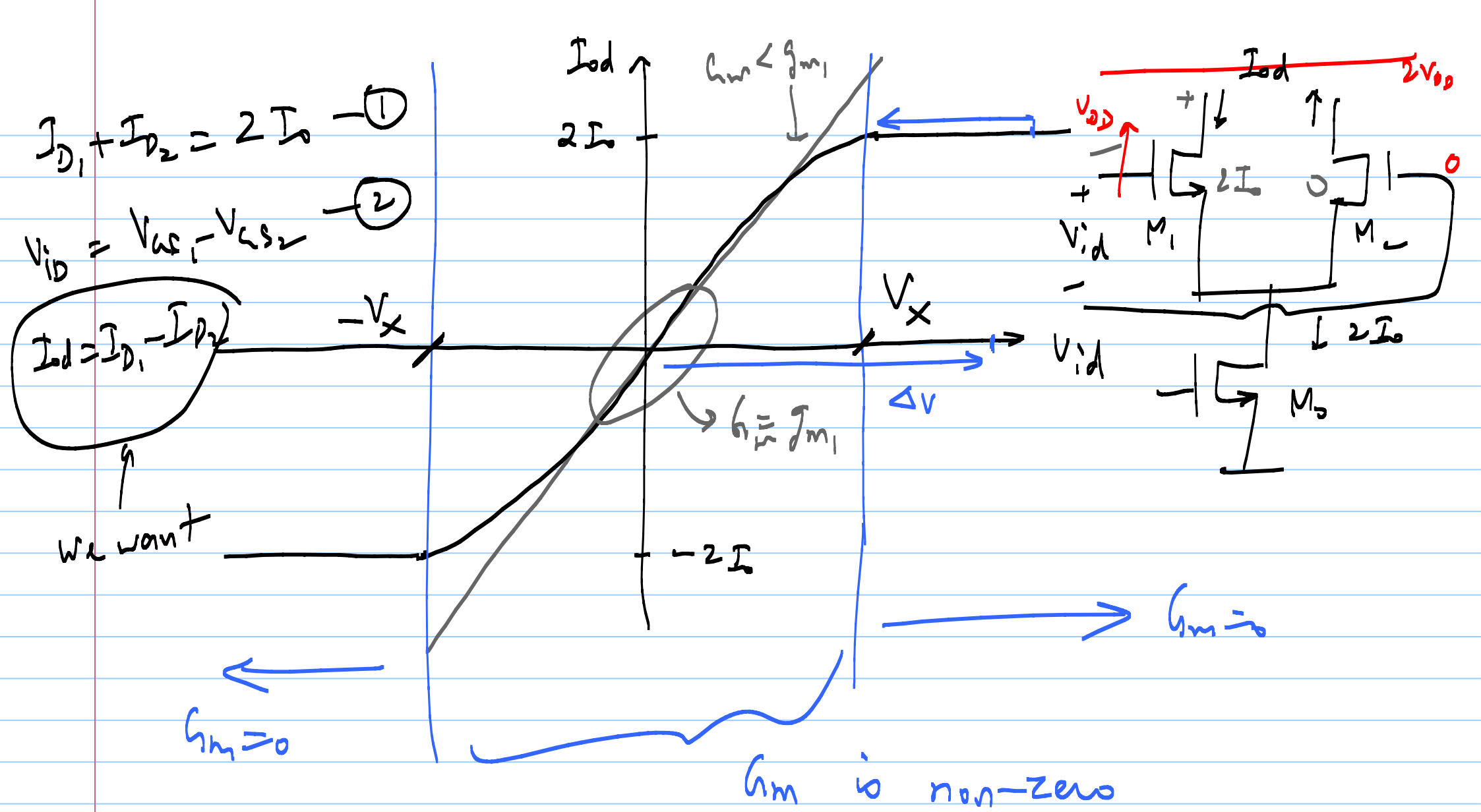
$$I_{max} = 2I_o$$

$$\text{Slew rate} = \frac{2I_o}{C_L}$$

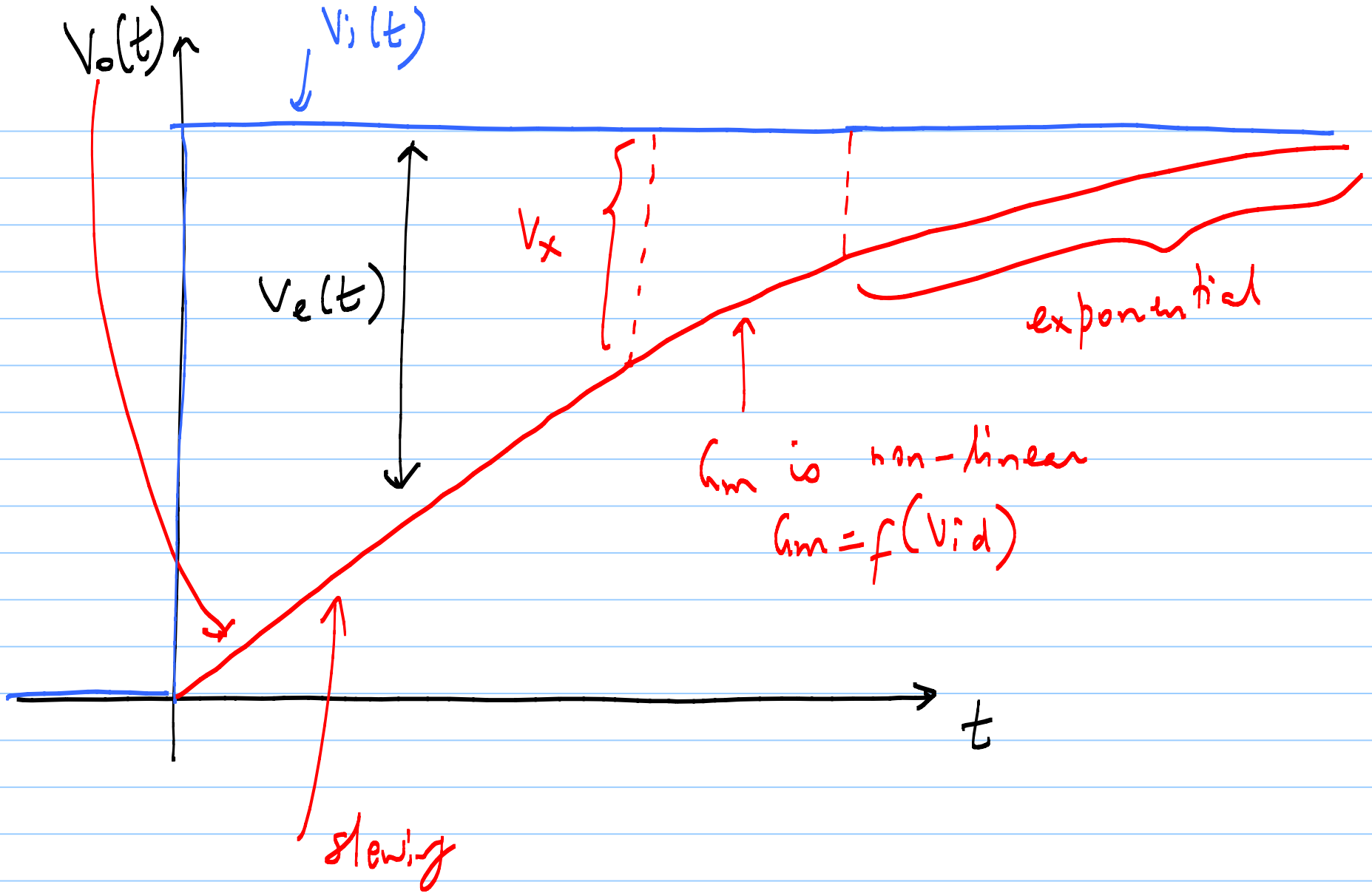
HW : find out V_{id} when $I_{od} = 2I_o$

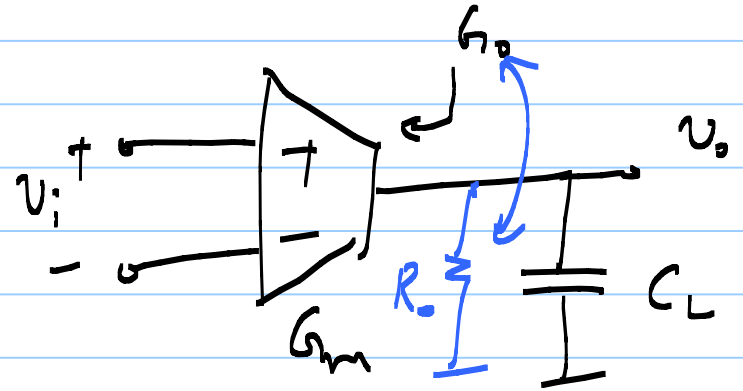



V_{DD}
M_3
M_4
$2I_0$
0
R_{out}
$2I_0$
V_A
V_B
M_1
M_2
M_0
C_L
ΔV_2
large ΔV
ΔV_3
linear
slope $= \frac{2I_0}{C_L}$
slope $= -\frac{2I_0}{C_L}$
@ $t = 0^+$, $V_e = \Delta V$



opamp will slew till V_e becomes small enough for $G_m > 0$


$V_o(t)$
$V_i(t)$
$V_e(t)$
V_x
exponential
G_m is non-linear
$G_m = f(V_{id})$
slewing
t

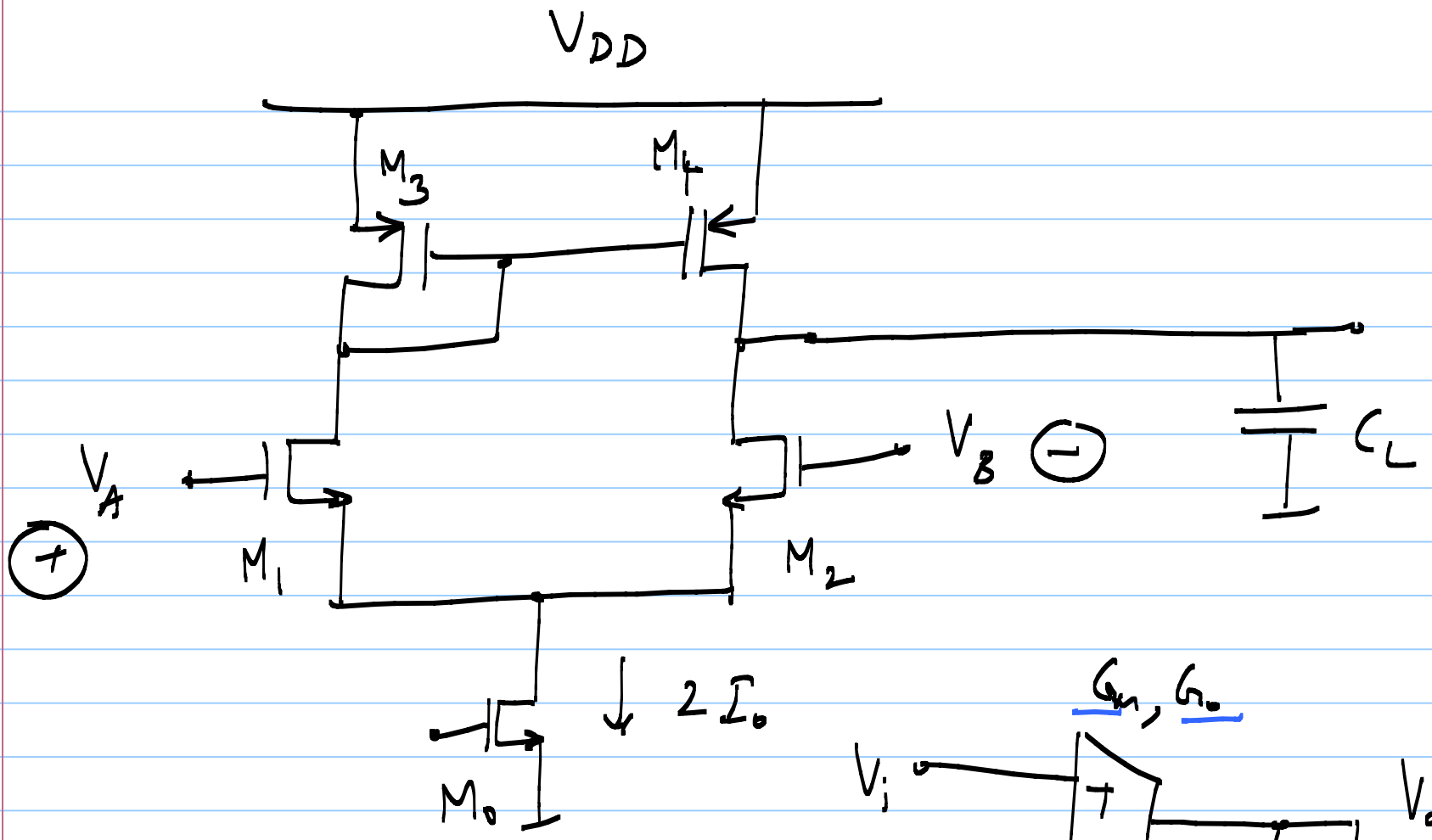


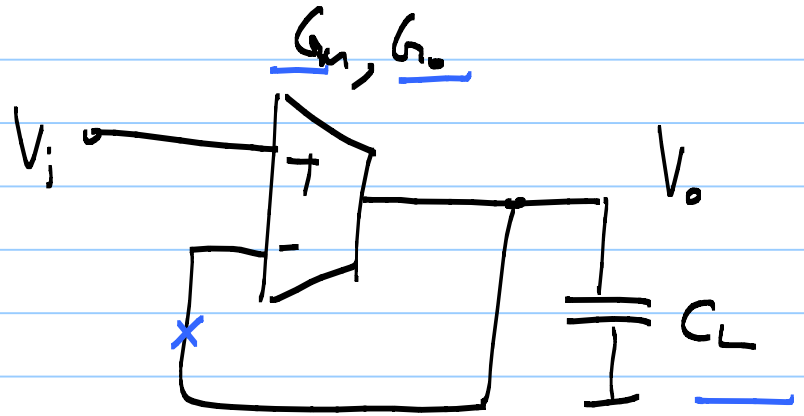
$$\text{DC gain} = \frac{G_m}{G_o} = G_m R_o$$

* Can only drive C_L loads
* we want $V_e(\infty) \rightarrow 0$ (steady state error)

$\Rightarrow$ DC gain — as large as possible

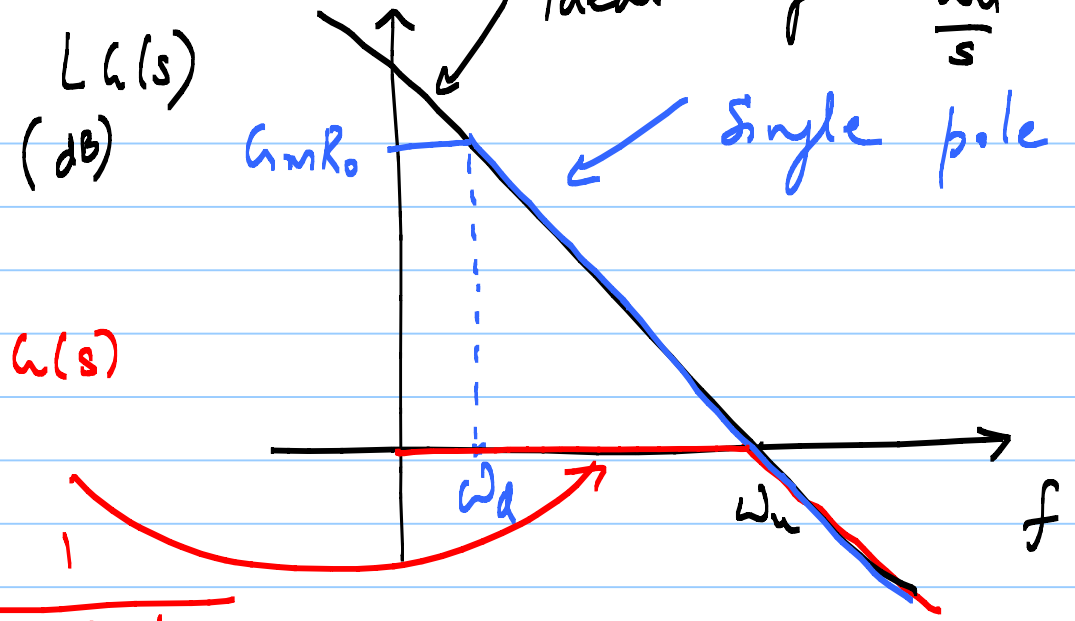
- Cascade amplifiers
  - 2 stage opamp
- Increase R_o
  - Telescopic opamp





$$\frac{V_o}{V_i}(s) = \frac{1}{1 + s/\omega_p}$$

$R_o = r_{ds_2} \| r_{ds_4}$; $G_m = g_{m_1}$



$\{\infty$ DC gain$\}$; $\omega_u = G_m/C_L$

Single pole system $\quad \frac{G_m R_o}{1 + s/\omega_d} = \frac{G_m R_o}{1 + s C_L R_o}$

$$\omega_d = \frac{1}{R_o C_L}$$

$$\omega_u = \frac{G_m}{C_L} \quad \{G_m R_o \text{ is large}\}$$

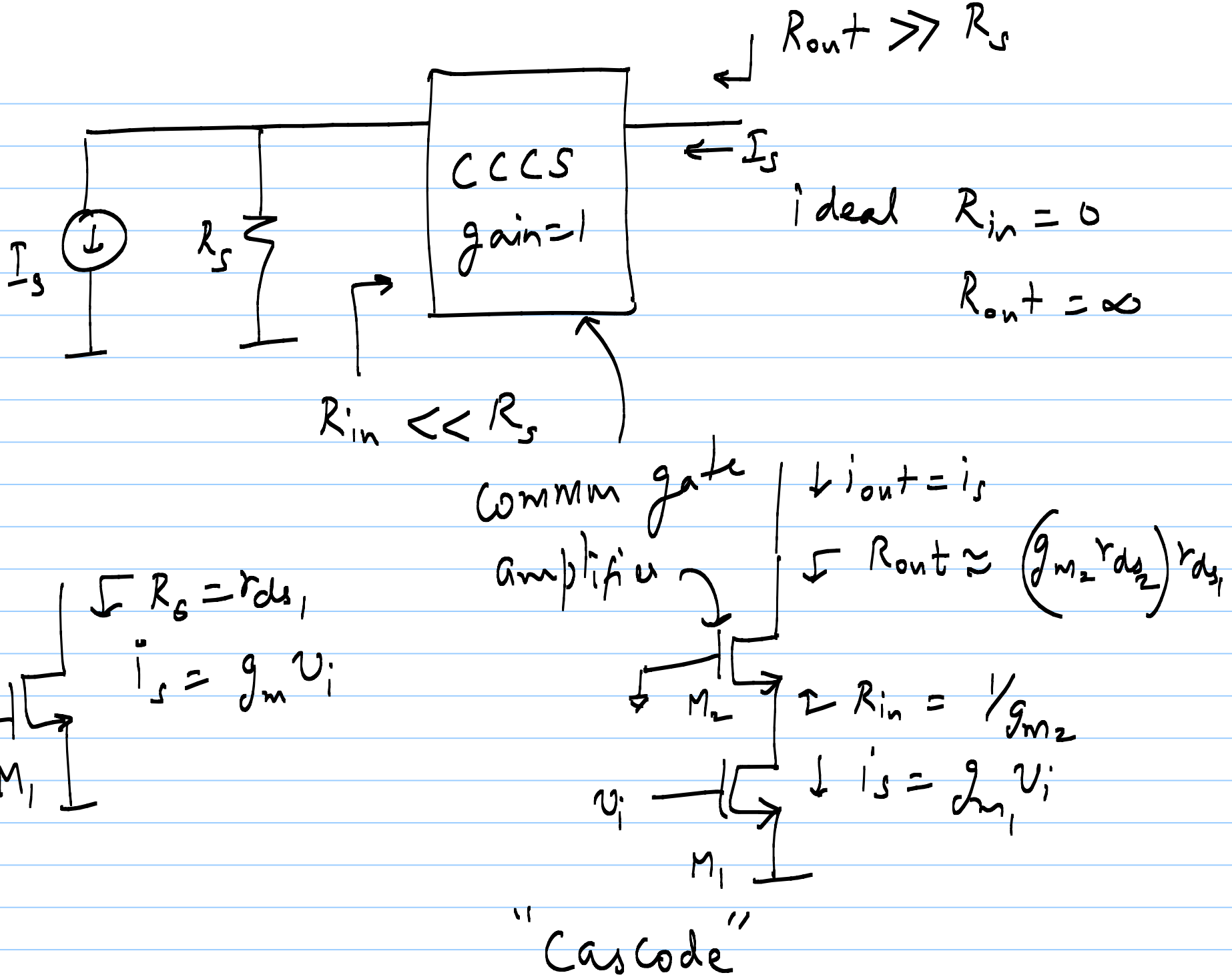
$C_L L_L(s)$

$$\frac{1}{1 + s C_L/G_m}$$

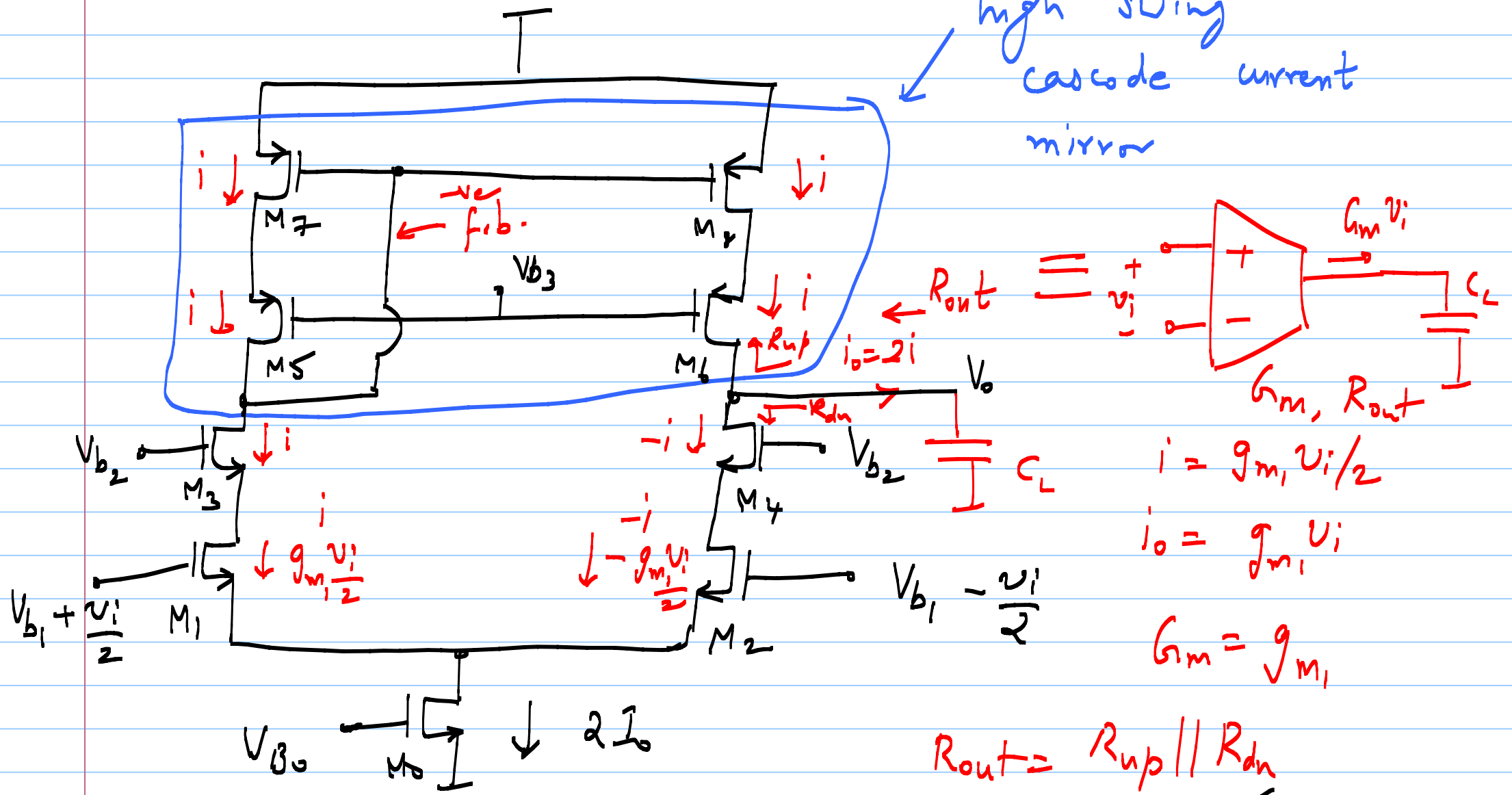
$$\frac{V_o}{V_i}(s) = \frac{1}{1 + s/\omega_p} = \frac{1}{f} \frac{L_L(s)}{1 + L_L(s)}$$

$$= 1 \cdot \frac{G_m R_o/(1 + s C_L R_o)}{1 + G_m R_o/(1 + s C_L R_o)} = \frac{G_m R_o}{1 + G_m R_o + s C_L R_o}$$

$$= \frac{1}{1/G_m R_o + 1 + s C_L/g_m} \Rightarrow \omega_p = \frac{G_m}{C_L}$$


$R_{out} \gg R_s$
CCCS
gain=1
I_s
I_s
R_s
$R_{in} \ll R_s$
ideal $R_{in} = 0$
$R_{out} = \infty$
common gate
amplifier
$i_{out} = i_s$
$R_{out} \approx (g_{m_2} r_{ds_2}) r_{ds_1}$
$R_G = r_{ds_1}$
$i_s = g_m v_i$
v_i
M_1
M_2
$R_{in} = 1/g_{m_2}$
$i_s = g_{m_1} v_i$
v_i
M_1
"Cascode"

Telescopic Opamp



$$R_{up} = (g_{m6} r_{ds6})\, r_{ds8}$$

$$R_{dn} = (g_{m4} r_{ds4})\, r_{ds2}$$

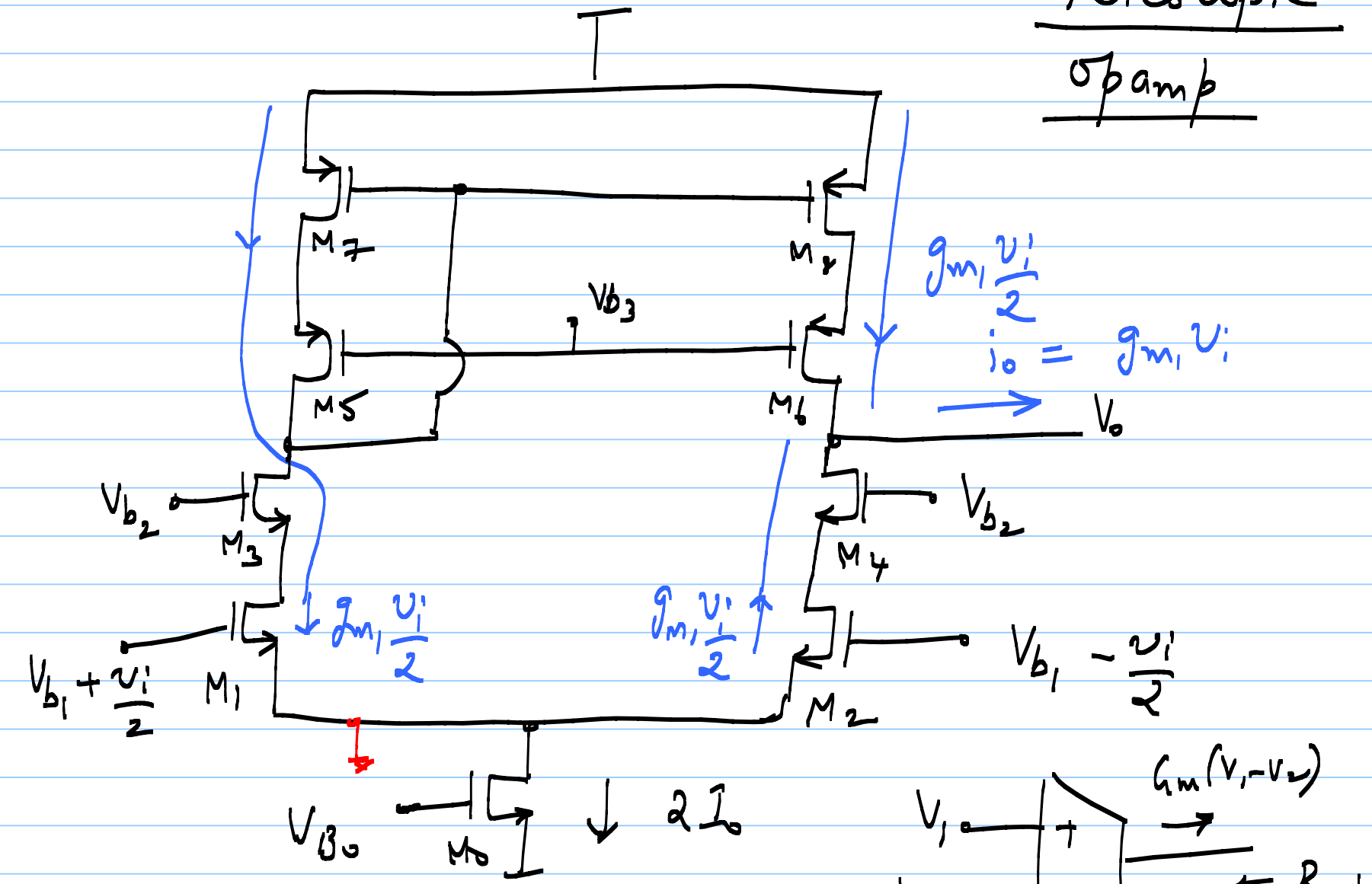
$$R_{out} = \left[(g_{m6} r_{ds6})\, r_{ds8}\right] \,||\, \left[(g_{m4} r_{ds4})\, r_{ds2}\right]$$

If No pmos cascode CM

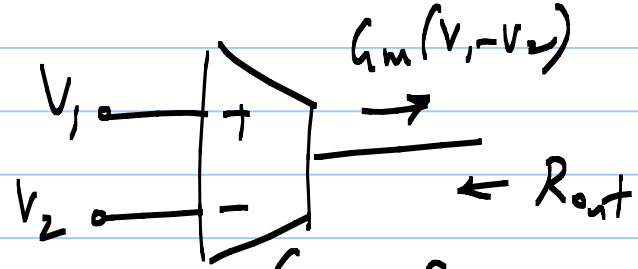
$$\Rightarrow R_{up} = r_{ds8}$$

$$R'_{out} = r_{ds8} \,||\, \left[(g_{m4} r_{ds4})\, r_{ds2}\right] \simeq r_{ds8}$$

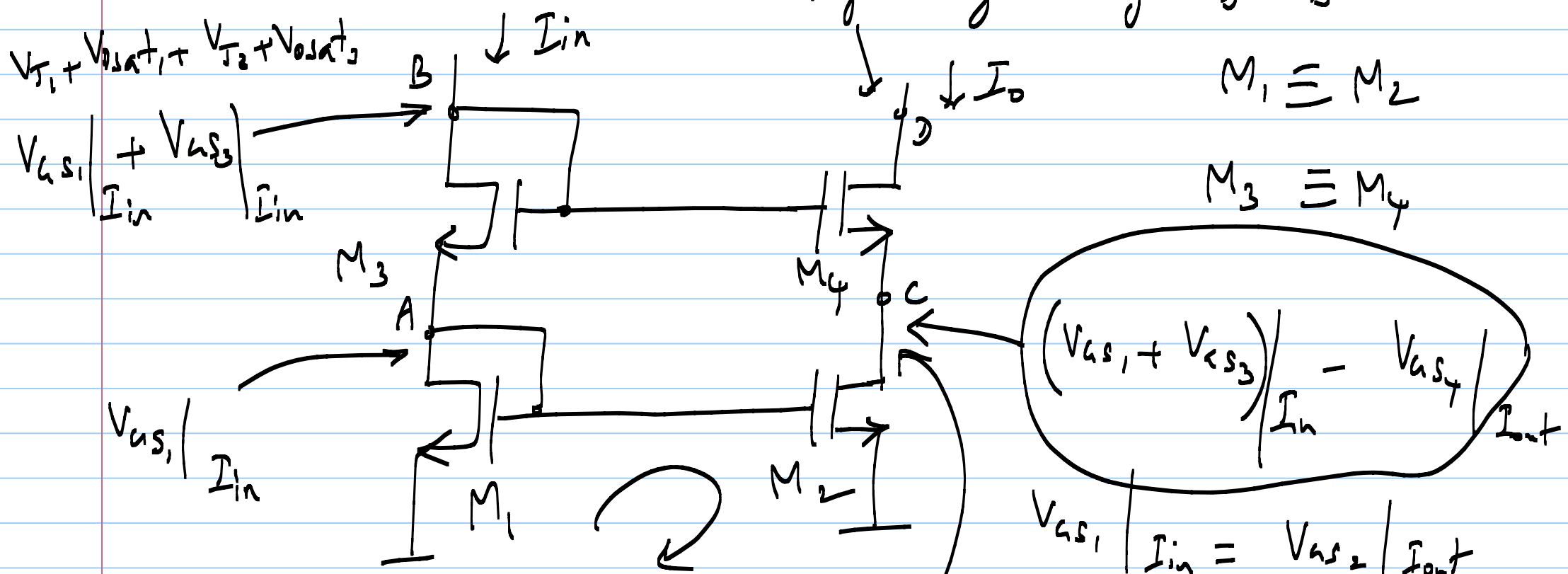
Telescopic opamp



$$R_{out} = \left(g_{m4} r_{ds4}\right) r_{ds2} \,||\, \left(g_{m6} r_{ds6}\right) \cdot r_{ds8}$$



Cascode CM



high enough e.g. $V_D = V_B$

$M_1 \equiv M_2$

$M_3 \equiv M_4$

$(V_{GS1}+V_{GS3})\big|_{I_{in}} - V_{GS4}\big|_{I_{out}}$

$V_{GS1}\big|_{I_{in}} = V_{GS2}\big|_{I_{out}}$

$I_{out} = I_{in}$

$\Rightarrow V_{GS3} = V_{GS4}$ ← (M_2 in sat.)

$= V_{GS1}$

① If M_2 is in triode

$\Downarrow$

$V_C < V_{GS1} - V_T \Rightarrow V_{GS4} > V_{GS3} \Rightarrow I_{out} > I_{in}$

$I_{out} < I_{in}$

① V_C is too high (M_2 is deep in sat.)

$\Rightarrow V_{GS_4} < V_{GS_3} \Rightarrow I_4 < I_3$

while $I_{out} \approx I_{in}$ ← conflicting

$V_C = V_A$ if $V_D = V_B$

$V_C \approx V_A$ if V_D is large enough to keep M_2, M_4 in sat. and λ's are reasonably low

What is min. value of V_D allowed?

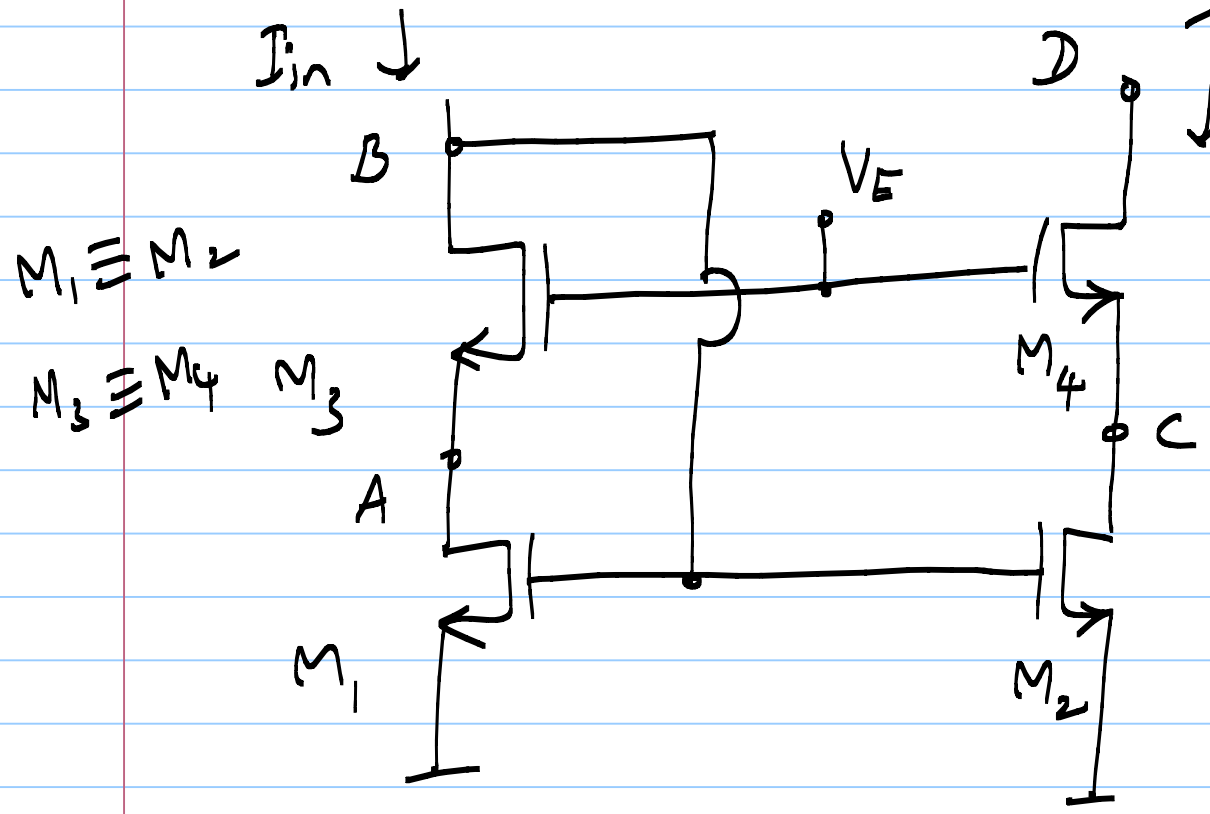
$V_{D\,min} = V_C + V_{DS_4\,min.}$

$$V_{D_{min.}} = V_{GS_2} + V_{DSat_4}$$

$$= V_{T_2} + V_{DSat_2} + V_{DSat_4}$$

High swing cascode CM

V_D is large enough to keep M_4, M_2 in sat.



$$V_B = V_{GS_1} = V_{GS_2}$$

$$V_A = V_C = V_E - V_{GS_{2,4}}$$

we want $V_C = V_A = V_{DSat_{1,2}}$

$$V_E = V_C + V_{GS_{2,4}}$$

$$= V_{T_4} + V_{DSat_4} + V_{DSat_2}$$

$$V_{D\min} = V_E - V_{t4}$$

$$= V_{Dsat_2} + V_{Dsat_4}$$

HW : 1) Calculate small signal r_{in} for the two cascode CMs

2) calculate r_{out} for both.

$$\approx (g_{m4} r_{ds4}) \cdot r_{ds2}$$

Telescopic Opamp Datasheet

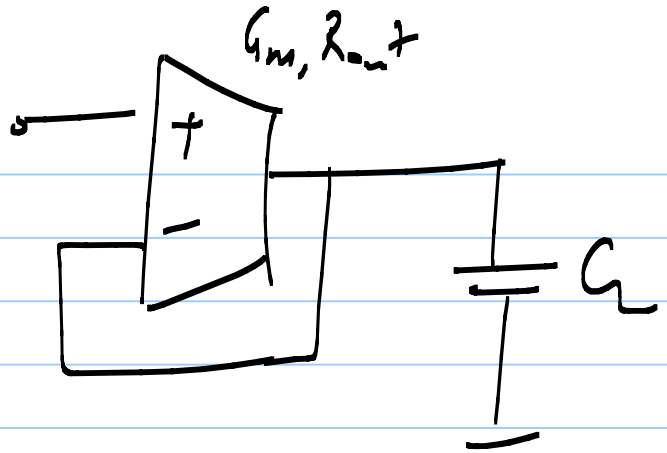
1) DC gain = $g_{m1} r_{out}$

much larger than 5-transistor OTA

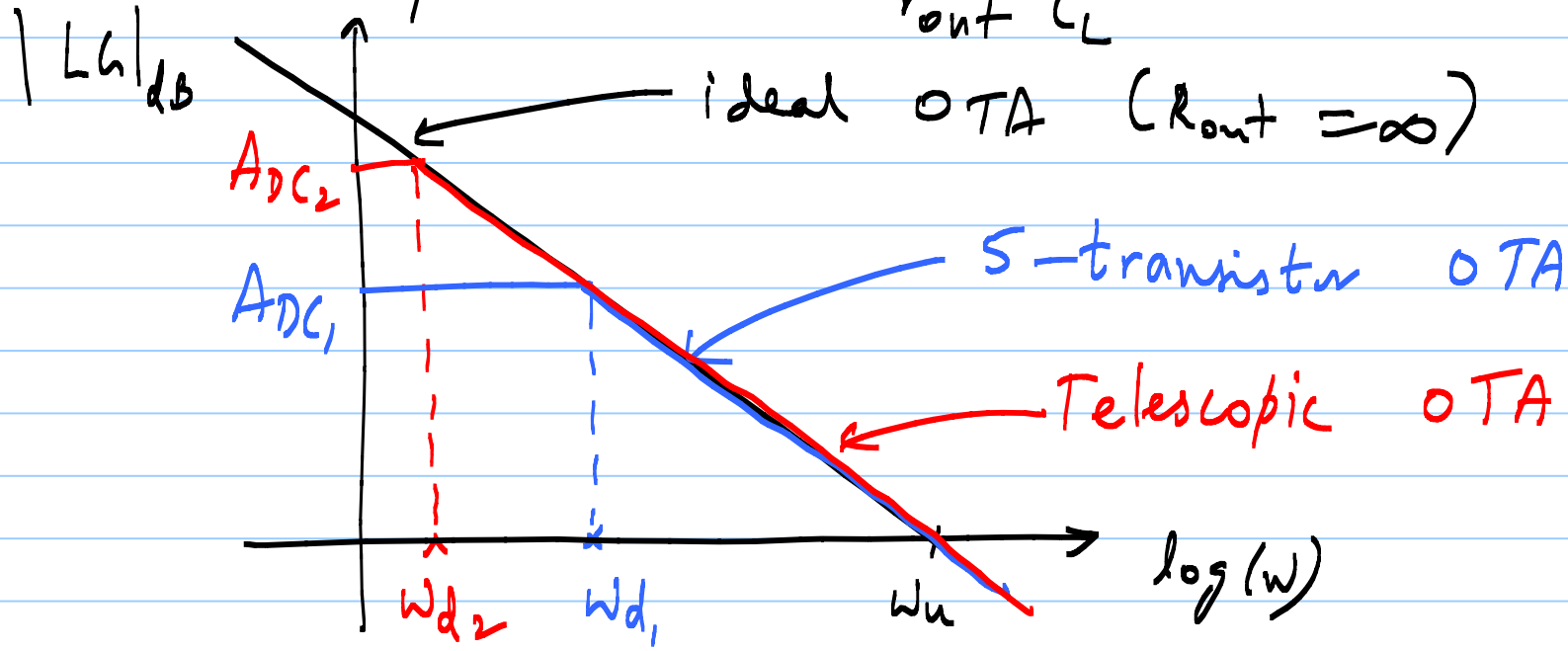
$$r_{out} = \left[(g_{m6} r_{ds6})\, r_{ds8}\right] \| \left[(g_{m4} r_{ds4})\, r_{ds2}\right]$$

2) $G_m = g_{m_1}$

3) $\omega_u = \dfrac{g_{m_1}}{C_L}$

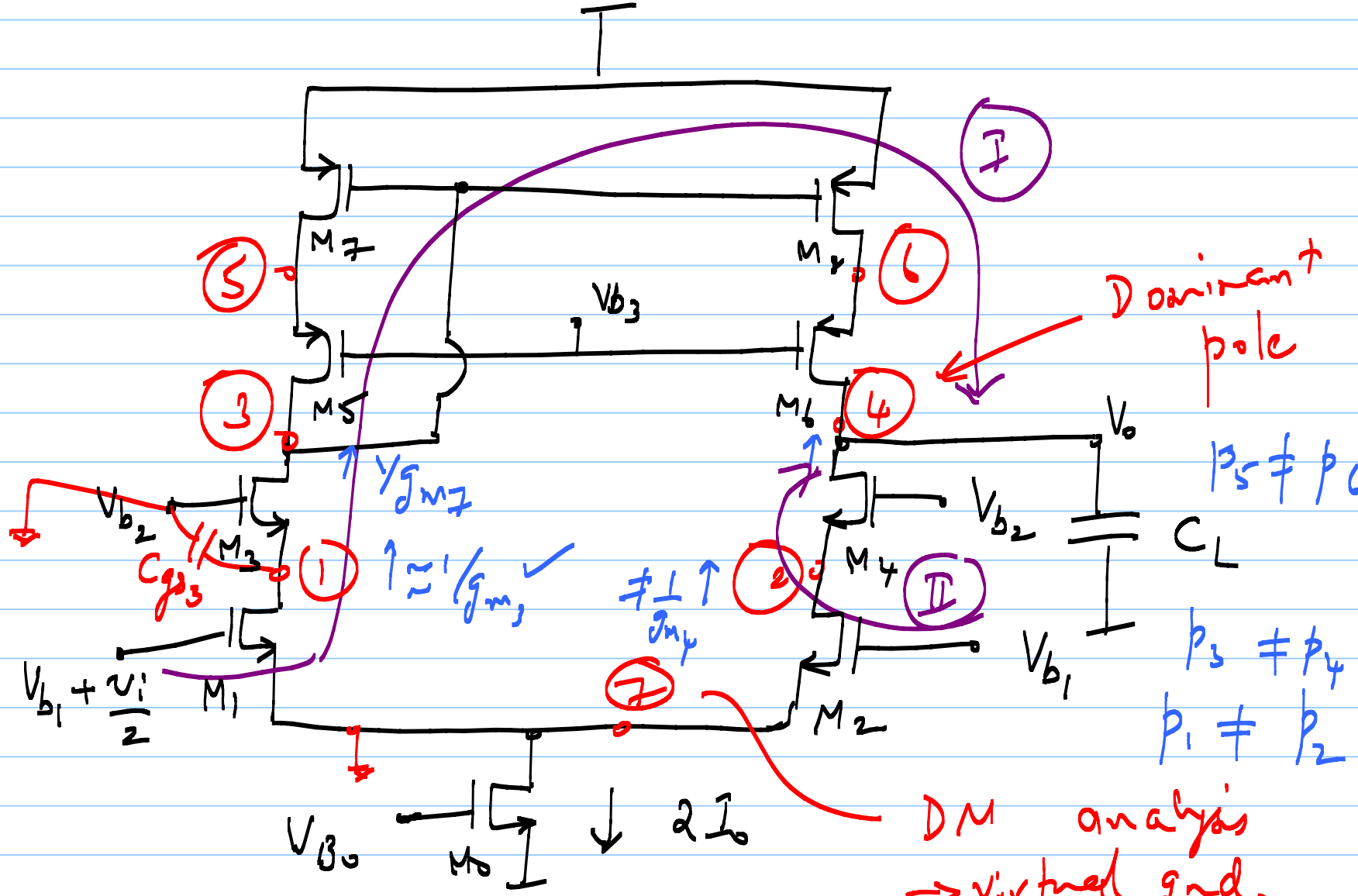


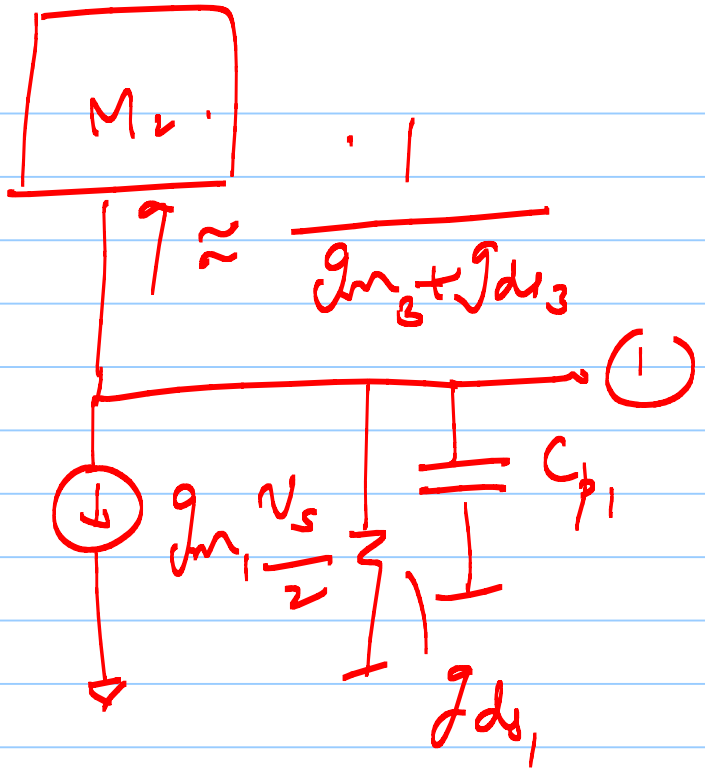
Dominant pole $\quad \omega_d = \dfrac{1}{r_{out} C_L}$



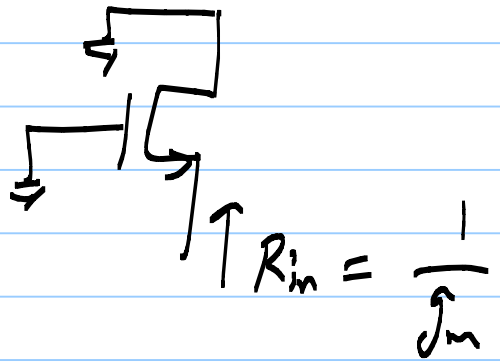
Hw: # ND poles & zeroes

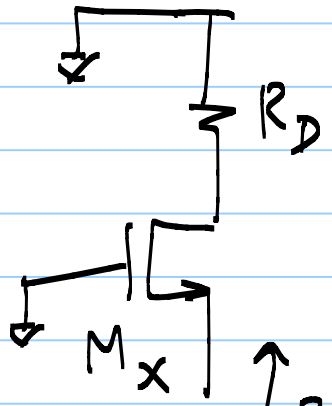
4) ND poles & zeroes



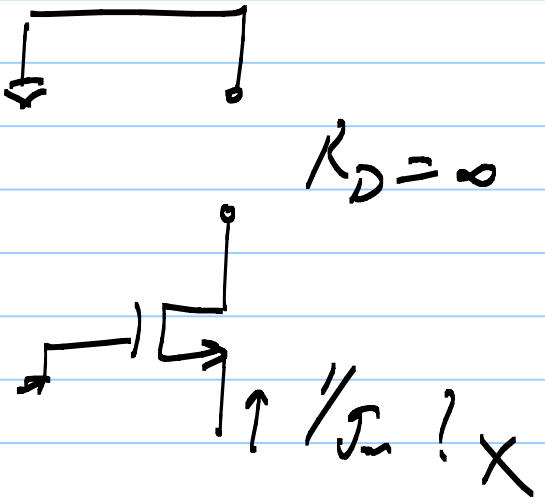


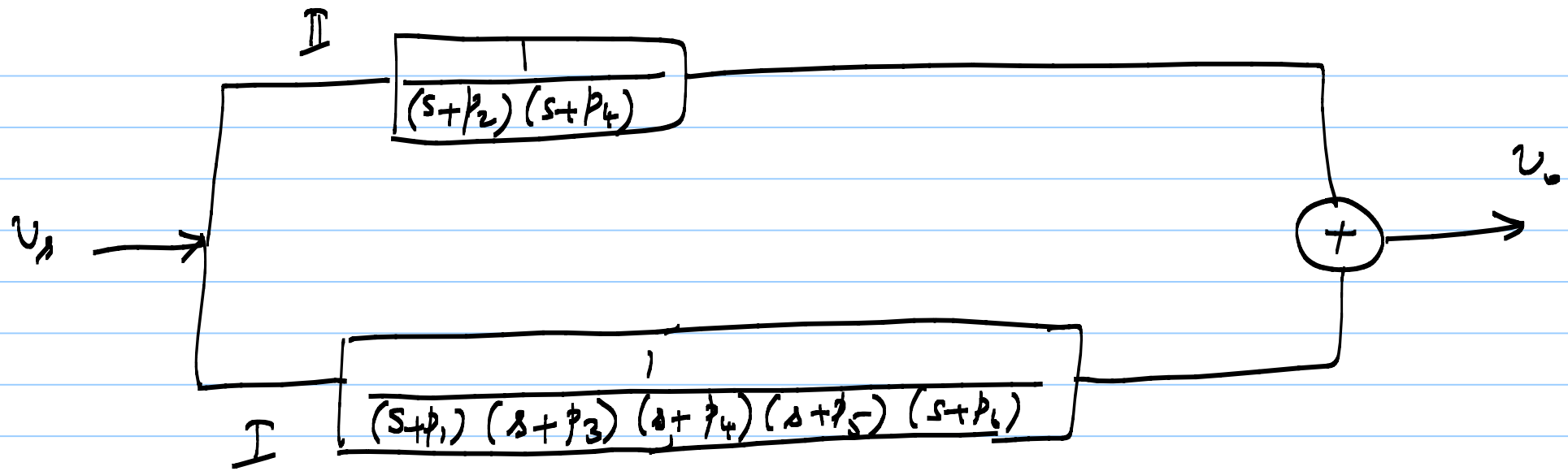
$$\omega_{p_1} \approx \frac{g_{m_3}+g_{ds_3}+g_{ds_1}}{C_{p_1}}$$





$$= f(g_m, r_{ds}, R_D)$$





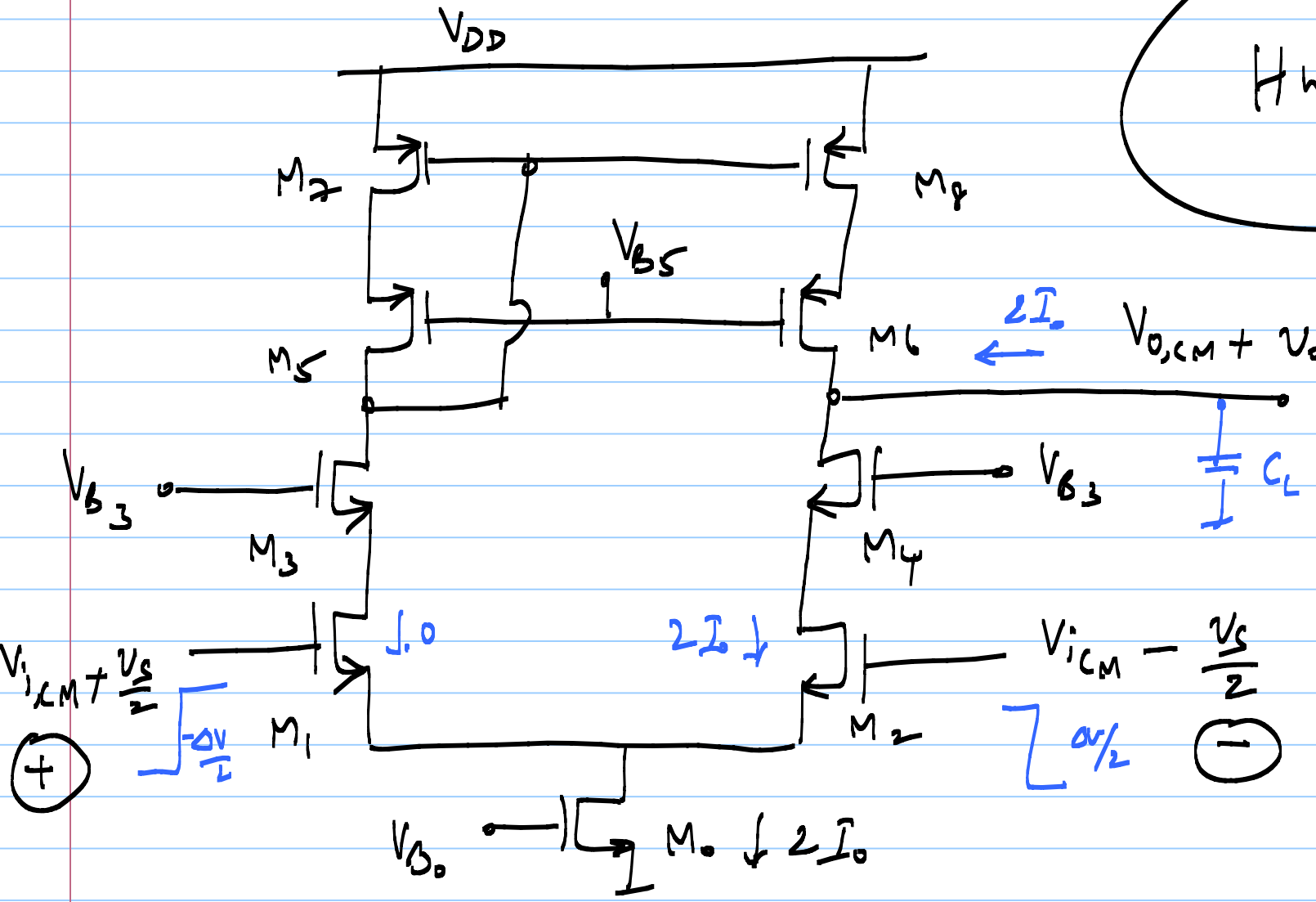
$$\frac{v_o}{v_i}(s) = \left(A_{DC}\right) \frac{\text{4 zeroes}}{\text{6 poles}}$$

ND poles = 5
ND zeroes = 4

} should be outside ω_n for good phase margin

min. & max. of

HW: V_{B3} & V_{B5}



5) I CMR: $V_{i,CM}\,(\text{min.}) = V_{DSat_0}\big|_{2I_0} + V_{GS_1}\big|_{I_0}$

$$V_{i,cm}\ (max.) = V_{B_3} - V_{GS_3}\Big|_{I_0} + V_{T_1}$$

6) <u>OCMR</u>

$$V_{o,cm}\ (min.) = V_{B_3} - V_{T_4}$$

$$V_{o,cm}\ (max.) = V_{B_5} + V_{T_6}$$

7) Noise $\overline{e_n}$

* @ low freq. $\overline{i_n} = 0$ $\{Z_{in} = \infty\}$

* No noise from M_0

* Noise from M_3, M_4, M_5, M_6 (Cascode devices) does not significantly contribute to $\overline{e_n}$ @ low frequencies. [HW]

$$\frac{\overline{e_n^2}}{\Delta f} = \frac{16kT}{3g_{m_1}} + \frac{16kT}{3g_{m_1}^2}\cdot g_{m_2}$$ ← same as 5-transistor 1-stage opamp

8) Offset: $\sigma_{os}^2 = \sigma_{V_{T1,2}}^2 + \left(\frac{g_{m7}}{g_{m1}}\right)^2 \sigma_{V_{T7,8}}^2$

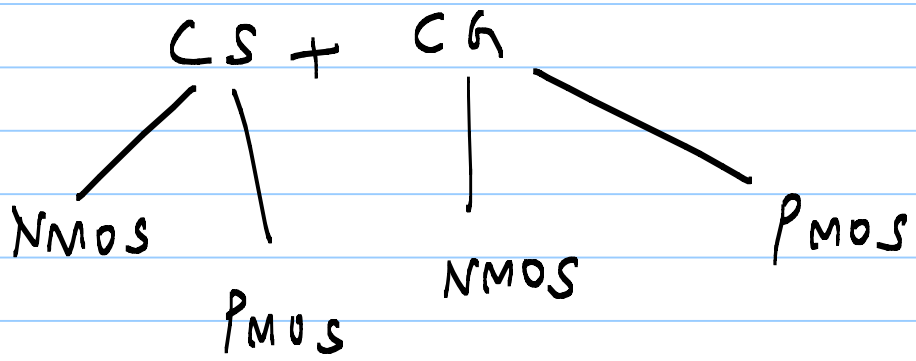
* ΔV_T of $M_3 - M_4$ & $M_5 - M_6$ } do not contribute significantly to $\overline{V_{os}}$ if $r_{ds1}, r_{ds2}, r_{ds7}, r_{ds8}$ are large

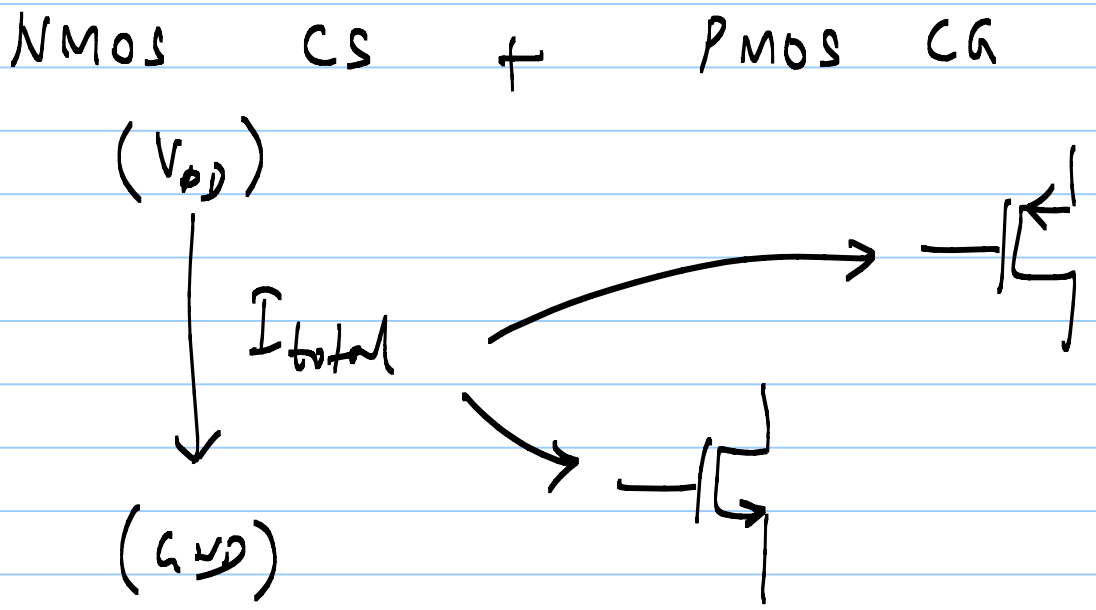
* Expect $\overline{V_{os}}$ to be same as that of 5-transistor 1-stage opamp.

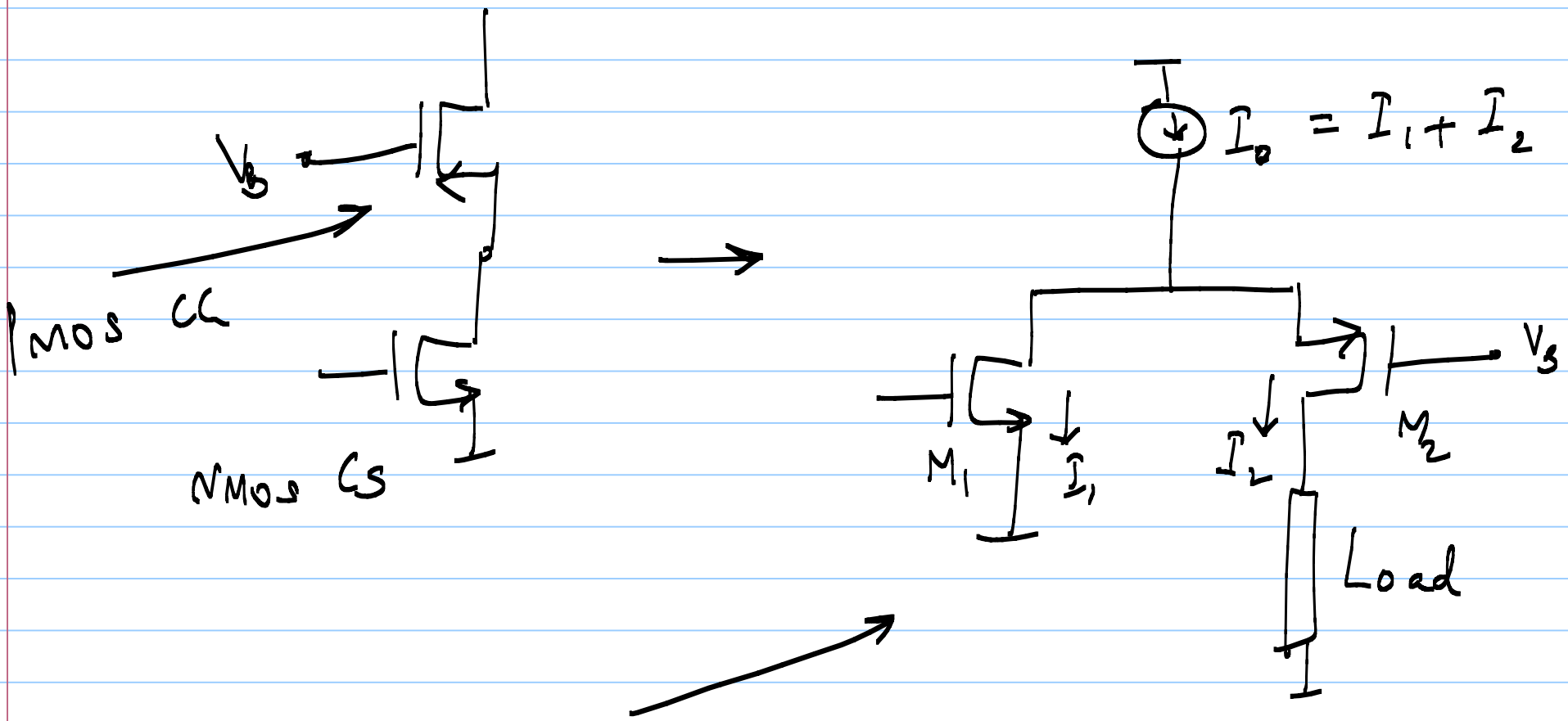
HW

9) slew rate : $\pm \frac{2I_0}{C_L}$

Cascode







"Folded Cascode" Amplifier

You can also have PMOS CS + NMOS CG

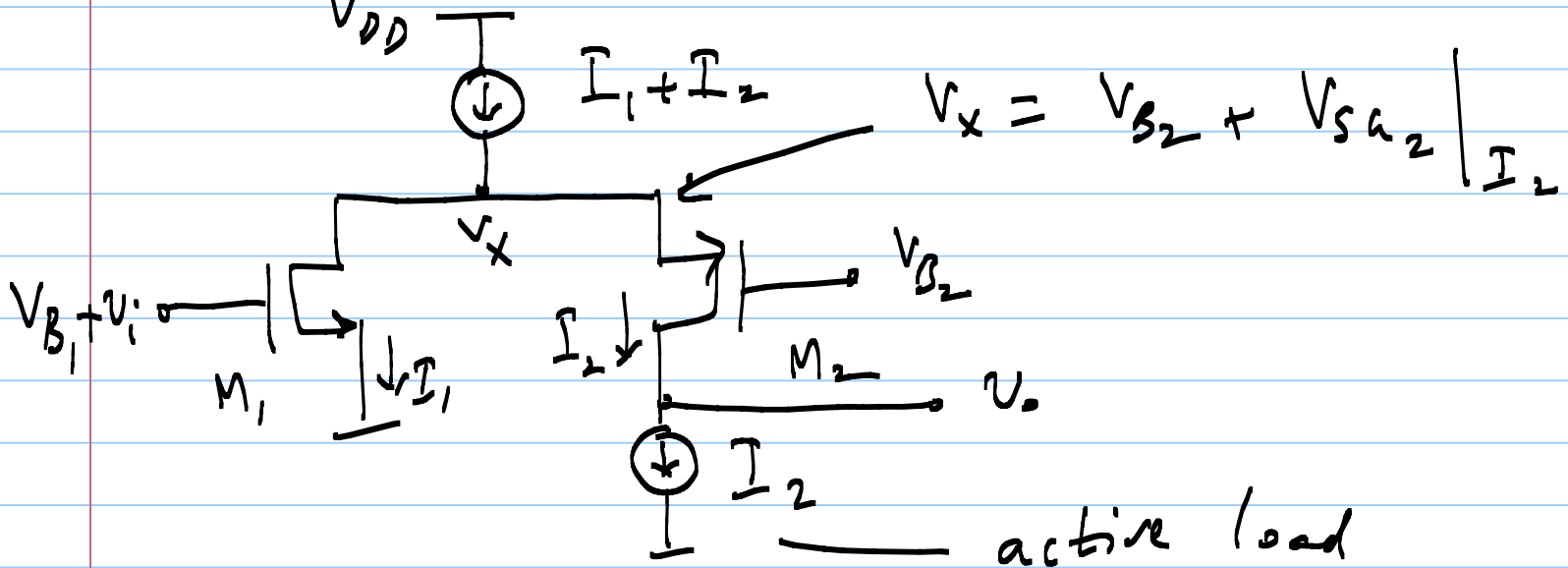
* Small-signal characteristics same as those of regular cascode amplifier

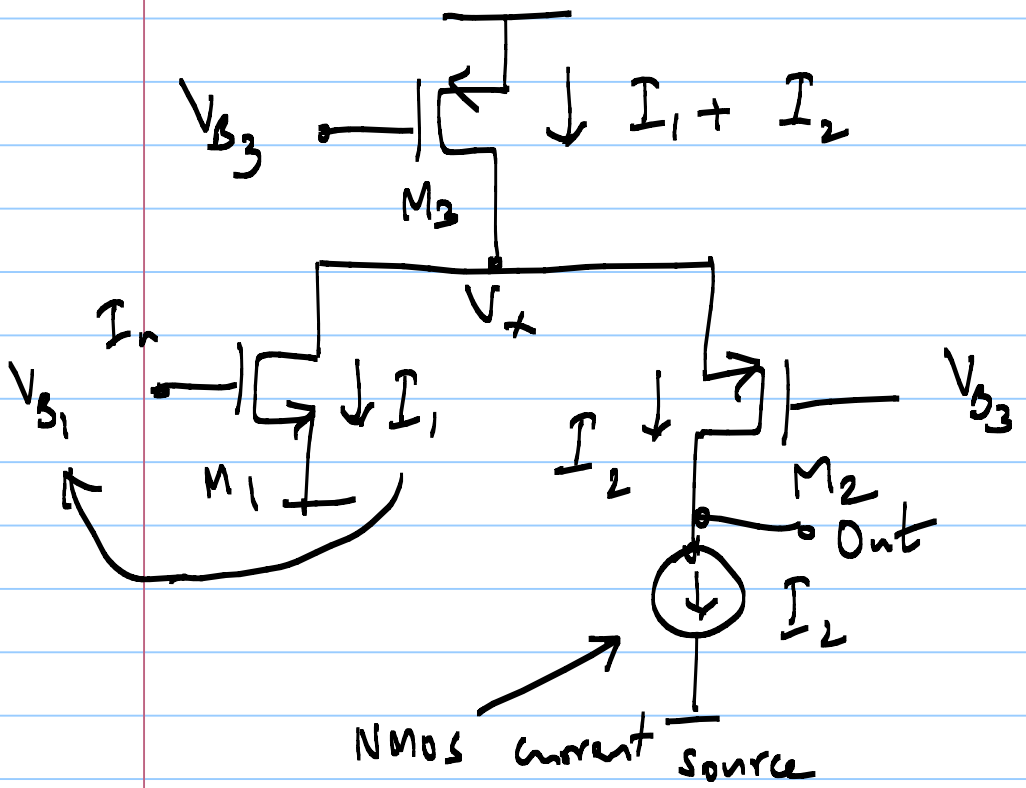
CS + CG — NMOS

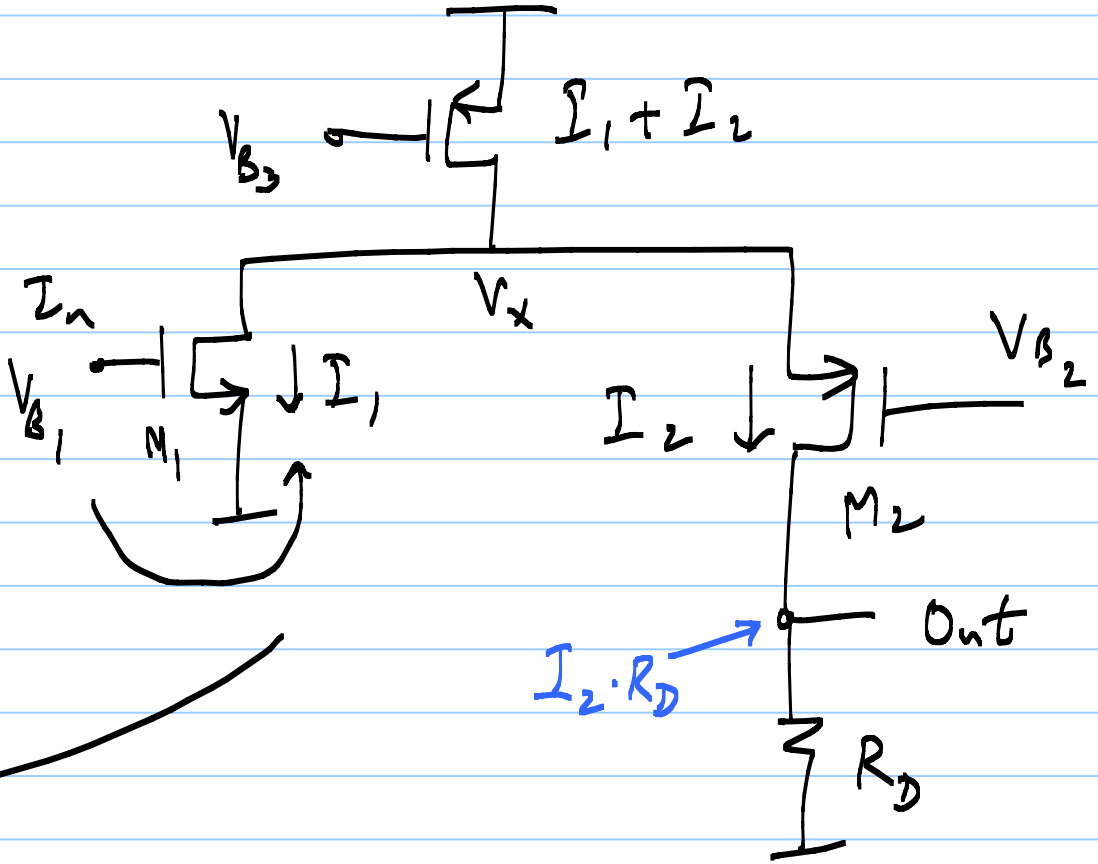
NMOS PMOS PMOS

4 types of cascode amplifiers

Folded Cascode



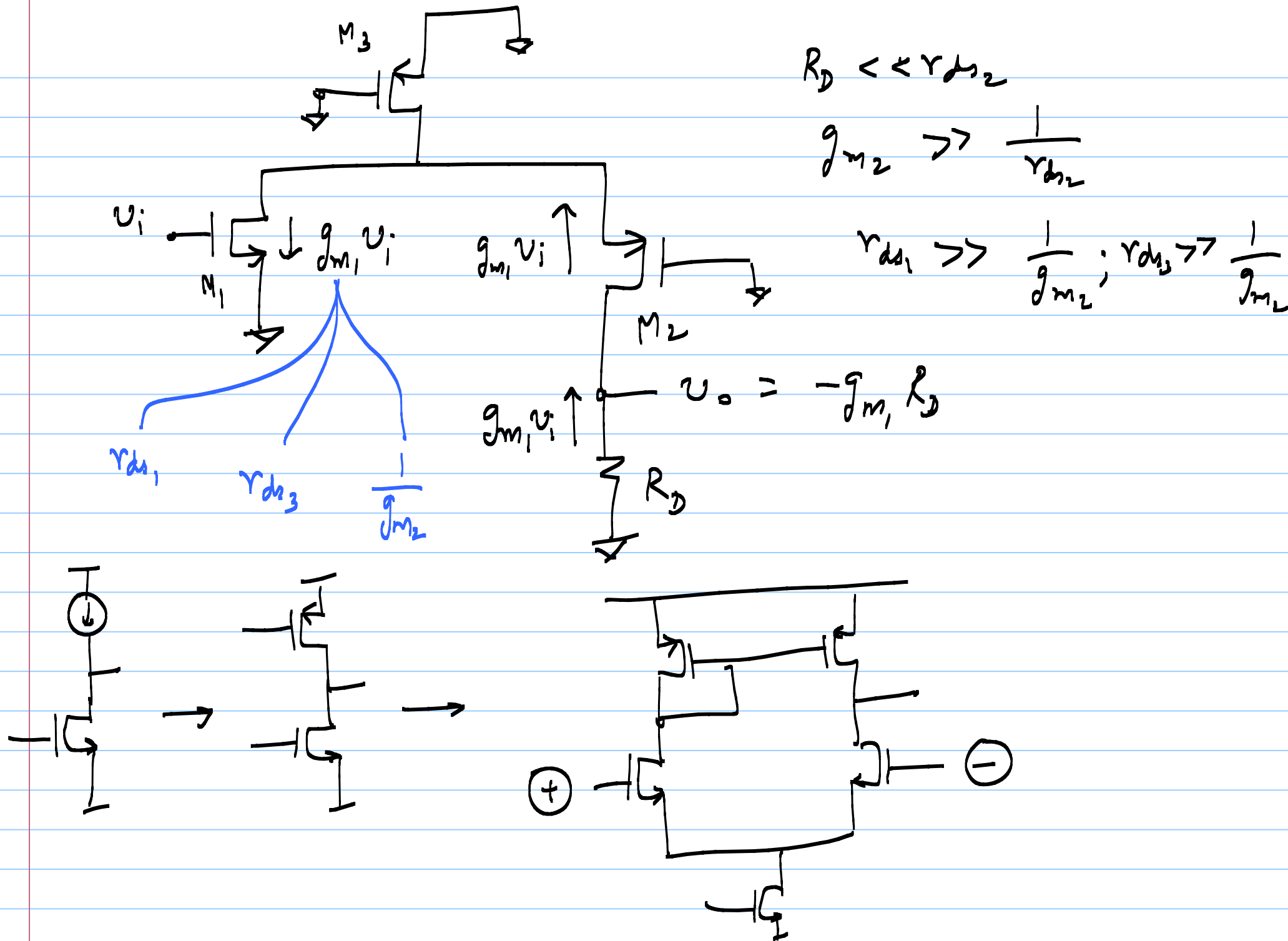


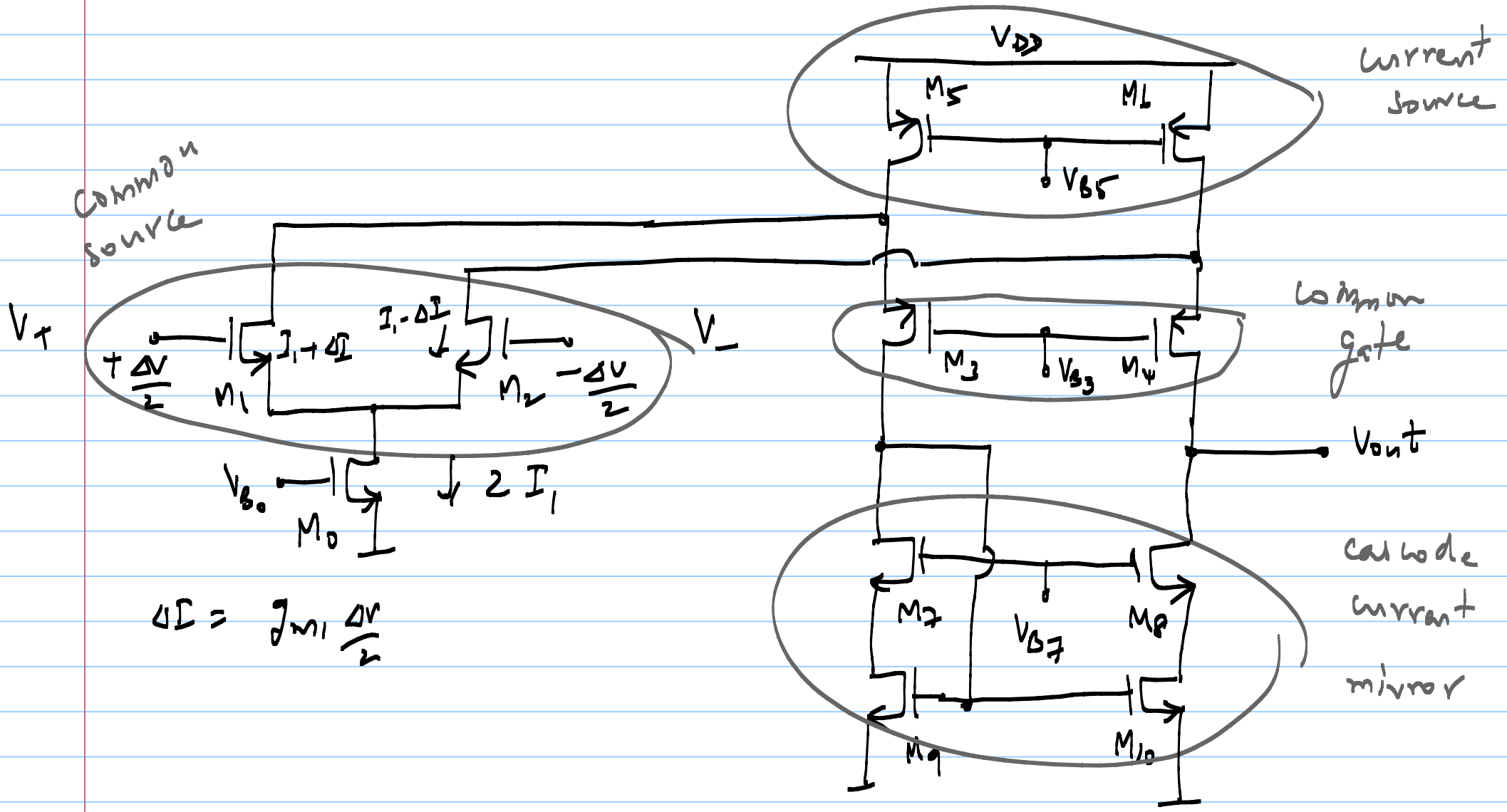


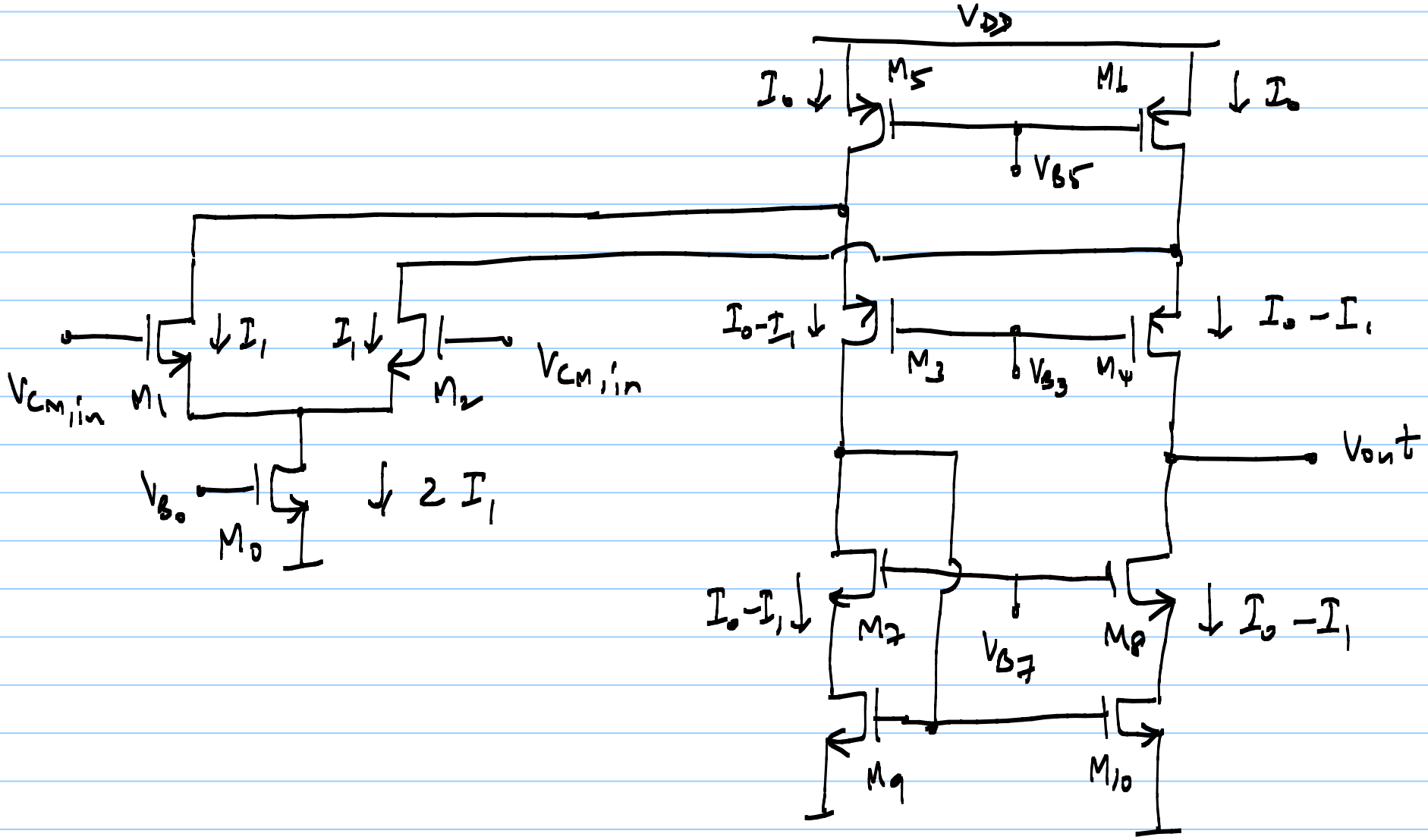
max V_x : $V_{B_3} + V_{T_p} = V_{DD} - V_{SD_{sat_3}}$

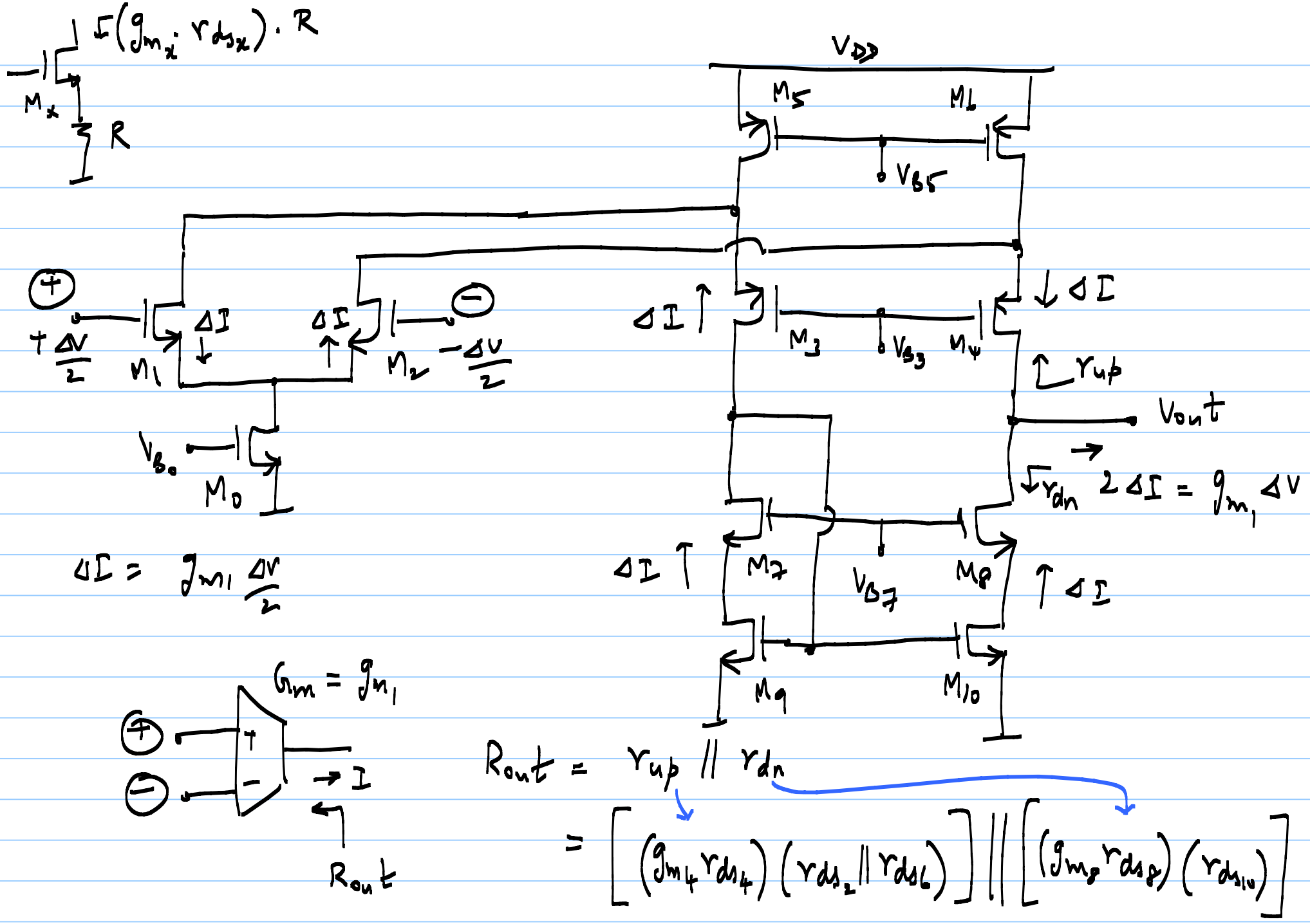
min. V_x : larger of $\{V_{D_{sat_1}},\ I_2 R_D + V_{SD_{sat_2}}\}$

} gives range of V_{B_2} that can be applied


M3
vi
M1
gm1vi
gm1vi
M2
gm1vi
RD
rds1
rds3
1/gm2
R_D << r_ds2
g_m2 >> 1/r_ds2
r_ds1 >> 1/g_m2 ; r_ds3 >> 1/g_m2
v_o = -g_m1 R_D
+
-


Common source
V_+
$+\frac{\Delta V}{2}$
$I_1+\Delta I$
$I_1-\Delta I$
M_1
M_2
$-\frac{\Delta V}{2}$
V_-
V_{B_0}
M_0
$2I_1$
$\Delta I = g_{m1}\frac{\Delta V}{2}$
V_{DD}
M_5
M_4
V_{B5}
current source
M_3
V_{B3}
M_4
common gate
V_{out}
M_7
V_{B7}
M_8
M_9
M_{10}
cascode current mirror


VDD
I0
M5
M4
I0
VB5
I1
I1
VCM,in
VCM,in
M1
M2
I0-I1
M3
VB3
M4
I0 - I1
VB0
2 I1
M0
Vout
I0-I1
M7
VB7
M8
I0 - I1
M9
M10


$\frac{1}{2}(g_{m_x} r_{ds_x}) \cdot R$
M_x
R
V_{DD}
M_5
M_6
V_{B5}
$+\frac{\Delta V}{2}$
M_1
ΔI
ΔI
M_2
$-\frac{\Delta V}{2}$
ΔI
M_3
V_{B3}
M_4
ΔI
r_{up}
V_{out}
V_{B0}
M_0
r_{dn}
$2\Delta I = g_{m_1} \Delta V$
$\Delta I = g_{m1} \frac{\Delta V}{2}$
ΔI
M_7
V_{B7}
M_8
ΔI
M_9
M_{10}
$G_m = g_{m_1}$
I
R_{out}
$R_{out} = r_{up} \parallel r_{dn}$
$= \left[(g_{m_4} r_{ds_4}) (r_{ds_2} \parallel r_{ds_6}) \right] \parallel \left[(g_{m_8} r_{ds_8}) (r_{ds_{10}}) \right]$

Process variations

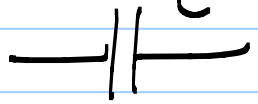
P V T — Variations

- P — process
- V — voltage
- T — temperature

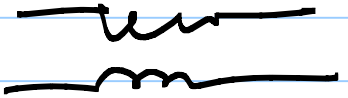
R



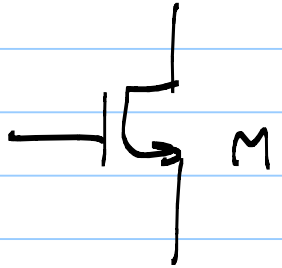
C





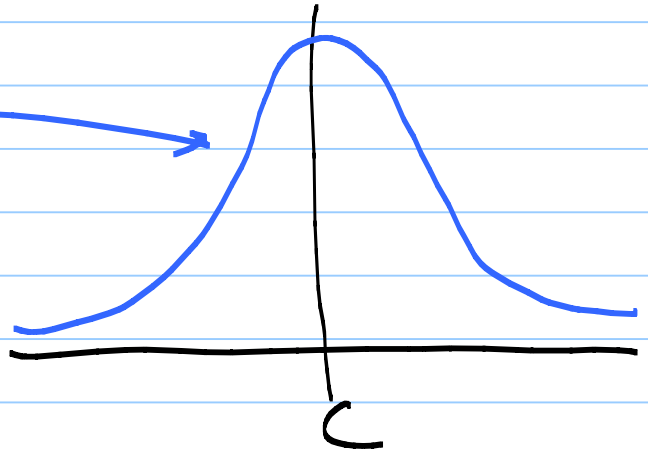


M

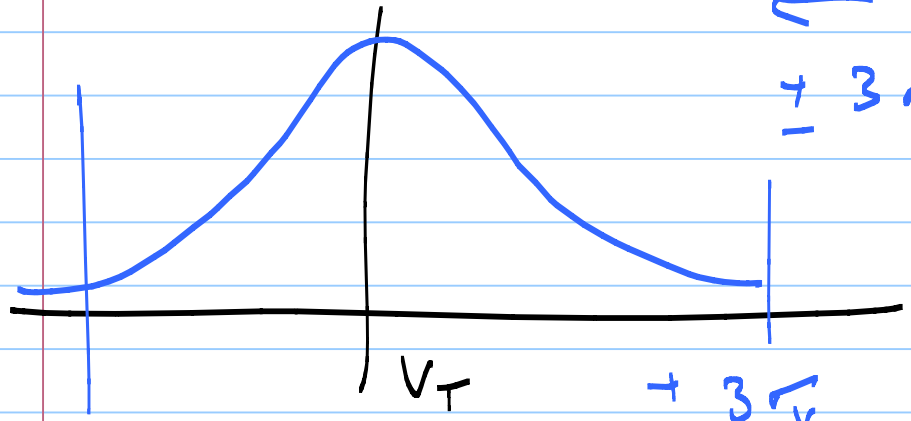




$\pm 3\sigma_R$



$\pm 3\sigma_C$



$\pm 3\sigma_{V_T}$

R ± 20% V_T ± 100mV

C ± 15% etc.

Process variations

Models @ Typical corner ← nominal value

Slow corner ← $+3\sigma$

Fast corner ← -3σ

Temperature variations

$$R = R_0(1 + \alpha T)$$

$$R_{diff.} = f(T)$$

V_{Tn}, V_{Tp} ...

μ_n, μ_p ...

operating temp. range

e.g. $-40^{\circ}C$ to $+100^{\circ}C$

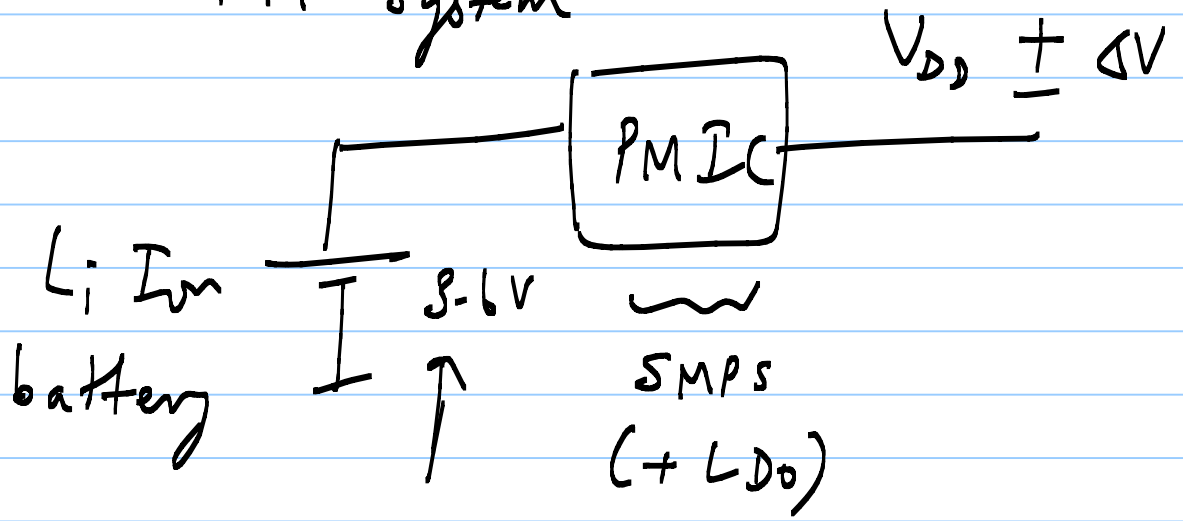
"ambient" temp.

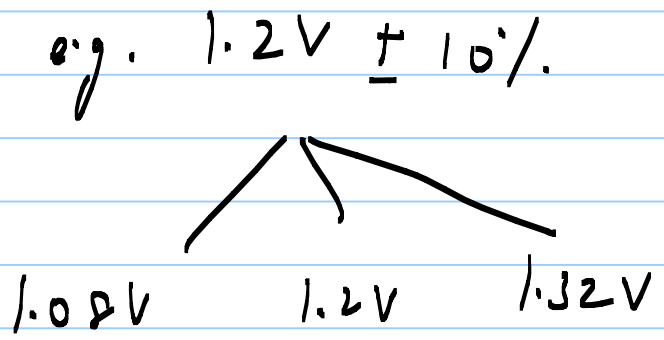
actual "junction" temp. is normally higher

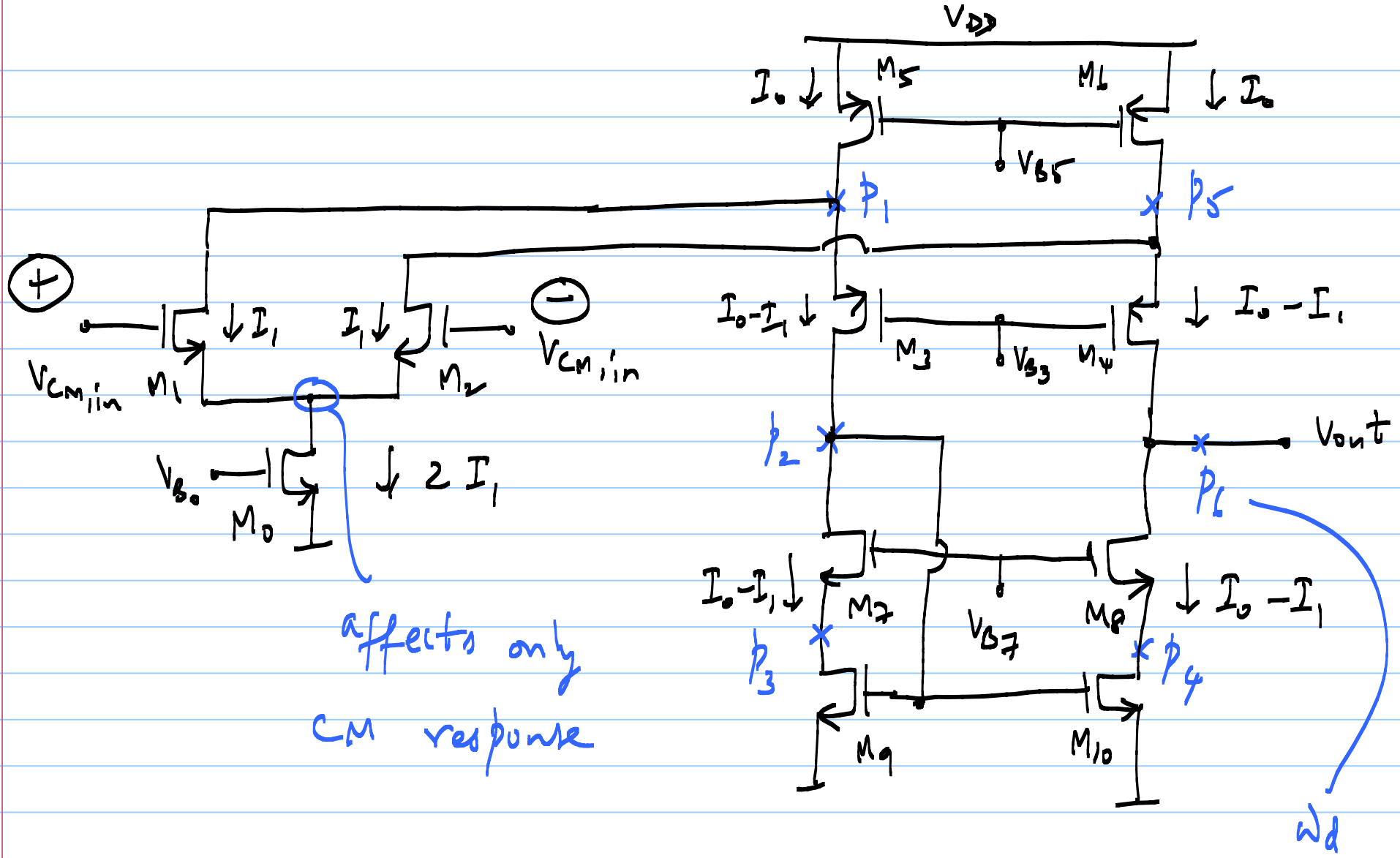
Voltage variation V_{DD} (power supply)

PMIC — power management IC

PM system






VDD
I0
M5
M6
I0
VB5
p1
p5
+
−
I1
I1
VCM,in
VCM,in
M1
M2
I0−I1
M3
VB3
M4
I0−I1
p2
Vout
VB0
M0
2 I1
p6
affects only
CM response
I0−I1
M7
VB7
M8
I0−I1
p3
p4
M9
M10
Wd

Folded Cascode opamp datasheet

$$G_m = g_{m_1}$$

$$r_{out} = r_{up} \,||\, r_{dn}$$

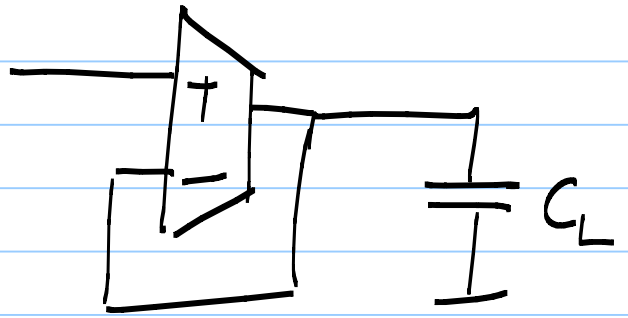
$$= \left[(g_{m_4} r_{ds_4}) (r_{ds_6} || r_{ds_2}) \right] || \left[(g_{m_8} r_{ds_8}) r_{ds_{10}} \right]$$

slightly smaller than that of Telescopic opamp

1) DC gain $= g_{m_1} r_{out}$

2) $G_m = g_{m_1}$

3) $\omega_u = \dfrac{g_{m1}}{C_L}$



Dominant pole $\omega_d = \dfrac{1}{r_{out} C_L}$

4) ND poles & zeroes

5 ND poles $p_1 - p_5$

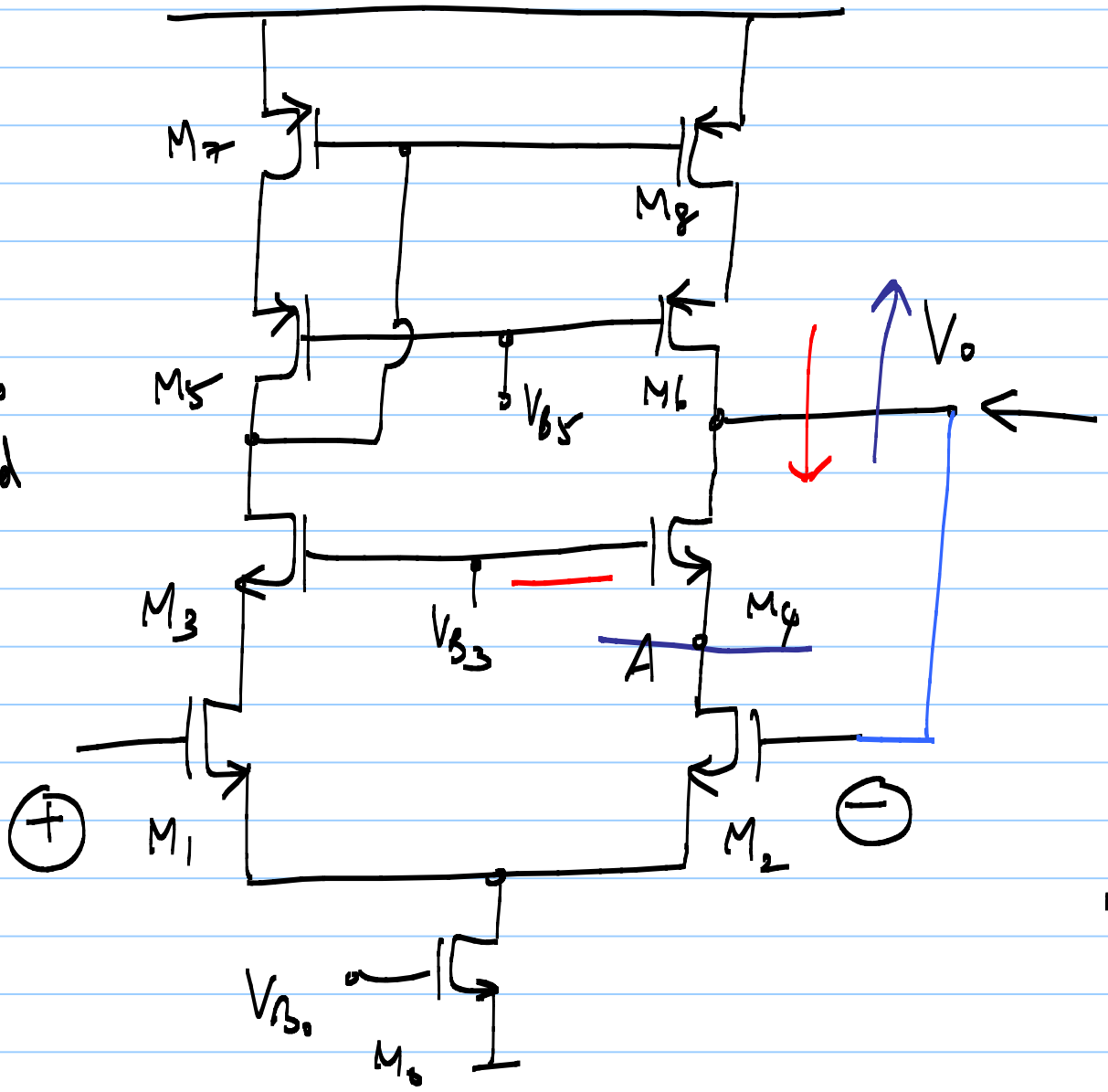
4 ND zeroes

5) $ICMR = \left\{ V_{Dsat_0}\Big|_{2I_1} + V_{GS_1}\Big|_{I_1} , V_{B_3} + V_{SG_3}\Big|_{I_0 - I_1} + V_{T_1} \right\}$

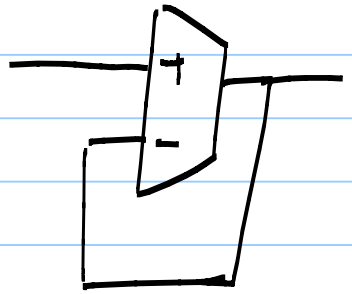
6) $OCMR = \left\{ V_{B_7} - V_{T_8} , V_{B_3} + V_{T_4} \right\}$

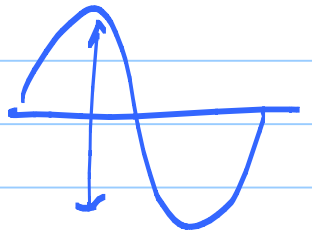
7) $\frac{\overline{e_n^2}}{\Delta f} = \frac{16kT}{3 g_{m_1}} + \frac{16kT}{3} \frac{g_{m_5}}{g_{m_1}^2} + \frac{16kT}{3} \frac{g_{m_9}}{g_{m_1}^2}$

* No noise from cascodes M_3, M_4, M_7 & M_8
* Noise is worse than 5-transistor & Telescopic OTAs



HW: check this for folded cascode opamp





$V_A = V_{B3} - V_{GS4}$

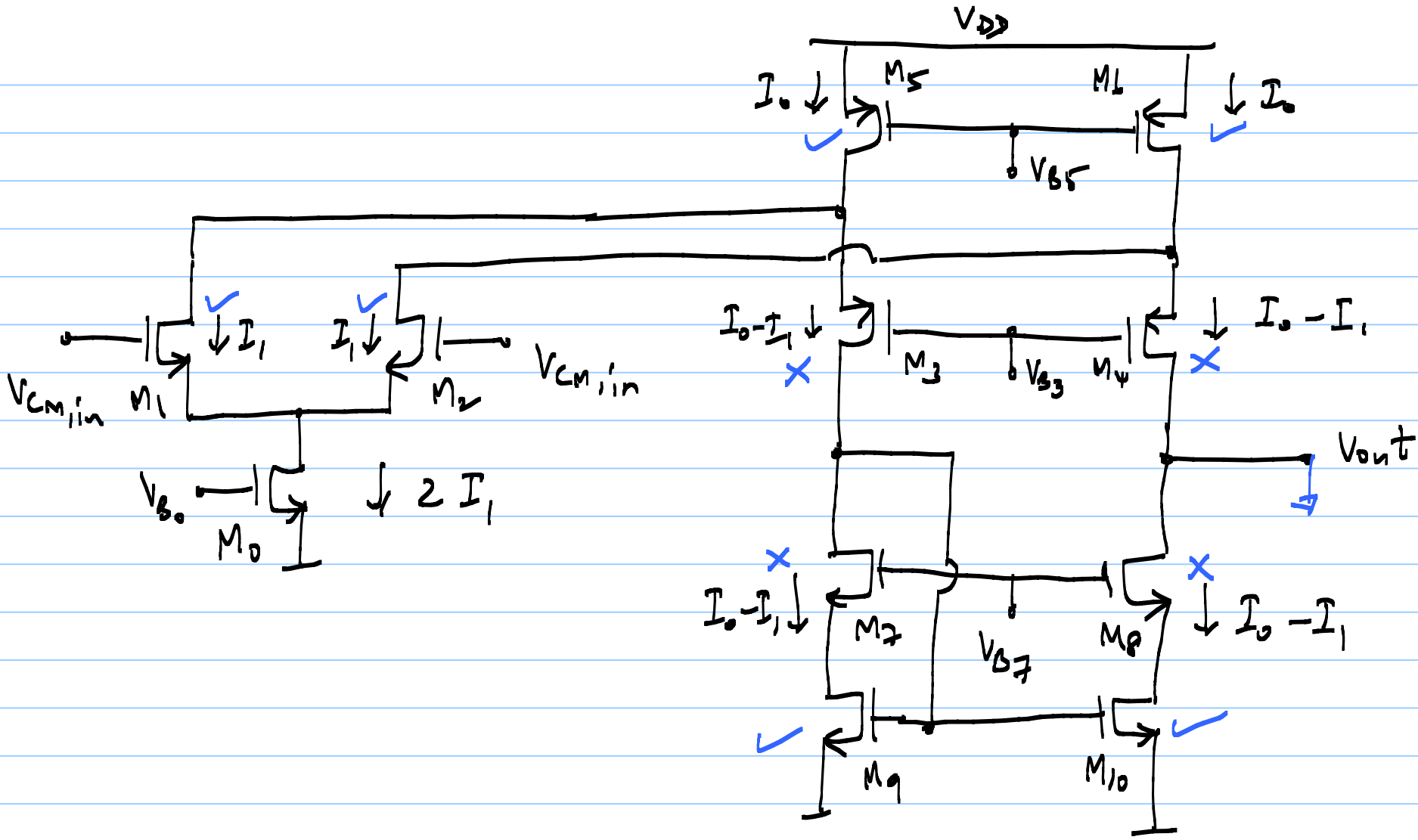
min. $V_o = V_{B3} - V_{T4}$ ✓

max. $V_o = V_A + V_{T2}$

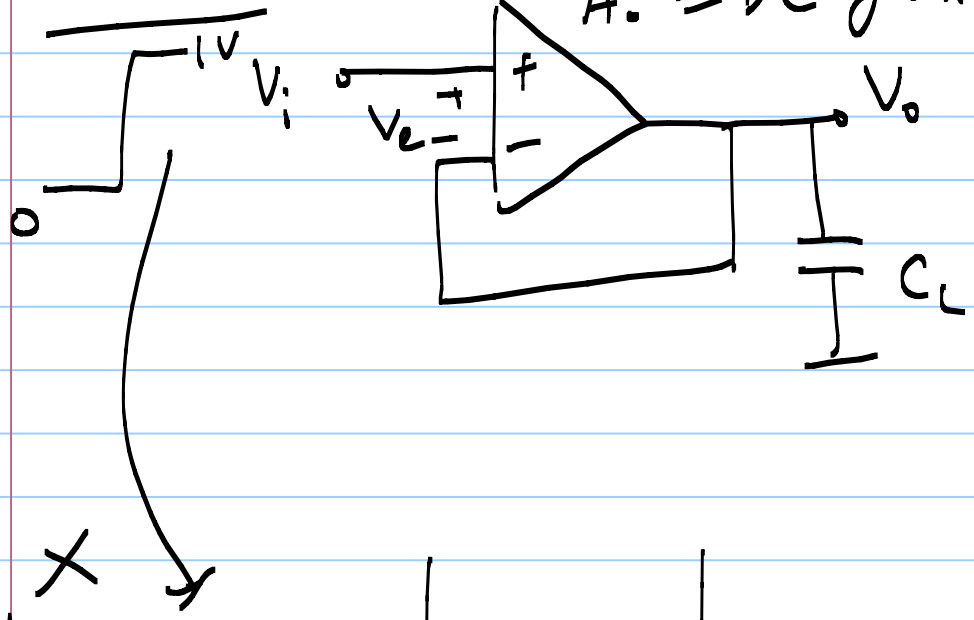
$= V_{B3} - V_{GS4} + V_{T2}$ ✓

max. $\Delta V_o = V_{o\,max} - V_{o\,min.} = V_{T2} - V_{DSat4}$

$V_{GS4} = V_{T4} + V_{DSat4}$


VDD
I0
M5
M4
I0
VB5
I1
I1
VCM,in
VCM,in
M1
M2
I0-I1
M3
VB3
M4
I0-I1
VB0
M0
2I1
Vout
I0-I1
M7
VB7
M8
I0-I1
M9
M10

Quiz 1

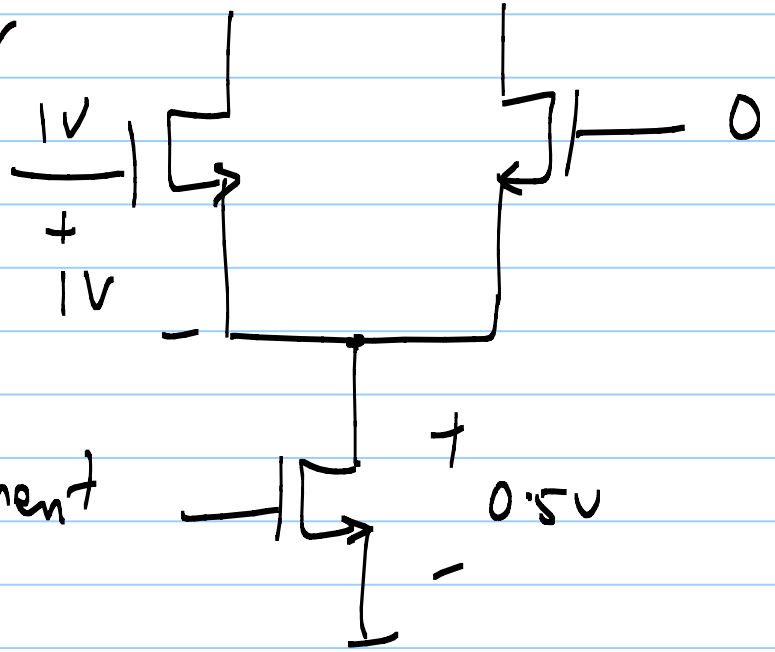


A_0 = DC gain

$$V_e = \frac{1}{1+A_0} \approx 20$$

$$= \frac{1}{21} = 47.6\,mV$$

X does not meet ICMR requirement



Key:

1) $A_0 = 20\ V/V$

2) $\frac{\overline{e_n}}{\sqrt{\Delta f}} = 10.5\ nV/\sqrt{Hz}$

3) $\overline{V_{os}} = 10\,mV$

4) $V_{in,CM}(max) = 4.5V$

5) Steady state error = 47.6mV

6) $f_p = -636.9$ MHz (LHP)

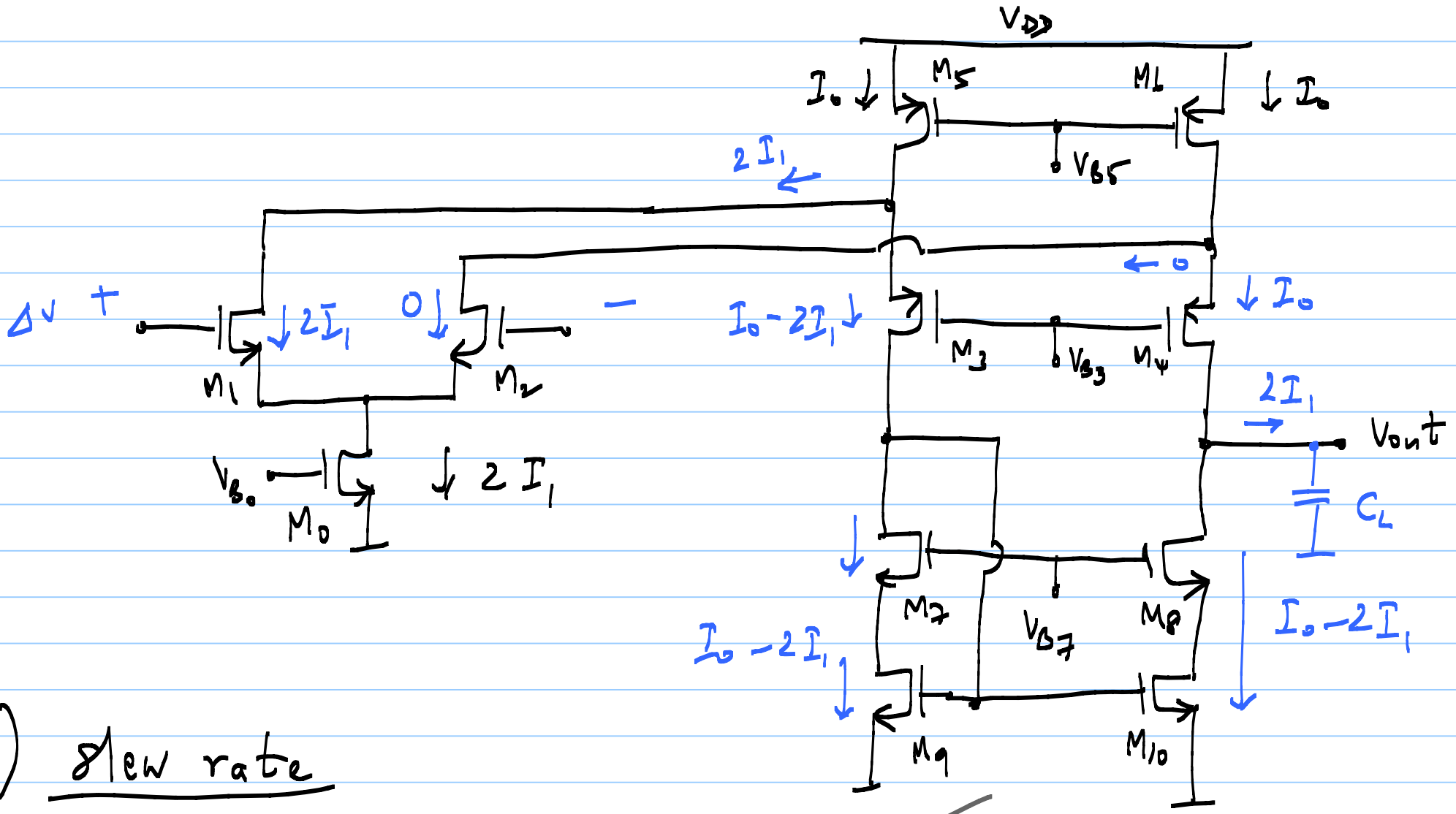
7) $f_z = 1273.9$ MHz (LHP)

8) $f_u = 63.69$ MHz

9) $C_L = 1pF$

10) $f_{BW} \approx 63.69$ MHz

p) offset $\sigma_{os}^2 = \sigma_{V_{T1-2}}^2 + \left(\frac{g_{m5}}{g_{m1}}\right)^2 \cdot \sigma_{V_{T5-6}}^2 + \left(\frac{g_{m9}}{g_{m1}}\right)^2 \cdot \sigma_{V_{T9-10}}^2$

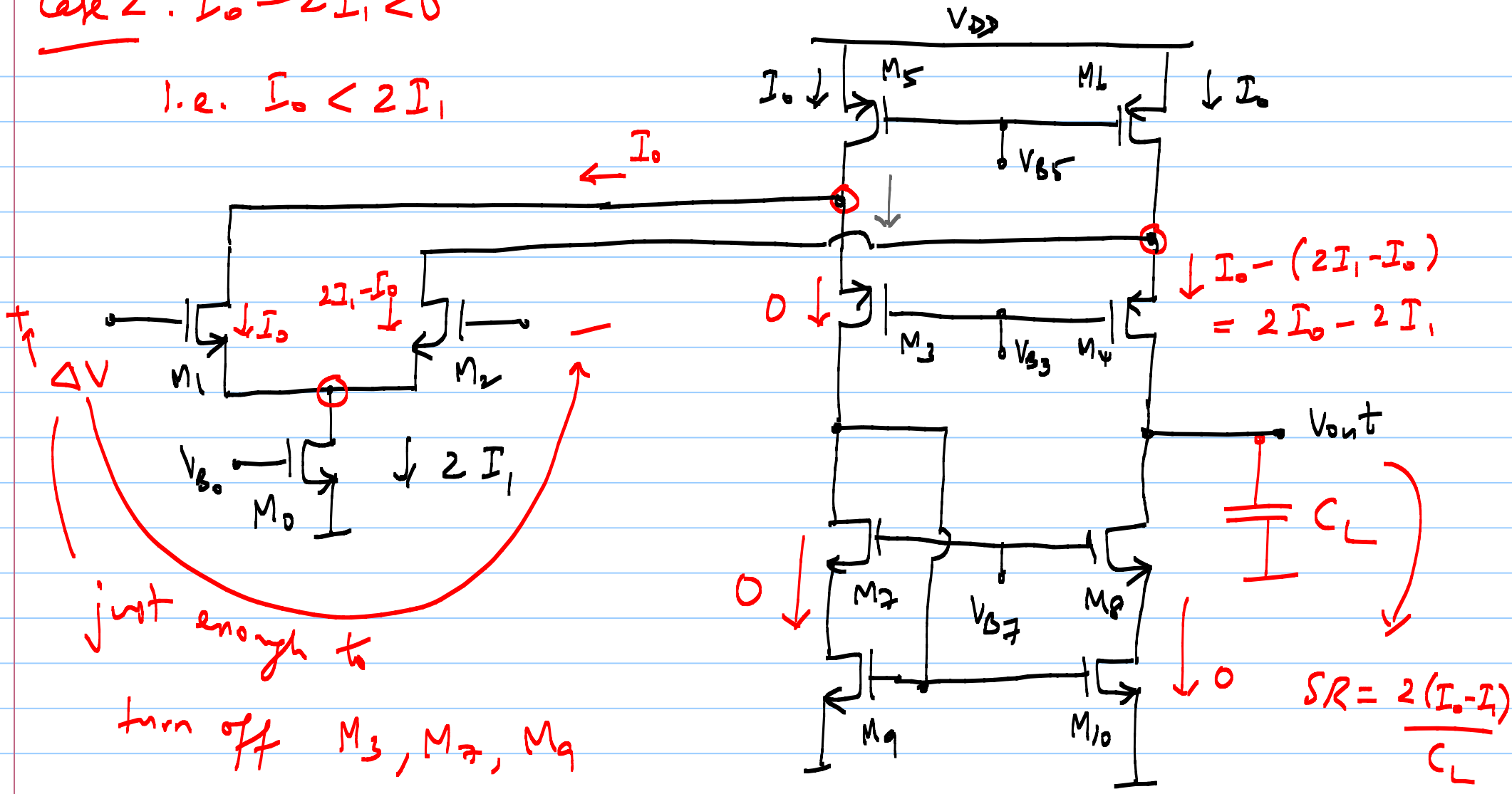


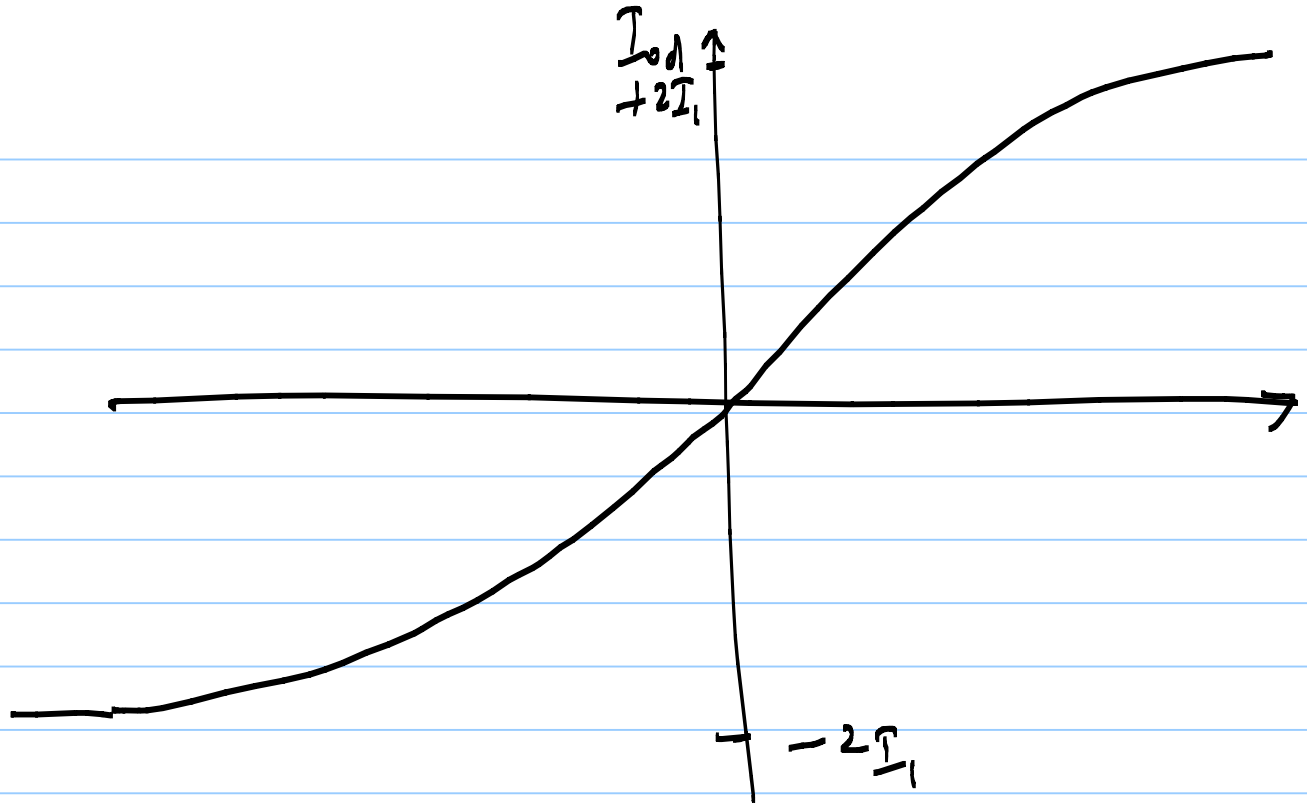
q) Slew rate

Case 1: $I_0 - 2I_1 > 0$ i.e. $I_0 > 2I_1 \Rightarrow SR = \pm 2I_1/C_L$

Case 2 : $I_o - 2I_1 < 0$

i.e. $I_o < 2I_1$



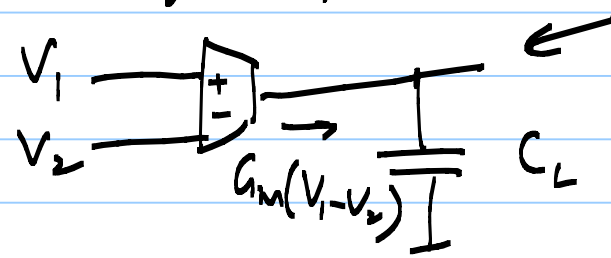


→ valid when M_1 & M_2 are in sat.

Two-stage opamps

5-transistor OTA
Telescopic
Folded Cascode

} 1-stage opamp

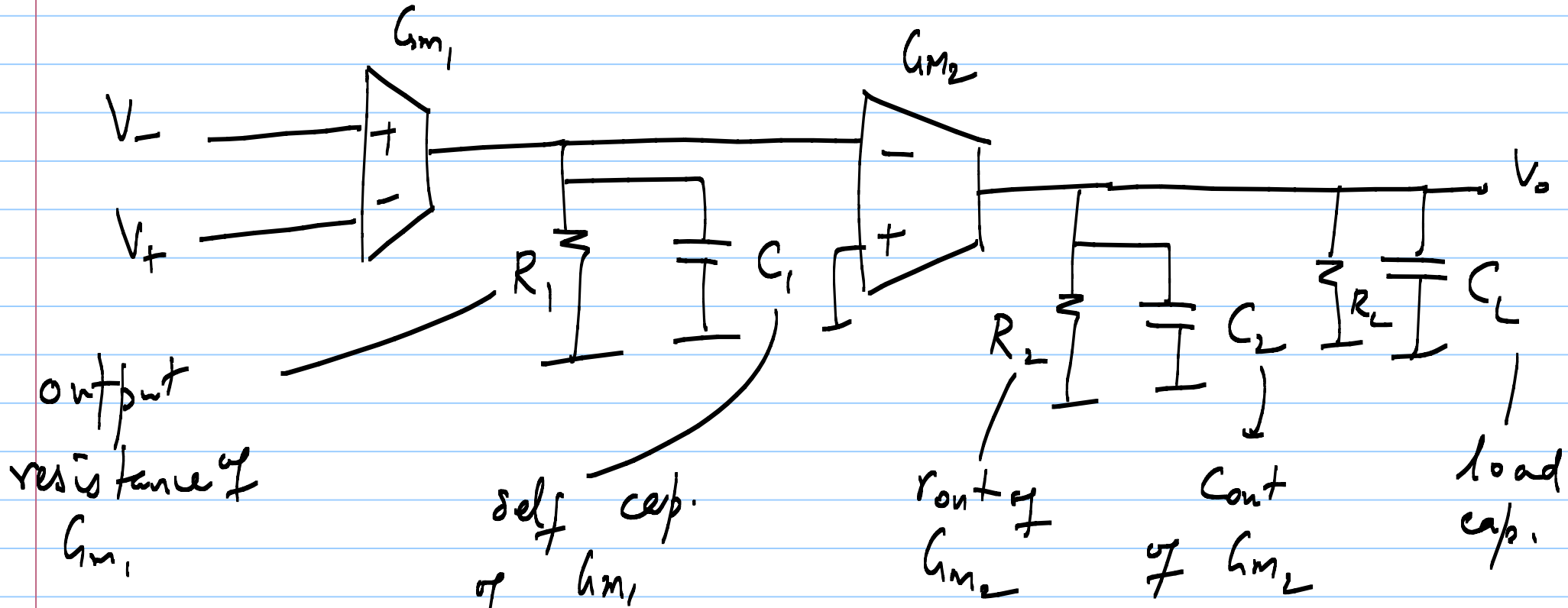


← output node is dominant

drawbacks : 1) limited gain

2) Inability to drive R loads

→ Cascade a second stage



* composite $C_2 = C_{out}$ of $G_{m_2} + C_L$
* composite $R_2 = r_{out}$ of $G_{m_2} \parallel R_L$

advantages: 1) More gain = $G_{m1}R_1 \cdot G_{m2}R_2$

2) Can drive R loads

→ gain of 1st stage is not effected by addition of R_L

Issue: 1) 2 explicit poles

$R_2 = R_{20} \| R_L$

$C_2 = C_{20} + C_L$

$$p_{10} = \frac{1}{R_1 C_1} \quad ; \quad p_{20} = \frac{1}{R_2 C_2}$$

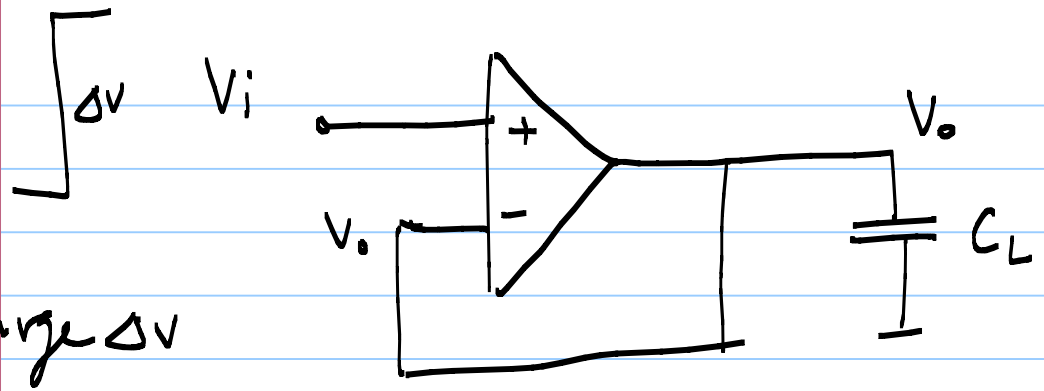
poles of loop gain

* p_{10} & p_{20} are likely to be close to each other

* High A_0 $\Rightarrow$ ringing in time domain response (low ζ, high Q)

* Will not have good phase margin by default

* Requires frequency compensation.



@ $t = 0^+$

$V_+(t=0^+) = V_i\ (t=0^+)$

$V_-(t=0^+) = V_o\ (t=0^+)$

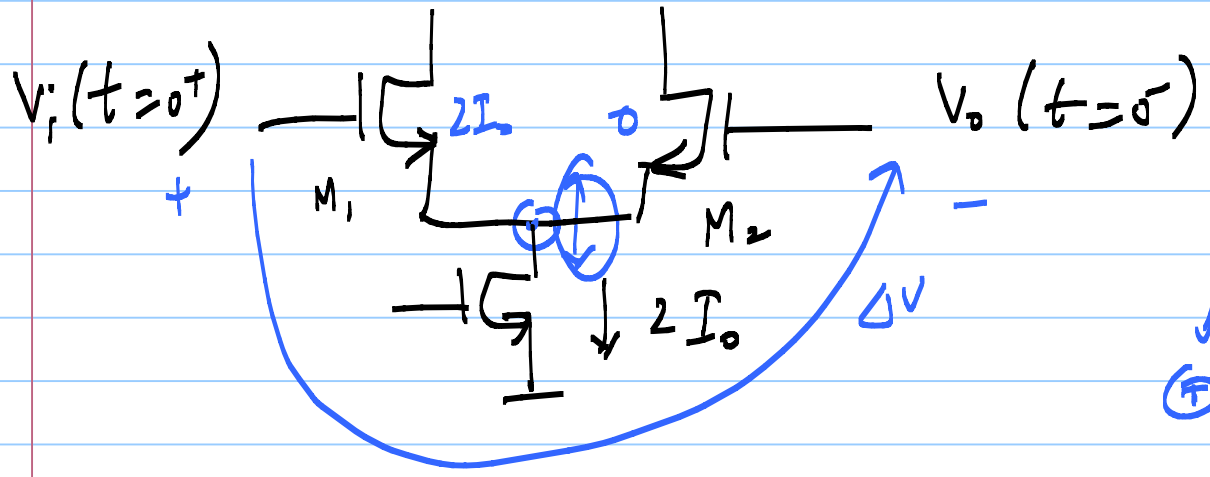
$= V_o\ (t=0^-)$

$V_e\ (t=0^+) = V_+(t=0^+) - V_-(t=0^+)$

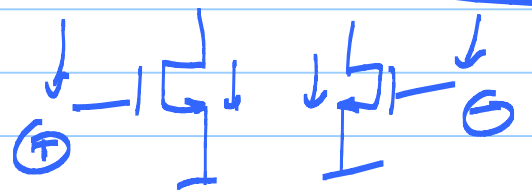
$= \Delta V$

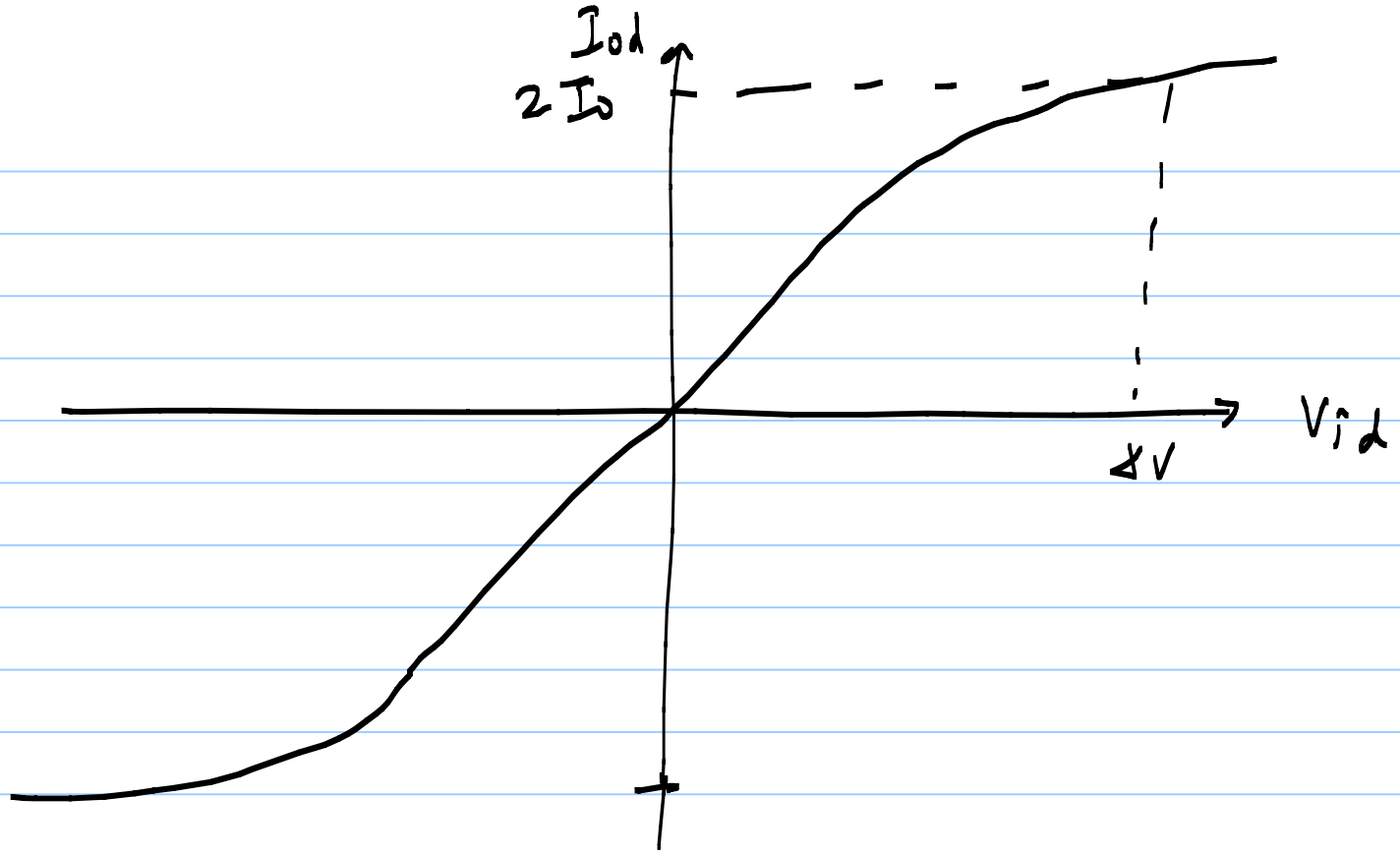
$I_{D_1}(t=0^-) = I_o = I_{D_2}(t=0^-)$

$V_{G_1}(t=0^-) \simeq V_{G_2}(t=0^-)$



$\left(V_{GS_1} - V_{GS_2}\right)\Big|_{t=0^+} = \Delta V$

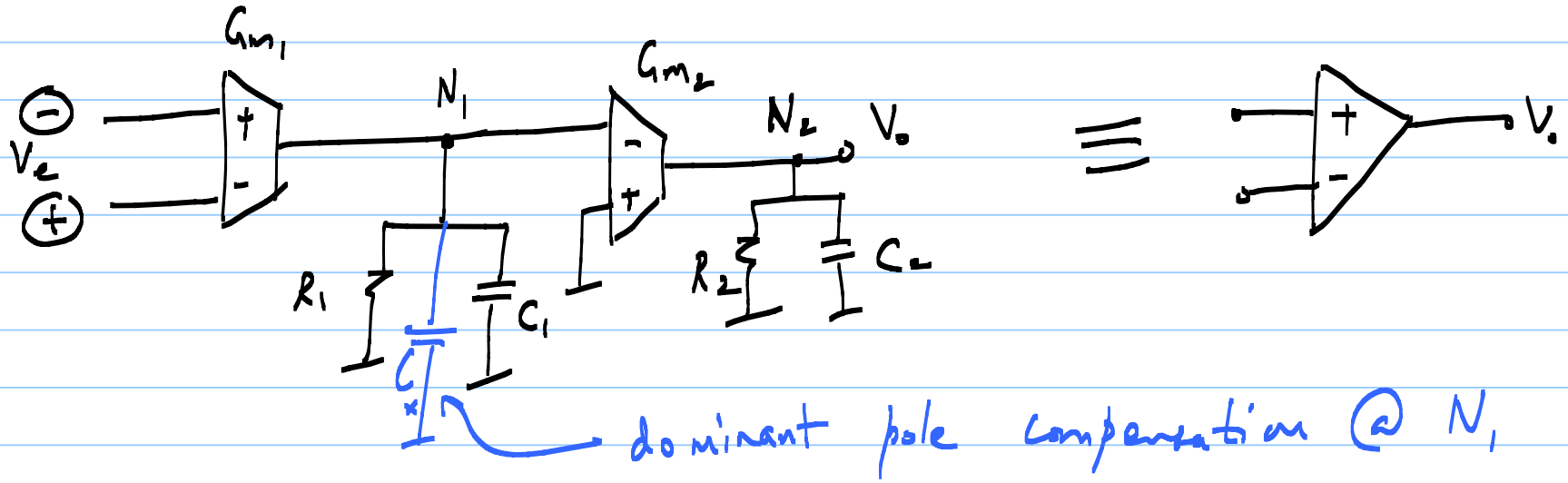


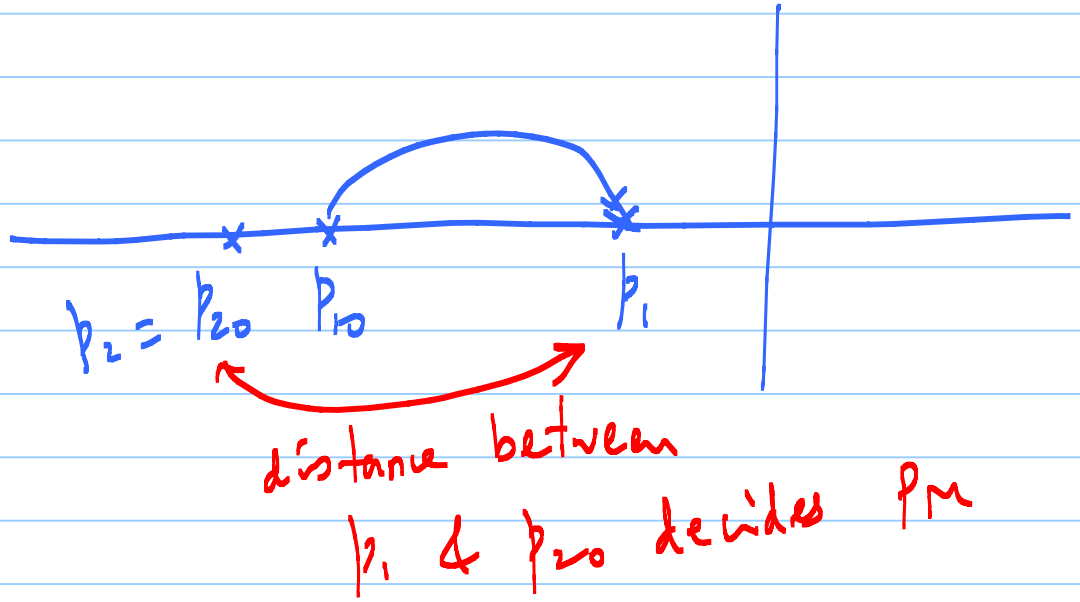


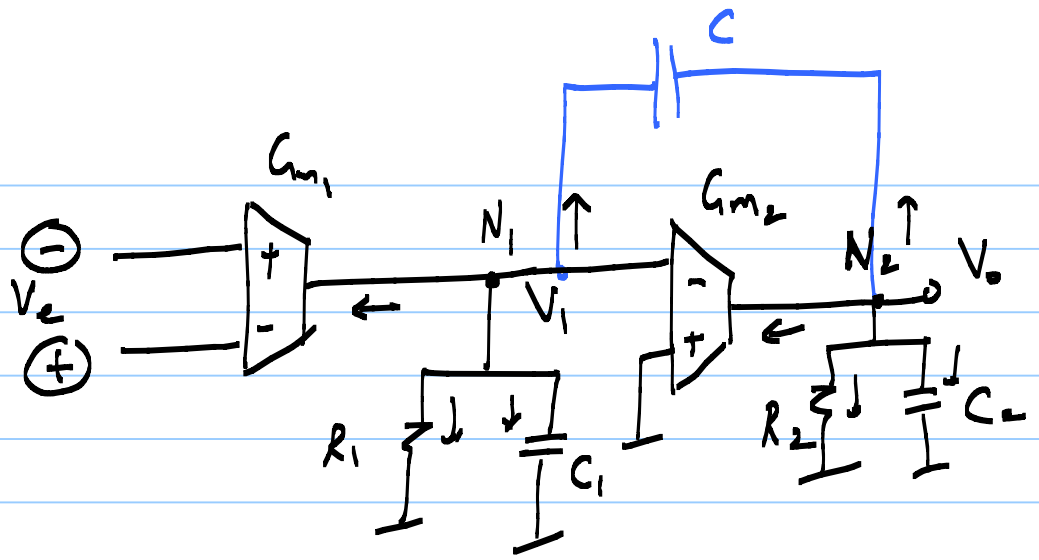
$$I_{od} = I_{D_1} - I_{D_2}$$

$$V_{id} = V_{G_1} - V_{G_2}$$

After Compensation : $p_1 << p_2$ i.e. $p_{10} \rightarrow p_1$







Miller Compensation

KCL @ N_1 :

$$G_{m_1}V_e + G_1V_1 + sC_1V_1 + (V_1 - V_o)sC = 0 \quad \text{— (A)}$$

KCL @ N_2

$$G_{m_2}V_1 + G_2V_o + sC_2V_o + (V_o - V_1)sC = 0 \quad \text{— (B)}$$

(A) $\Rightarrow$ $V_1 = \dfrac{sCV_o - G_{m_1}V_e}{G_1 + s(C + C_1)}$ $\Rightarrow$ plug into (B)

$$\frac{V_o}{V_e}(s) = A_{dc} \frac{1 \text{ zero}}{2 \text{ poles}} \frac{N(s)}{D(s)}$$

assuming G_{m1} & G_{m2} are ideal

$$= A_0 \frac{(1 - s/z)}{(1 + s/p_1)(1 + s/p_2)}$$

$$A_0 = A_1 \cdot A_2 = \frac{G_{m1}}{G_1} \cdot \frac{G_{m2}}{G_2}$$

RHP zero

$$z = +\frac{G_{m2}}{C}$$

$$N(s) = (1 - sC/G_{m2})$$

$$D(s) = 1 + \frac{s}{p_1} + \frac{s}{p_2} + \frac{s^2}{p_1 p_2}$$

1) we know $p_1 \ll p_2 \Rightarrow$ coefficient of $s \simeq \frac{1}{p_1}$

(p_1: dominant pole)

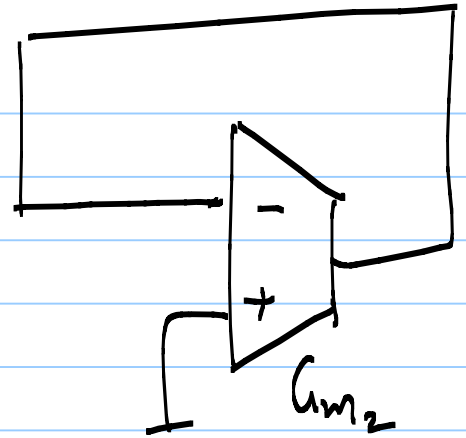
2) $G_{m_1}, G_{m_2} \gg G_1, G_2$ [intrinsic gains are large]

$C \sim C_1$; $C \sim C_2$ (miller cap)

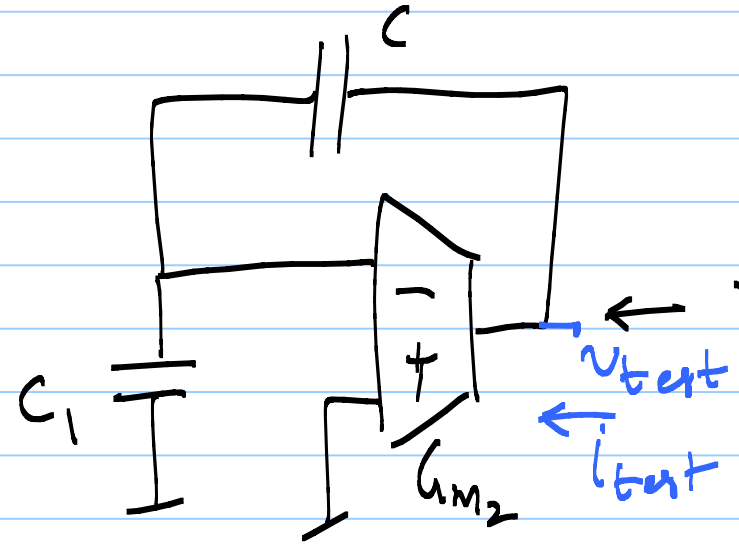
Result : $p_1 \simeq \frac{G_1 G_2}{C G_{m_2}} = \frac{1}{R_1} \cdot \frac{1}{C \cdot (G_{m_2} R_2)}$

($G_{m_2} R_2$: gain of 2nd stage)

p_2 :



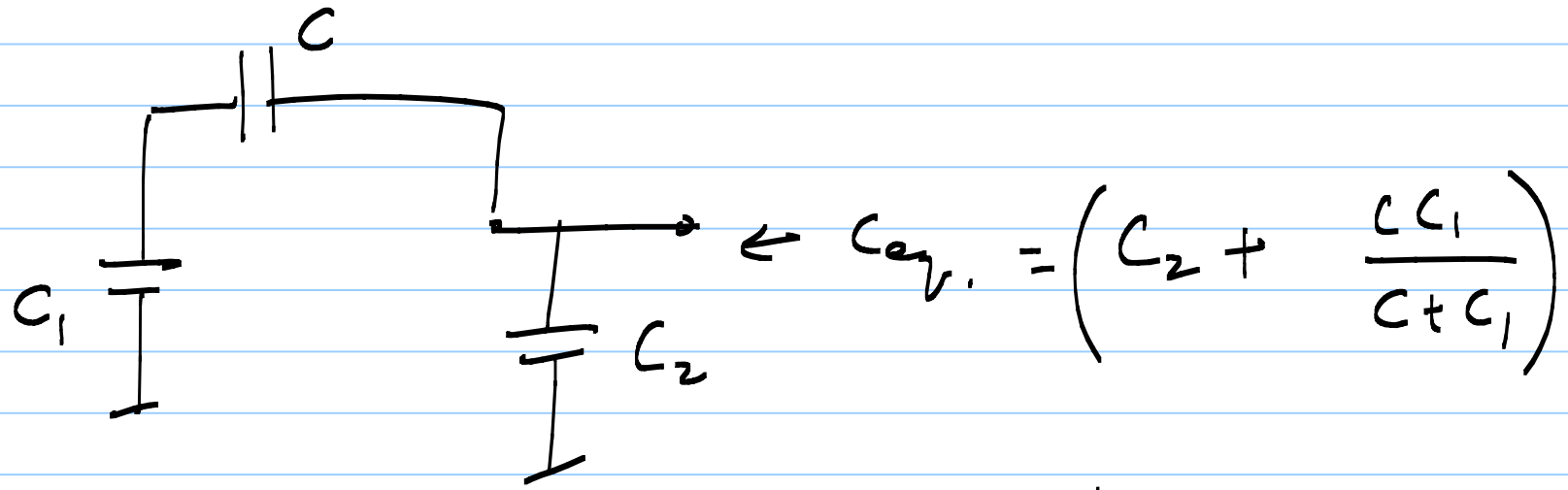
$\leftarrow r_{out} = \frac{1}{G_{m_2}}$

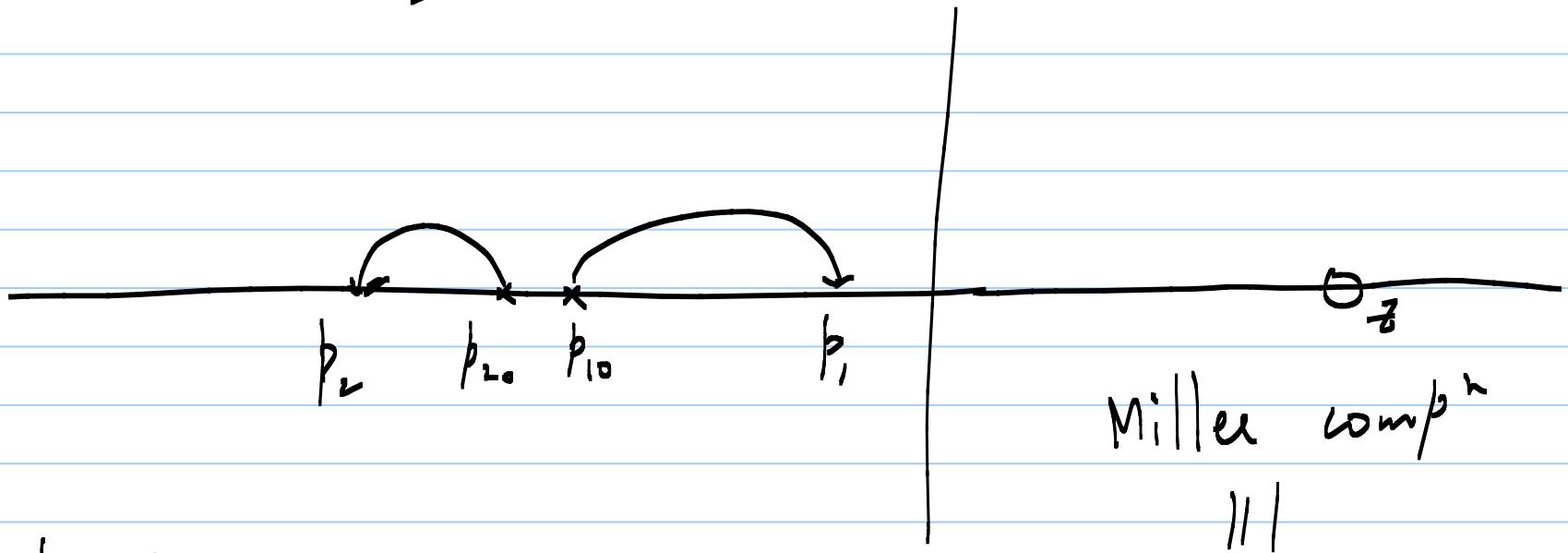


$\leftarrow r_{out}' = \left(\frac{C + C_1}{C}\right) \cdot \frac{1}{G_{m_2}} << R_2$

$C_{eq} = ?$

$$p_2 = \frac{1}{r_{out}' \; C_{eq.}}$$





Miller compⁿ

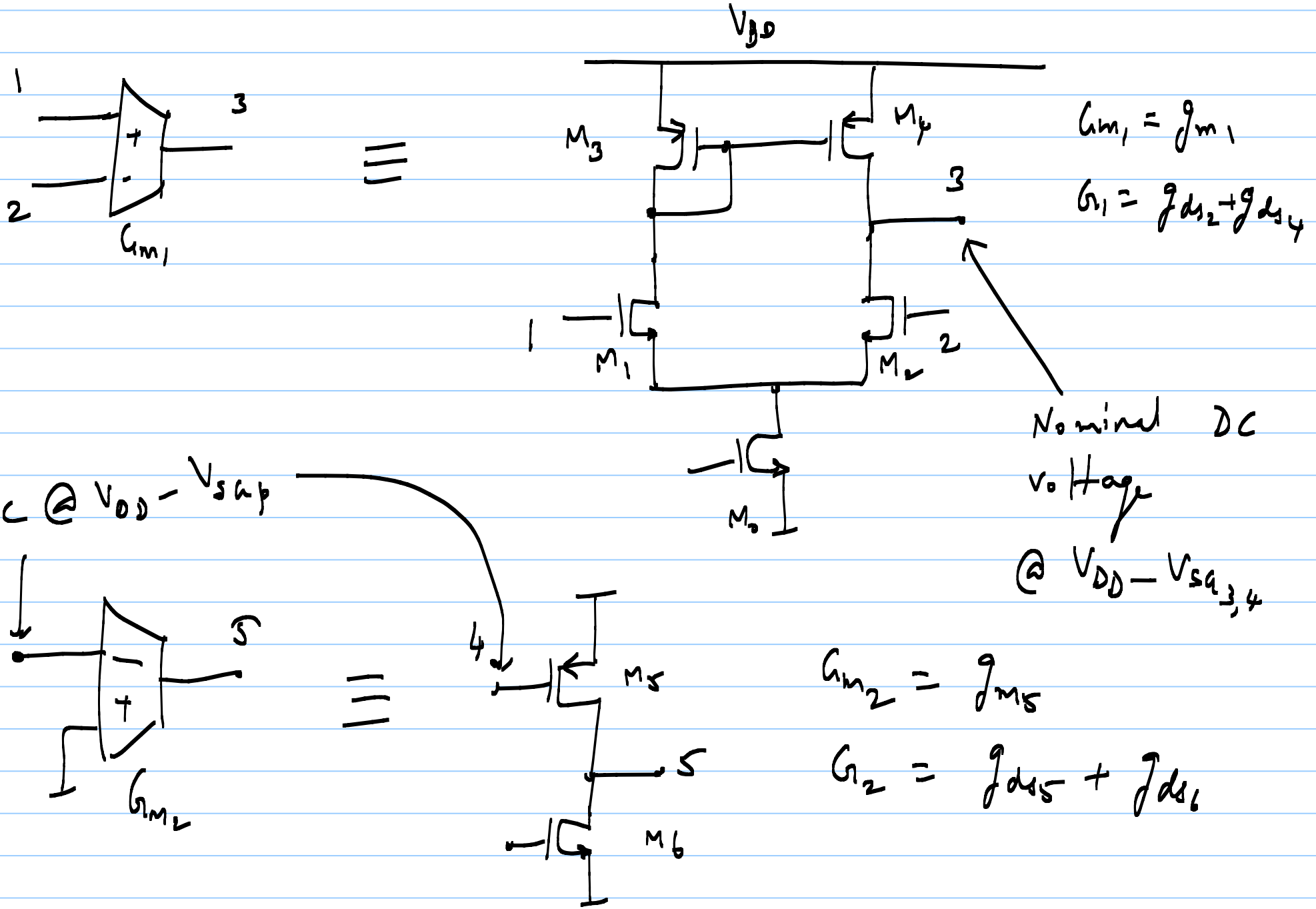
|||

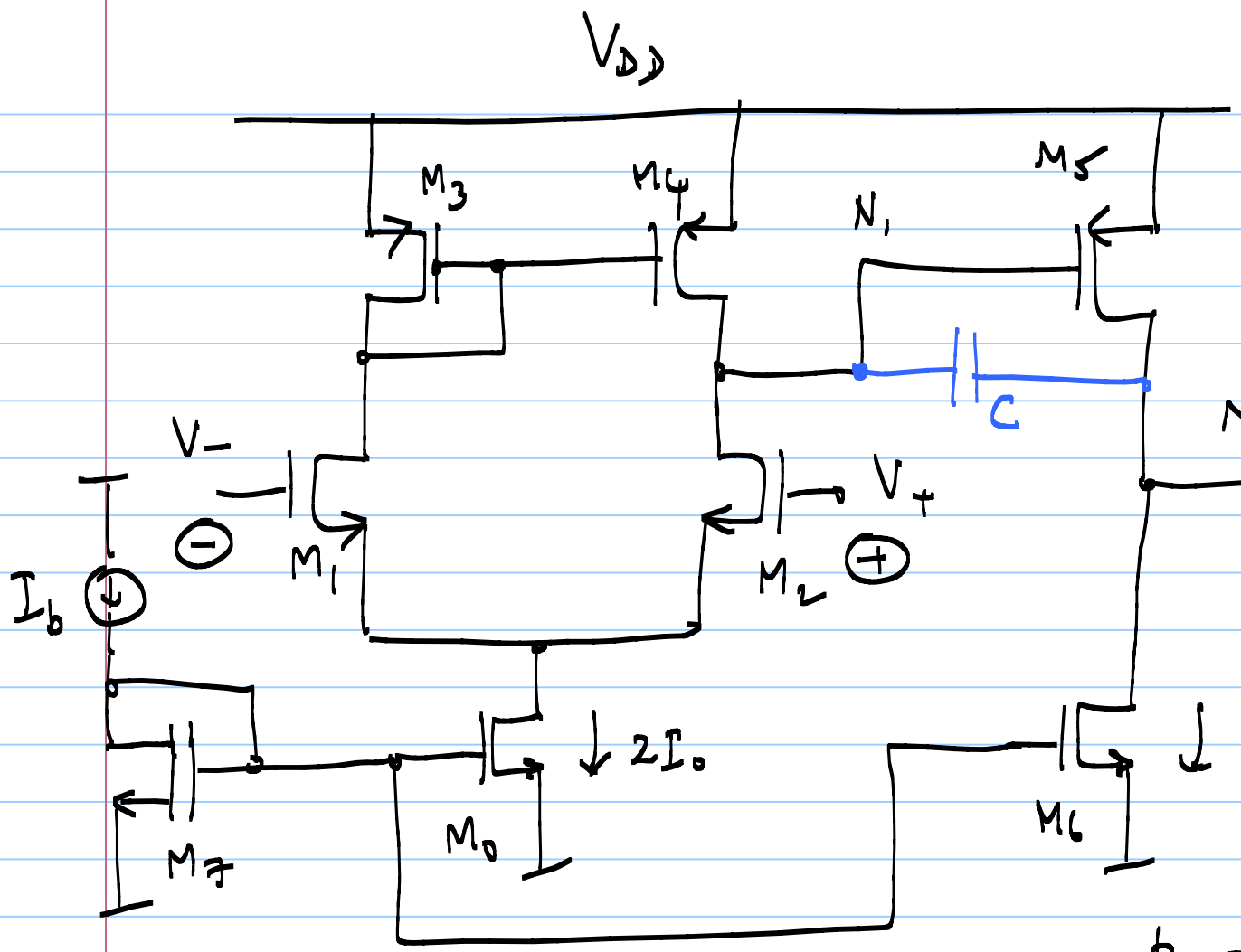
pole splitting

compⁿ

$p_2 \gg p_{2a}$

Lecture 25


1
2
3
G_{m_1}
V_{DD}
M_3
M_4
3
1
M_1
M_2
2
M_0
Nominal DC voltage @ $V_{DD} - V_{SG_{3,4}}$
$G_{m_1} = g_{m_1}$
$G_1 = g_{ds_2} + g_{ds_4}$
DC @ $V_{DD} - V_{SG_3}$
4
5
G_{m_2}
4
M_5
5
M_6
$G_{m_2} = g_{m_5}$
$G_2 = g_{ds_5} + g_{ds_6}$



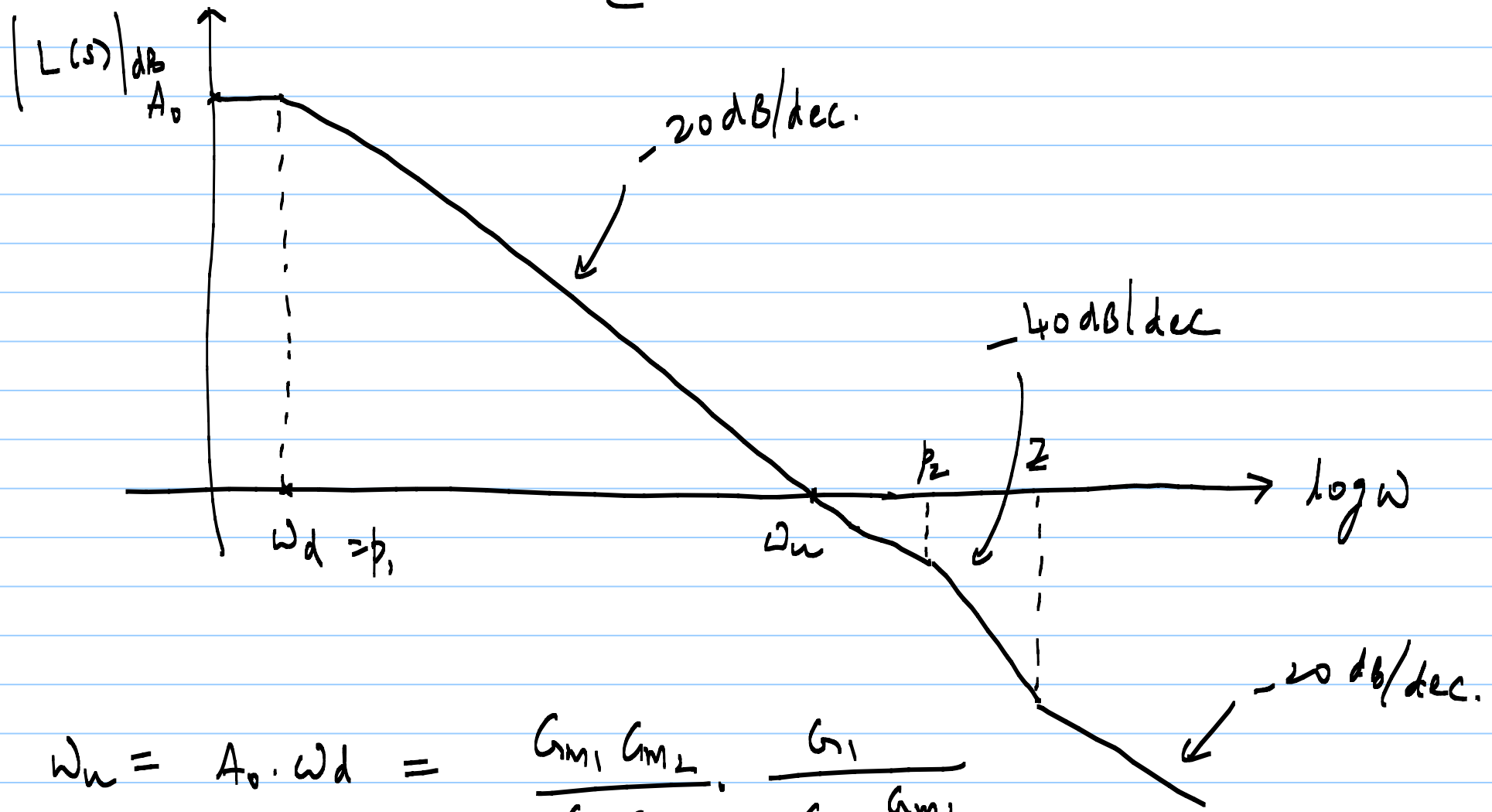
$$C_1 = C_{db_2} + C_{db_4} + C_{sb_5}$$

$$C_2 = C_{db_5} + C_{db_6} + C_L$$

$$p_{10} = \frac{G_1}{C_1} \; ; \; p_{20} = \frac{G_2}{C_2}$$

$$p_1 \approx \frac{G_1}{C_1 + \frac{G_{m_2} \cdot C}{G_2}} \; ; \; p_2 \approx \frac{G_{m_2} \frac{C}{C + C_1}}{C_2 + \frac{C C_1}{C + C_1}}$$

Zero $\quad z = +\dfrac{G_{m_2}}{C}$



$$\omega_u = A_0 \cdot \omega_d = \frac{G_{m_1} G_{m_2}}{G_1 G_2} \cdot \frac{G_1}{C \cdot \frac{G_{m_2}}{G_2}}$$

$$= \frac{G_{m_1}}{C}$$

1) $z \gg \omega_u$: $\frac{G_{m2}}{C} \gg \frac{G_{m1}}{C} \Rightarrow G_{m2} \gg G_{m1}$

$$\boxed{g_{m5} \gg g_{m1}}$$

2) $$V_{DD} - V_{SG_{3,4}}\Big|_{I_0} = V_{DD} - V_{SG_5}\Big|_{I_5}$$

$$V_{SG_{3,4}}\Big|_{I_0} = V_{SG_5}\Big|_{I_5}$$

$$\sqrt{\frac{2I_0}{\mu_p C_{ox}\left(\frac{W}{L}\right)_3}} = \sqrt{\frac{2I_5}{\mu_n C_{ox}\left(\frac{W}{L}\right)_5}}$$

$$\frac{I_0}{(W/L)_{3,4}} = \frac{I_5}{(W/L)_5}$$

equal current densities

3) $L_{3,4} = L_5$

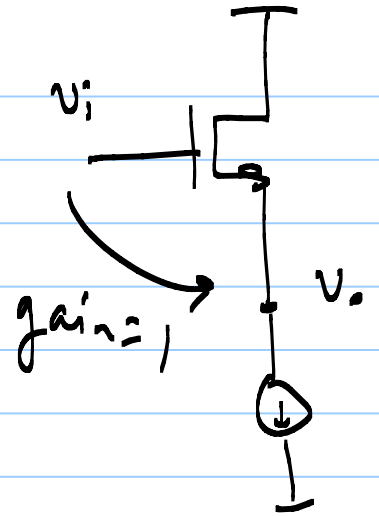
4) size M_7, M_0, M_L for proper current mirror action.

2-stage opamp datasheet

1) $G_m = G_{m_1} R_1 \cdot G_{m_2} = \dfrac{g_{m1}}{g_{ds_2} + g_{ds_4}} \cdot g_{m5}$

2) DC gain $A_0 = \underbrace{\dfrac{g_{m1}}{g_{ds_2} + g_{ds_4}}}_{A_1} \cdot \underbrace{\dfrac{g_{m5}}{g_{ds_5} + g_{ds_6} + G_L}}_{A_2}$

G_L: if load resistance exists. $R_L = \frac{1}{G_L}$



can try VCVS if R_L is v. small

3) $\omega_u = \frac{G_{m1}}{C} = \frac{g_{m1}}{C}$; $\omega_d = \frac{g_{ds2} + g_{ds4}}{C \cdot \frac{g_{m5}}{g_{ds5} + g_{ds6}}}$

4) ND poles & zeroes

p_2 @ output node — LHP pole

z @ $+g_{m5}/C$ — RHP zero

p_3 & z_2 from $M_3 - M_4$ current Mirror

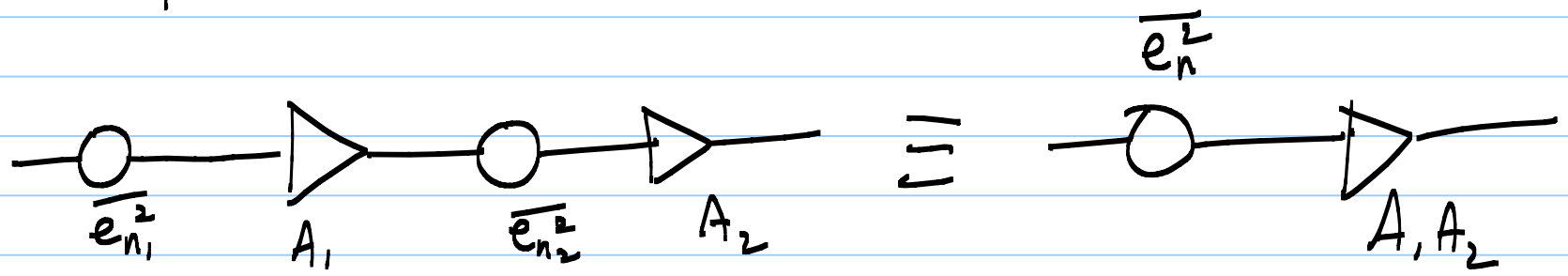
5) ICMR

$$V_{in,CM} = \left\{ V_{GS_1}\Big|_{I_0} + V_{Dsat_0}\Big|_{2I_0} \;,\; V_{DD} - V_{SG_3}\Big|_{I_0} + V_{T_1} \right\}$$

6) OCMR

$$V_{out,CM} = \left\{ V_{Dsat_4} \;,\; V_{DD} - V_{Dsat_5} \right\}$$

7) $\overline{e_n^2}/\Delta f$



$$\overline{e_n^2} \cdot (A_1 A_2)^2 = \overline{e_{n_1}^2} (A_1 A_2)^2 + \overline{e_{n_2}^2} (A_2)^2$$

$$\frac{\overline{e_n^2}}{\Delta f} = \frac{\overline{e_{n_1}^2}}{\Delta f} + \frac{1}{A_1^2} \cdot \frac{\overline{e_{n_2}^2}}{\Delta f}$$

$$\overline{e_{n_2}^2}/\Delta f = \frac{\frac{8kT}{3}(g_{m5} + g_{m6})}{g_{m5}^2}$$

$$= \frac{8kT}{3}\left[\frac{1}{g_{m5}} + \frac{g_{m6}}{g_{m5}^2}\right]$$

$$\frac{\overline{e_{n_1}^2}}{\Delta f} = \frac{16kT}{3}\left[\frac{1}{g_{m_1}} + \frac{g_{m_3}}{g_{m_1}^2}\right]$$

A_1 is large
$g_{m_2} \gg g_{m_1}$

$\overline{e_{n_2}^2}/\Delta f$ contribution to $\overline{e_n^2}/\Delta f$ is negligible

$$\frac{\overline{e_n^2}}{\Delta f} \simeq \frac{\overline{e_{n_1}^2}}{\Delta f} = \frac{16kT}{3}\left[\frac{1}{g_{m_1}} + \frac{g_{m_3}}{g_{m_1}^2}\right]$$

8) $\overline{V_n}$: $\sigma_{OS}^2 = ?$ HW

18/3/2021

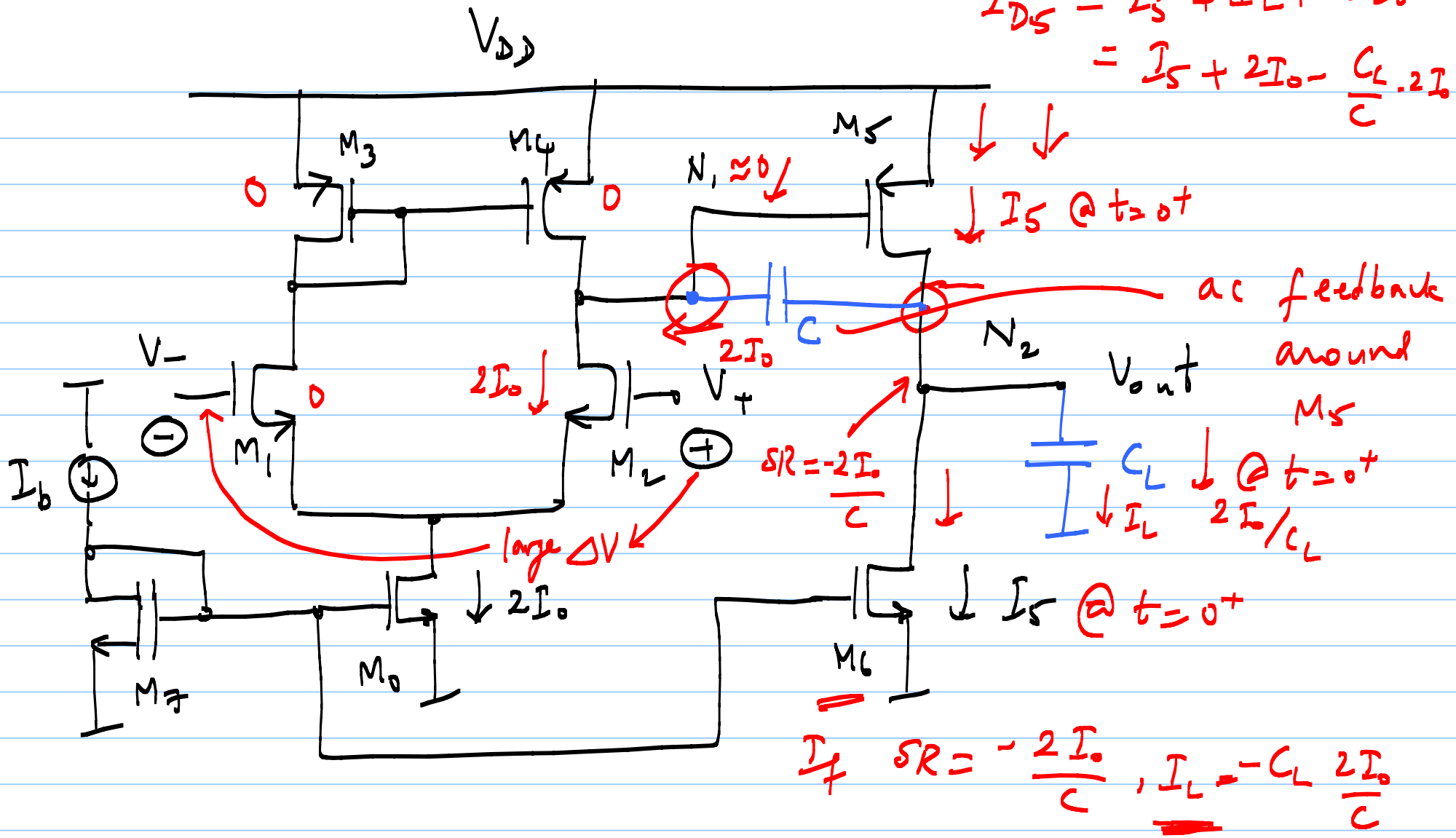
Lecture 26

8) Offset $\overline{V_{os}}$ / σ_{os}^2

$$\sigma_{os}^2 = \sigma_{V_{T1,2}}^2 + \left(\frac{g_{m3}}{g_{m1}}\right)^2 \sigma_{V_{T3-4}}^2$$

9) Slew rate $= \pm \frac{2I_o}{C}$

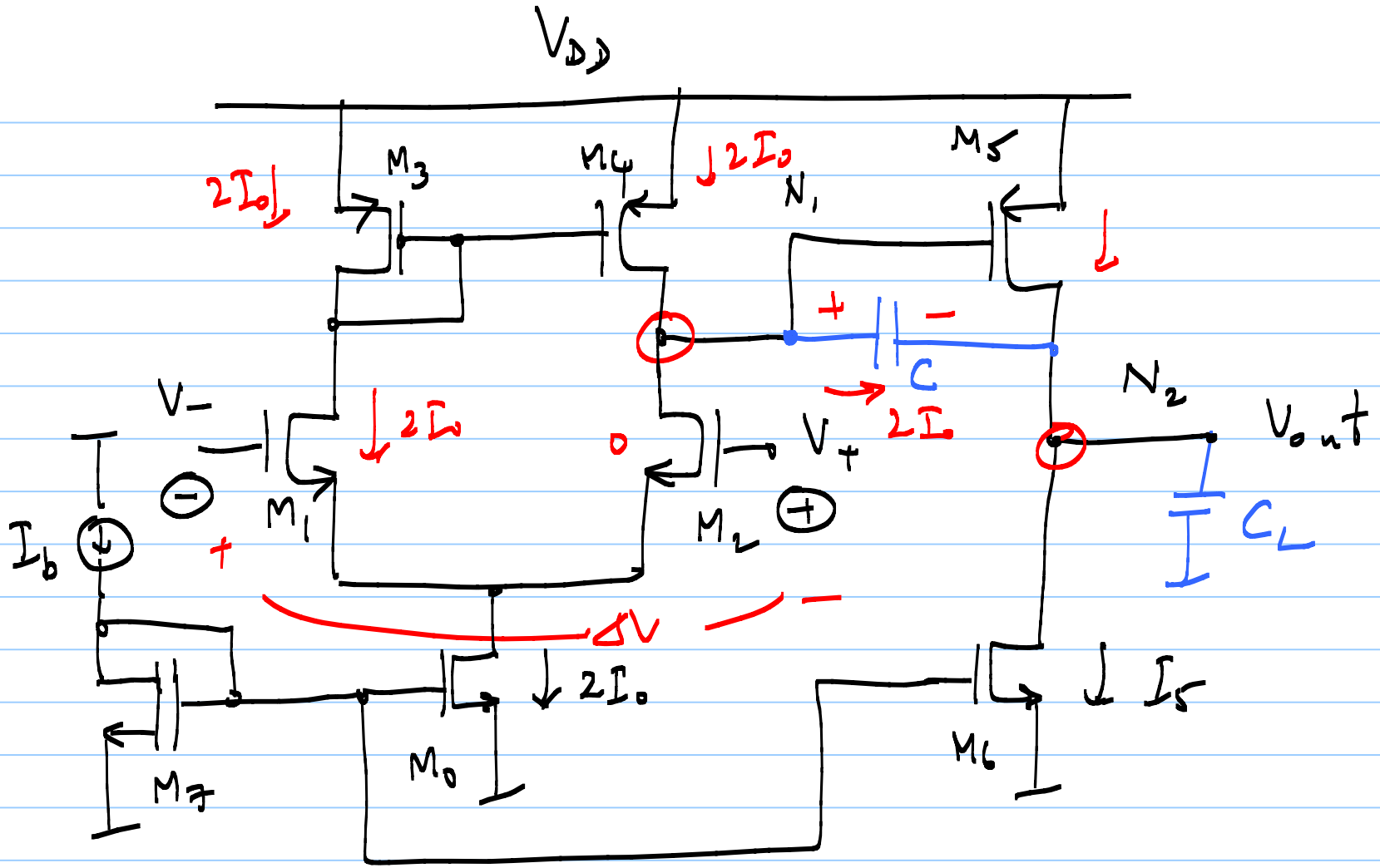
* AC f.b. around M_5
* $g_{m5} \gg g_{m1}$; $I_5 \gg I_o$



$I_{D5} = I_5 + I_L + 2I_0$

$= I_5 + 2I_0 - \frac{C_L}{C} \cdot 2I_0$

If $SR = -\frac{2I_0}{C}$, $I_L = -C_L \frac{2I_0}{C}$


V_{DD}
M_3
M_4
$2I_0$
$2I_0$
N_1
M_5
I_0
+
−
C
$2I_0$
V_-
$2I_0$
0
V_+
$2I_0$
N_2
V_{out}
M_1
M_2
C_L
I_0
+
−
ΔV
$2I_0$
I_5
M_7
M_0
M_6

2-stage opamp — Design example

Specs: $A_0 \geq 80\,dB = 10^4$; $V_{opp} \geq 4V$;

$f_u \geq 5\,MHz$; $SR \geq 10V/\mu s$; ~~$V_{in,cm}$~~ $V_{in,cm}^{min.} = 1V$

unity gain $PM \geq 60^\circ$; $C_L = 10pF$

MOSFET parameters: $\mu_n C_{ox} = 50\,\mu A/V^2$; $\mu_p C_{ox} = 25\,\mu A/V^2$

$V_{Tn} = V_{Tp} = 1V$; $V_{DD} = 5V$;

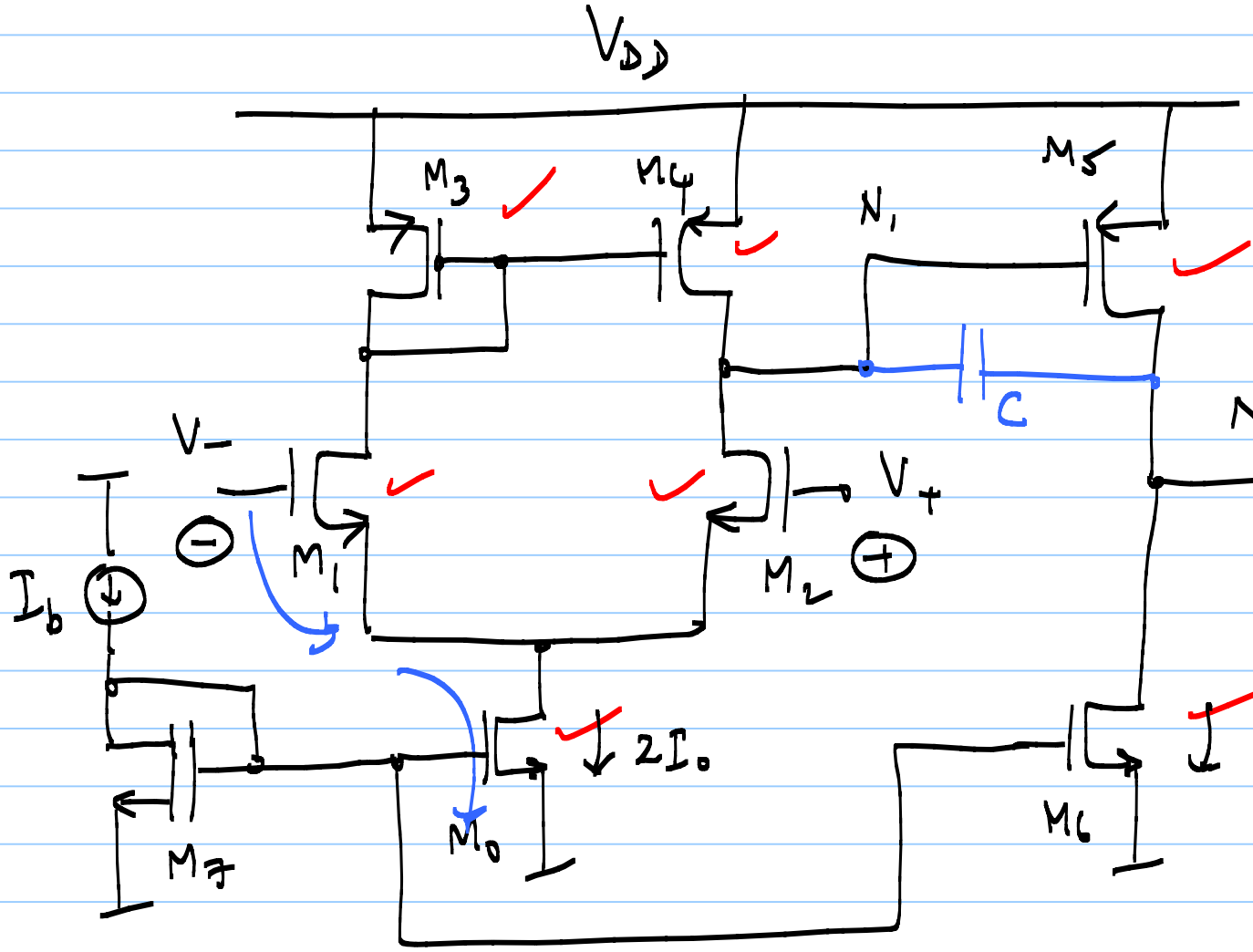
$L_{min} = 2\,\mu m$; $C_{ox} = 1.5\,fF/\mu m^2$ — gate area

$(\lambda L)_p = 0.1\,\mu m/V$; $(\lambda L)_n = 0.04\,\mu m/V$;

λL = constant

$r_{ds} = \frac{1}{\lambda I}$

} larger r_{ds} $\longleftrightarrow$ longer L



Implicit assumption:

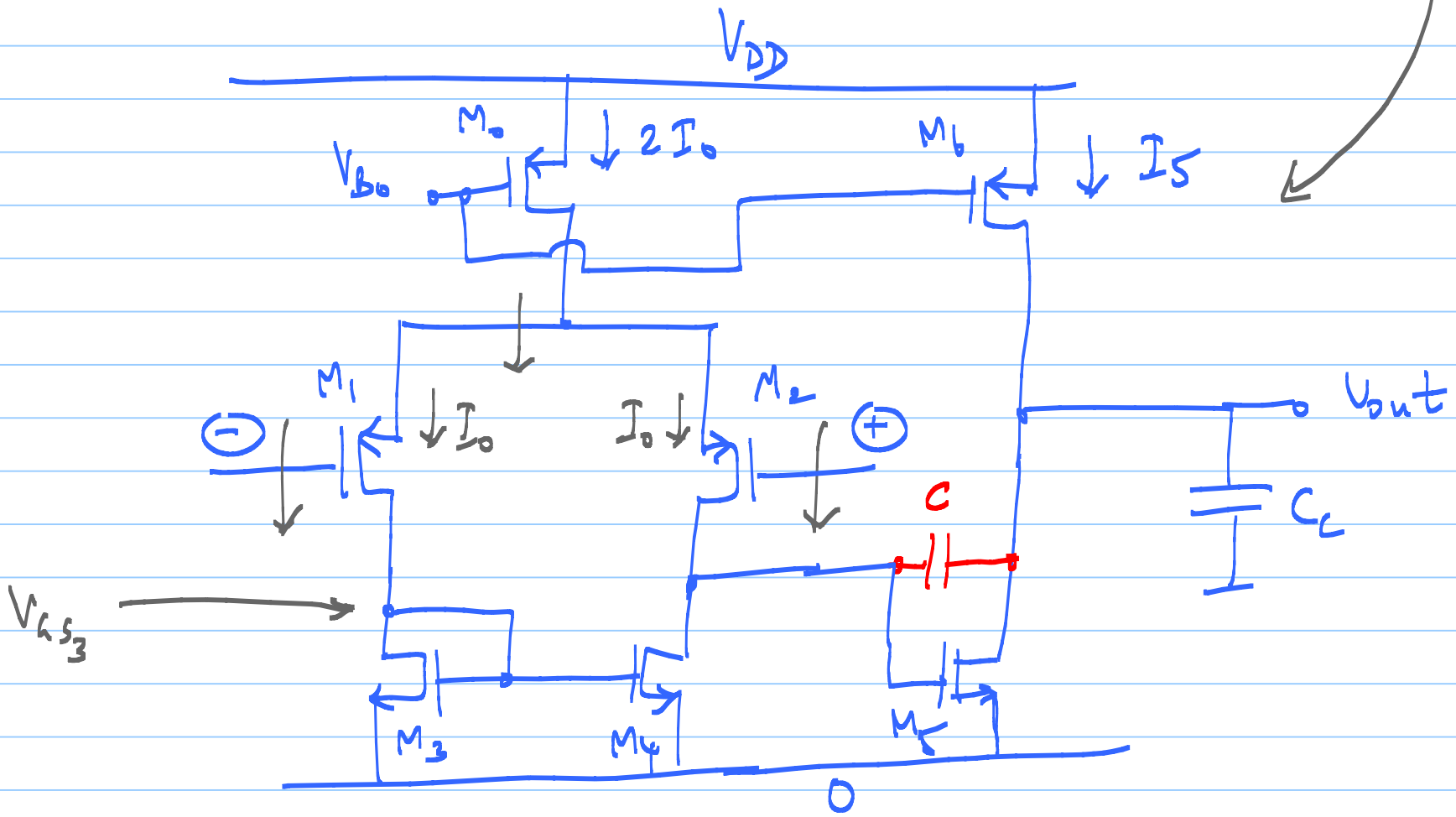
min. $V_{in,cm} > 0$

hence > 1V (V_{TN})

min $V_{in,cm}$

$= V_{T1} + V_{DSat1} + V_{Dsat0}$

If min. $V_{in,cm}$ required = 1V → Use PMOS input pair



min. $V_{in,CM} = V_{GS_3} - V_{T_1} = V_{T_3} + V_{DSat_3} - V_{T_1}$

$$SR = \frac{2\,I_0}{C_L} \quad \text{or} \quad \frac{2\,I_0}{C}$$

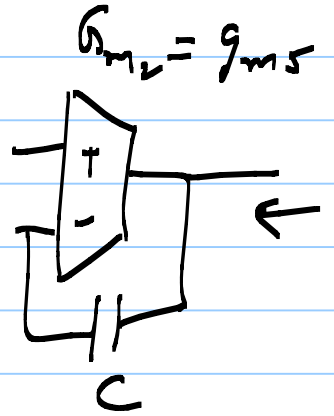
↑

Choose this & check at the end that $C < C_L$.

1) $SR = \frac{2I_0}{C_L}$

$10V/\mu s = \frac{2I_0}{10pF} \Rightarrow 2I_0 = 100\mu A$

$I_{D_{1,2}} = 50\mu A$



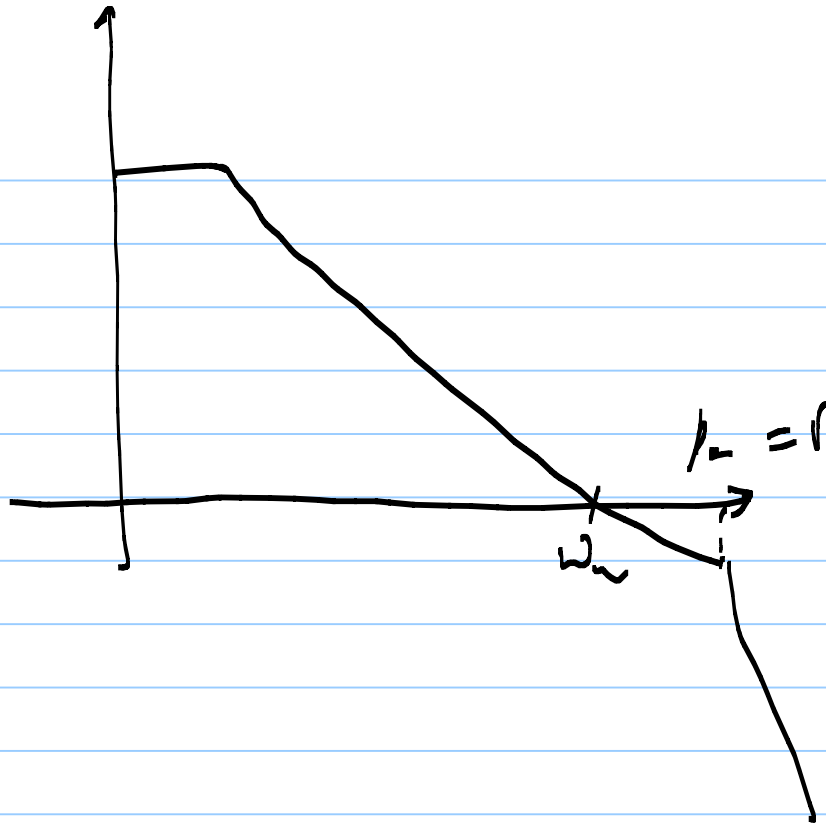
2) $z \gg \omega_u \Rightarrow g_{m_5} \gg g_{m_1}$

e.g. set $g_{m_5} = 10\, g_{m_1}$

3) $C_1 \simeq 0 \quad (C_1 \ll C)$; $PM = 60°$

$p_2 = -g_{m_5}/C_L$

$|p_2| = \sqrt{3}\,\omega_u$



$\omega_u = \sqrt{3}\,\omega_u$ for $PM = 60°$

$$\frac{g_{m5}}{10pF} = \sqrt{3} \times 2\pi \times 5 \times 10^6$$

$$g_{m5} = 543.8\ \mu S \Rightarrow g_{m1} = 54.4\ \mu S$$

4) $$\Rightarrow g_{m1} = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1 \cdot I_{D1}} \Rightarrow \left(\frac{W}{L}\right)_1 = 0.6$$

5) $\omega_u = \frac{g_{m1}}{C} \Rightarrow C = \frac{g_{m1}}{\omega_u} = \frac{54.4\,\mu S}{6.28 \times 5 \times 10^6}$

$= 1.7\,pF$

check :

$SR = \frac{2I_o}{C} = \frac{100\mu A}{1.7pF} = 58.8\ V/\mu s$

6) $A_o = 10^4$; $V_{o\,pp} = 4V$

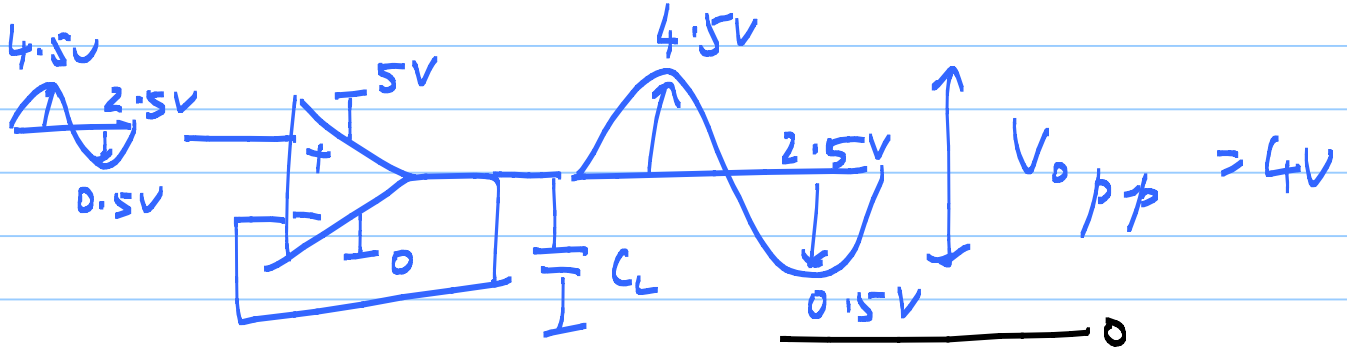
$A_o = (g_{m1}\, r_{out_1})(g_{m5}\, r_{out_2})$

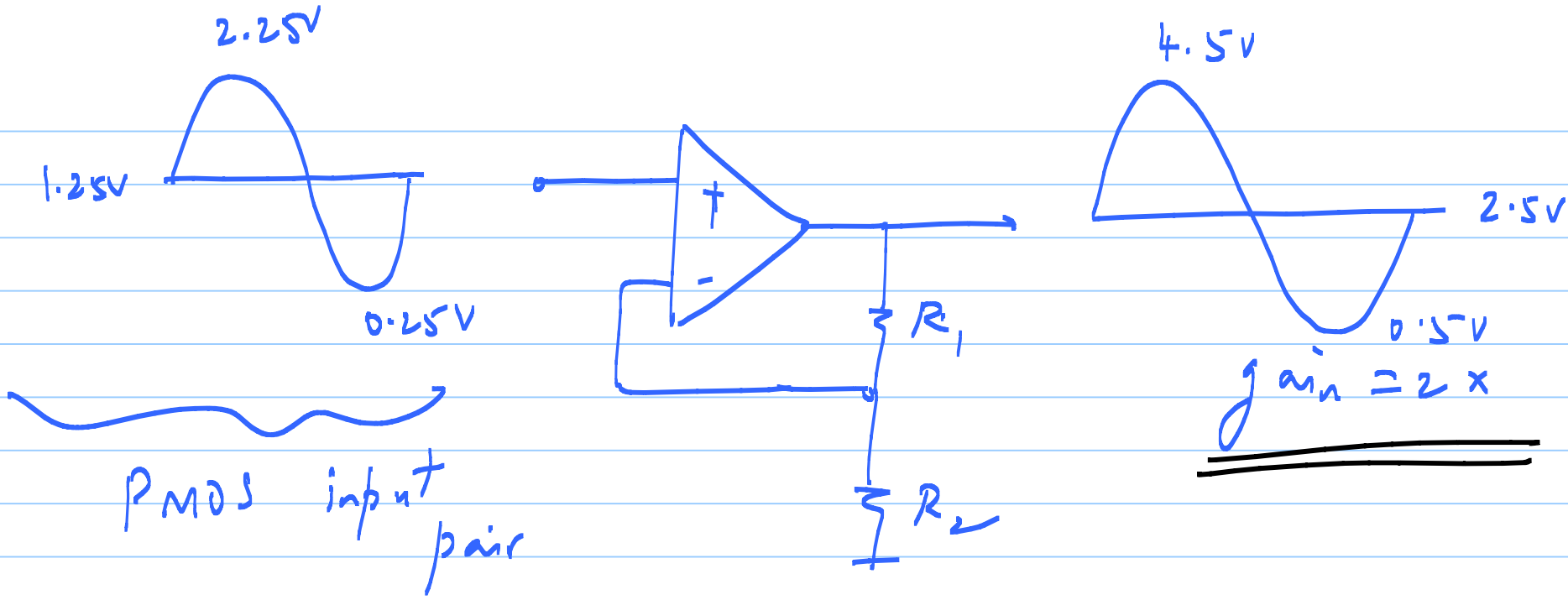
$r_{out} \leftarrow r_{ds} \leftarrow L, I_D$

L_1, L_2, L_3, L_4 (1, 2, 3)

W_3, W_4, W_5, L_5

W_6, L_6 (4, 5)





$$V_{opp} = 5V \Rightarrow V_{Dsat_5} = V_{Dsat_6} = 0.5V$$

M_5 :

$$\left.\begin{aligned} g_{m5} &= 543.8\,\mu S \\ V_{Dsat_5} &= 0.5V \end{aligned}\right\} \quad I_5 = 135\,\mu A \qquad \left(\frac{W}{L}\right)_5 = 43.2$$

7)

$$I_6 = I_5$$

$$V_{Dsat_6} = V_{Dsat_5} \qquad \left(\frac{W}{L}\right)_6 = \frac{1}{2}\left(\frac{W}{L}\right)_5 = 21.6$$

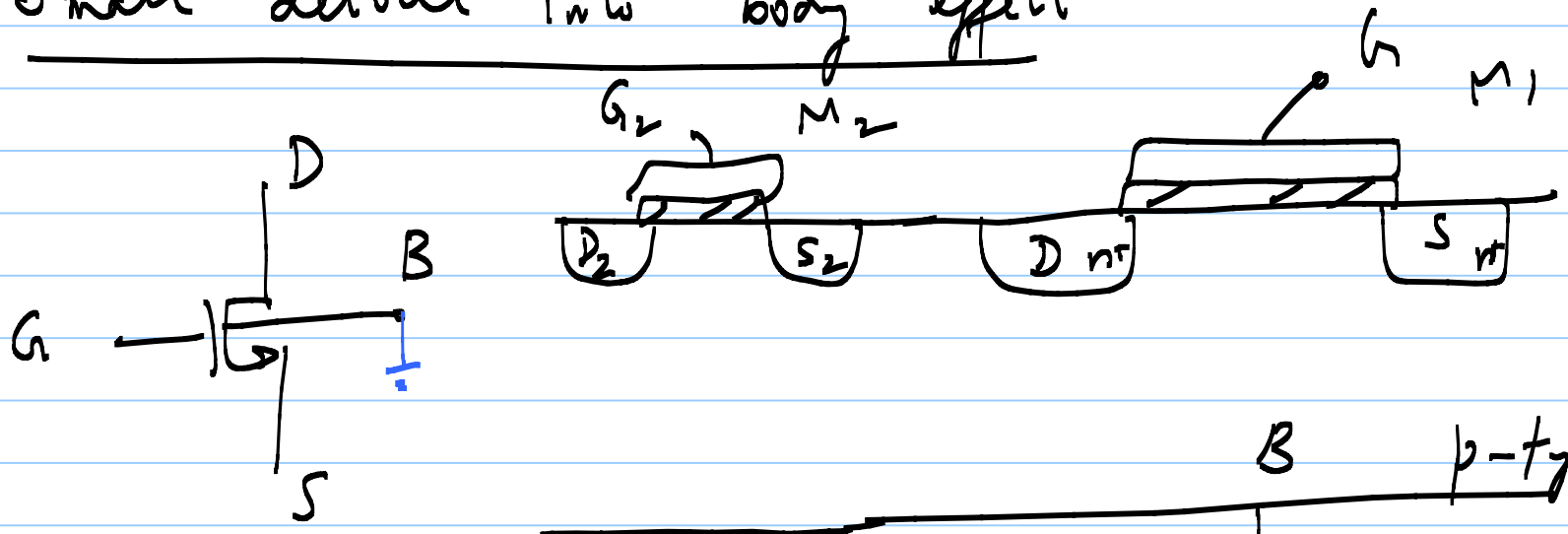
8) $$\left(\frac{W}{L}\right)_0 = \frac{2I_0}{I_6} \cdot \left(\frac{W}{L}\right)_6 = \frac{100\mu A}{135\mu A} \times 21.6 = 16$$

9) $$\frac{I_0}{(W/L)_3} = \frac{I_5}{(W/L)_5}$$

$$\left(\frac{W}{L}\right)_{3,4} = \frac{I_0}{I_5} \cdot \left(\frac{W}{L}\right)_5 = \frac{50\mu A}{135\mu A} \cdot 43.2$$

$$= 16$$

Small detour into body effect

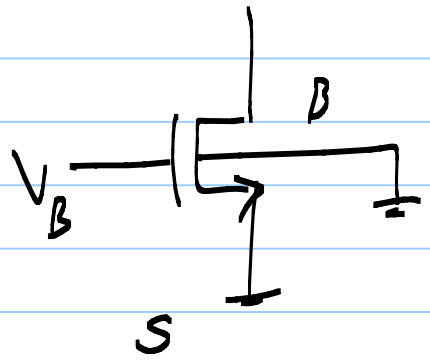


$$V_T = V_{T_0} + \gamma\left[\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F}\right]$$

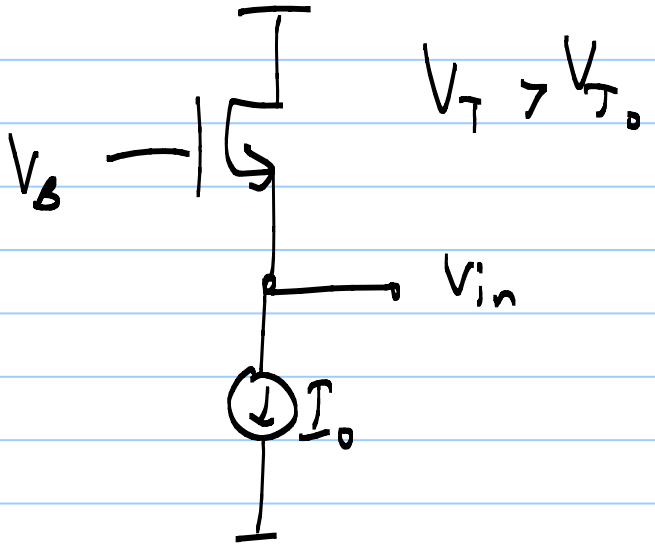
If $V_{SB} > 0 \Rightarrow V_T > V_{T_0}$

(i.e. $V_S > V_B$)

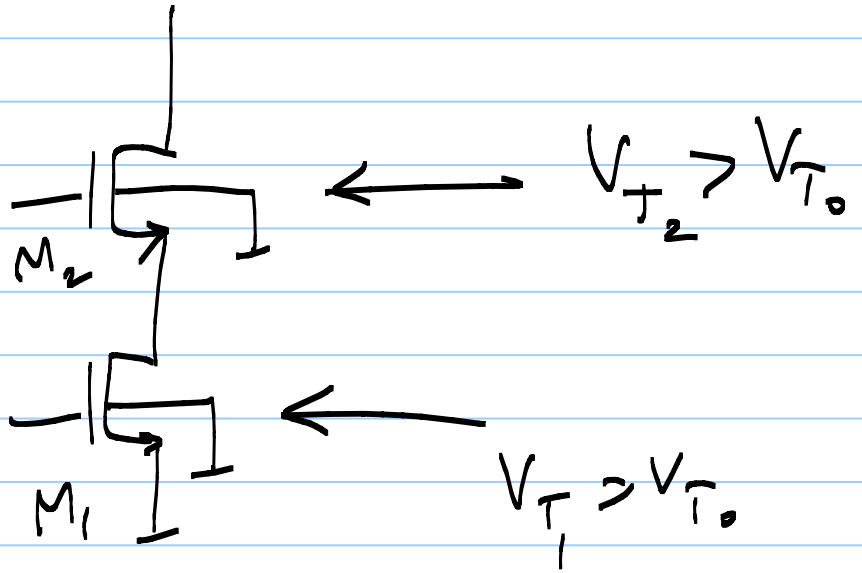
If $V_{SB} = 0 \Rightarrow V_T = V_{T_0}$



$V_T > V_{T_0}$

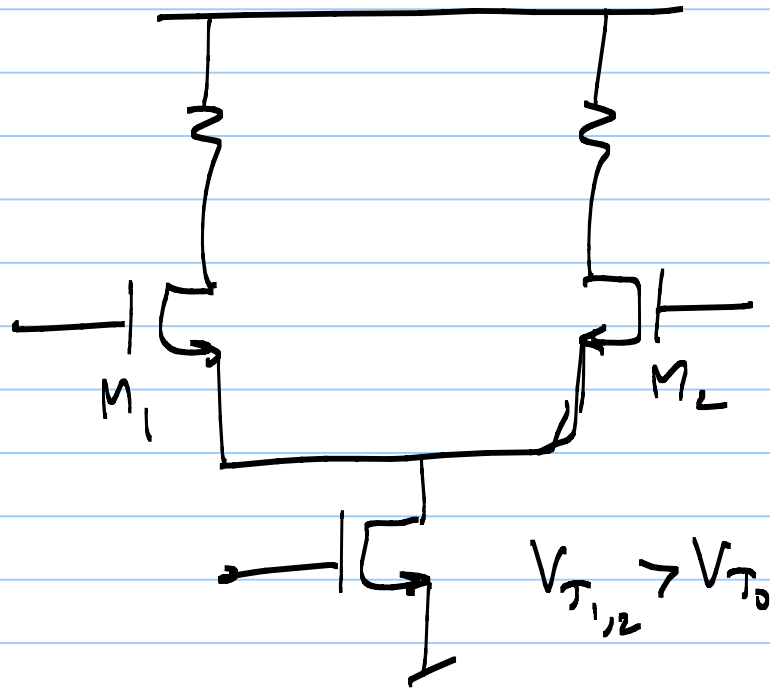


$V_T = V_{T_0}$

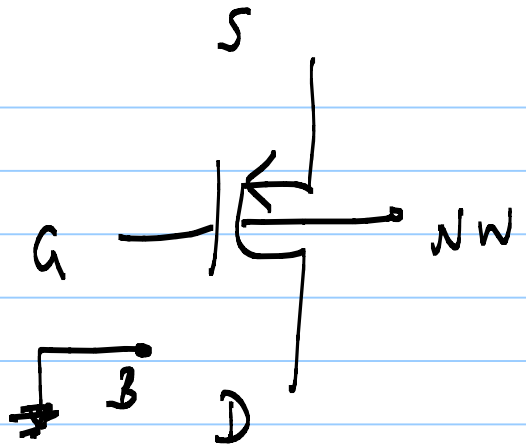


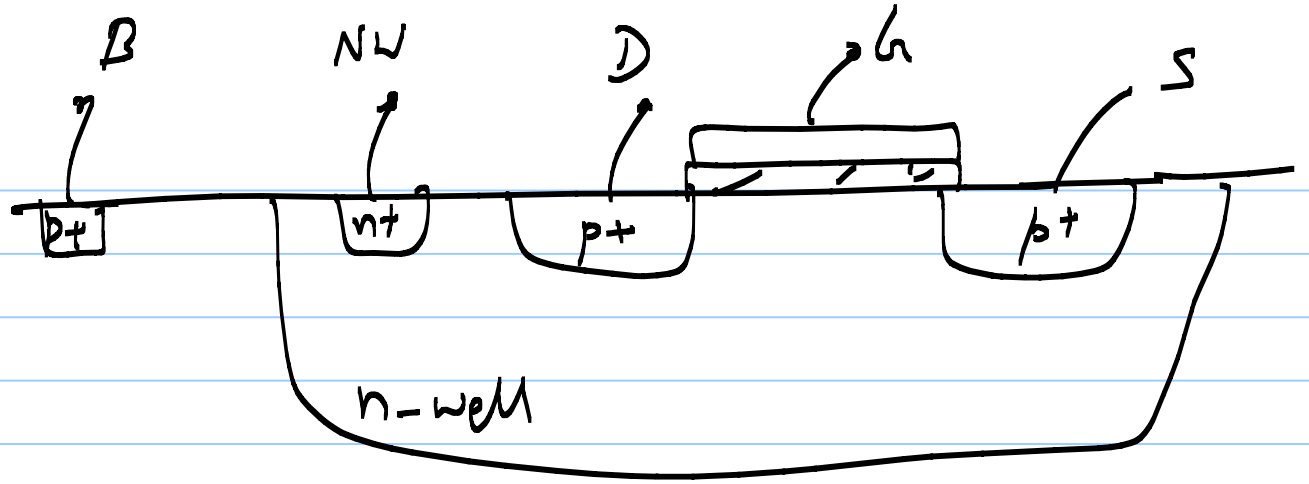
$V_{T_2} > V_{T_0}$

$V_{T_1} > V_{T_0}$

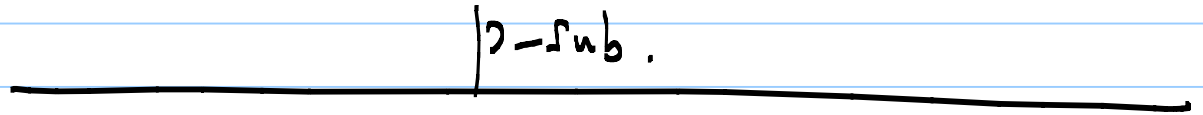


$V_{T_{1,2}} > V_{T_0}$

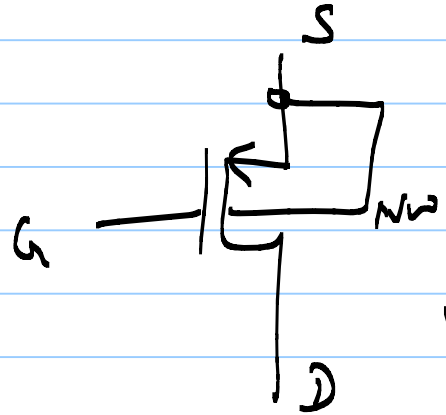




* each pmos can have its own individual nwell.

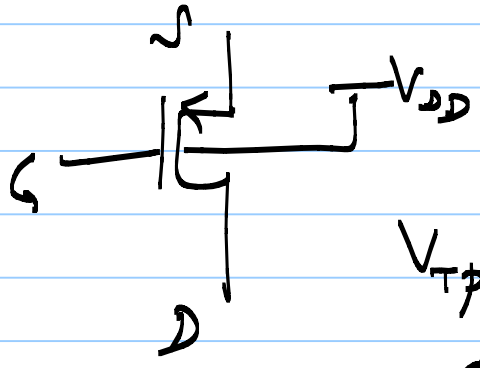


* Nwell can be shorted to source or tied to V_{DD}

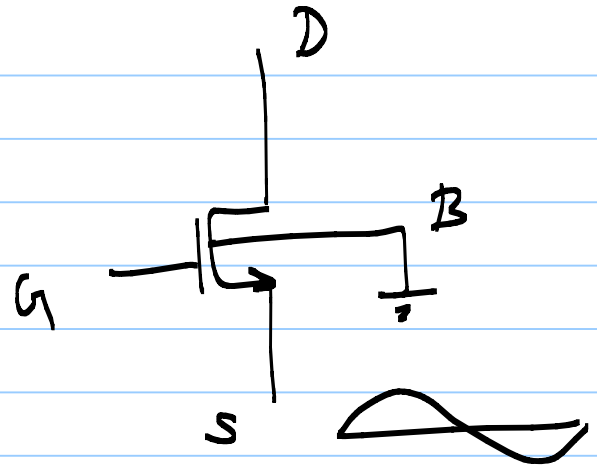


or

$V_{Tp} = V_{Tp_0}$



V_{Tp} can be different from V_{Tp_0}.



$$I_D = \frac{1}{2}\underbrace{\mu_n C_{ox}\left(\frac{W}{L}\right)}_{\beta}(V_{GS}-V_T)^2(1+\lambda V_{DS})$$

g_m g_{mbs} r_{ds}

$$\frac{\partial I_D}{\partial V_{SB}} = \frac{1}{2}\beta \times 2(V_{GS}-V_T)(-1)\times\frac{\partial V_T}{\partial V_{SB}}$$

(assume λ is small)

$$= \underbrace{-\beta(V_{GS}-V_T)}_{-g_m}\frac{\partial V_T}{\partial V_{SB}} = -g_m\frac{\partial V_T}{\partial V_{SB}}$$

$$\frac{\partial I_D}{\partial V_{SB}} = \frac{\partial I_D}{\partial V_T}\cdot\frac{\partial V_T}{\partial V_{SB}} - \frac{\partial I_D}{\partial V_{GS}}$$

$$V_T = V_{T_0} + \gamma\left[\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F}\right]$$

$$\frac{\partial V_T}{\partial V_{SB}} = \frac{\gamma}{2} \cdot \frac{1}{\sqrt{2\phi_F + V_{SB}}}$$

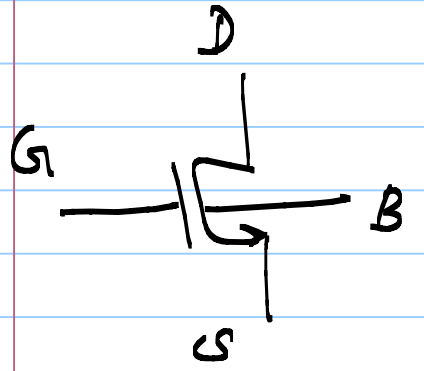
V_{SB} @ op. pt.

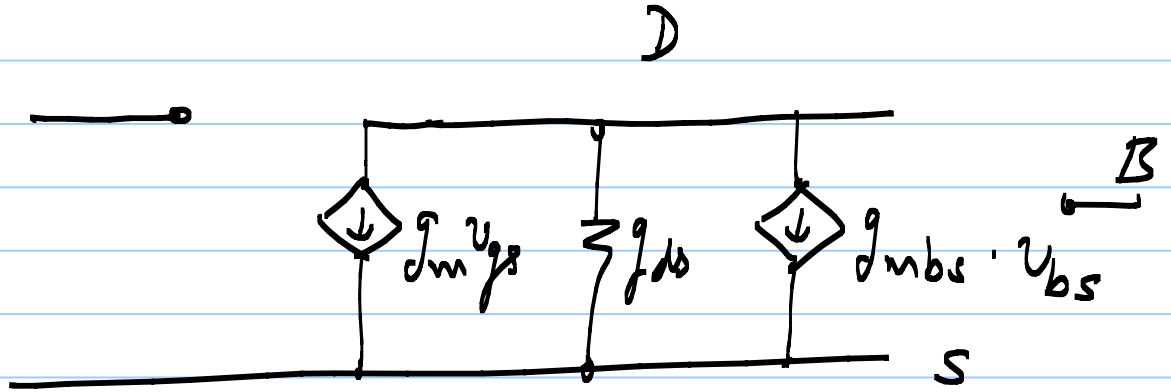
$$\frac{\partial I_D}{\partial V_{SB}} = \frac{-g_m \gamma}{2} \cdot \frac{1}{\sqrt{2\phi_F + V_{SB}}} = -\alpha g_m$$

$g_m \rightarrow g_{mbs} \rightarrow g_{ds}$

$$g_{mbs} = \frac{\partial I_D}{\partial V_{BS}} = \alpha g_m$$

normally $\alpha << 1$





Back to opamp design example

10) $L_0 - L_6$

$L_0 = L_6$; $L_1 = L_2$; $L_3 = L_4 = L_5$

$A_0 = 80\,dB = 10^4$

$A_1 \cdot A_2 = 10^4$

$g_{m_1} r_{out_1} \cdot g_{m_5} r_{out_2} = 10^4$

(I) More gain to first stage (if you have R_L)

(II) $g_{m5} = 10 \times g_{m1}$ so set $r_{out_1} = 10 \times r_{out_2}$

may not be practical

$$r_{out_1} = r_{ds_2} \parallel r_{ds_4} \quad \text{— PMOS}$$

$$r_{out_2} = r_{ds_5} \parallel r_{ds_6} \quad \text{— NMOS}$$

For $L = L_{min}$; $\lambda_n = 0.02\,/V$; $\lambda_p = 0.05/V$

Set all $L = L_{min} = 2\mu m$

$$r_{ds_{1,2}} = \frac{1}{\lambda_n I_{D_1}} = 1\,M\Omega$$

$$r_{ds_{3,4}} = \frac{1}{\lambda_p I_{D_3}} = 0.4\,M\Omega$$

$$r_{ds_5} = \frac{1}{\lambda_p I_{D_5}} = 148\,k\Omega$$

$$r_{ds_6} = \frac{1}{\lambda_n I_{D_5}} = 370\,k\Omega$$

$$A_1 = g_{m1}\left(r_{ds2} \| r_{ds4}\right) = 15.5 \ \ V/V$$

$$A_2 = g_{m2}\left(r_{ds5} \| r_{ds6}\right) = 57.5 \ \ V/V$$

$$A_0 = A_1 A_2 = 891 \ \ V/V$$

* Make PMOS devices 2.5x longer (λ_p/λ_n)

so that $r_{up} = r_{dn}$

i.e. $L_3 = L_4 = L_5 = 5\mu m$

$r_{ds5} = 370k\Omega$

$r_{ds34} = 1M\Omega$

$r_{out_1} = 0.5\,M\Omega$

$r_{out_2} = 185\,k\Omega$

$A_1 = g_{m_1}(0.5\,M\Omega) = 27.2\ V/V$

$A_2 = g_{m_2}(185\,k\Omega) = 100.6\ V/V$

$A_0 = 2700\ V/V$

* You can choose to Make $A_1 = 100$

$\Rightarrow r_{out_1} \simeq 4x \simeq 2\,M\Omega$

$\Rightarrow r_{ds_2} = 4\,M\Omega \Rightarrow L_{1,2} = 8\,\mu m$

$r_{ds_4} = 4\,M\Omega \Rightarrow L_{3,4} = 20\,\mu m$

But: $L_5 = 20\,\mu m \Rightarrow r_{ds5} = 1.48\,M\Omega$

$r_{ds4} = 370\,k\Omega$

r_{out2} is slightly larger

A_2 slightly larger than 100 V/V

HW: find new A_1, A_2 & A_0

$A_1 \simeq 104$ V/V

$A_2 \simeq 150$ V/V

11) Check $C_{gs5} << A_2 \cdot C$

12) Check pole & zero from $M_3, M_4 \Rightarrow \omega_n$

13) Recheck position of p_2 (ND pole)

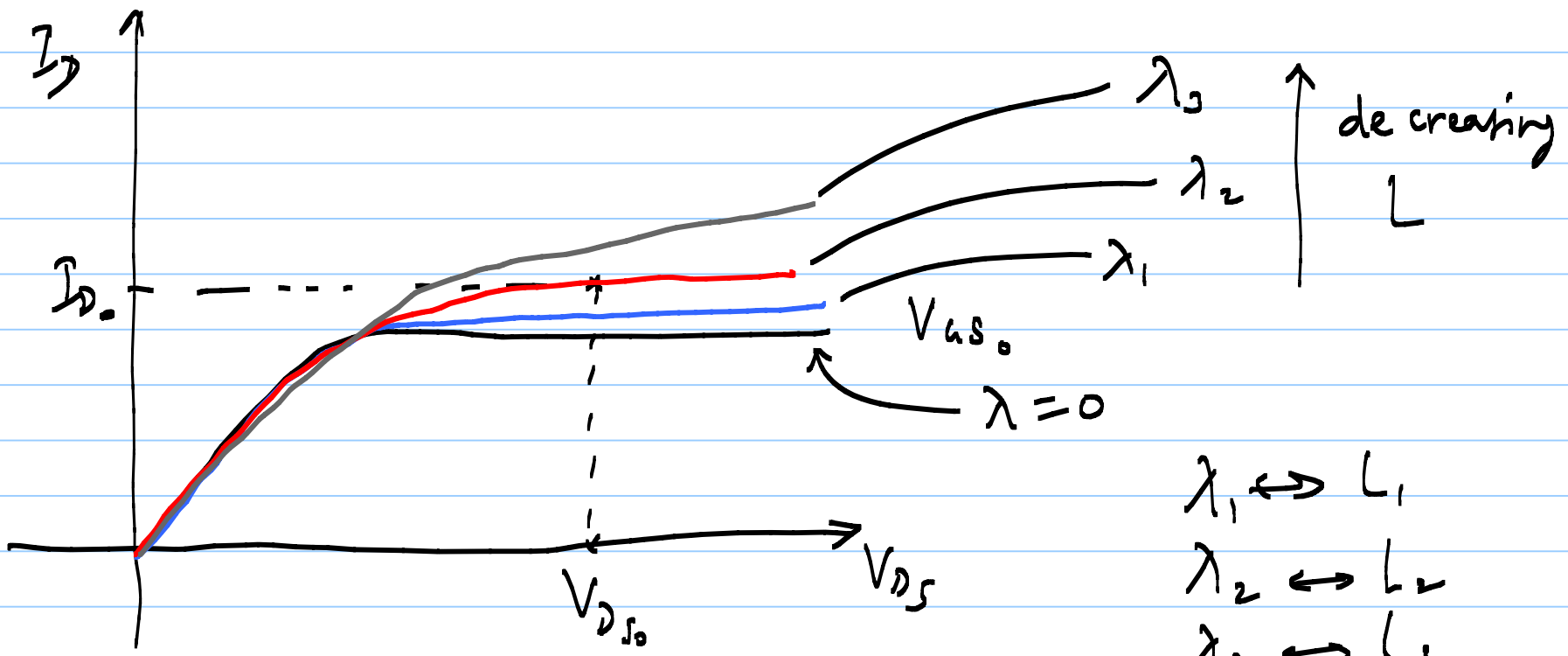
with C_{gs5}

14) Check if anything changes due to

body effect $V_T > V_{T0}$

W & L can be > Wmin & Lmin resp

@ steps of 0.01μm, 0.05μm etc.



$\lambda_1 \leftrightarrow L_1$

$\lambda_2 \leftrightarrow L_2$

$\lambda_3 \leftrightarrow L_3$

$$\left.\frac{\partial I_D}{\partial V_{DS}}\right|_{(I_{D_0}, V_{DS_0})} = \frac{1}{r_{ds}}$$

* $\mu_n, C_{ox}, \left(\frac{W}{L}\right), V_T$ constant
* $V_{DS_0}, V_{GS_0}, I_{D_0}$ kept constant
* Change only L

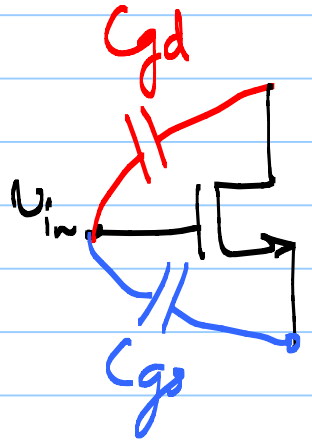
$$\frac{\Delta L}{L} = \lambda V_{DS} \qquad \{ \text{i.e. } \lambda L = \text{constant} \}$$

In any process, you cannot change

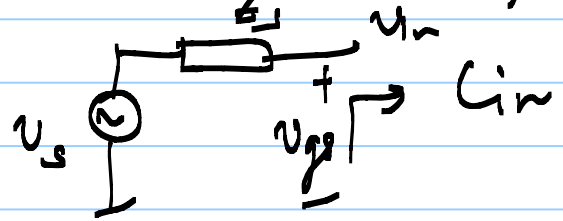
* L_{min}, W_{min}.
* μ, C_{ox}, V_T, ϕ_F

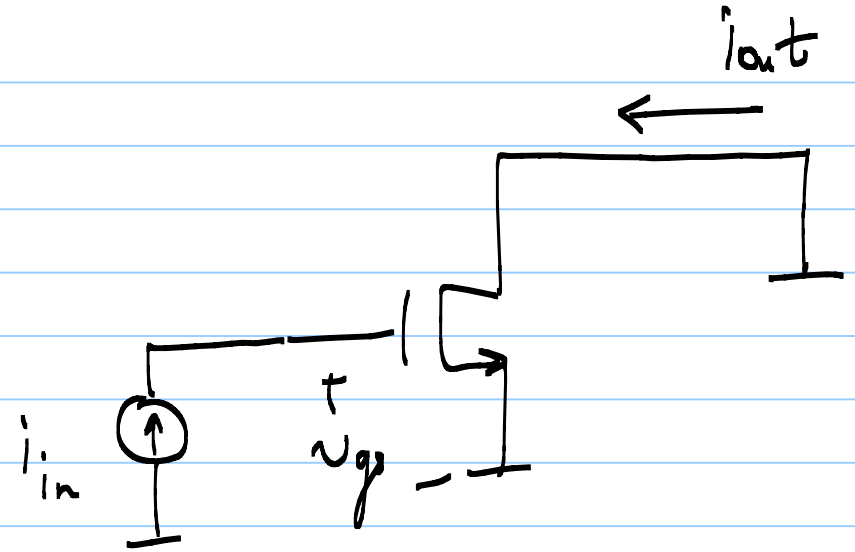
Transit frequency f_T, ω_T

$Z_{in} \neq \infty$ →



C_{gs} & C_{gd} effect freq. response





$$v_{gs} = i_{in} \cdot \frac{1}{j\omega(C_{gs} + C_{gd})}$$

$$i_{out} = g_m \cdot v_{gs}$$

$$= \frac{g_m}{j\omega(C_{gs}+C_{gd})} \cdot i_{in}$$

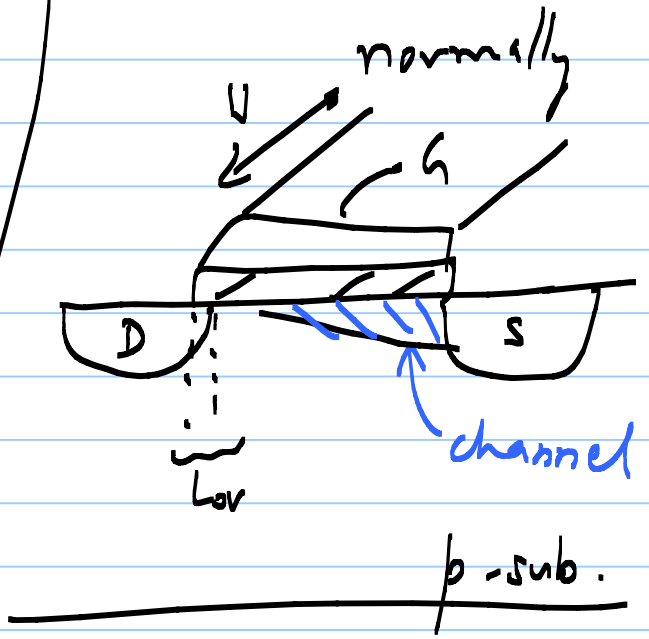
current gain $\left| \frac{i_{out}}{i_{in}} \right| = \frac{g_m}{\omega(C_{gs}+C_{gd})}$

find out ω at which $\left| \frac{i_{out}}{i_{in}} \right| = 1$

ω_T

$f_T = \omega_T / 2\pi$

$$\omega_T = \frac{g_m}{C_{gs} + C_{gd}} \simeq \frac{g_m}{C_{gs}} \quad \text{if } C_{gd} << C_{gs}$$



$$C_{gs} = \frac{2}{3} WL\, C_{ox}$$

$$C_{gd} = WL_{ov}\, C_{ox}$$

$$g_m = \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_T)$$

$$\omega_T \simeq \frac{\mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_T)}{\frac{2}{3} WL\, C_{ox}}$$

$$\boxed{\omega_T \simeq \frac{3}{2} \frac{\mu_n (V_{GS} - V_T)}{L^2}}$$

If you want high speed:

* Use NMOS devices $(\mu_n > \mu_p)$

* Use small L {L = L_{min} gives largest ω_T}

→ but smaller L ⟹ smaller r_{ds} ⇒ smaller gains

* Design for large $(V_{gs} - V_T)$

→ Smaller swings

Short-channel Effects

As technology scales, L_{min} ↓, Electric fields inside device ↑

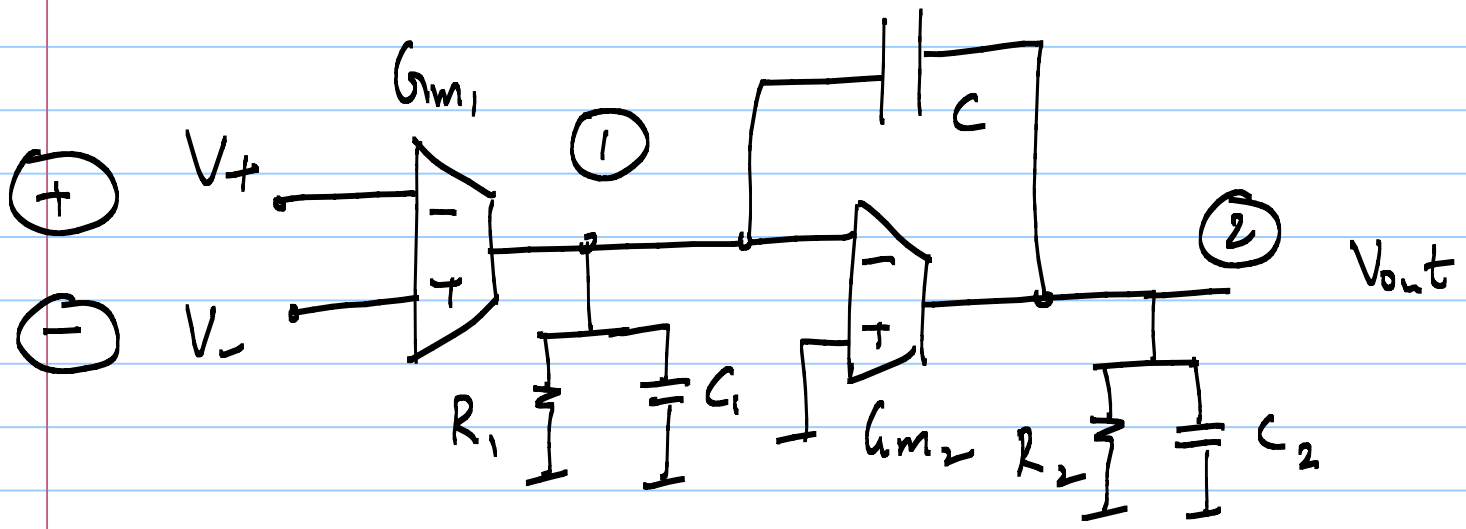
e.g. velocity = $\mu_n \mathcal{E}$ is not true anymore

"velocity saturation"

$$I_D = \frac{W C_{ox}}{2} (V_{GS} - V_T) \cdot V_{sat}$$

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \frac{W C_{ox}}{2} \cdot V_{sat}$$

$$\omega_T \simeq \frac{g_m}{C_{gs}} = \frac{\frac{W C_{ox}}{2} \cdot V_{sat}}{\frac{2}{3} W L C_{ox}} = \frac{3}{4} \frac{V_{sat}}{L}$$

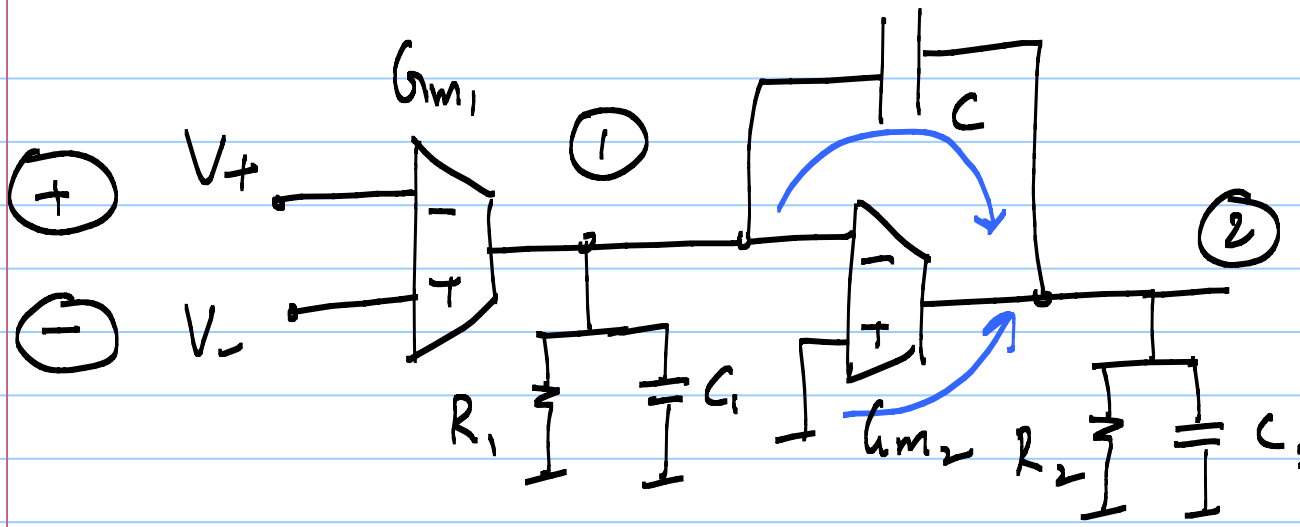


Miller compensated 2-stage opamp

$$p_1 \approx \frac{-1}{R_1 \cdot (G_{m_2} R_2 C)}$$

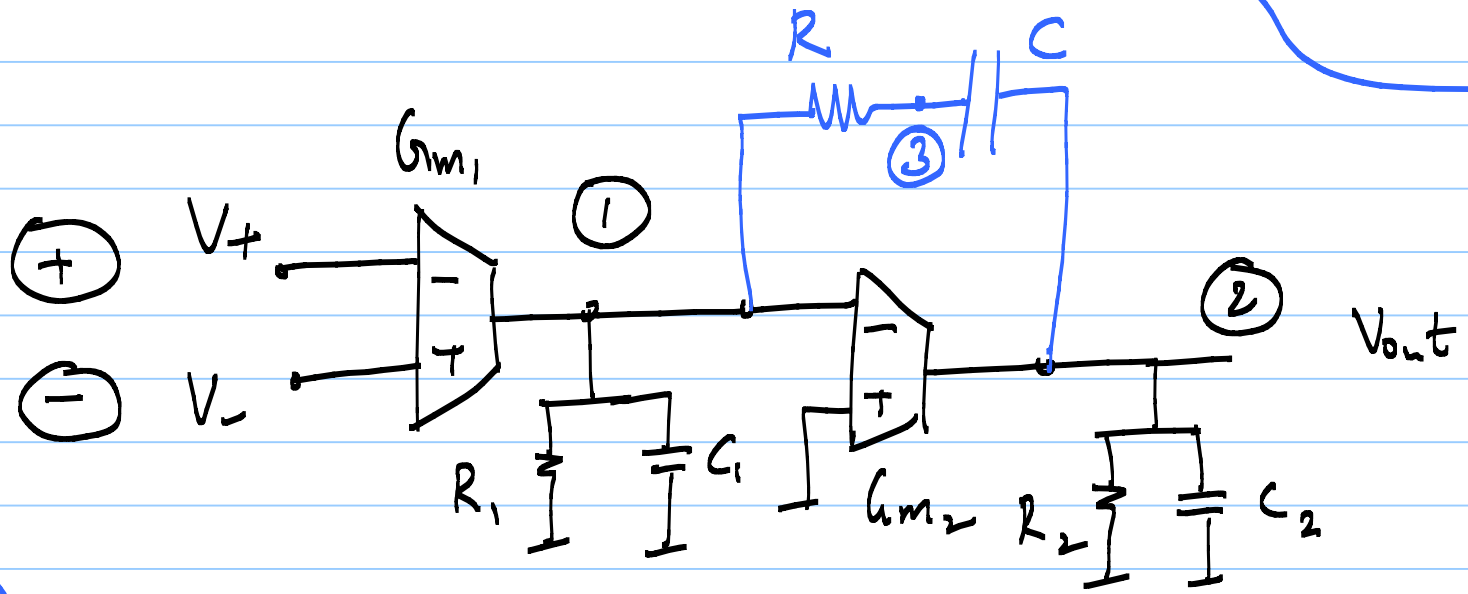
$$z = + \frac{G_{m_2}}{C}$$

$$p_2 \approx \frac{-1}{\left[R_2 \,||\, \left(\frac{C}{C+C_1}\right)\frac{1}{G_{m_2}}\right] \cdot \left[C_2 + \frac{CC_1}{C+C_1}\right]}$$



Move Zero:

change amplitude/phase of signal in one of these paths



HW

$p_1' =$, $p_2' =$, $p_3' = ?$, $z' = ?$

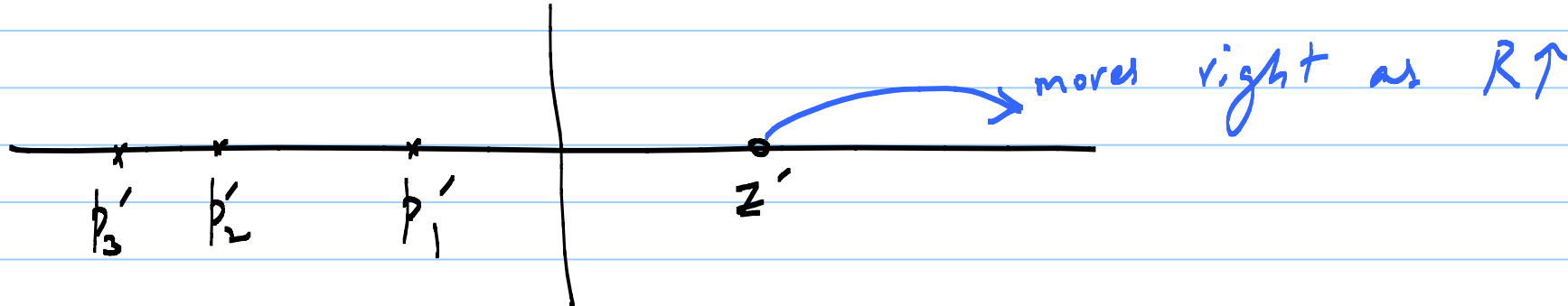
$$p_1' \approx \frac{-1}{R_1 (G m_2 R_2) C}$$

$$p_2' \approx \frac{-G m_2}{C_1 C_2 + C C_1 + C C_2}$$

$$p_3' \approx \frac{-1}{R C_1}$$

$$z' = \frac{+1}{C\left(\frac{1}{G m_2} - R\right)}$$

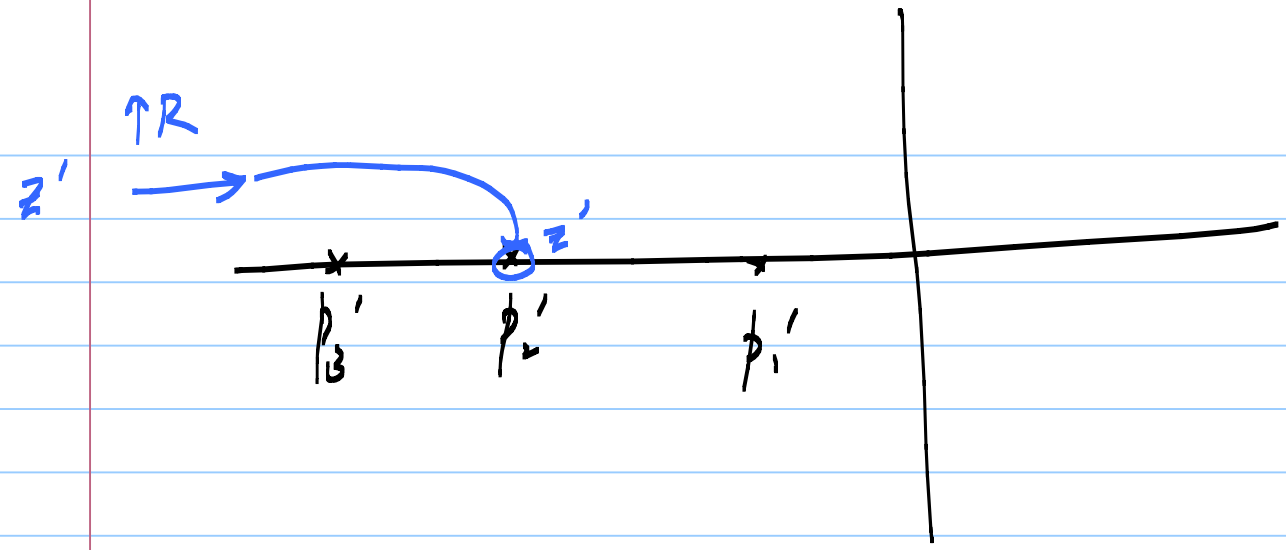
RHP pole only if $R < \frac{1}{G m_2}$



1) Set $R = \frac{1}{G_{m_2}} = \frac{1}{g_{m5}} \Rightarrow z' = \infty$

in this case $p_3' = \frac{-1}{RC_1} = -\frac{G_{m_2}}{C_1} = \frac{-g_{m5}}{C_1}$

2) $R > \frac{1}{G_{m_2}} \Rightarrow z' = \frac{-1}{C\left(R - \frac{1}{G_{m_2}}\right)} \rightarrow$ LHP zero



Set $z' = p_2'$

$$\frac{1}{C\left(\frac{1}{GM_2} - R\right)} = \frac{-GM_2}{\sum C_i C_j}$$

Can be used to determine R required.

New poles of system $= p_1'$ & p_3'

(N_0 p_2', N_0 z') → further apart ⇒ higher PM

If PM' is too high ⇒ move p_1' to a larger value (reduce Miller cap C)

⇒ wider closed loop BW {larger ω_n}

We want to set $R = f\left(\frac{1}{g_{m_5}}\right)$

* passive resistor & MOSFET transconductance are not related

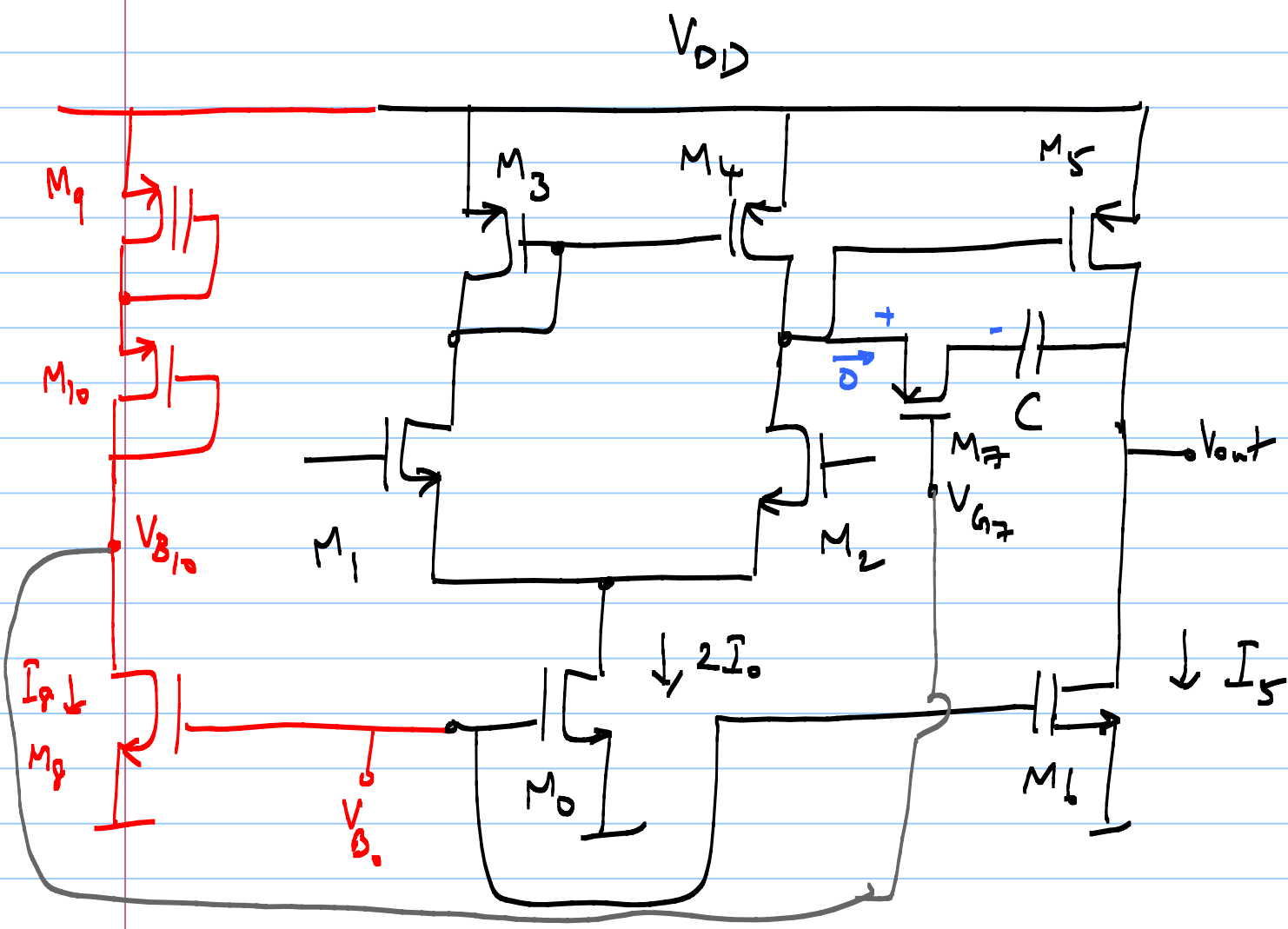
* over PVT variations

  g_m in slow, typical & fast corners of MOSFET

  R " " " " Resistor

* We want R to track g_m or vice versa

1) Use MOSFET in triode regim to generate R



M_7 is in triode

$$r_{ds_7} = \frac{1}{g_{ds_7}}$$

$$g_{ds_7} = \frac{\partial I_{DS_7}}{\partial V_{DS_7}}$$

$$I_{DS_7} = \mu_p C_{ox} \left(\frac{W}{L}\right)_7 \left((V_{SG_7} - V_T) \cdot V_{DS_7} - \frac{V_{DS_7}^2}{2} \right)$$

$$g_{ds_7} = \mu_p C_{ox} \left(\frac{W}{L}\right)_7 \left(V_{SG_7} - V_T - V_{DS_7} \right)$$

(V_{DS_7} → 0)

$$= \mu_p C_{ox} \left(\frac{W}{L}\right)_7 \left(V_{SG_7} - V_T \right)$$

* Choose $g_{ds_7} = \frac{1}{R}$ required value as nominal [choice of $\left(\frac{W}{L}\right)_7$]

* Generate V_{SG_7} so that PVT variations are compensated

* $V_{G_7} = V_{DD} - V_{SG_5} - V_{SG_7}$

* $V_{B10} = V_{DD} - V_{SG9} - V_{SG10}$

* Choose $V_{SG9} = V_{SG5}$ by design

$$\frac{I_5}{(W/L)_5} = \frac{I_8}{(W/L)_9}$$

* Choose $V_{SG7} = V_{SG10}$ by design

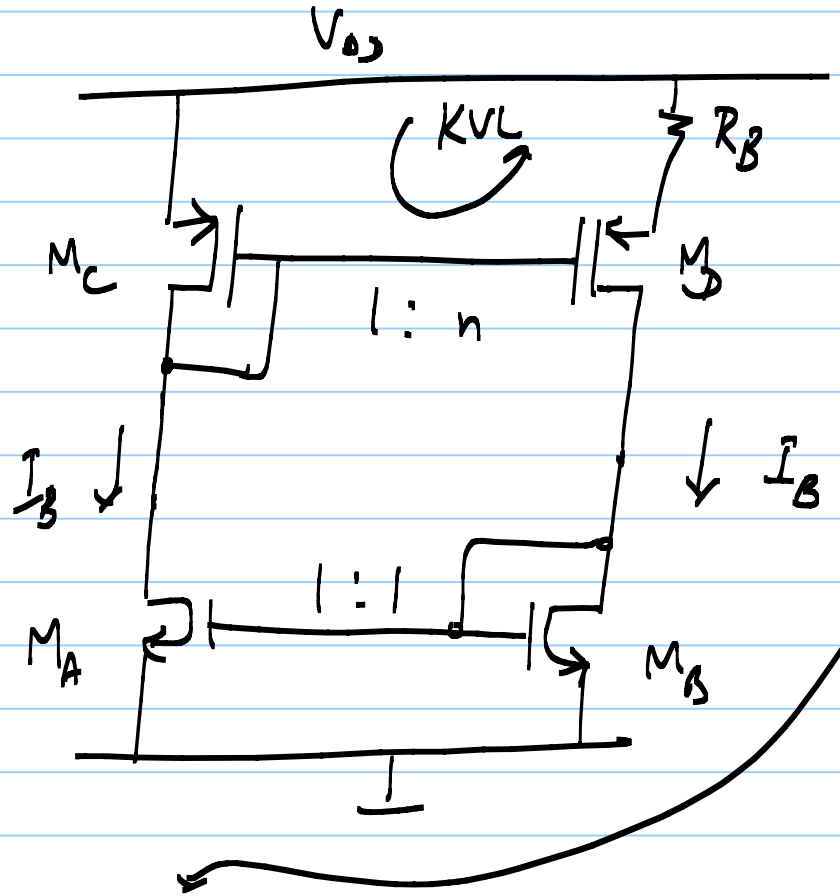
i.e. $I_7 \left(\frac{W}{L}\right)_7 = \left(\frac{W}{L}\right)_{10}$

$$g_{ds7} = g_{m10}$$

* V_{G7} adjusts itself over PVT so that g_{ds7} tracks g_{m10} & g_{m5}.

2) Case 2: generate $g_{m5} \propto \frac{1}{R}$

Bias generator:



$$V_{SG_C} = V_{SG_D} + I_B R_B$$

$$V_{T_C} + V_{ov_C} = V_{T_D} + V_{ov_D} + I_B R_B$$

also: $V_{ov_C} = \sqrt{n}\, V_{ov_D}$

$$V_{ov_C} = \frac{V_{ov_C}}{\sqrt{n}} + I_B R_B$$

$$g_m = \frac{2 I_{bias}}{V_{ov}}$$

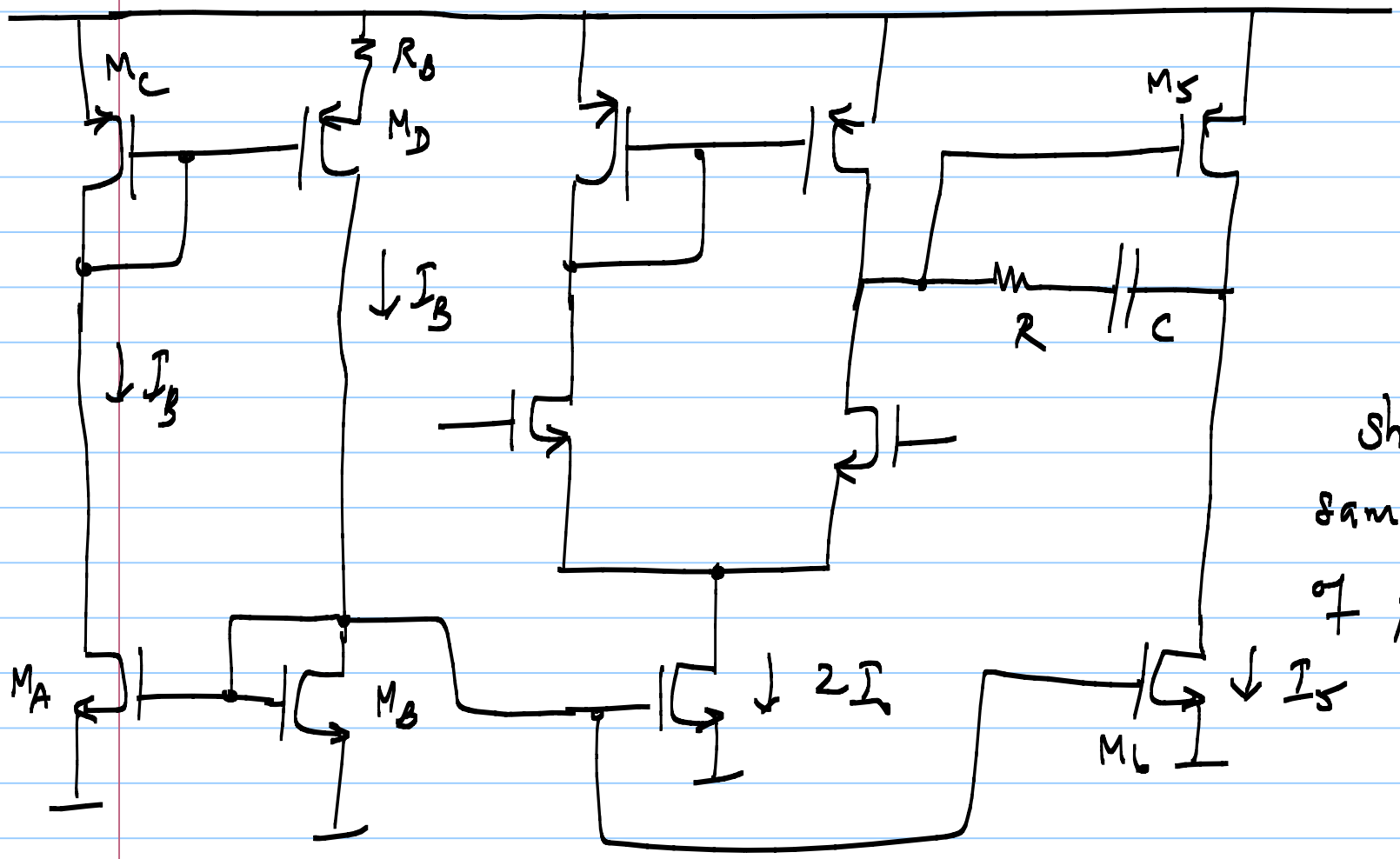
$$\frac{2I_B}{g_{m_c}} = \frac{1}{\sqrt{n}} \frac{2I_B}{g_{m_c}} + I_B R_B$$

If $I_B \neq 0$: { circuit might require startup portion }

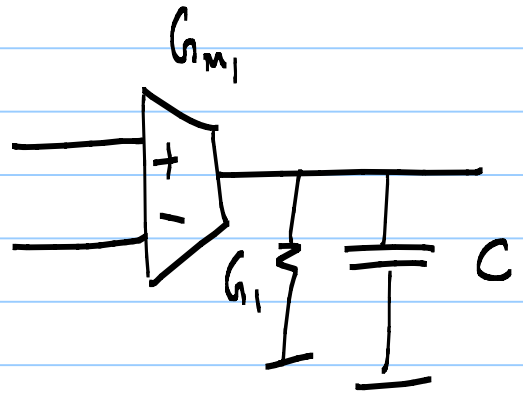
$$\frac{2}{g_{m_c}} = \frac{1}{\sqrt{n}} \frac{2}{g_{m_c}} + R_B$$

$$g_{m_c} = \frac{2(1 - 1/\sqrt{n})}{R_B}$$ ← mirror I_B into M_5 so that $g_{m_5} \propto \frac{1}{R_B}$

Note: $I_B \propto \frac{1}{R_B^2}$



R &
R_B
Should be
same type
of passive Res.



one-stage (differential pair w/ active load)

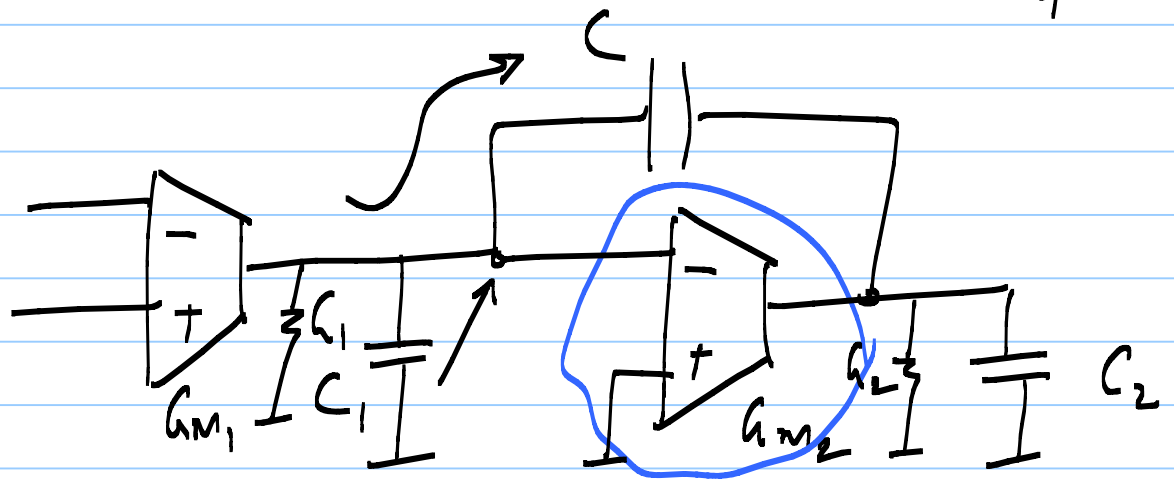
$$\omega_u = \frac{G_{m_1}}{C}$$

$$\text{DC gain} = G_{m_1}/G_1$$

Telescopic opamp

folded cascode

$G_{m_2} \to 7 G_{m_1}$



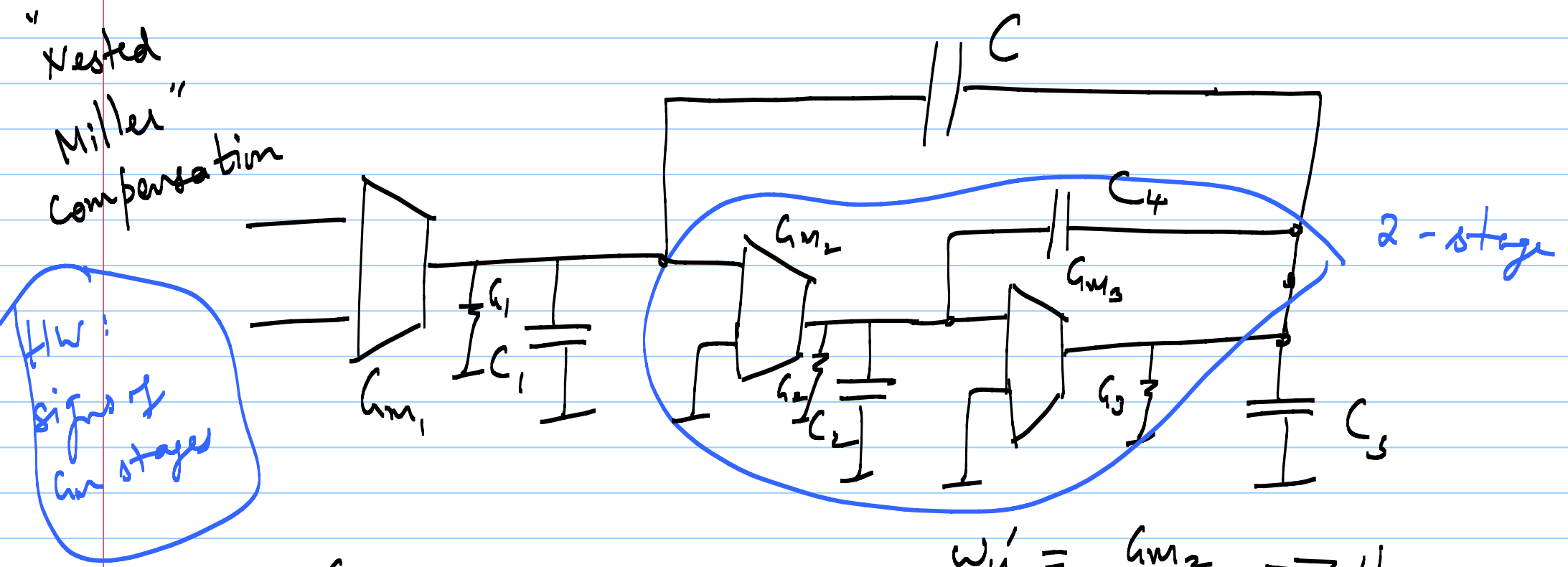
$$\omega_u = \frac{G_{m_1}}{C}$$

$$A_0 = \frac{G_{m_1} G_{m_2}}{G_1 G_2}$$

Even larger A — 3 stages

↳ replace G_{m_2} itself with a 2-stage opamp

"nested Miller" compensation

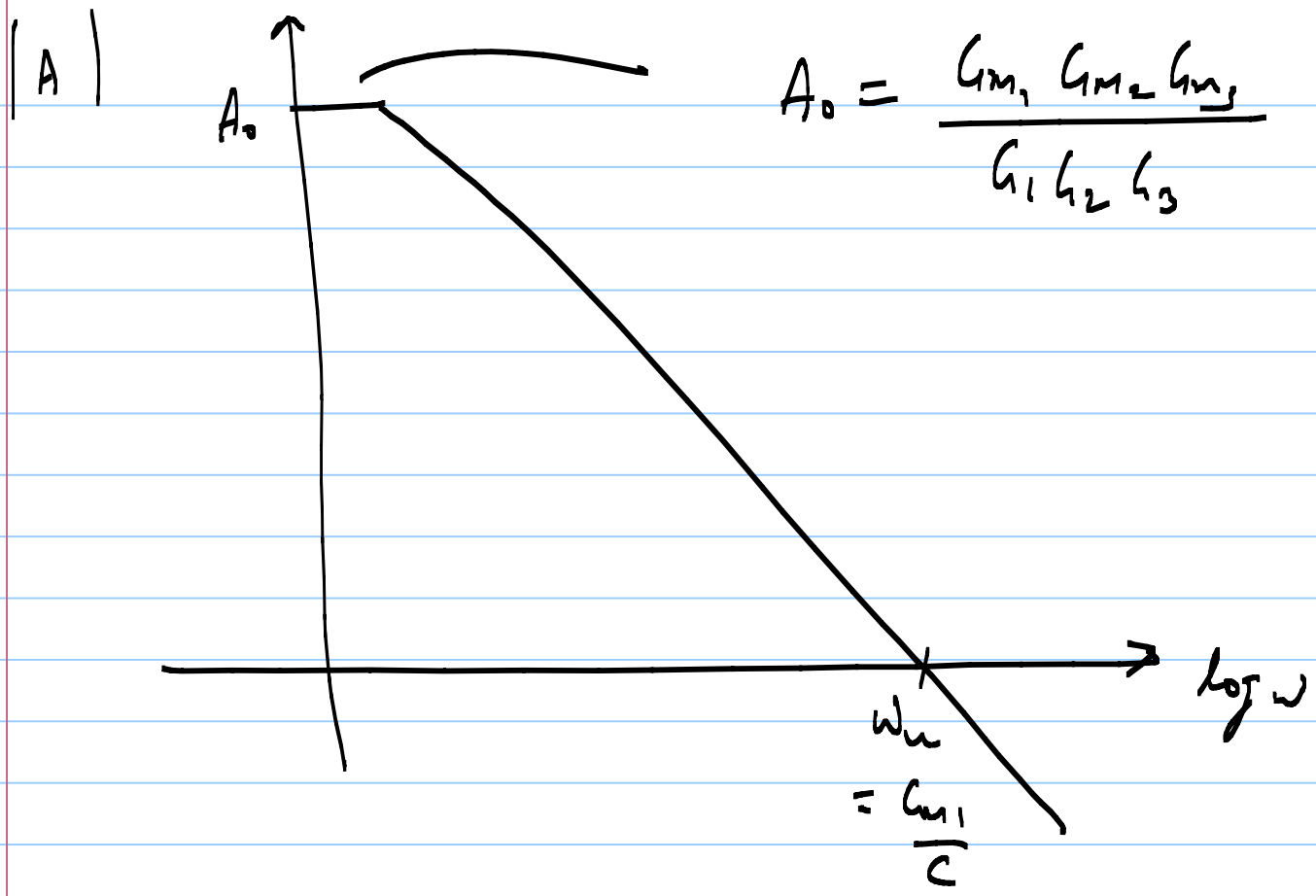


HW: signs of Gm stages

$$\omega_u = \frac{G_{m1}}{C}$$

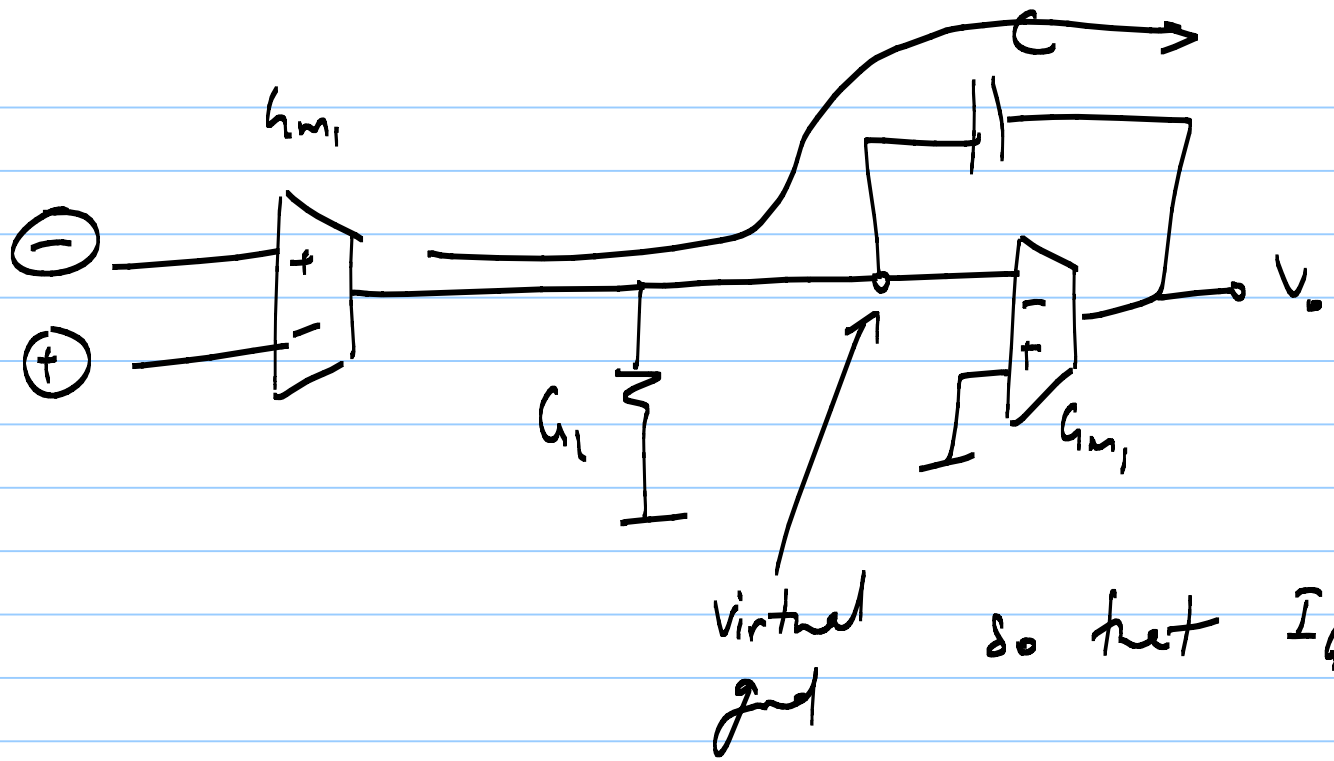
$$\omega_u' = \frac{G_{m2}}{C_4} \gg \omega_u$$

$\Rightarrow G_{m2}, G_{m3} \gg G_{m1}$

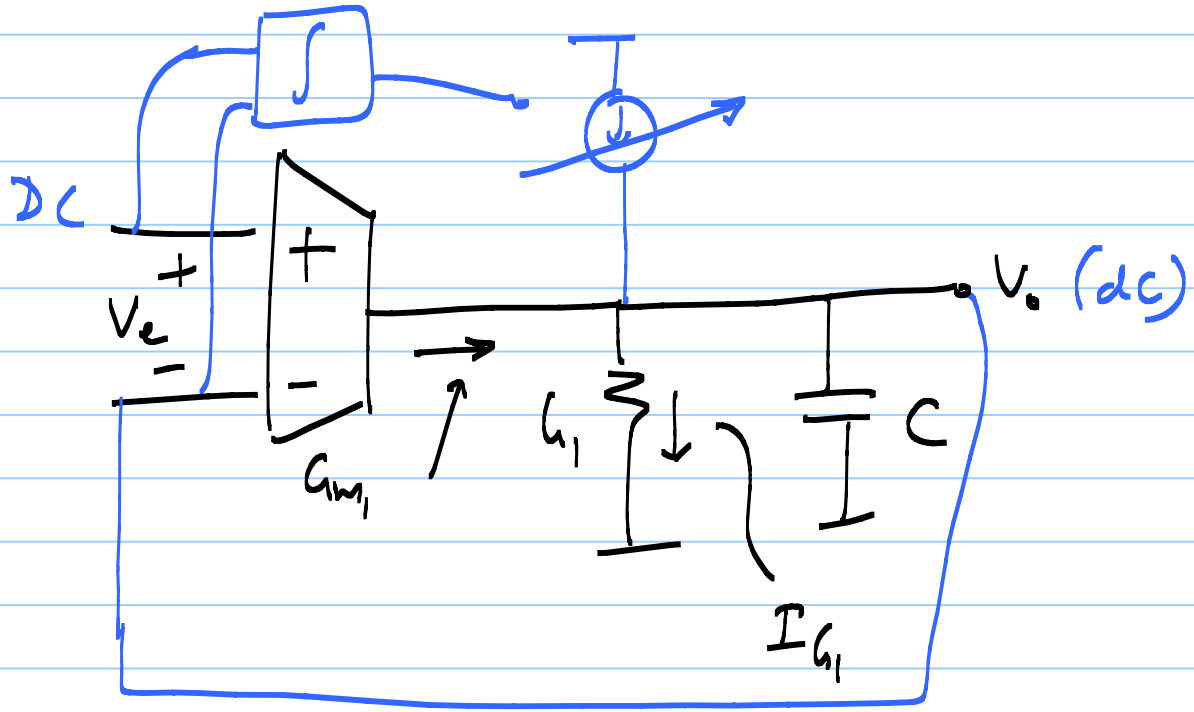


$$A_0 = \frac{G_{m1} G_{m2} G_{m3}}{G_1 G_2 G_3}$$

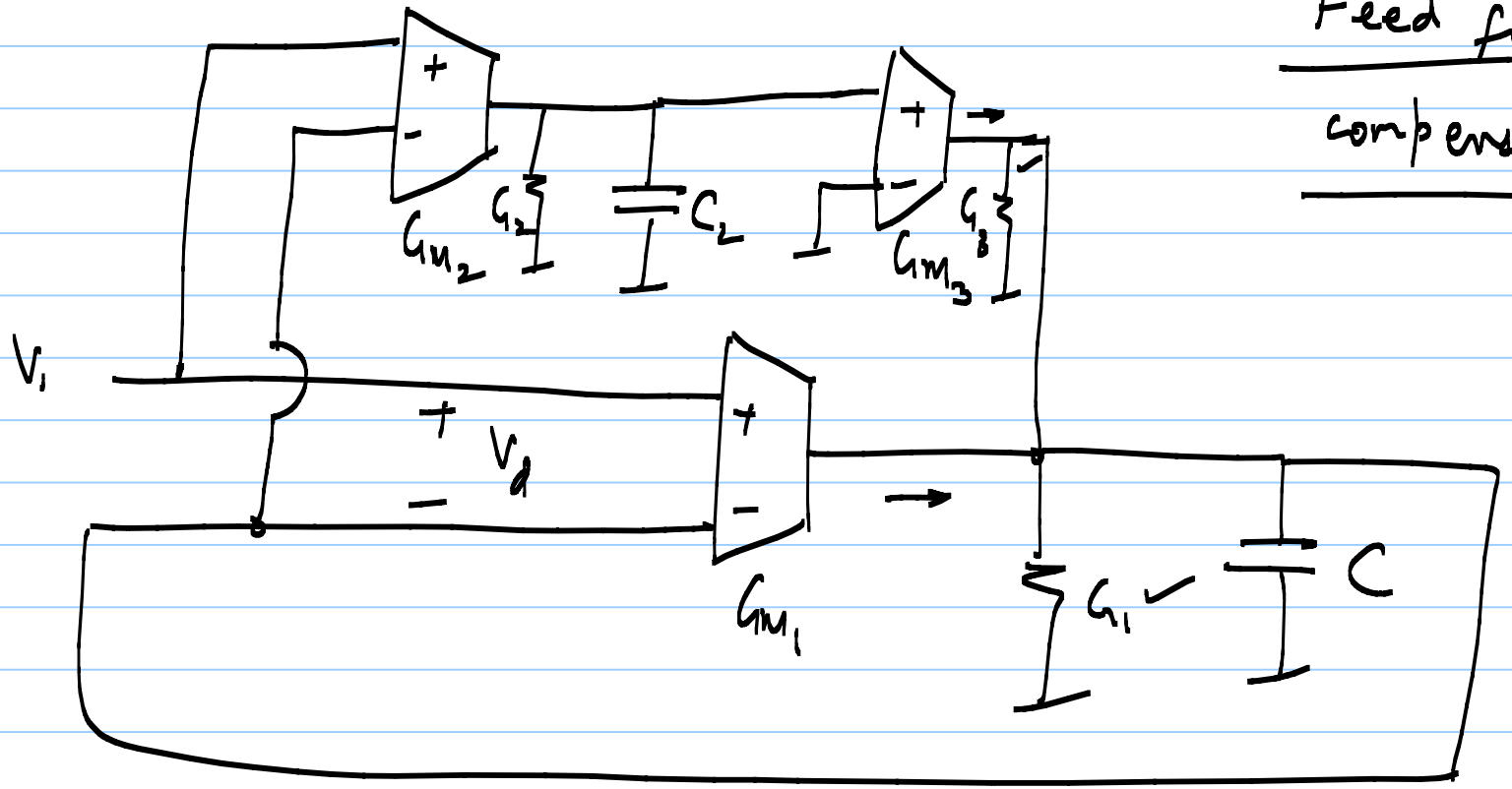
steady state error → DC gain → # stages with speed constraint



so that $I_{G_1} = 0$



"Feed forward compensated opamp"



[Will discuss stability later]

2 paths to output

$$V_o = \frac{G_{m_1} V_d + G_{m_3} \cdot \frac{G_{m_2}}{G_2} \cdot V_d}{G_1 + G_3}$$

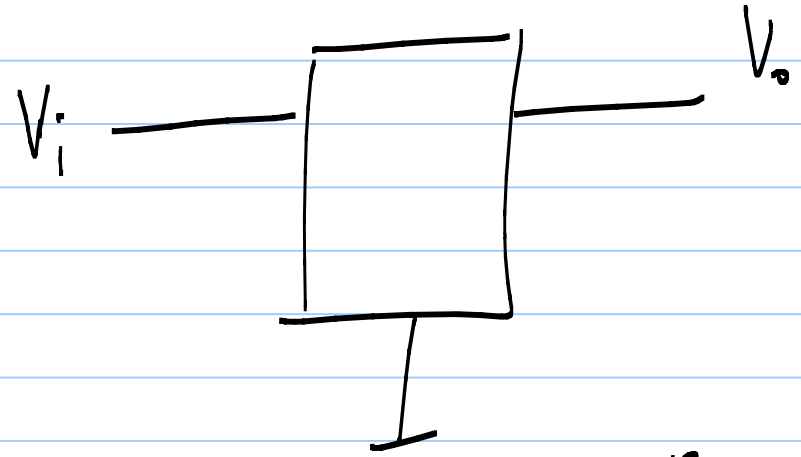
$$V_o/V_d = \frac{G_{m_1} + G_{m_3} \cdot G_{m_2}/G_2}{G_1 + G_3}$$

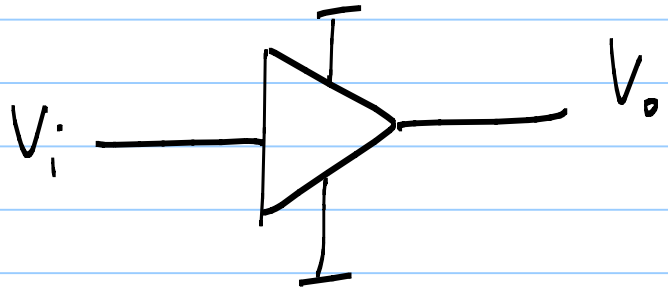
⇓

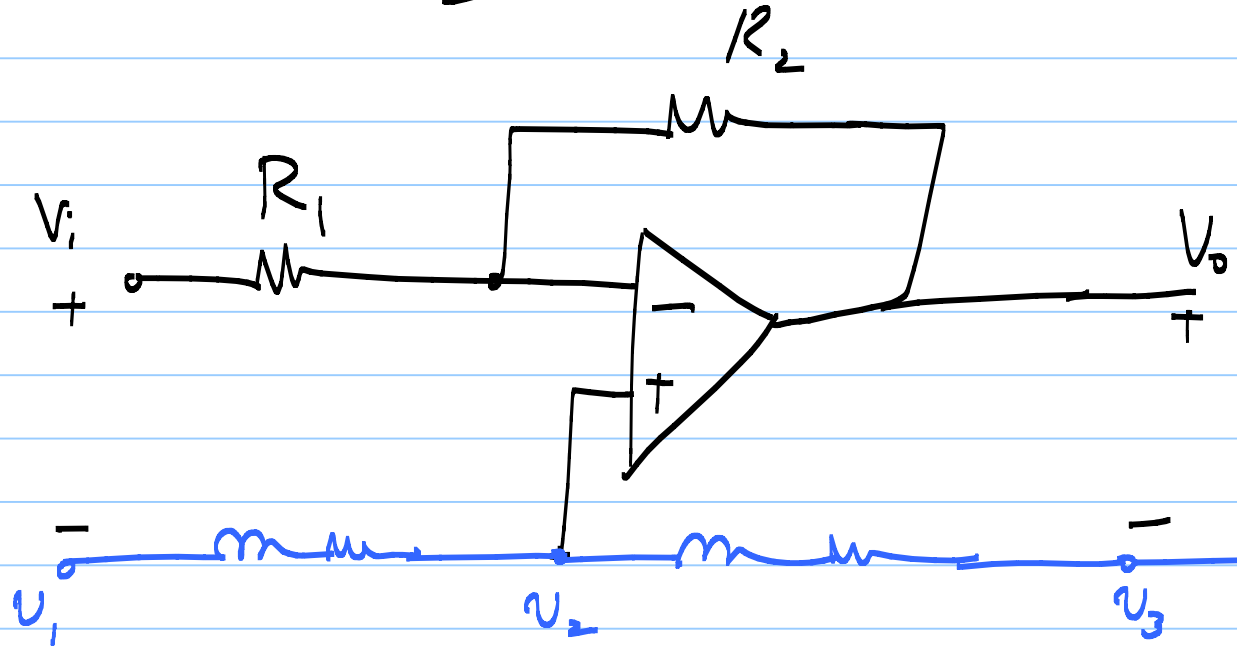
Zero

(Complicated step response)

Fully Differential opamps

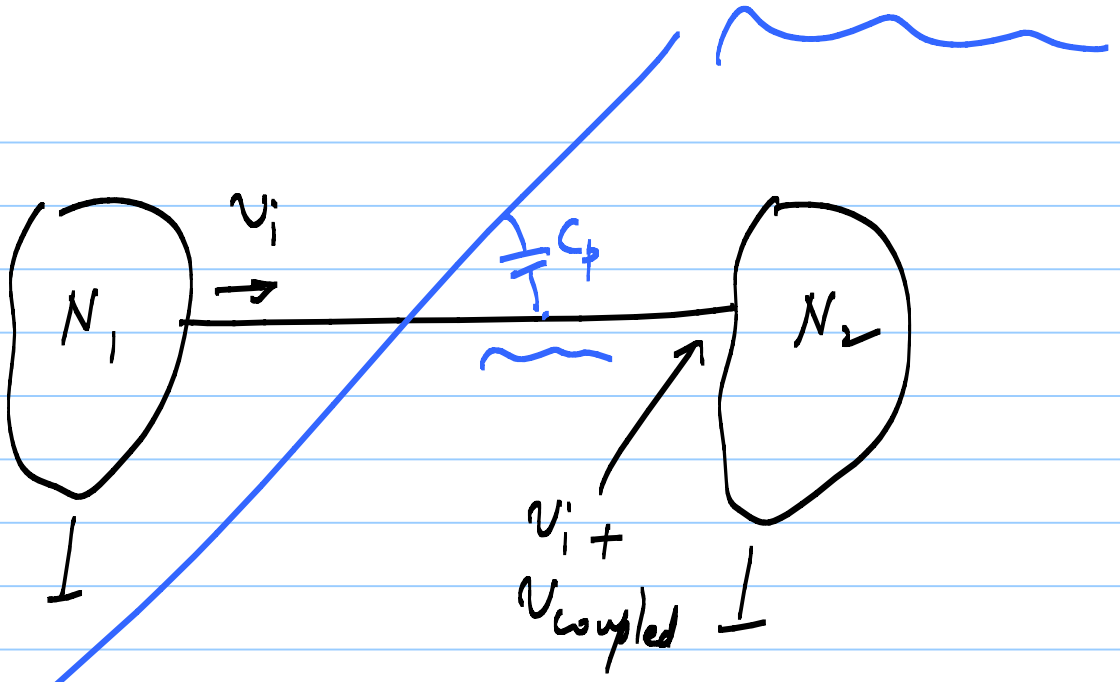


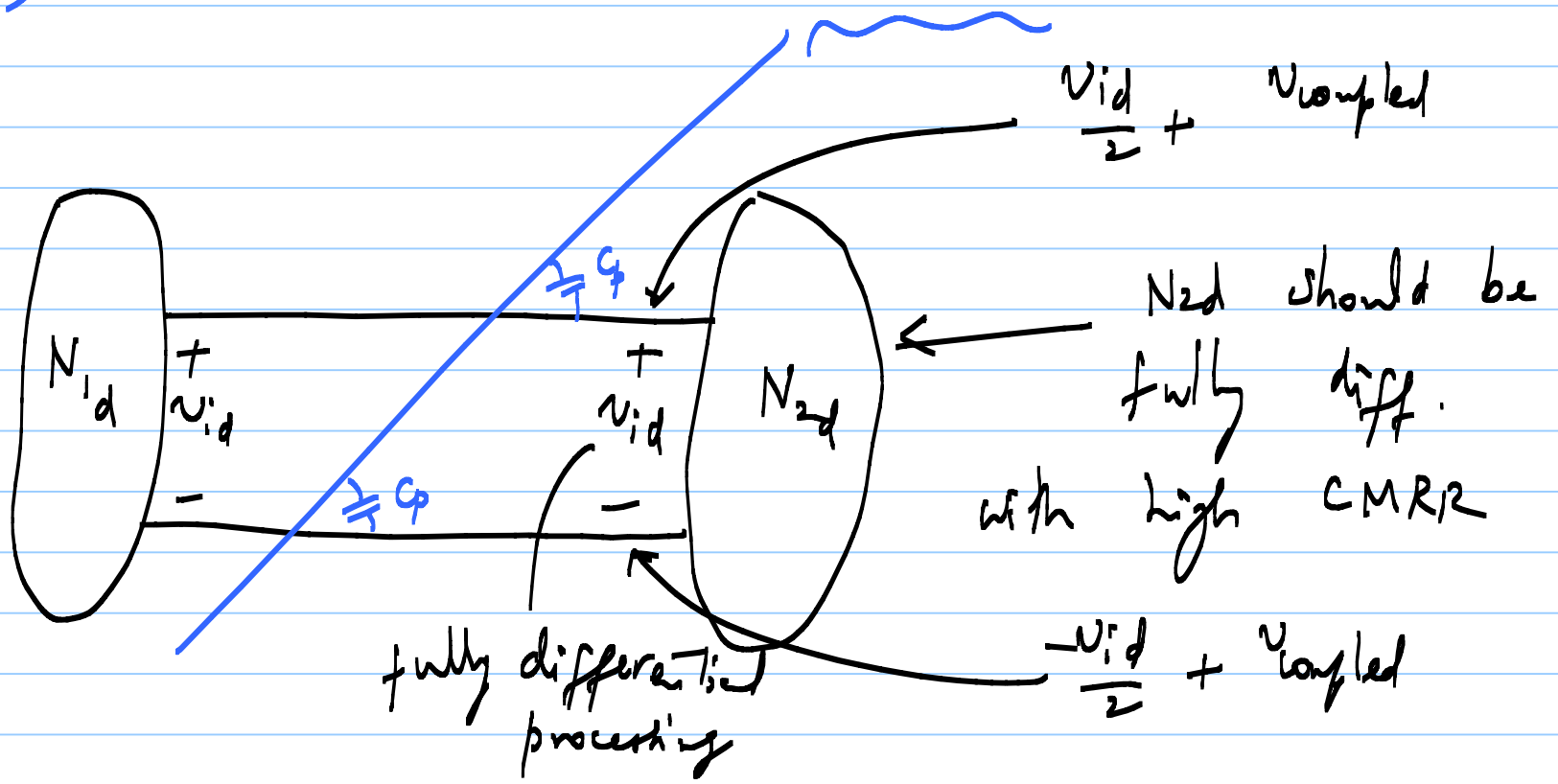


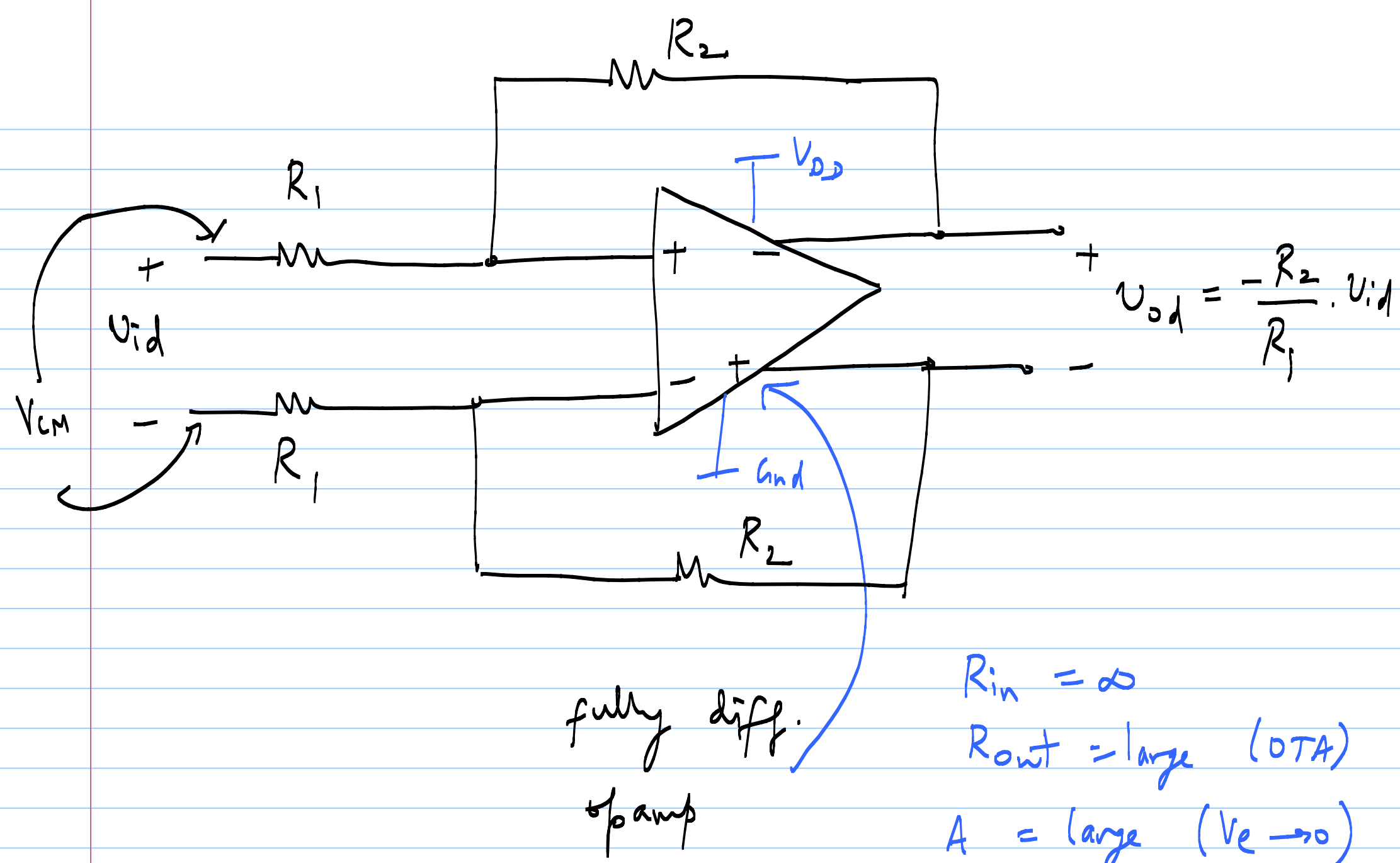


$$\frac{V_o}{V_i} = -\frac{R_2}{R_1} + (\dots)$$

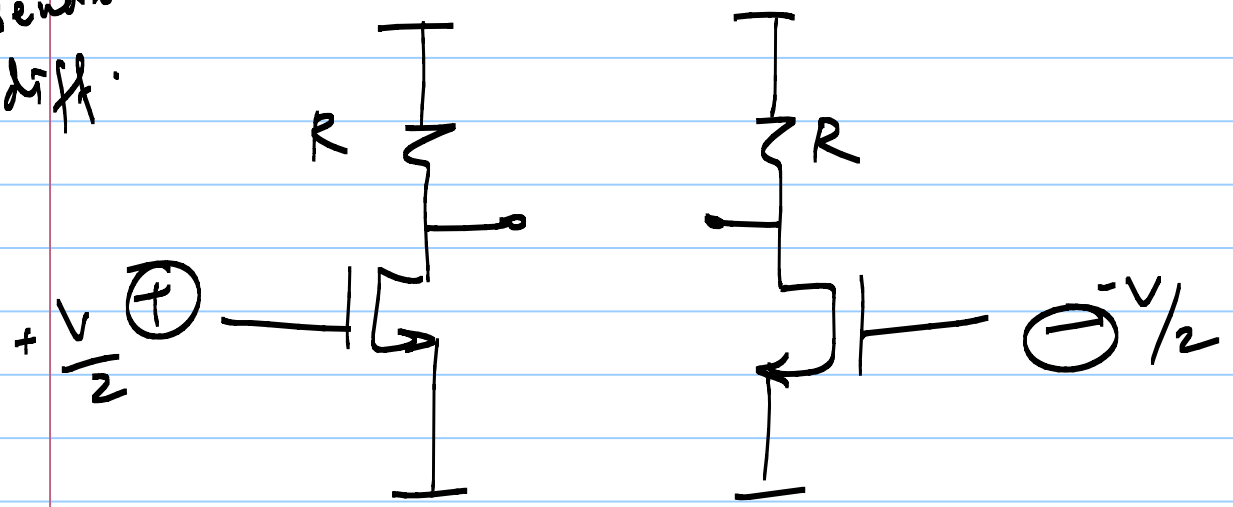
i_{grd} = can come from many circuits


v_i
C_p
N_1
N_2
v_i +
$v_{coupled}$


$\frac{v_{id}}{2}$ + $v_{coupled}$
C_p
N_{1d}
+
v_{id}
−
C_p
+
v_{id}
−
N_{2d}
N_{2d} should be fully diff. with high CMRR
fully differential processing
$-\frac{v_{id}}{2}$ + $v_{coupled}$

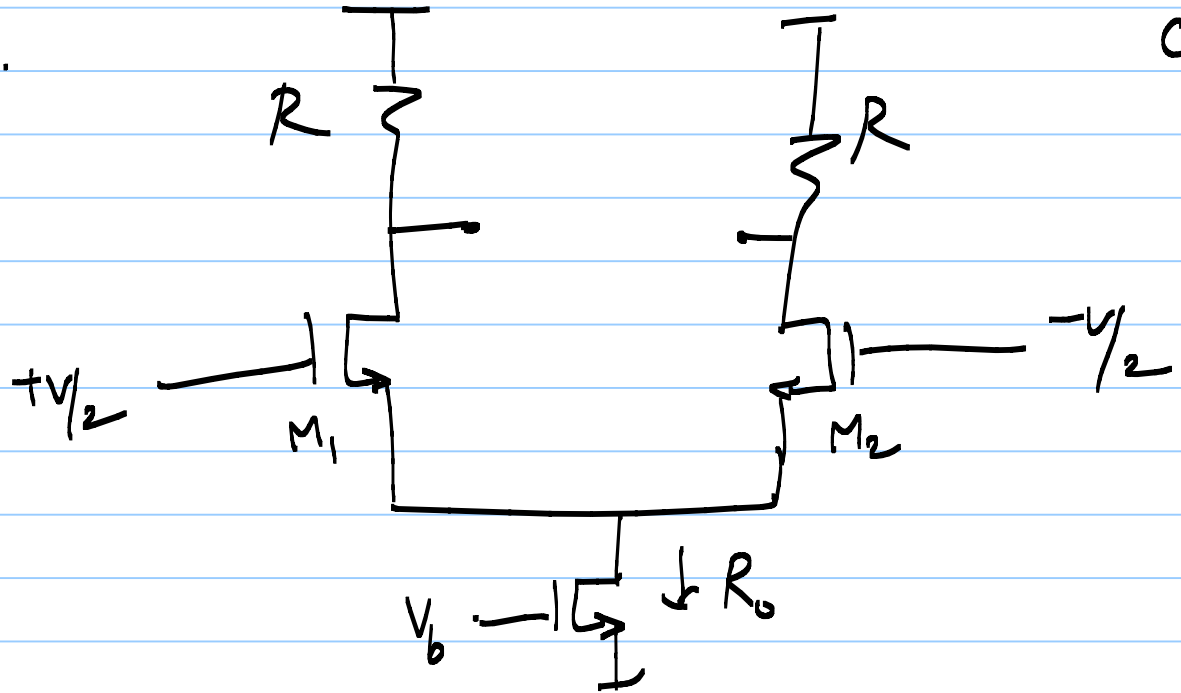

R2
R1
VDD
+
vid
−
VCM
+
−
R1
R2
Gnd
+
−
vod = −(R2/R1) · vid
fully diff. opamp
Rin = ∞
Rout = large (OTA)
A = large (Ve → 0)

Pseudo diff.

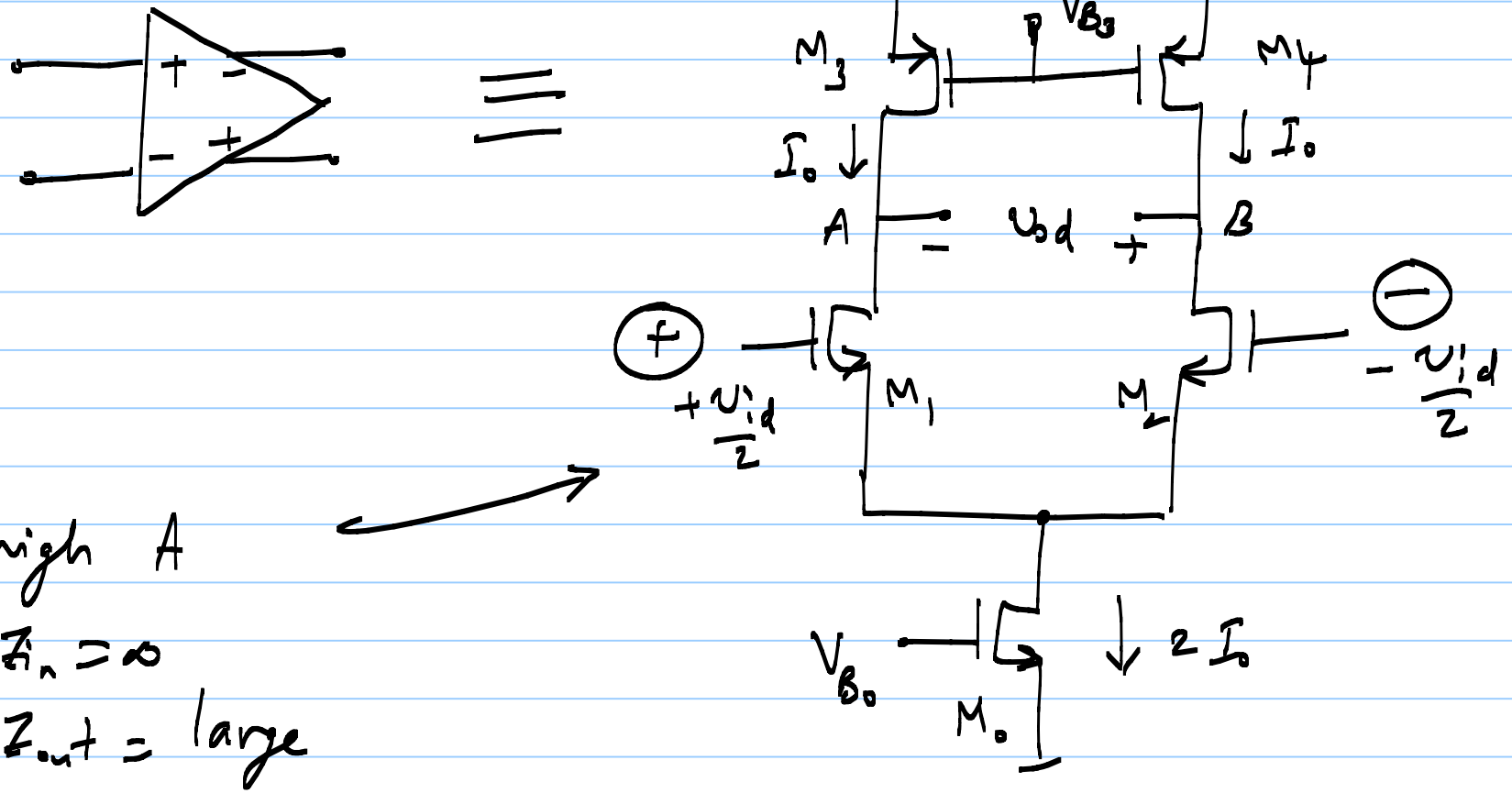


← CMRR = 1 (0 dB)

Fully diff.



CMRR is high $(1+2g_{m_1}R_0)$



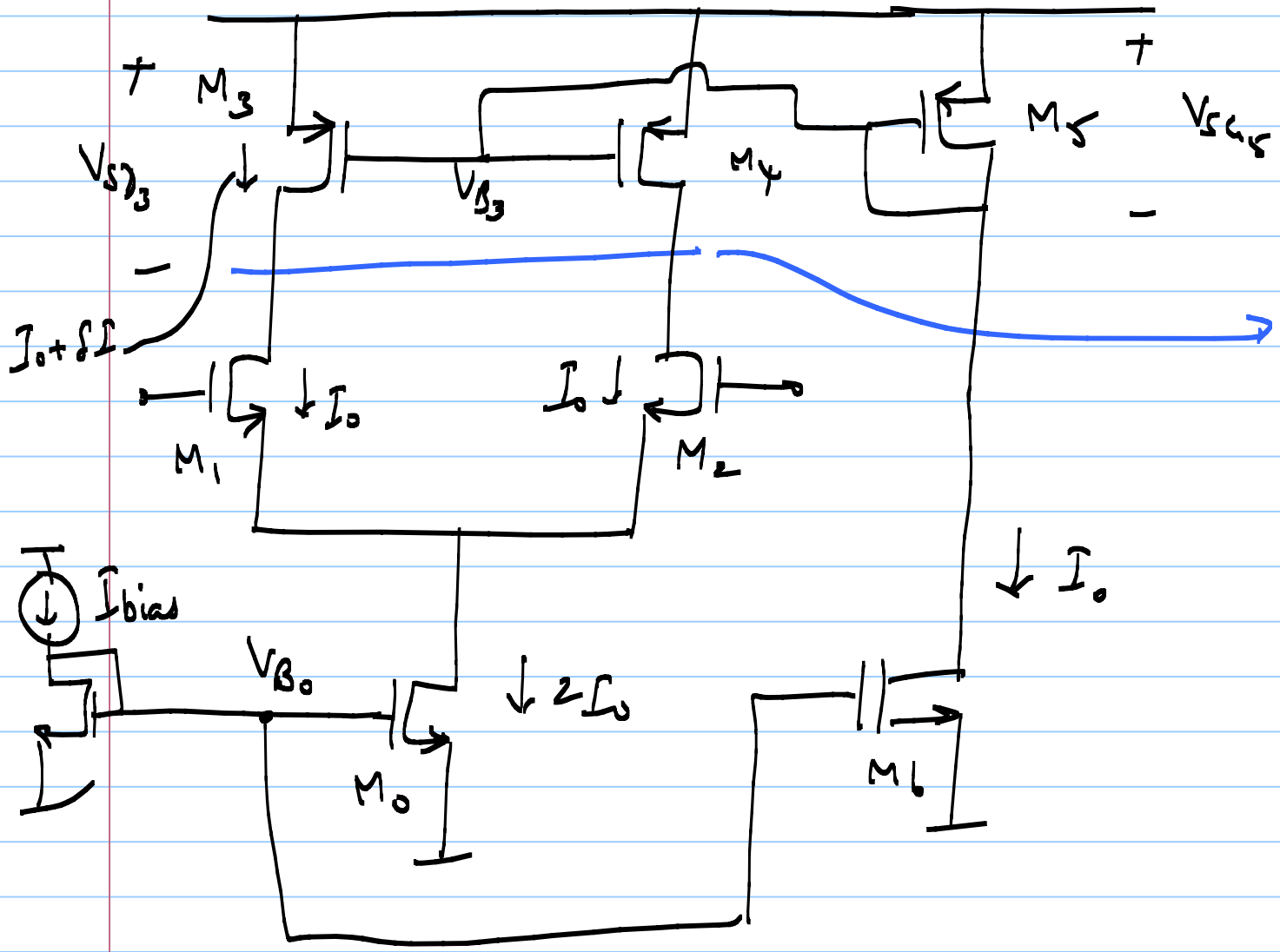
high A

$Z_{in} = \infty$

Z_{out} = large

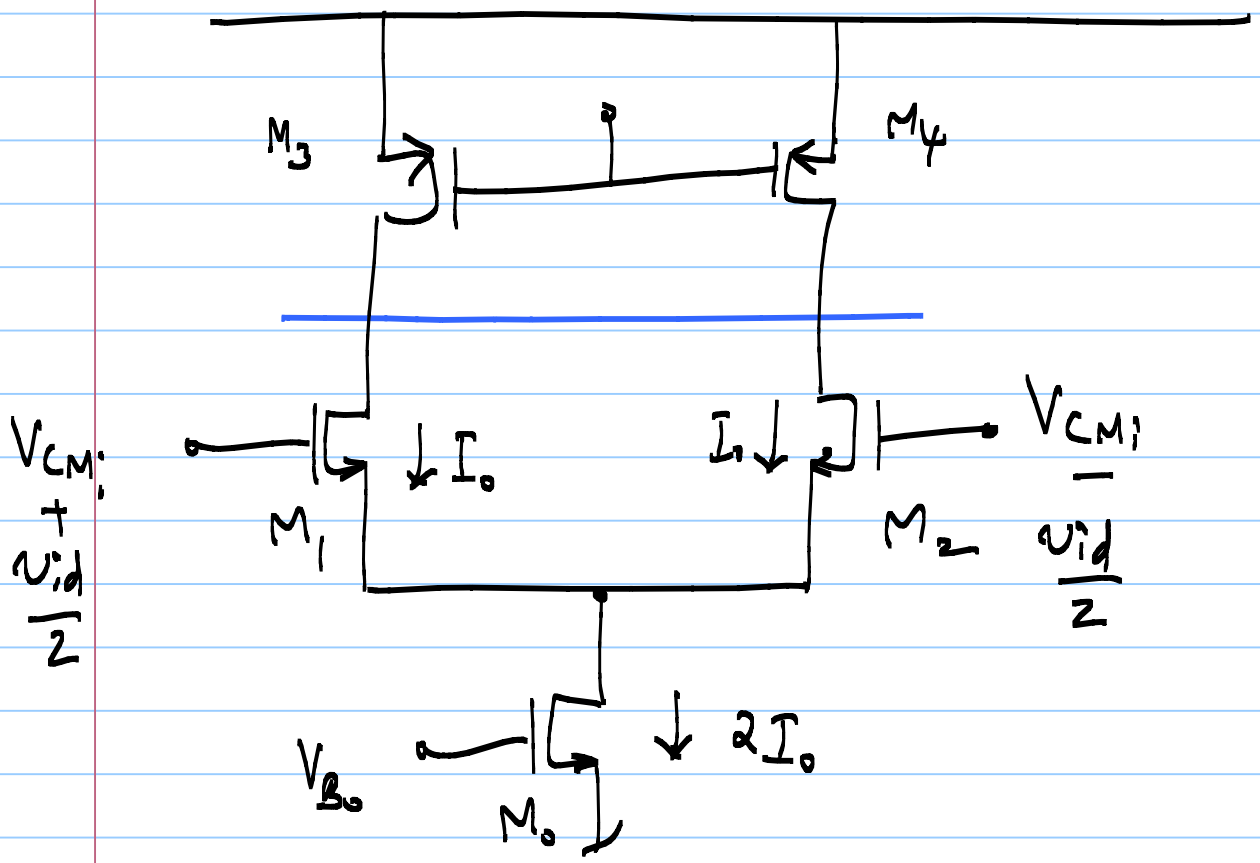
One stage FD OTA

* V_{B_0} & V_{B_3} can not be set independently



DC voltage is not very well defined

Use negative f.b. to set the bias point



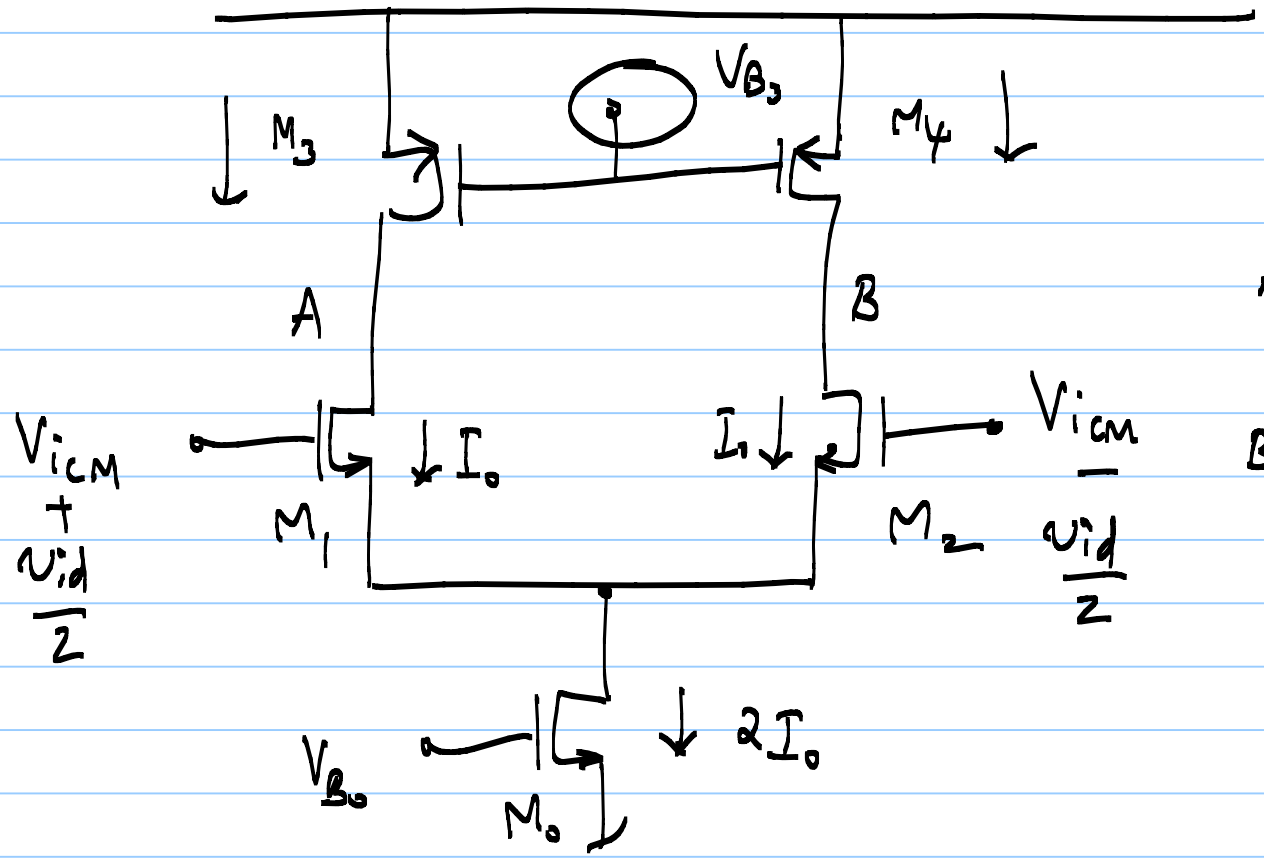
* Set $V_{DD} \sim V_{B3}$ using neg. f.b.

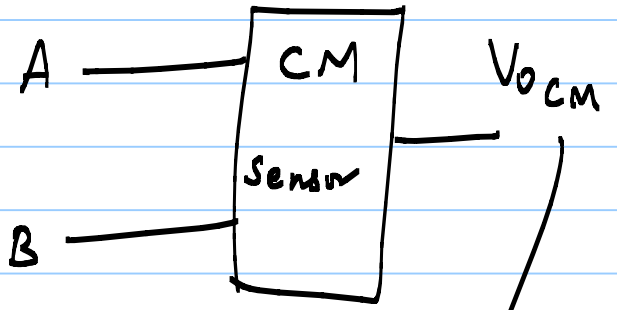
*

1) Measure $I_{D3} - I_{D1}$ use f.b.

2) short G & D of M_3 & M_4

3) Compare V_{D3} & V_{D4} of f.b. to V_{B3}

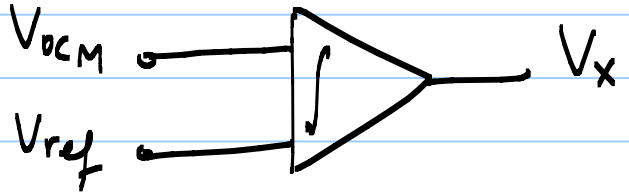




Output CM Voltage

step 1: Sense V_{OCM}

step 2: Compare with desired value 'V_{ref}'

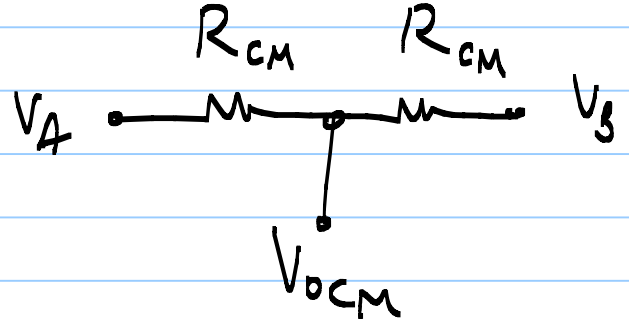


connect V_x so that $V_{OCM} \rightarrow V_{ref}$

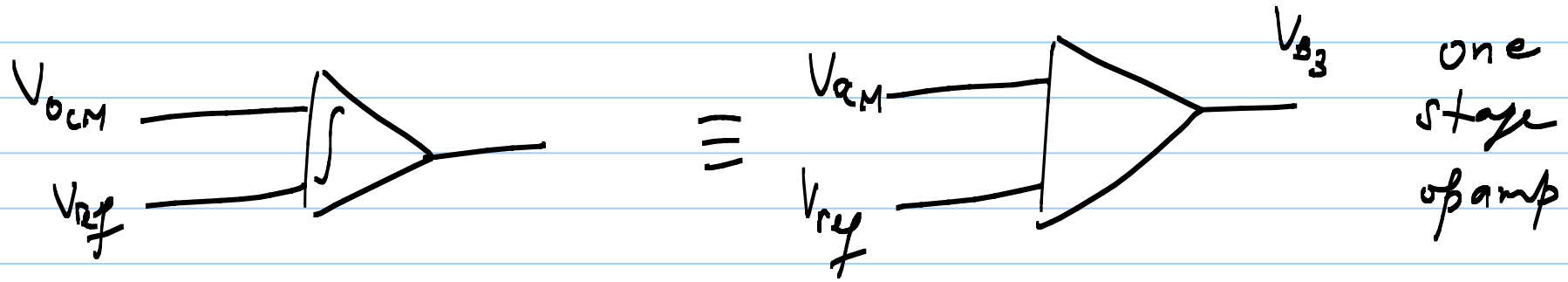
should affect

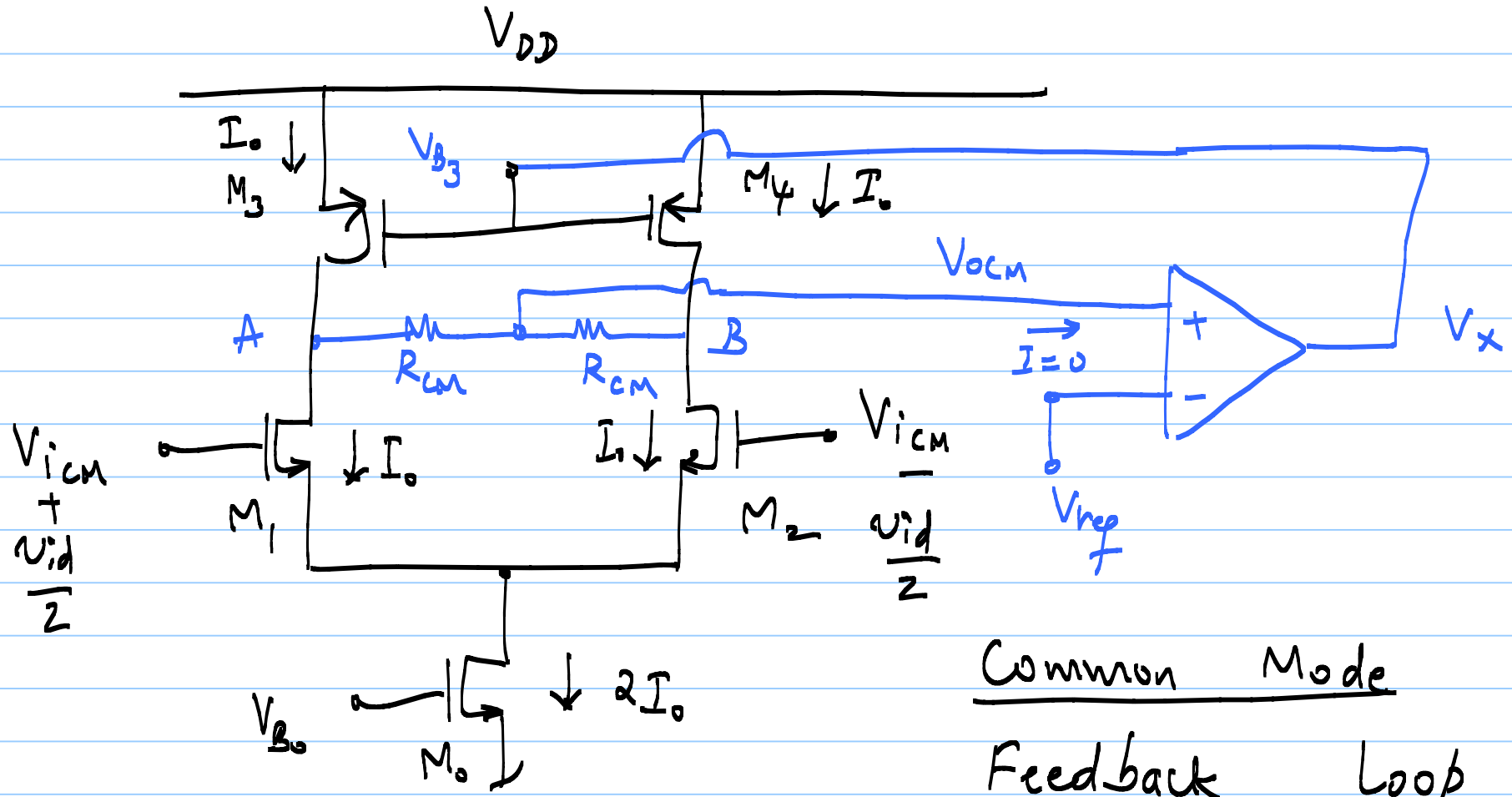
In this case $V_x \leftrightarrow V_{B_3}$ affects V_{OCM}

CM Sensor:



$$\frac{V_A + V_B}{2} = V_{OCM}$$





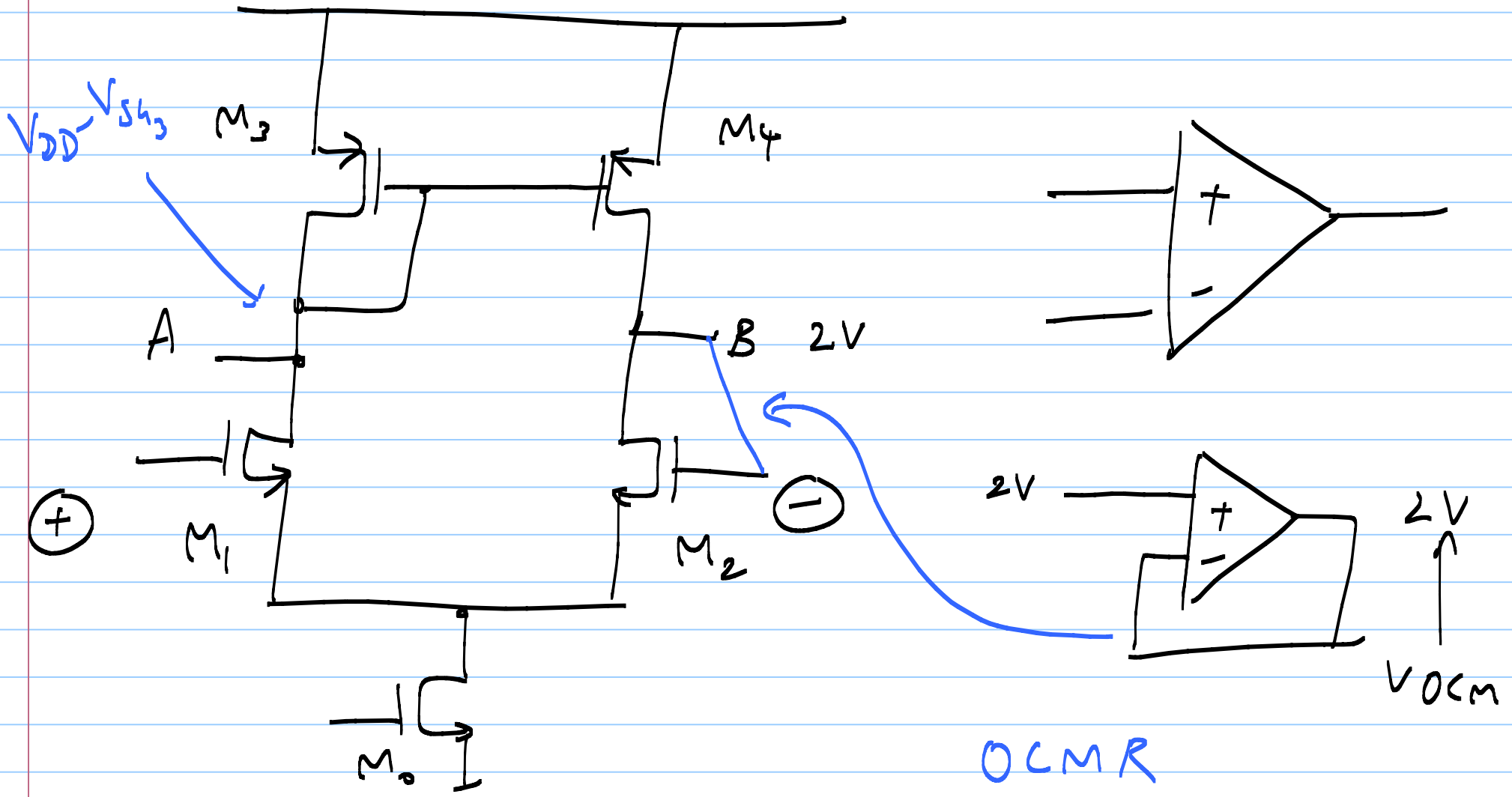
Common Mode Feedback Loop

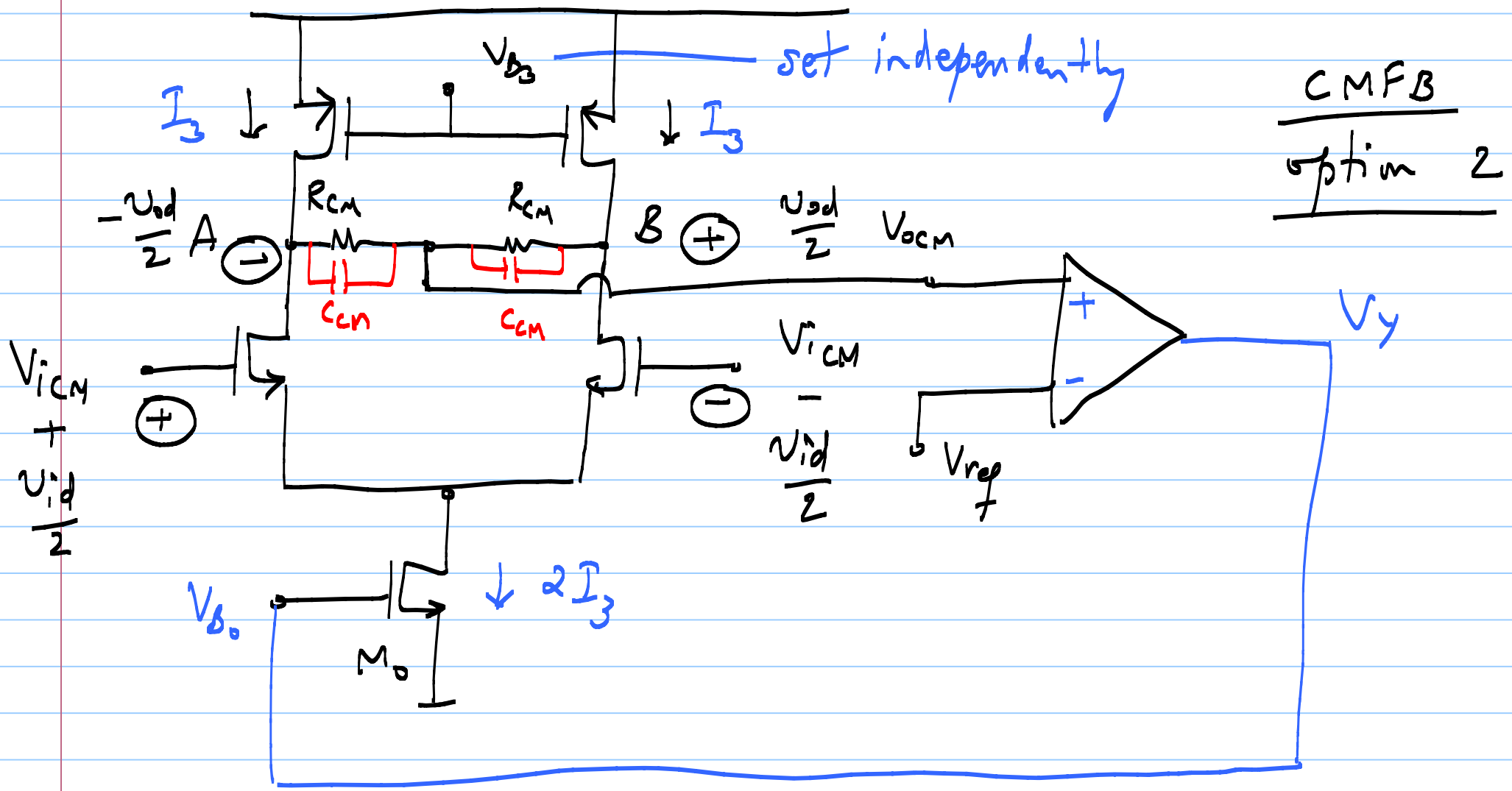
"CMFB"

$V_{OCM} = V_{ref}$

$I_{D3} = I_{D4} = I_0$

Choose V_{ref} so that all devices are in sat.


$V_{DD}-V_{SG_3}$
M_3
M_4
A
B
2V
+
M_1
−
M_2
M_0
+
−
2V
+
−
2V
V_{OCM}
OCMR



CMFB

optim 2

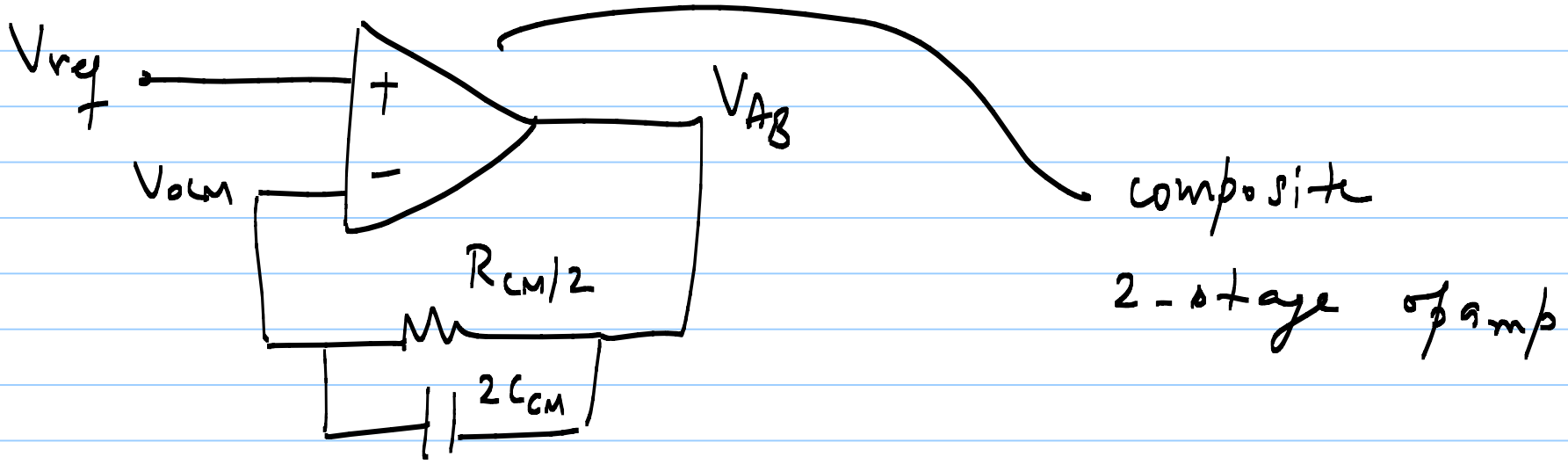
$$\left.\begin{array}{l}\text{differential mode}\\ \text{gain}\end{array}\right\} = \frac{v_{od}}{v_{id}} = g_{m_1}\left(r_{ds_2} \| r_{ds_4} \| R_{CM}\right)$$

* Choose R_{CM} large enough so as to not affect DM gain

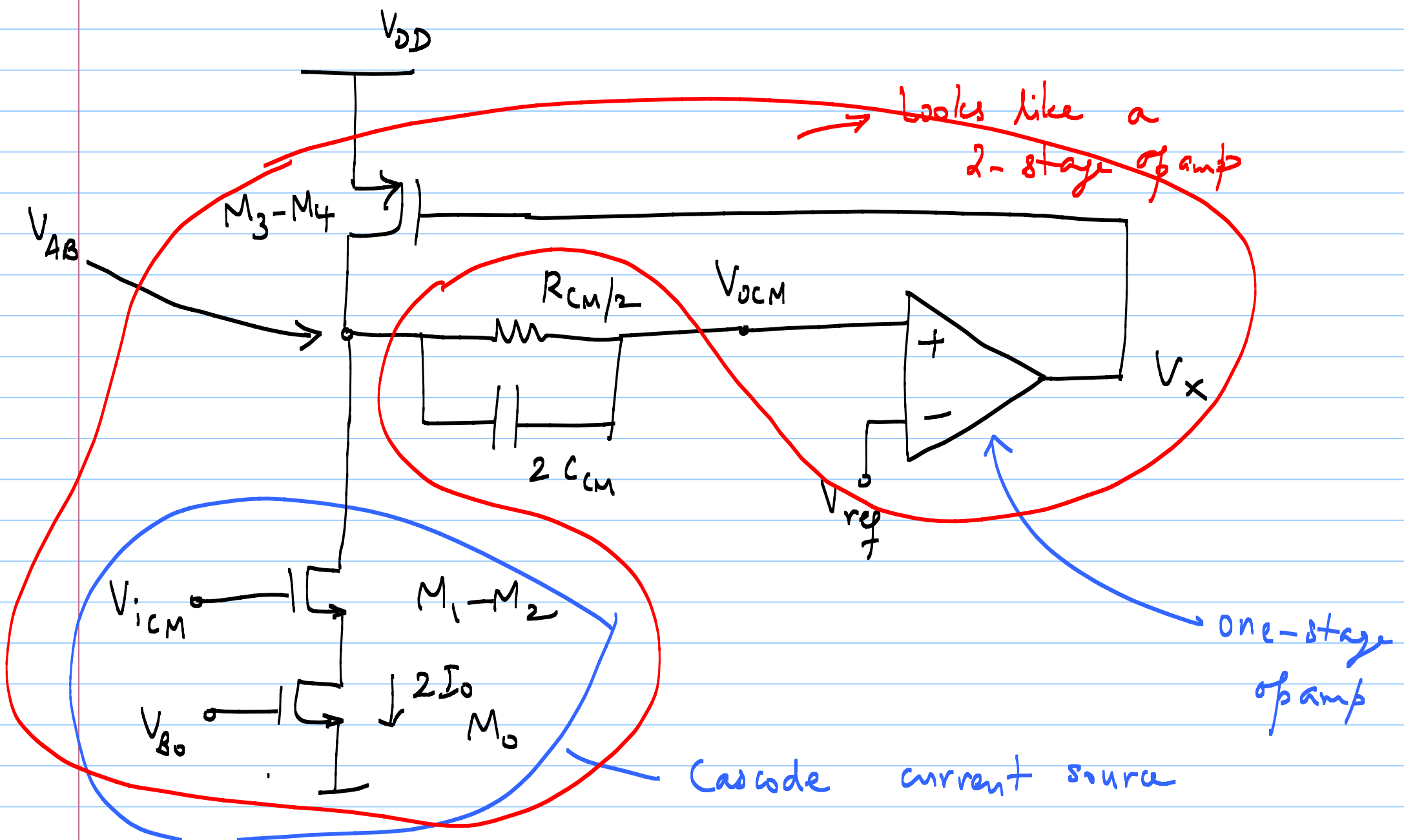
$$R_{CM} \gg r_{ds_2} \| r_{ds_4}$$

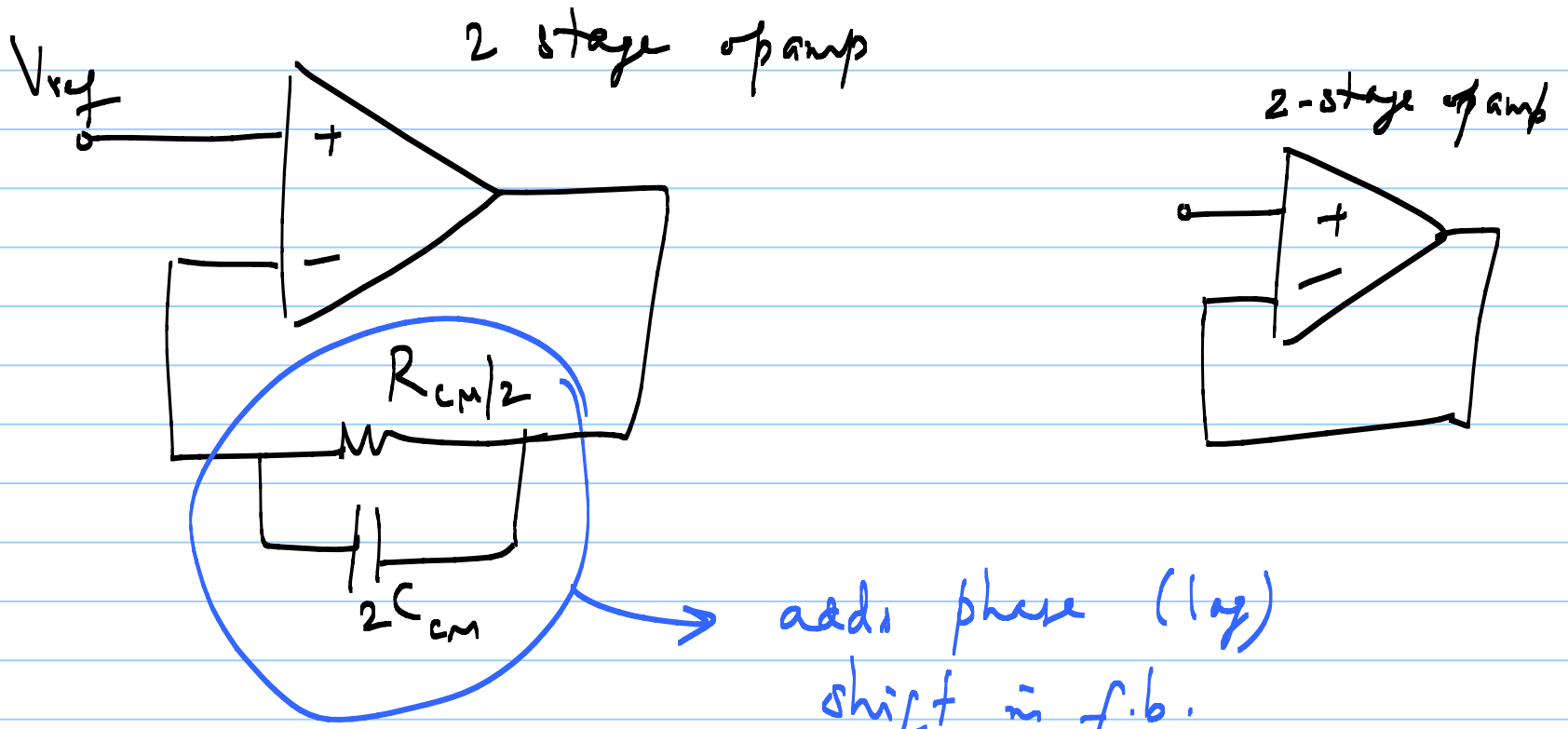
↑ may be of the order of MΩ

* R_{CM} may likely have significant parasitic cap. C_{CM}



CM equivalent circuit of CMFB option 1

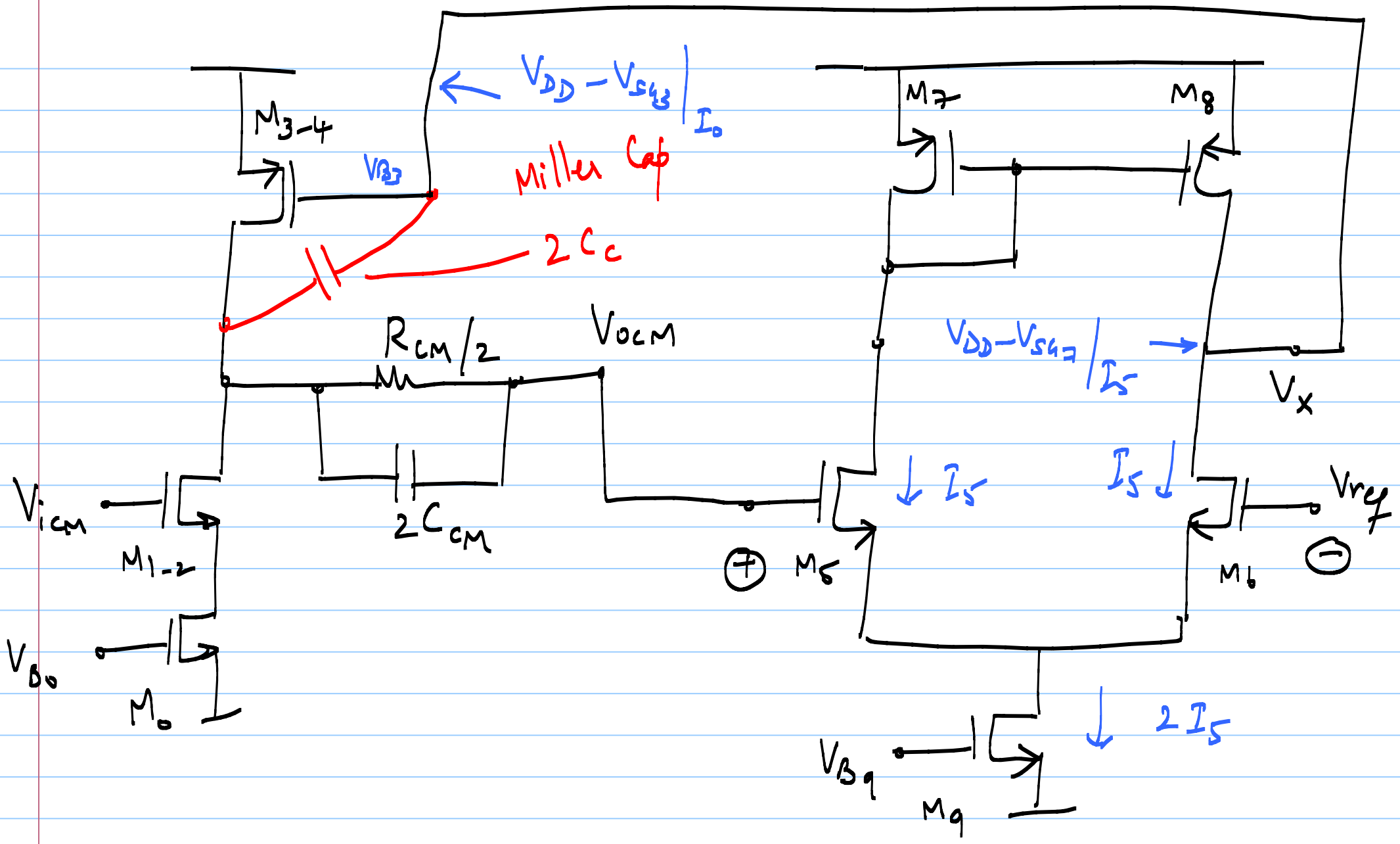




adds phase (lag) shift in f.b.

$\Rightarrow$ PM of f.b. loop degrades

* 2-stage CMFB loop needs to be compensated
* Phase lag from R_{CM} makes PM worse


$V_{DD}-V_{SG3}\,/\,I_0$
M_{3-4}
V_{B3}
Miller Cap
$2C_c$
$R_{CM}/2$
V_{OCM}
$V_{DD}-V_{SG7}\,/\,I_5$
M_7
M_8
V_x
V_{ICM}
M_{1-2}
$2C_{CM}$
I_5
I_5
V_{ref}
M_5
M_6
V_{B0}
M_0
$2I_5$
V_{B1}
M_1

CMFB loop :

$$\omega_{u_{CM}} = \frac{g_{m5}}{2C_c}$$

$$DC\ gain = g_{m5}\left(r_{ds6} \,||\, r_{ds8}\right) \cdot g_{m3-4}\left[r_{ds3-4} \,||\, \left(g_{m1-2} \cdot r_{ds1-2} \cdot r_{ds0}\right)\right]$$

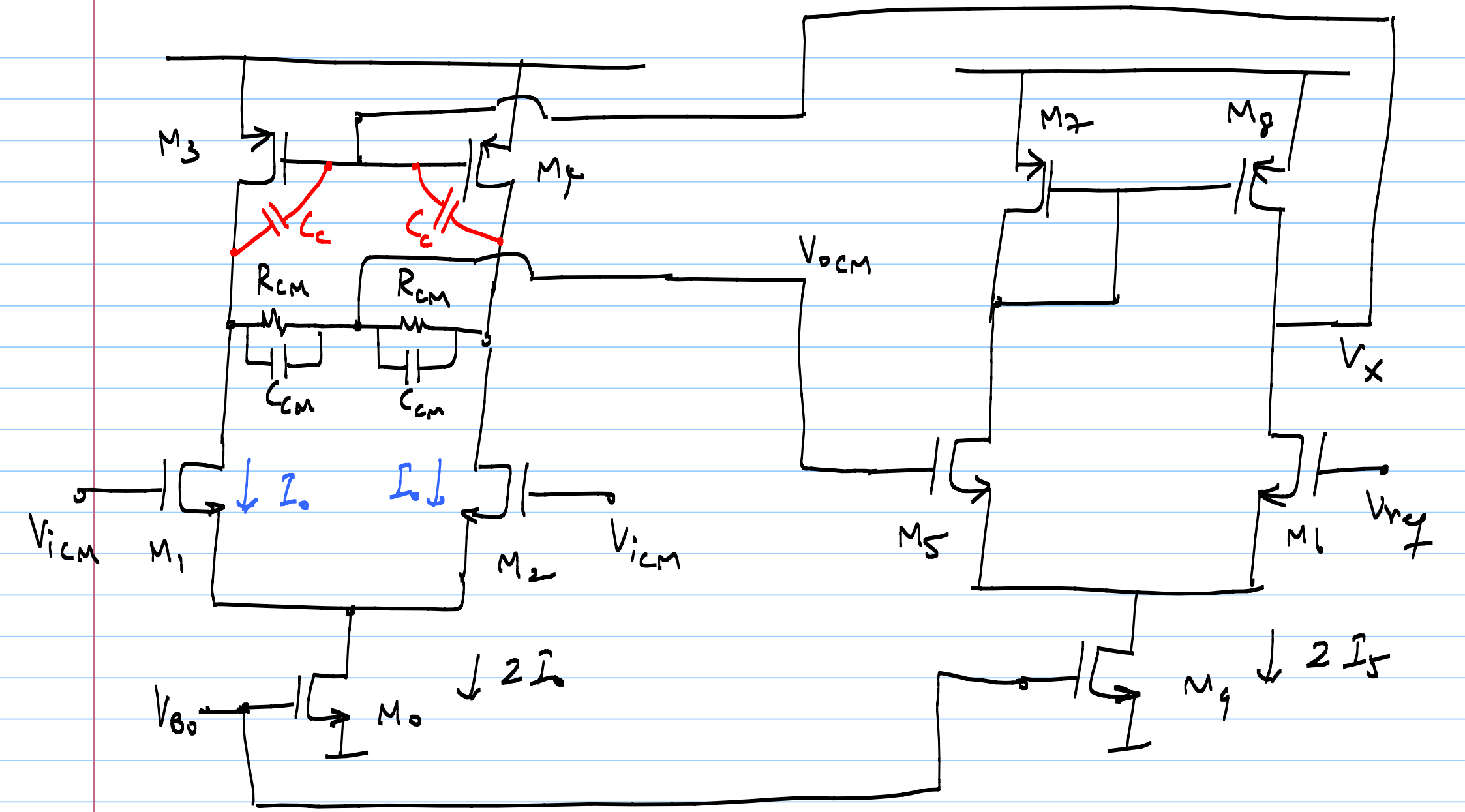
* V_x likes to be at $V_{DD} - V_{SG7}\big|_{I_5}$

* V_{B3} likes to be at $V_{DD} - V_{SG3}\big|_{I_0}$

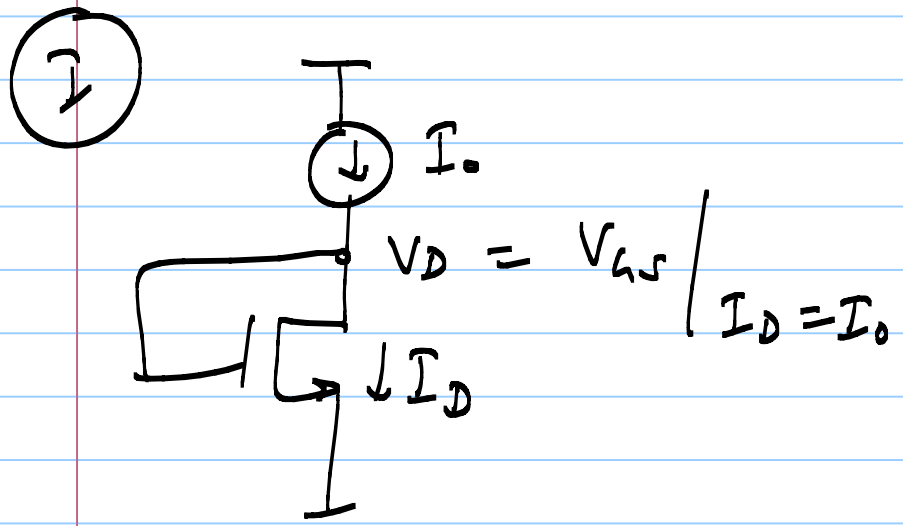
$\therefore$ Choose $\dfrac{I_5}{(W/L)_7} = \dfrac{I_0}{(W/L)_3}$

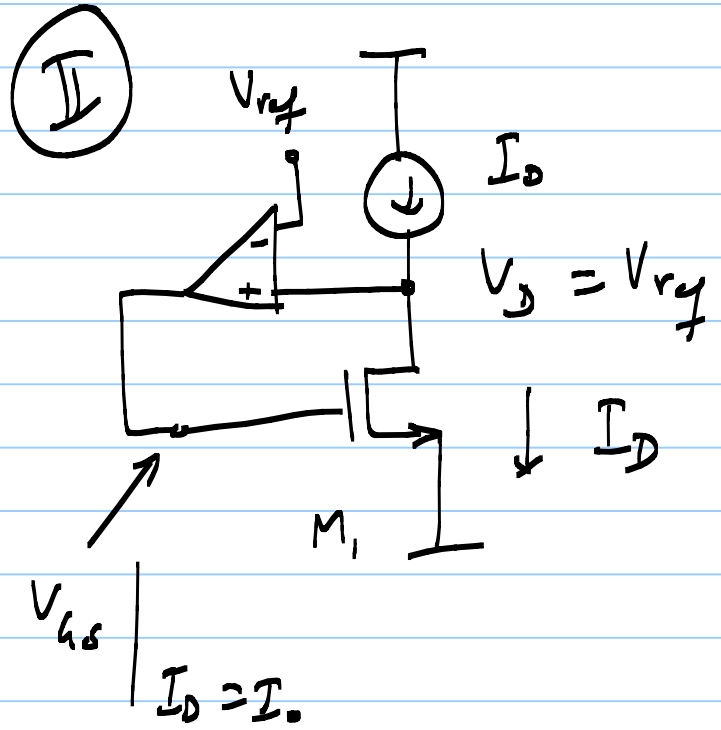
Normally $I_5 << I_0$

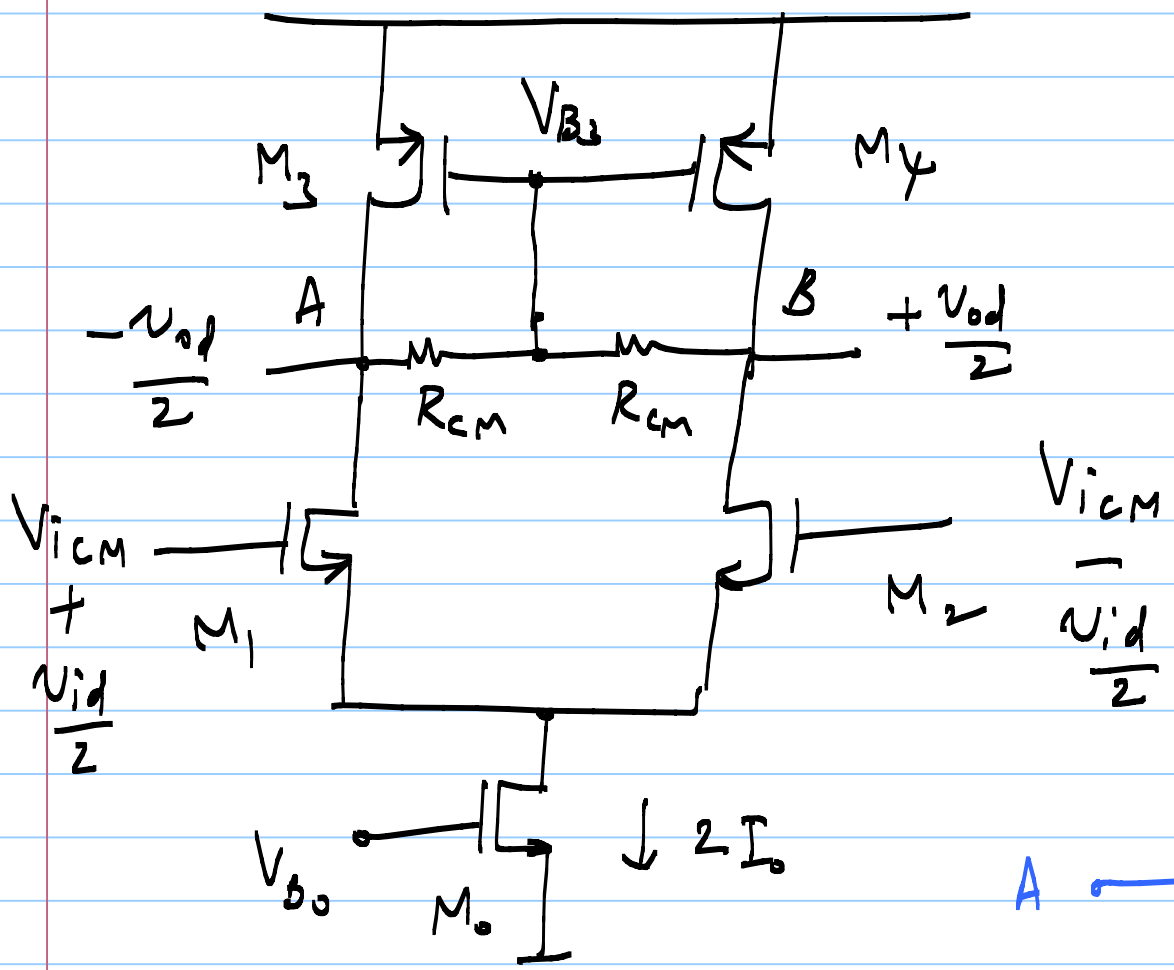
also $L_{3,4} = L_{7,8}$


M3
M4
Cc
Cc
RCM
RCM
CCM
CCM
VoCM
M7
M8
Vx
I0
I0
ViCM
M1
M2
ViCM
M5
M6
Vreg
2I0
VB0
M0
M9
2I5

Neg. fb. biasing of current mirror:



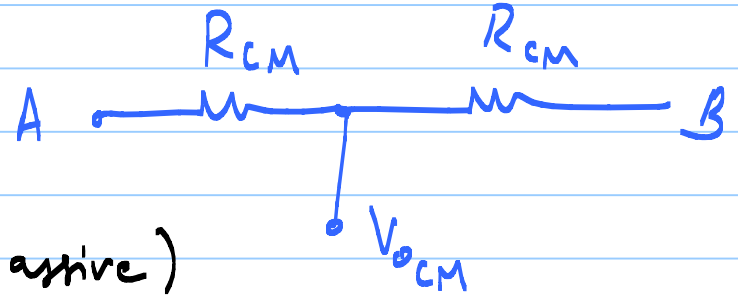




$$V_{B_3} = V_{OCM} = V_{DD} - V_{SG_3}\Big|_{I_0}$$

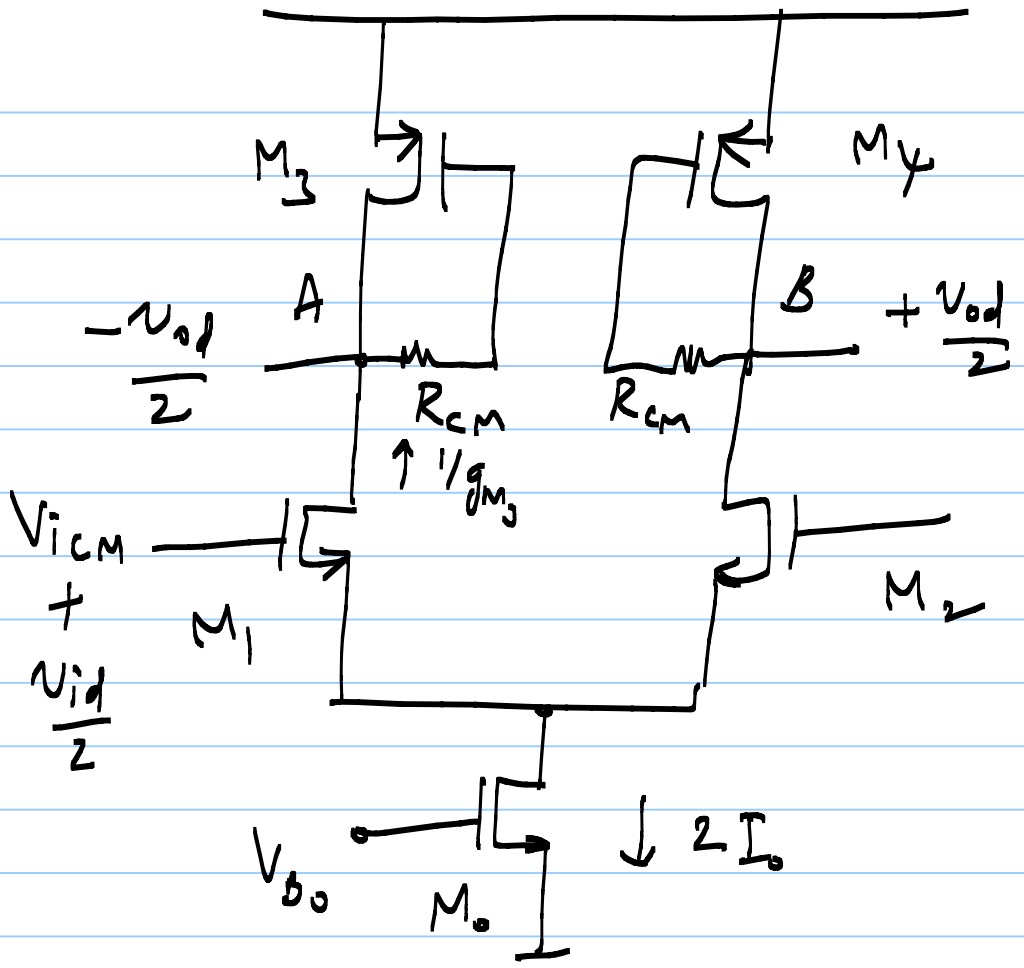
$$\frac{v_{od}}{v_{id}} = g_{m_1}\left(r_{ds_2} \| r_{ds_4} \| R_{CM}\right)$$

CM sensor



(passive)

* linear CM detector
* ↓ gain ; parasitic cap.



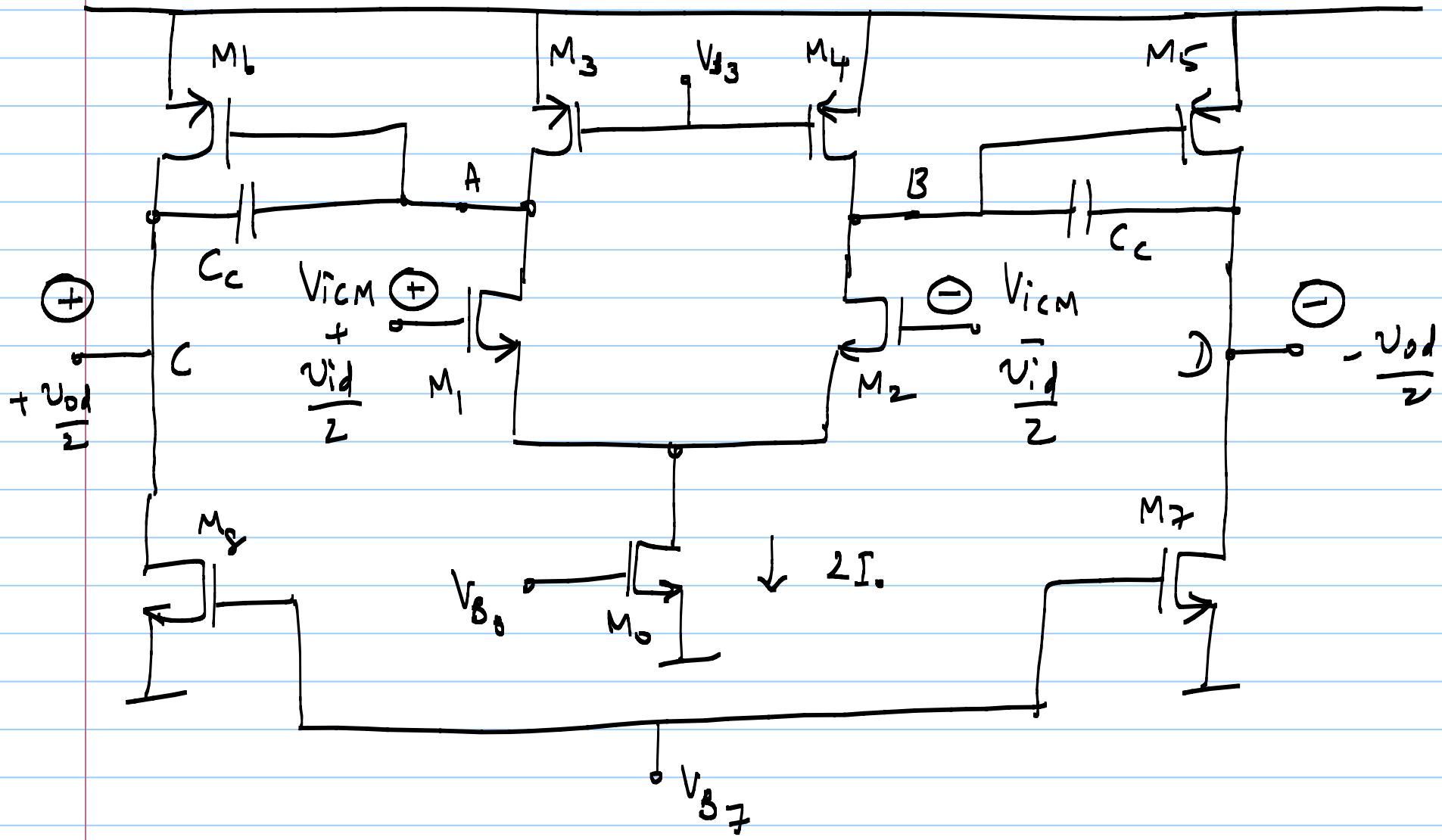
Same

Not a CMFB in reality

$$V_{oCM} = V_{DD} - V_{SG_3}\Big|_{I_0}$$

$$\frac{v_{od}}{v_{id}} \simeq \frac{g_{m_1}}{g_{m_3}}$$

Fully Differential 2-stage opamp



* C_c - compensation cap for FD feedback amplifier

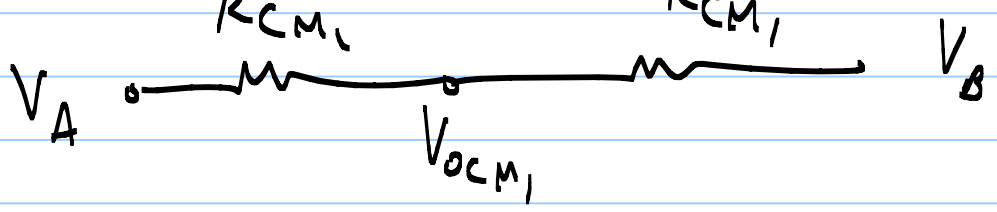
* Need to set

$$V_{OCM_1} = \frac{V_A + V_B}{2}$$ ← CM @ output of stage 1

$$V_{OCM} = \frac{V_C + V_D}{2}$$ ← CM @ output of stage 2

* V_{OCM_1}

1)

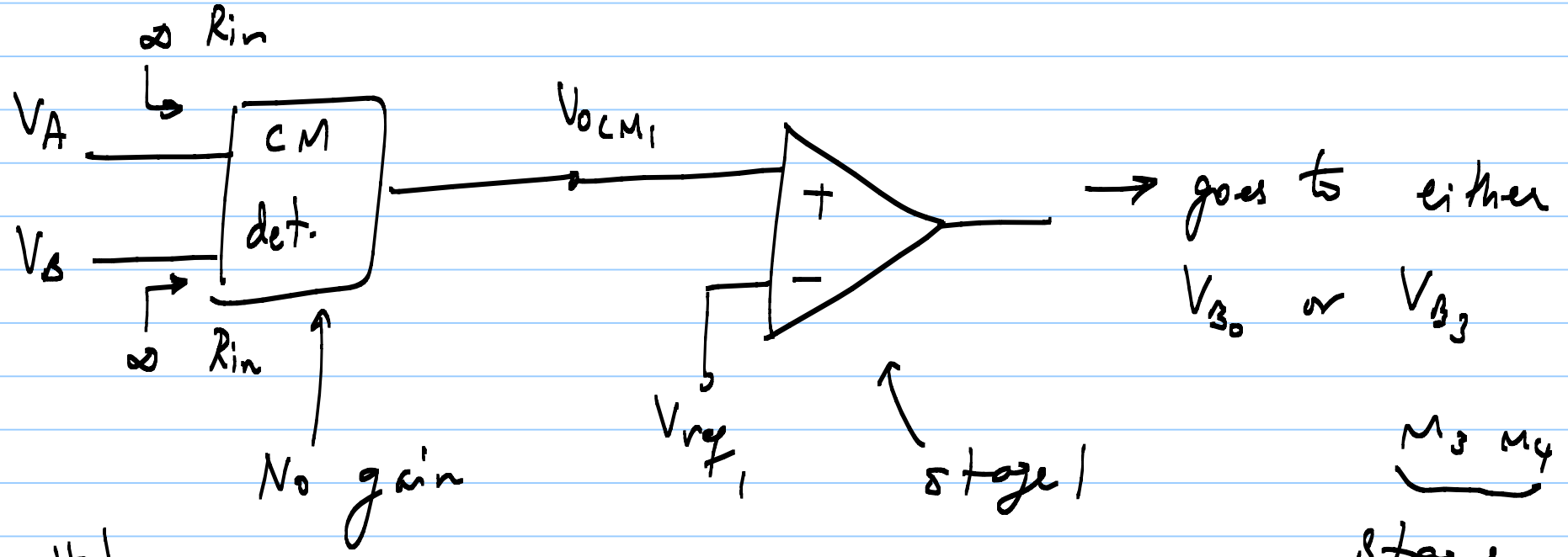


→ Reducing gain of stage 1

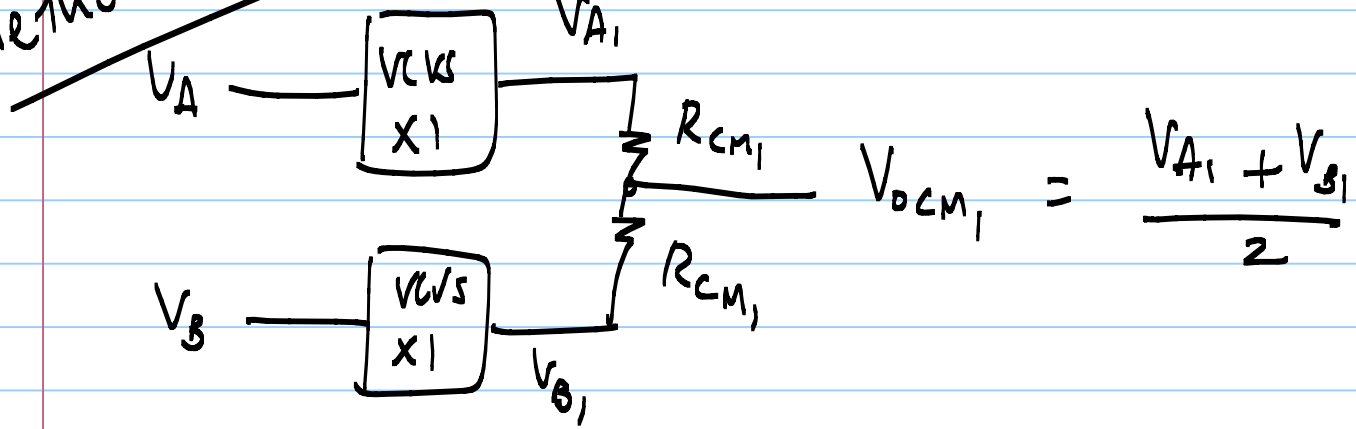
} stage 1 gives gain when driving R_L (res. load)

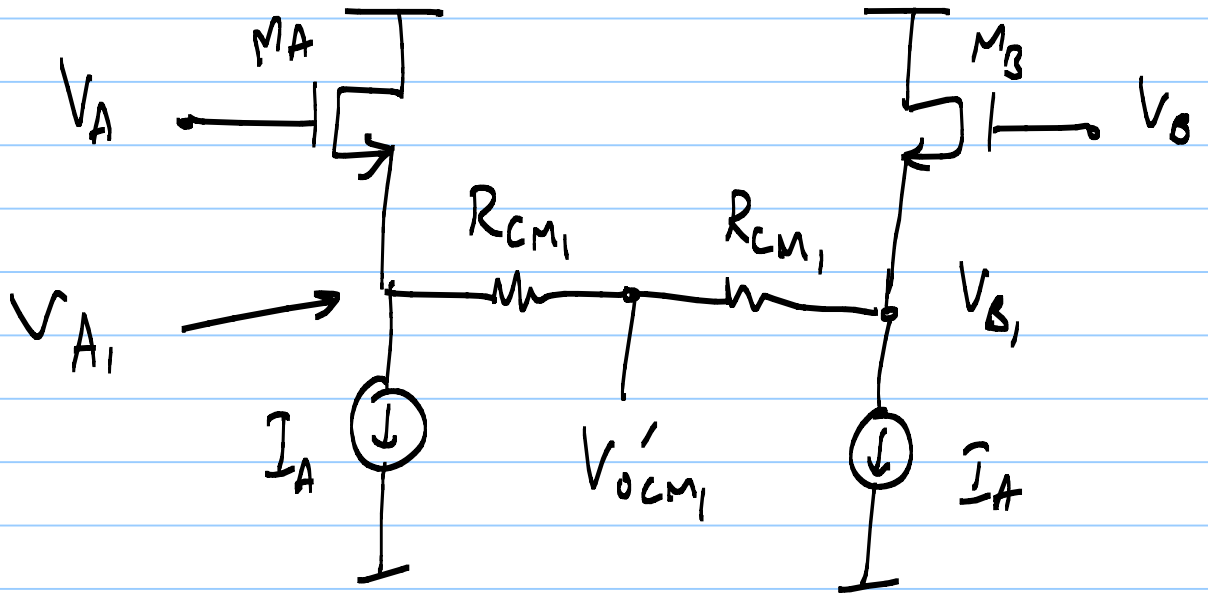
2) swings @ A & B are low — few 10^s of mV

→ can use active CM detectors



Method #1



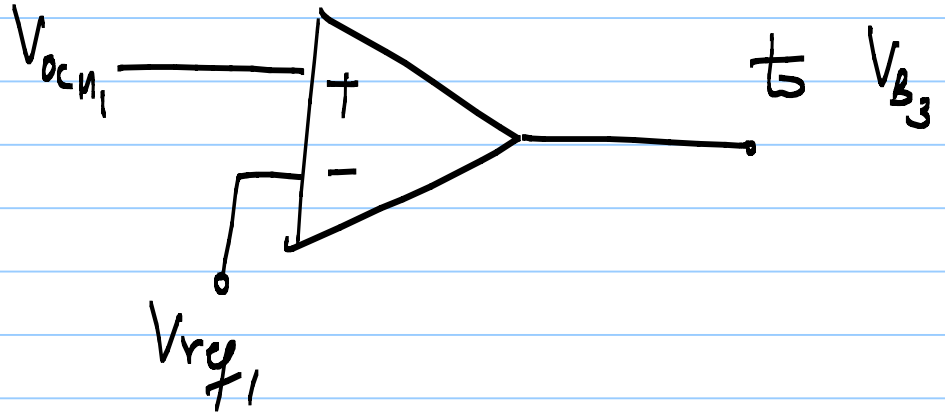


$$V'_{OCM_1} = \frac{V_{A_1} + V_{B_1}}{2}$$

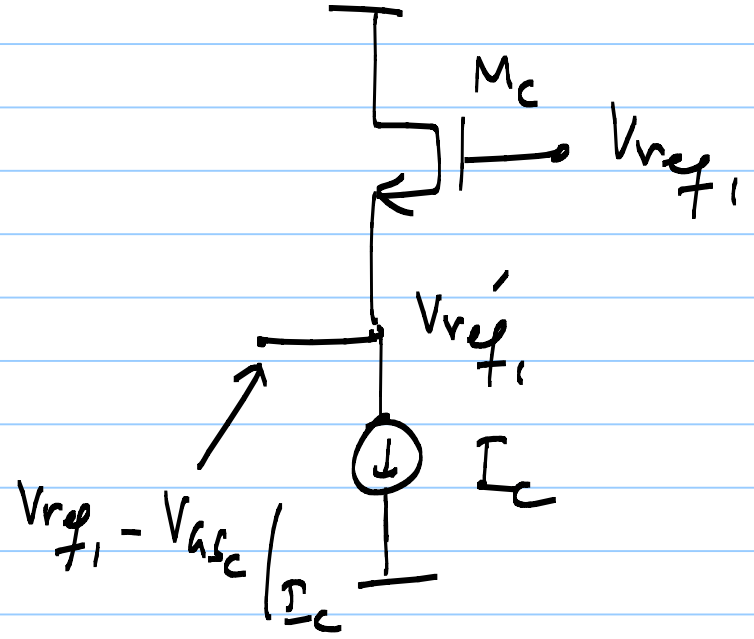
$$= V_{OCM_1} - V_{GS_{A_B}}\Big|_{I_A}$$

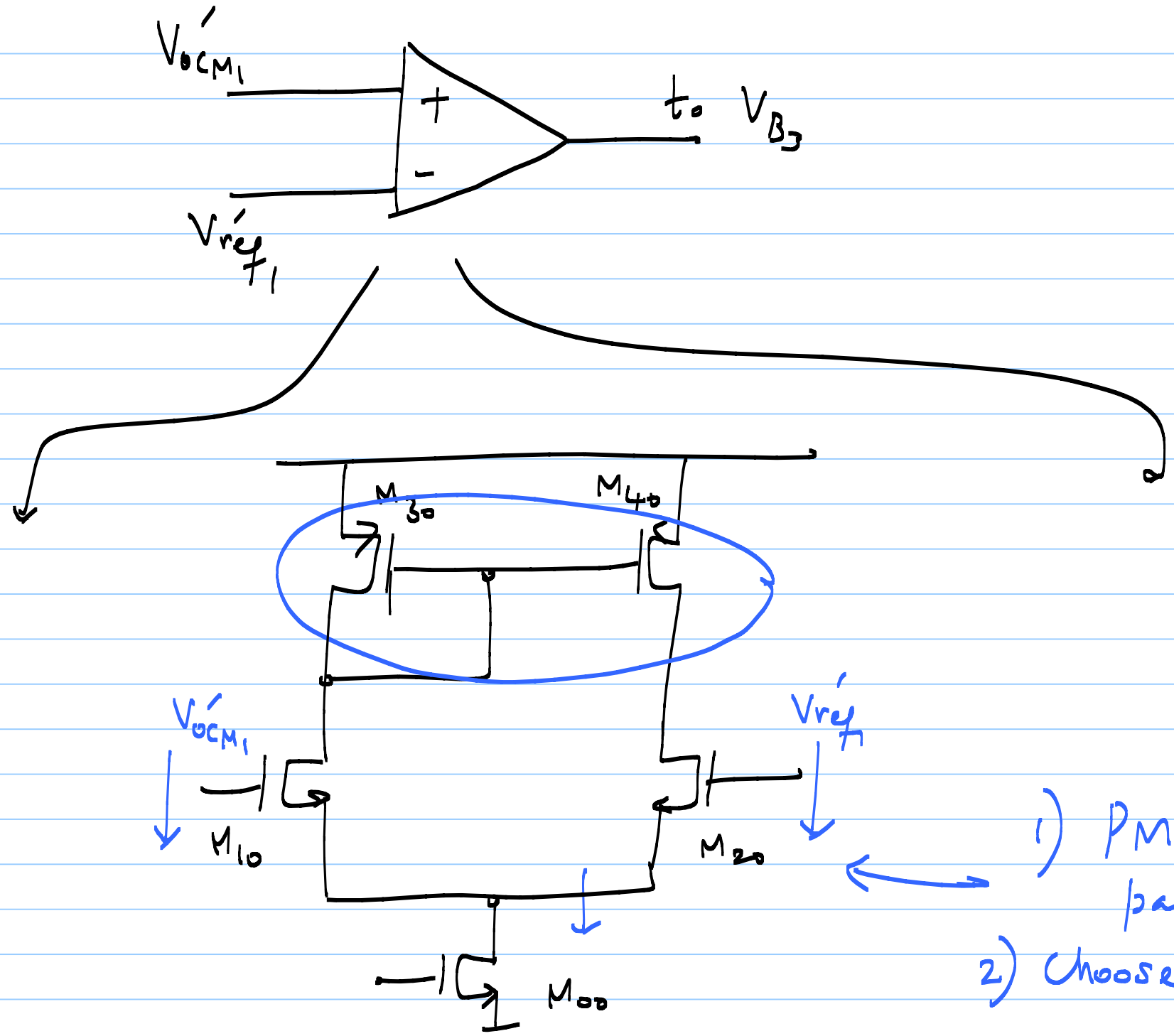
$$V_{A_1} = V_A - V_{GS_A}$$

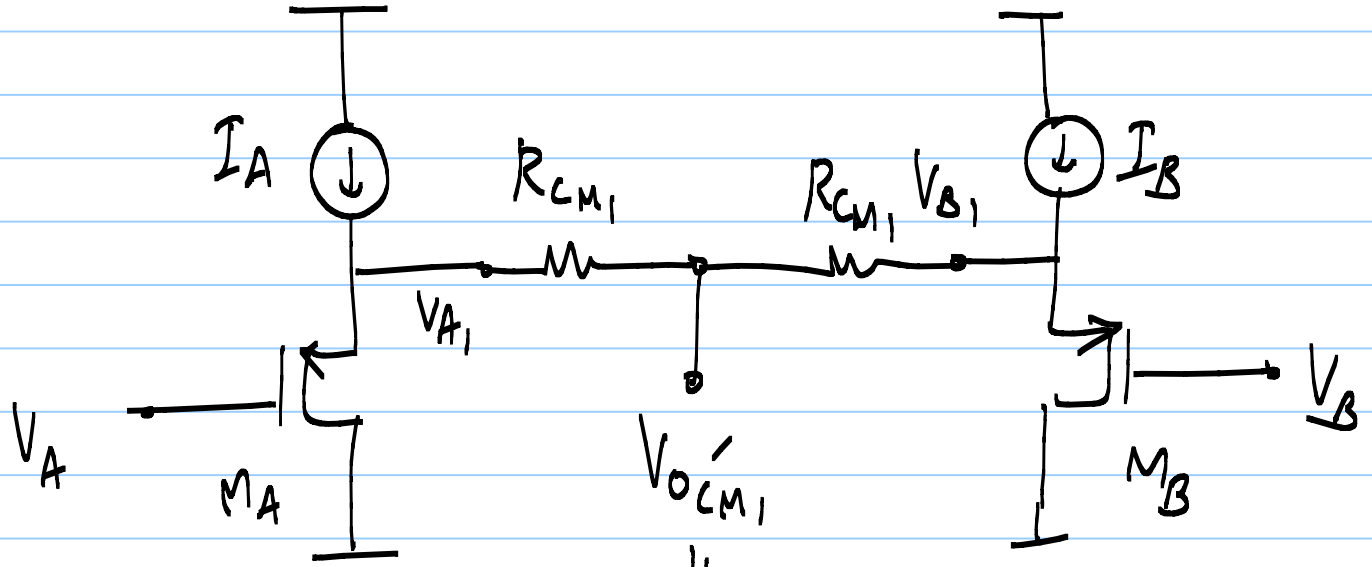
$$V_{B_1} = V_B - V_{GS_B}$$



Ensure $V_{GS_C}\Big|_{I_C} = V_{GS_A}\Big|_{I_A}$

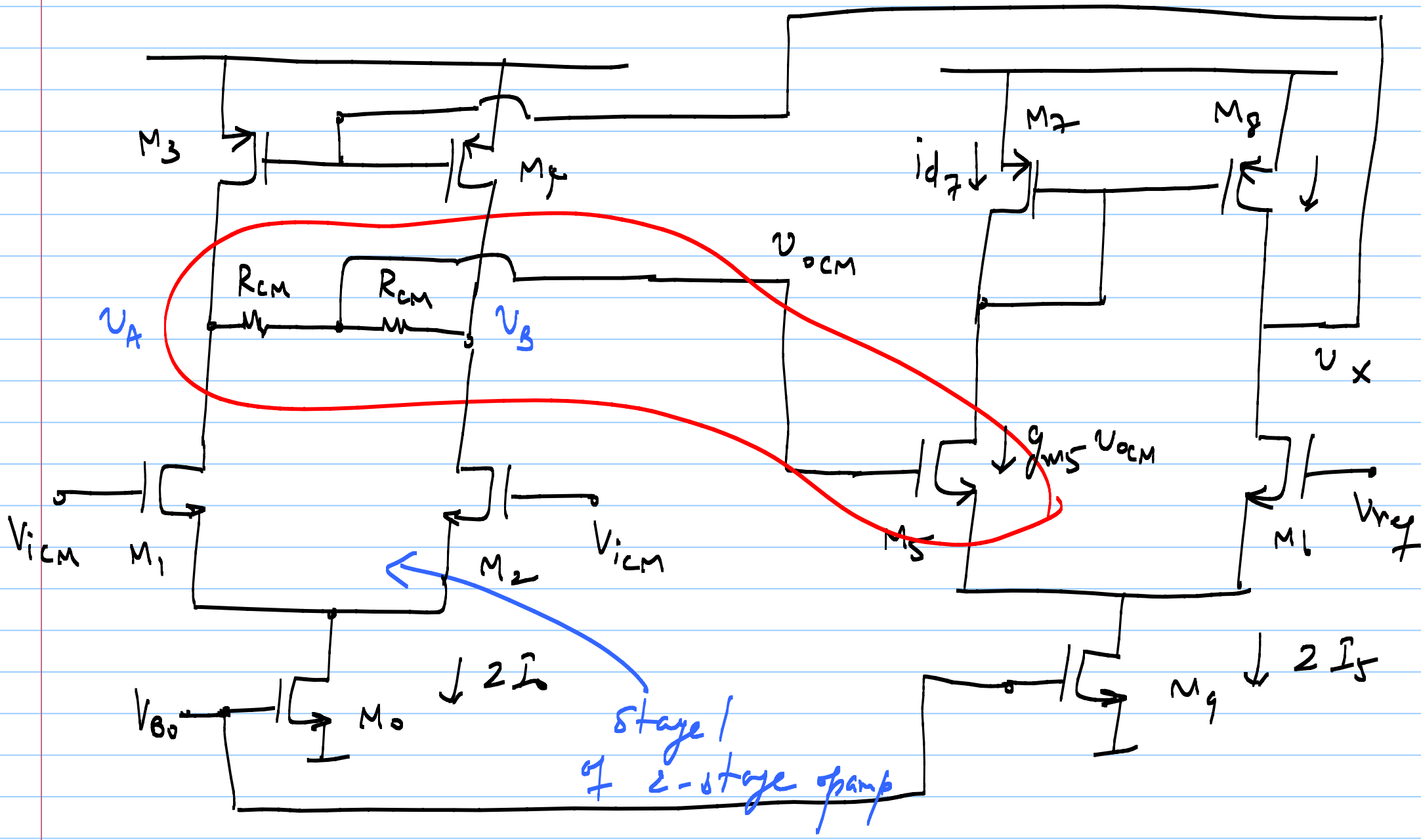



V_{OCM_1}'
+
−
to V_{B_3}
V_{ref_1}'
M_{30}
M_{40}
V_{OCM_1}'
V_{ref_1}'
M_{10}
M_{20}
M_{00}
1) PMOS input pair
2) Choose PMOS VCVS



$$\frac{V_A + V_B}{2} + V_{SG_A}\Big|_{I_A}$$

Method #2


M3
M4
RCM
RCM
vA
vB
vocm
id7
M7
M8
vx
gm5 vocm
M5
M6
Vreg
VicM
M1
M2
VicM
2I
VBo
M0
M9
2I5
stage 1
of 2-stage opamp

* combination of 2 R_{CM}'s and M_5

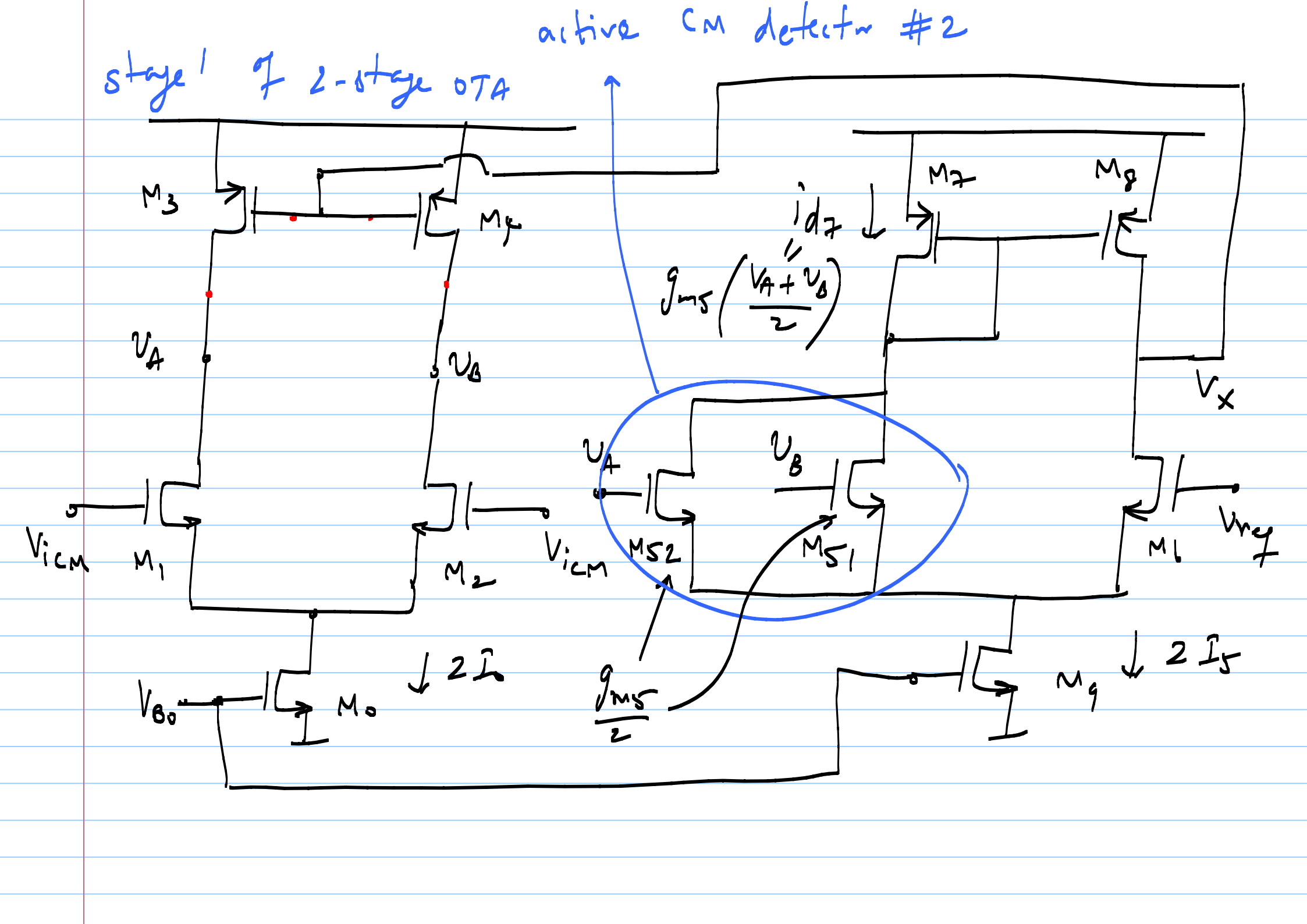
$$\rightarrow \quad i_{d_5} = g_{m5} \cdot v_{ocm}$$

$$= g_{m5} \left(\frac{v_A + v_B}{2} \right) = i_{d_7}$$

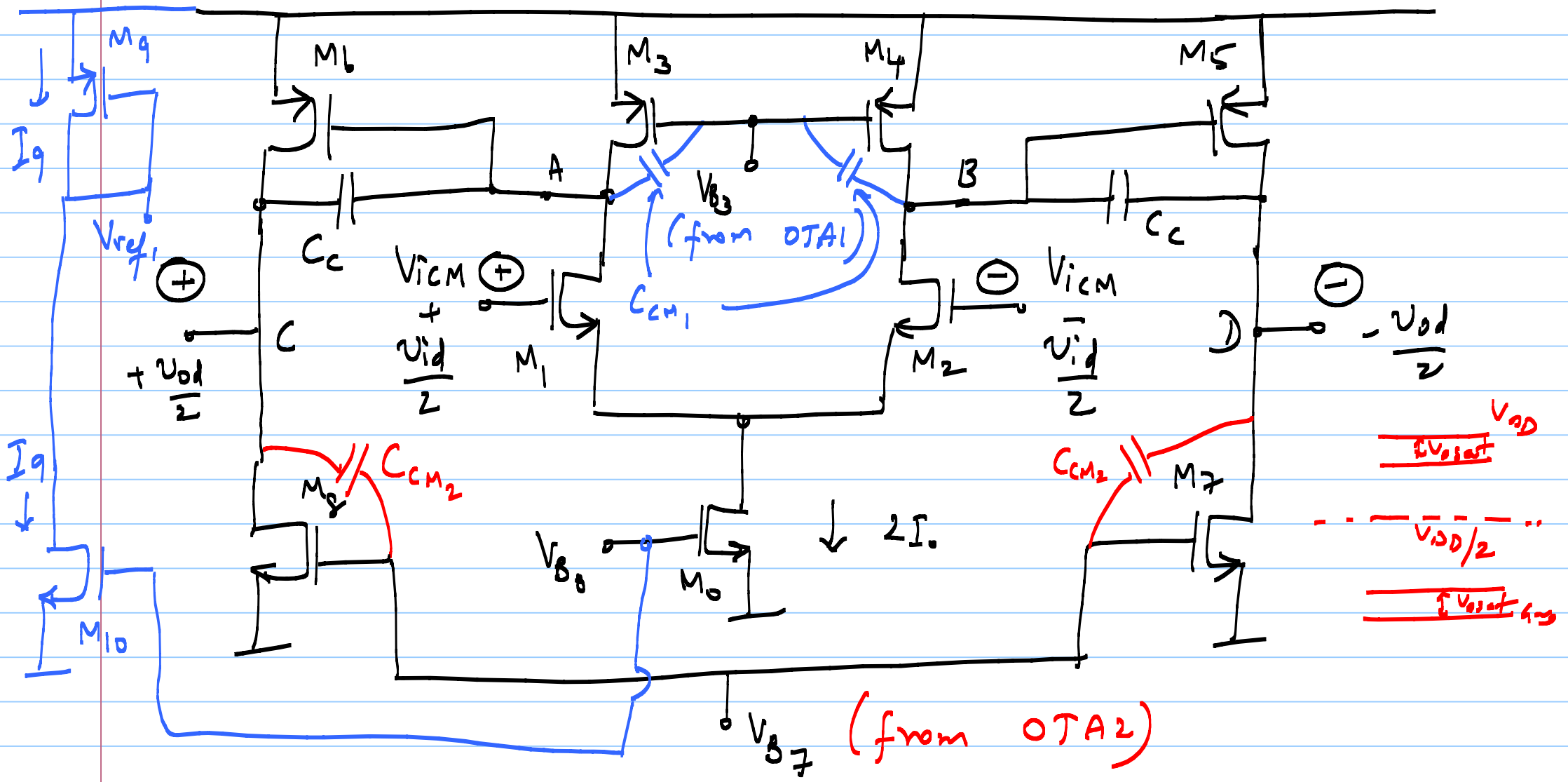
* Alternative way:

→ generate $\frac{g_{m5} \cdot v_A}{2}$, $\frac{g_{m5} v_B}{2}$

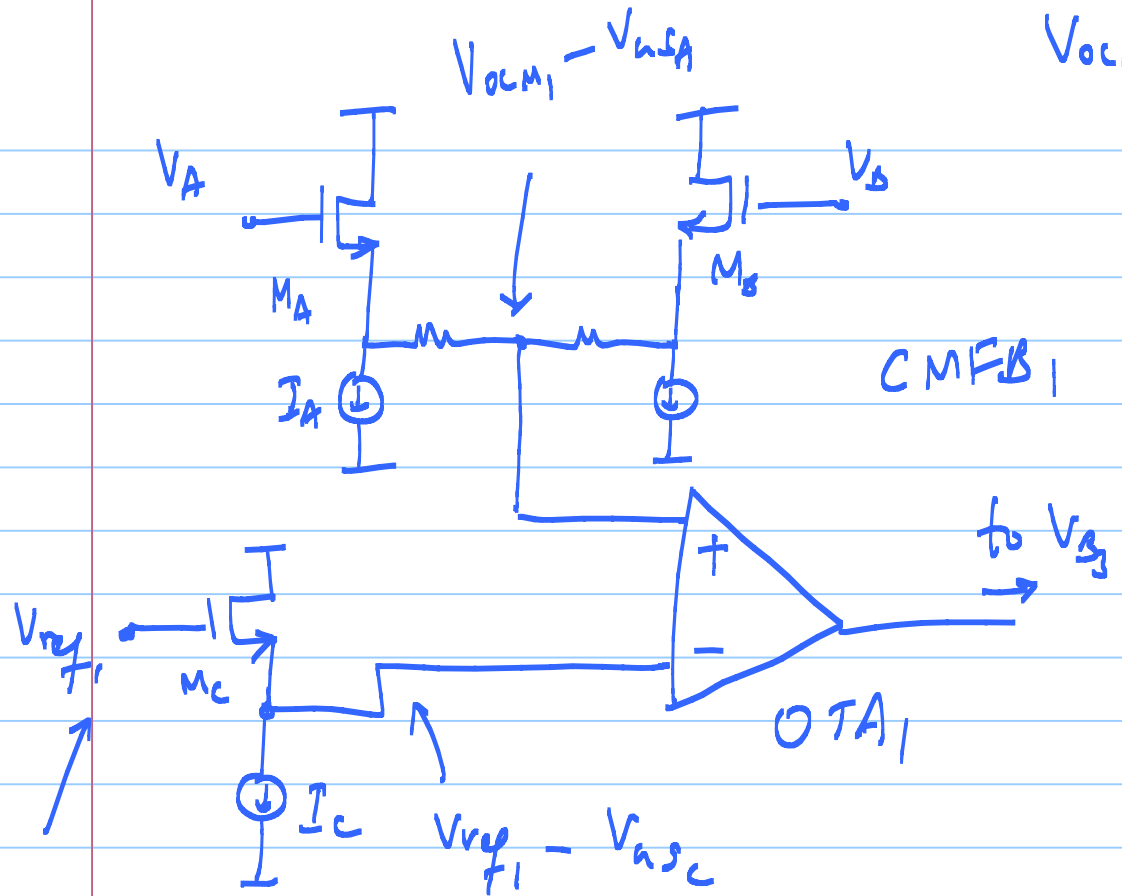
→ add them


stage 1 of 2-stage OTA
active CM detector #2
M3
M4
M7
M8
i_{d7}
$g_{m5}\left(\frac{V_A+V_B}{2}\right)$
V_A
V_B
V_X
V_{icM}
M1
M2
M5
M6
V_{B0}
M0
$2I_0$
$\frac{g_{m5}}{2}$
$2I_5$

METHOD #1



$$V_{OCM} = \frac{V_C + V_D}{2} \rightarrow \text{set to } V_{DD}/2 = V_{ref}$$

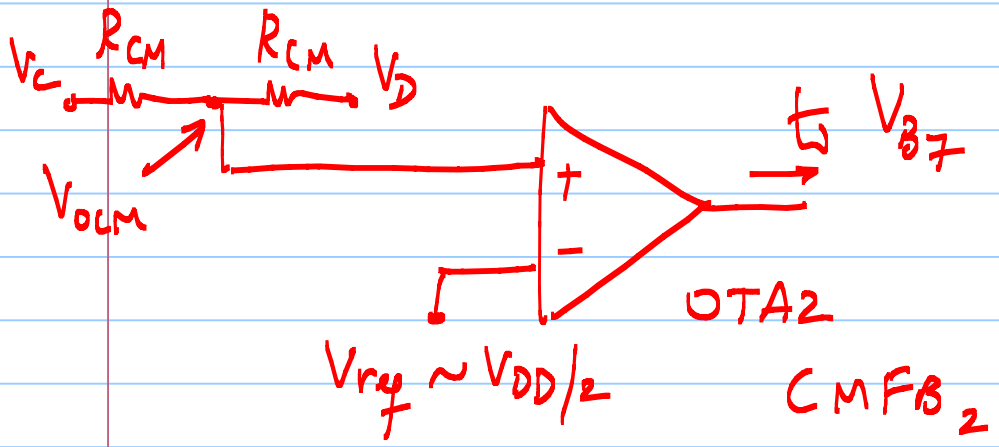


$$V_{OCM_1} = \frac{V_A + V_B}{2}$$

$$V_{ref_1} = V_{DD} - V_{SG_4}/I_4$$

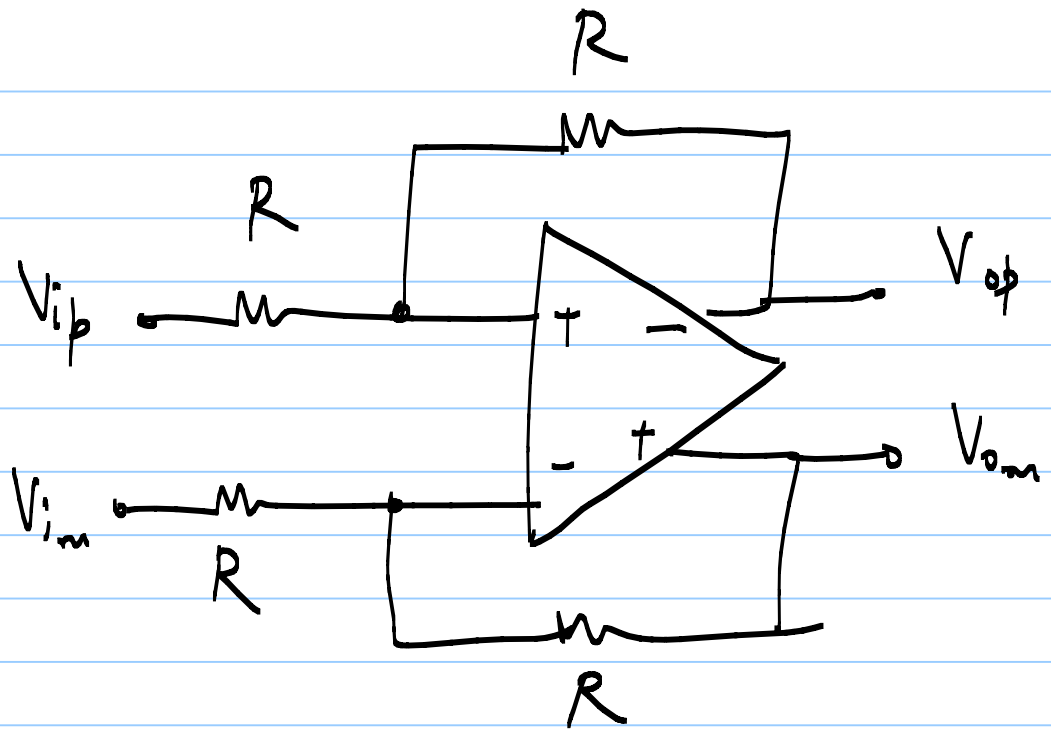
set $V_{ref_1} = V_{DD} - V_{SG_5}/I_5$ by design

$$\frac{I_4}{(W/L)_4} = \frac{I_5}{(W/L)_5}$$

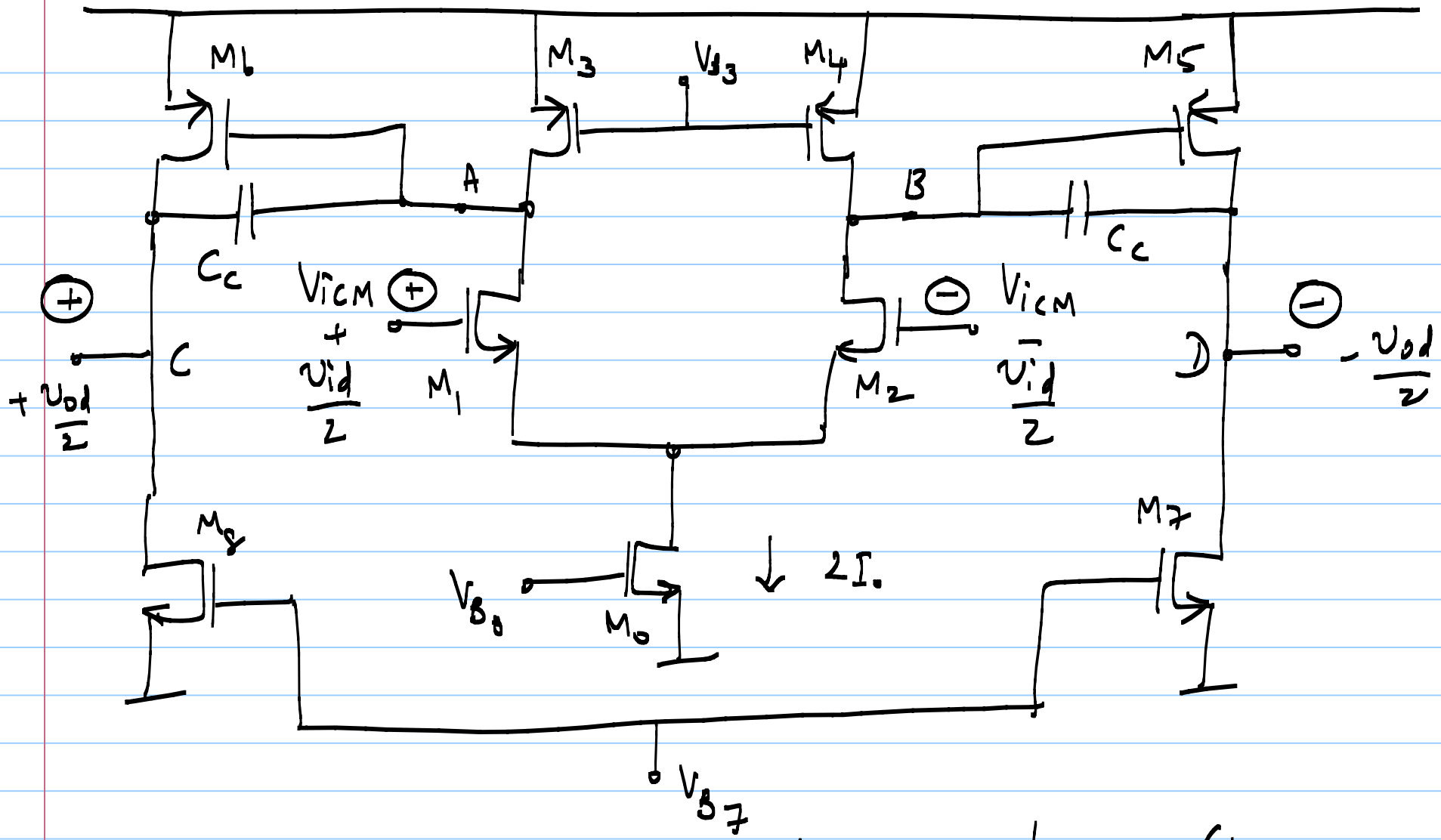


C_{CM_1} – compensatin cap for CMFB loop 1

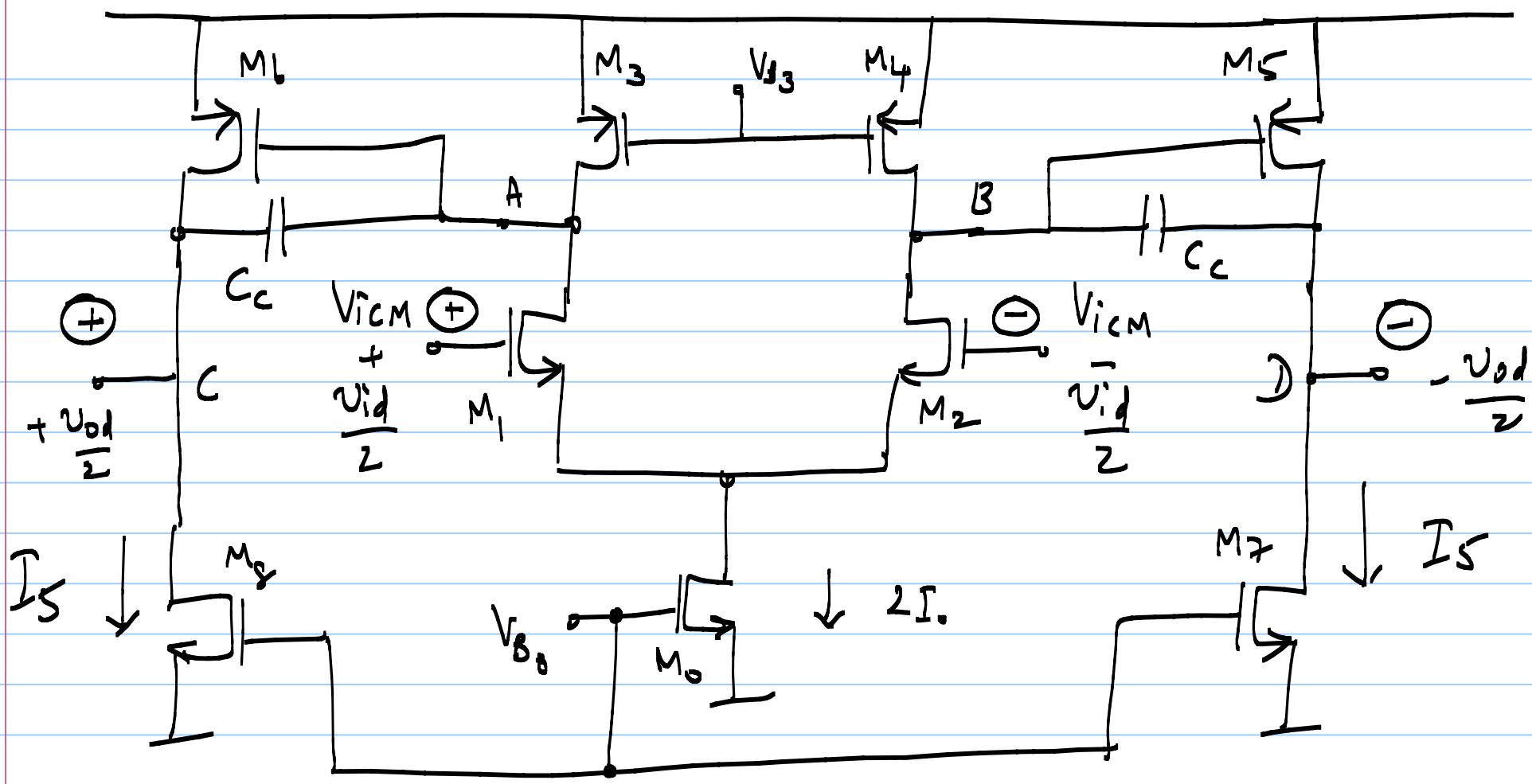
C_{CM_2} – compensatin cap fw CMFB loop 2

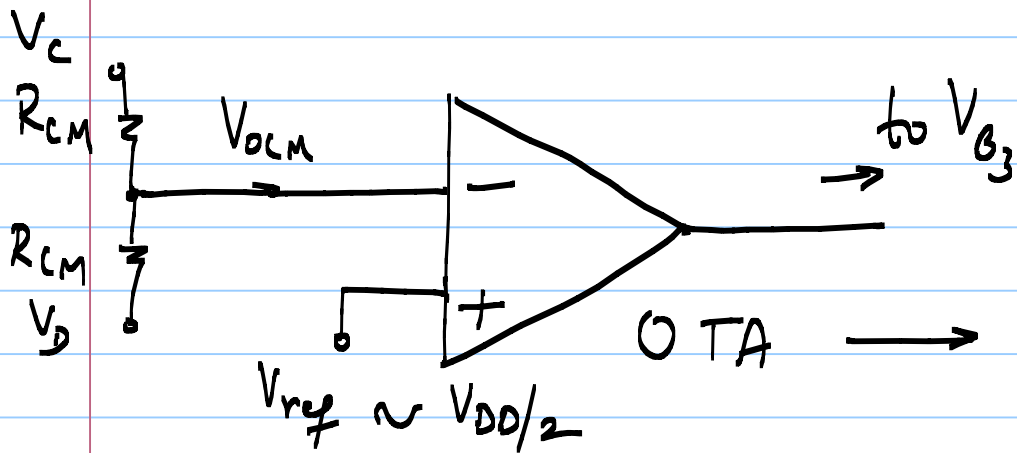

R
R
V_{ip}
+
−
V_{op}
V_{im}
−
+
V_{om}
R
R

METHOD #2



* M_5 & M_6 — pseudo diff. amplifier (has CM gain)
* Changing V_{OCM_1} also changes V_{OCM}





3 − stage CMFB loop

OTA ⟶ does not need much gain

* CMFB loop will

→ set $V_{ocm} = \frac{V_C + V_D}{2} = V_{ref}$

→ set $V_{ocm,} = \frac{V_A + V_B}{2} = V_{DD} - V_{SG_5}|_{I_5}$

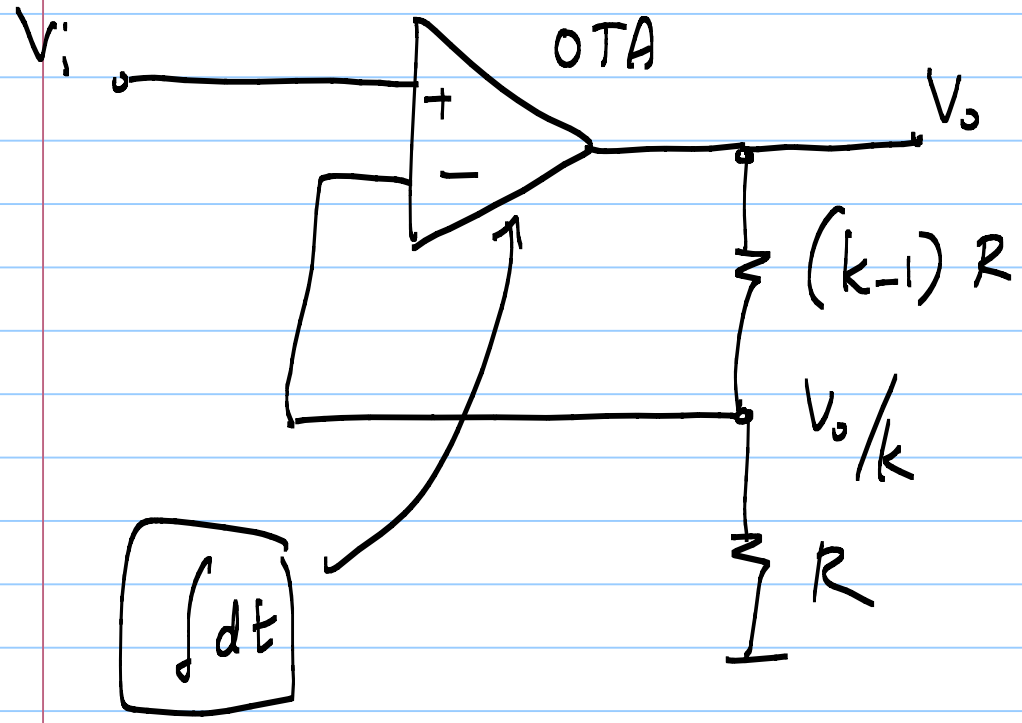
* C_c (compensation cap of FD opamp)

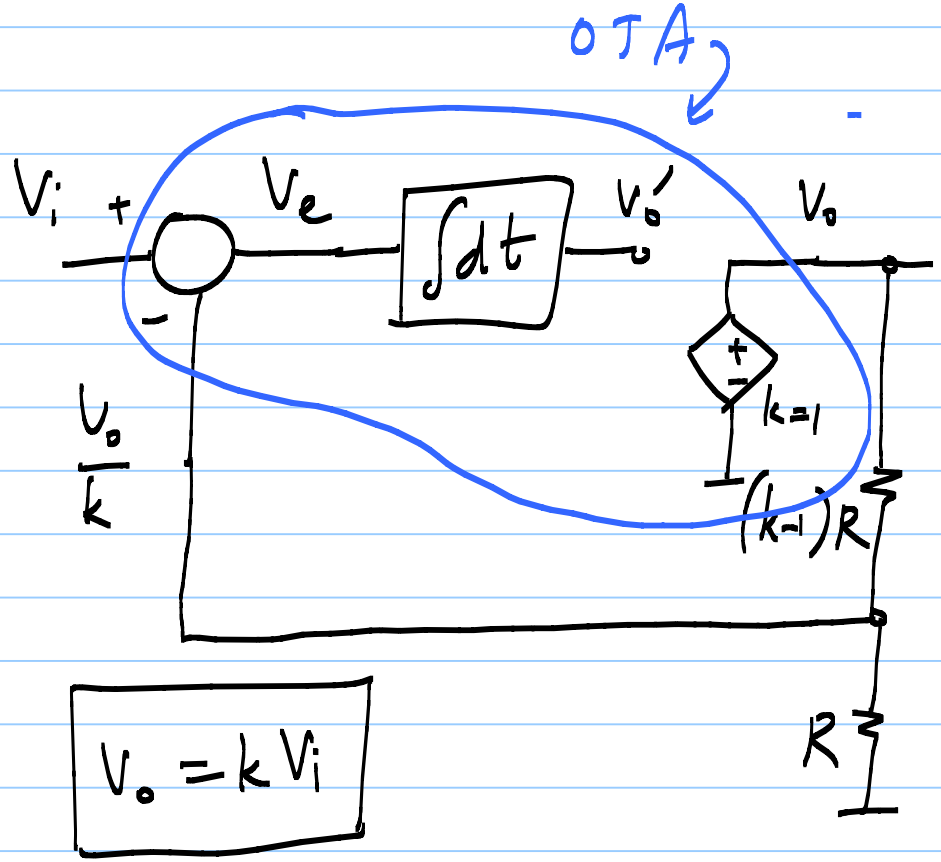
can also be designed to compensate the 3- stage CMFB loop, in some cases.

Phase Locked Loops (PLLs)

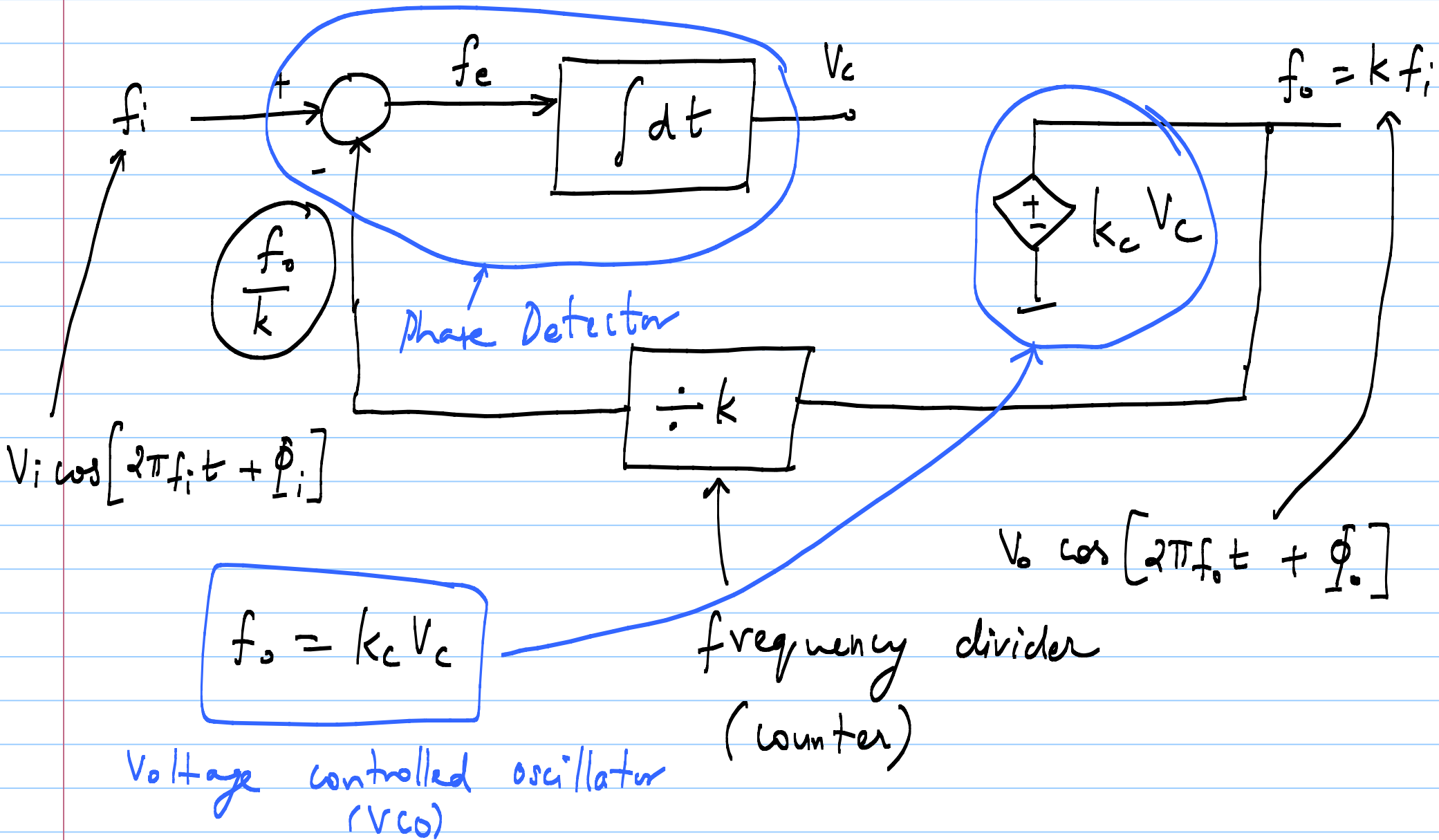
phase is being set via a negative f.b. loop

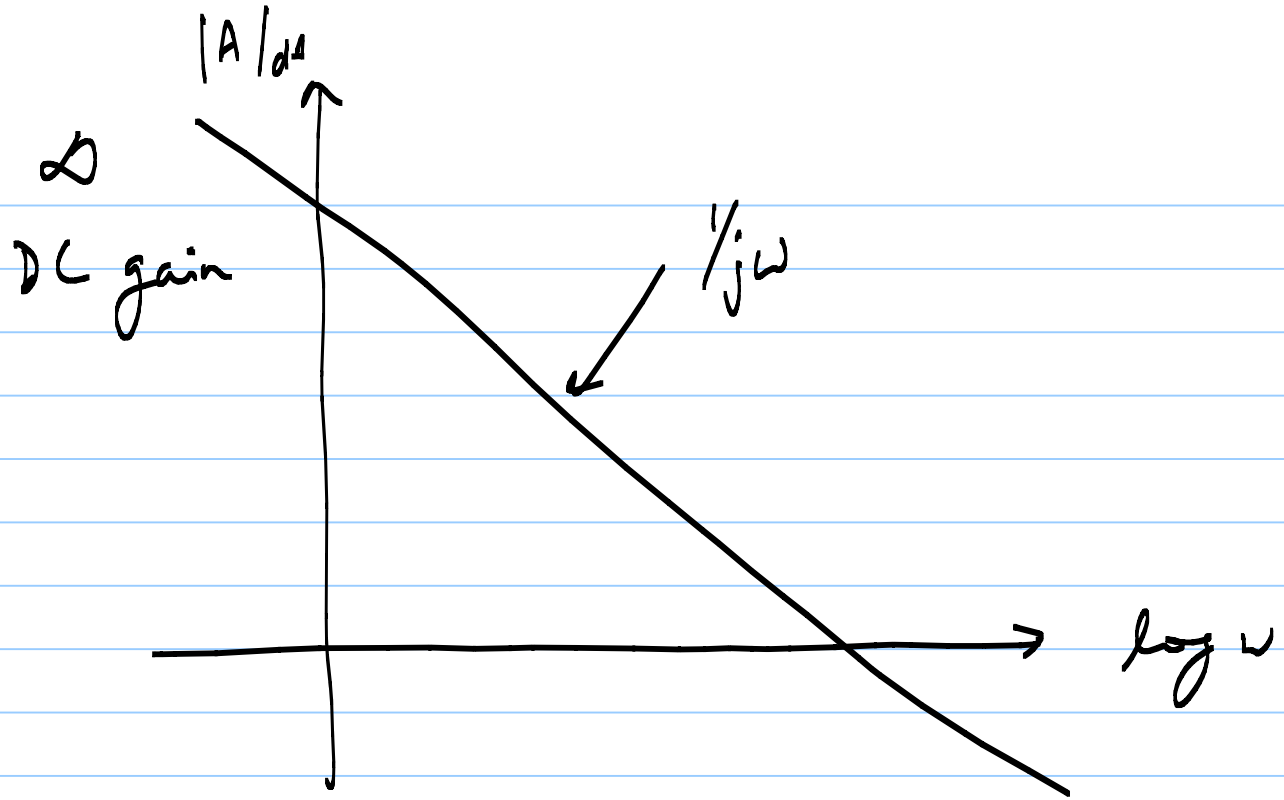
Voltage Amplifier of gain k

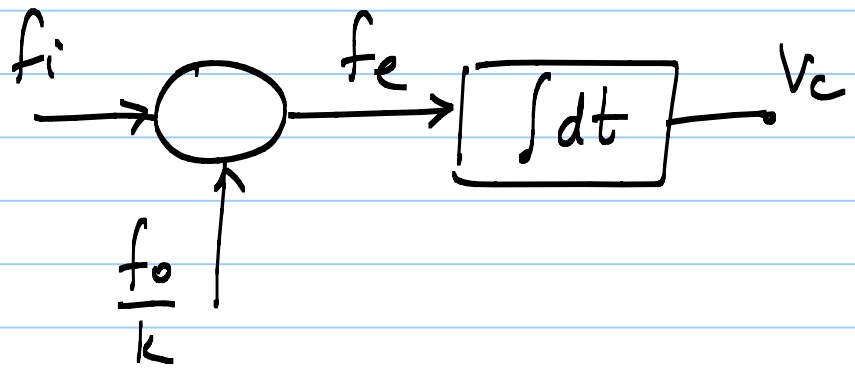




Frequency multiplier of gain k (k = integer)

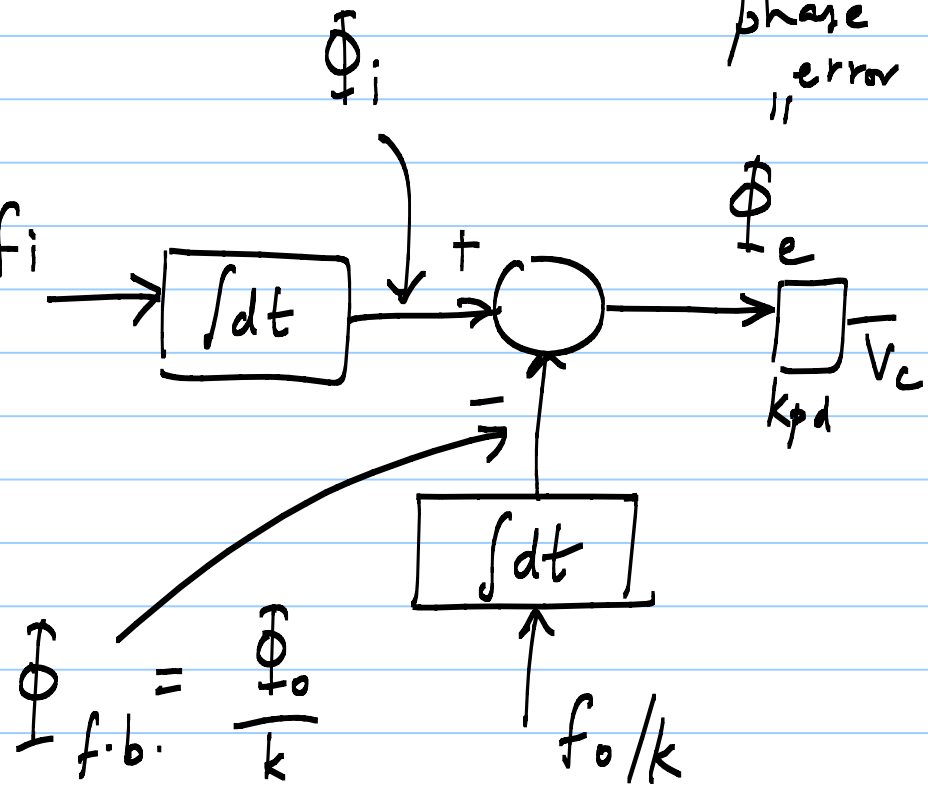




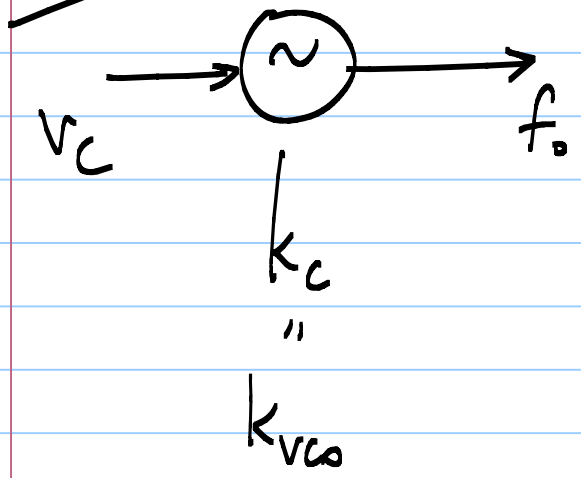


"Phase Detector"

≡



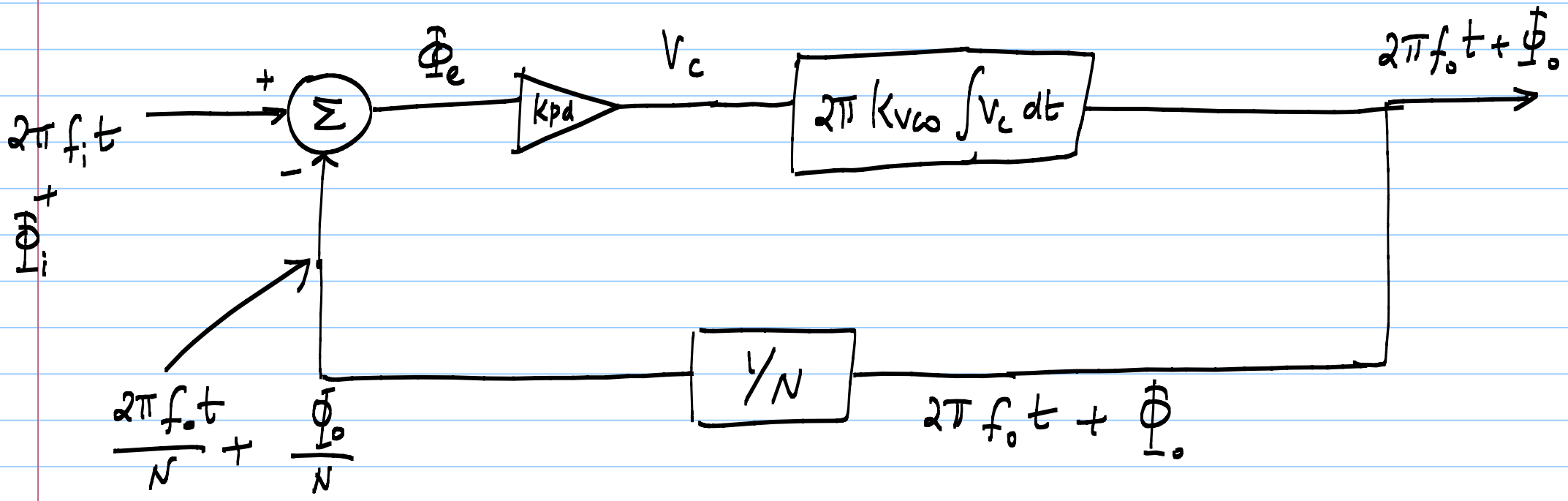
VCO



$\cos \omega_0 t = \cos(2\pi f_0 t)$

f_0 linearly related to V_c

PLL model (gain = N) in the "phase domain"



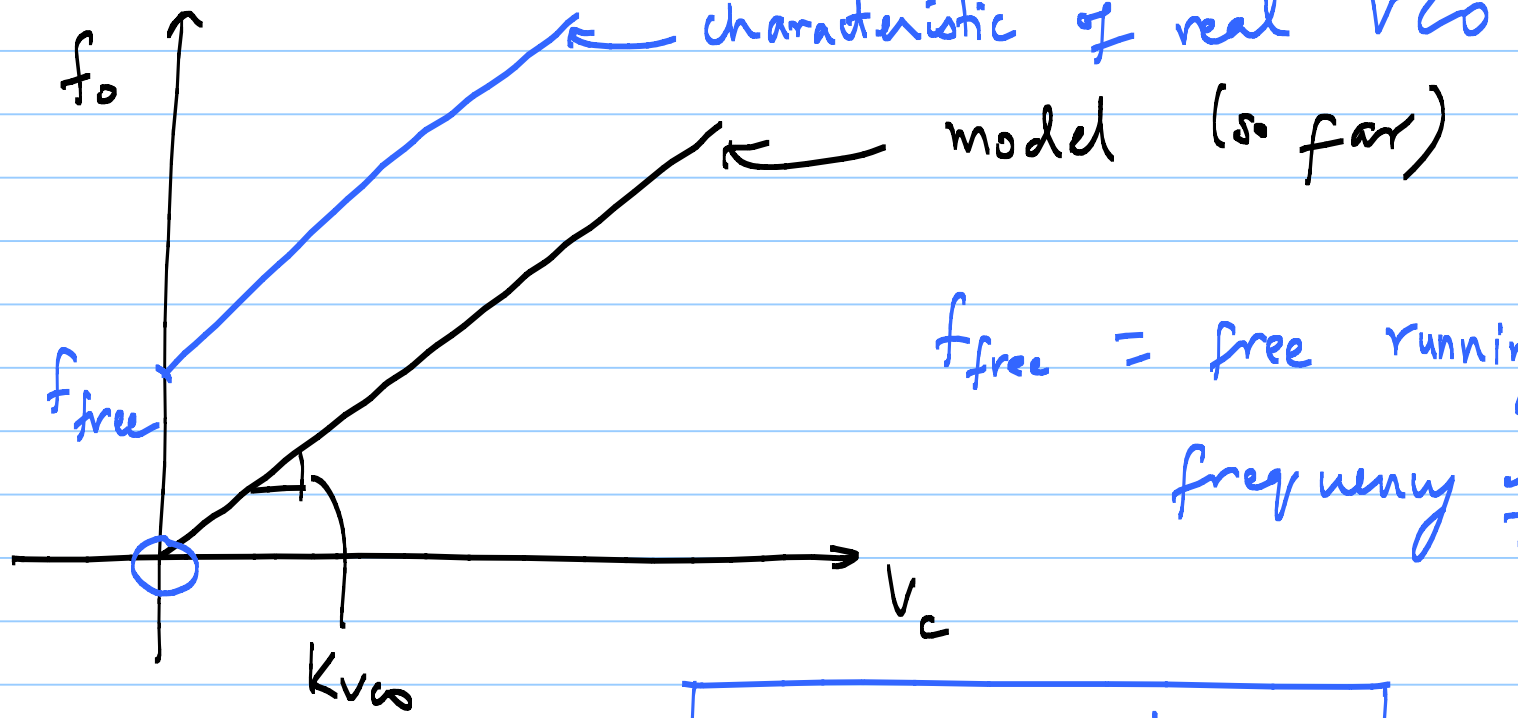
VCO model



$$\cos(\omega_o t) = \cos(2\pi f_o t)$$

$$f_o = k_{vco} \cdot V_c \quad ; \quad \omega_o = 2\pi K_{vco} \cdot V_c$$

units of MHz/V



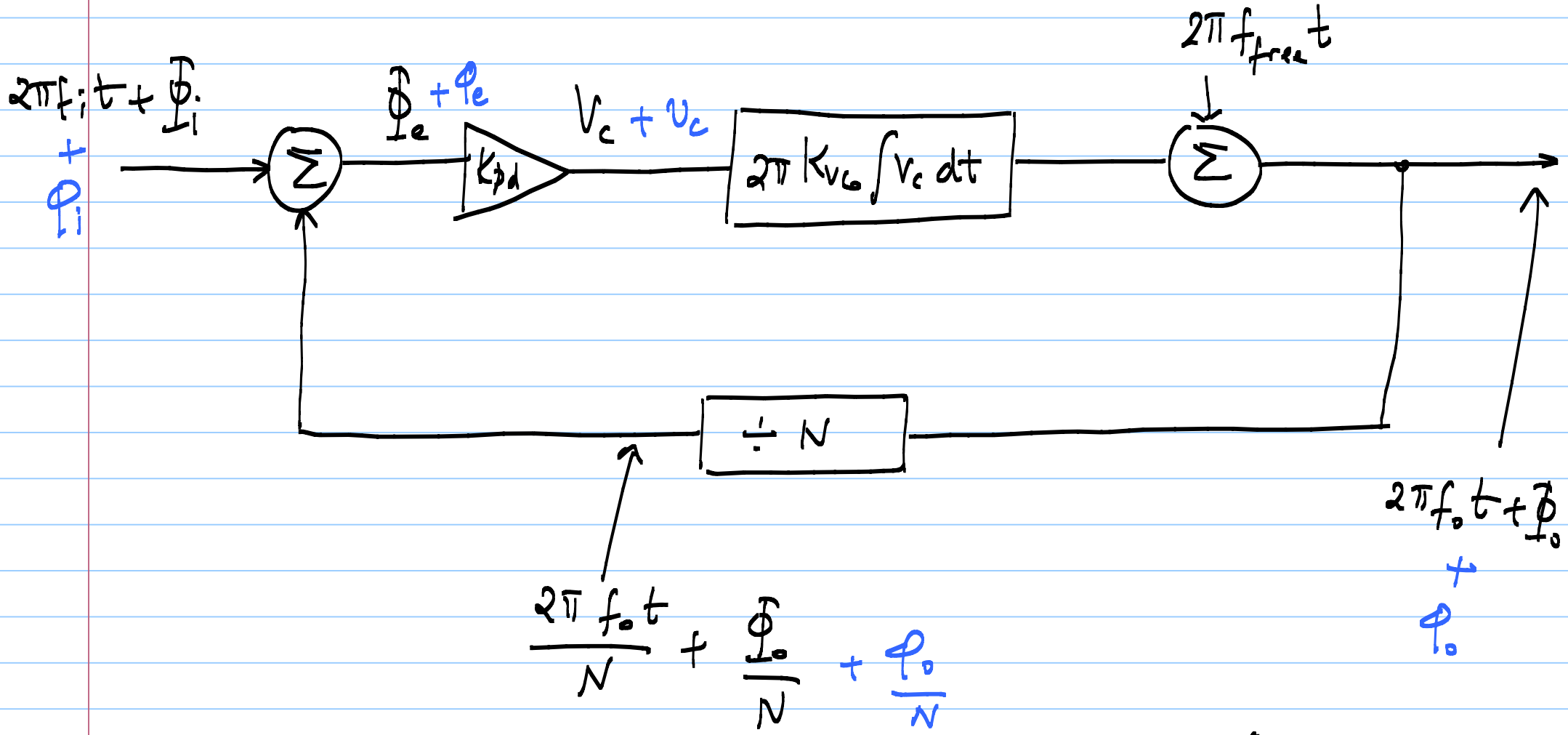
← characteristic of real VCO

← model (so far)

f_{free} = free running frequency of VCO

actual model of VCO : $\boxed{f_o = f_{free} + K_{vco} \cdot V_c}$

Phase domain model



* Quantities in black – 'operating point' quantities
* Now assume an incremental input is applied to PLL input = φ_i

In steady state (before ϕ_i is applied):

$$f_o = f_i \cdot N$$

$$V_c = K_{pd} \cdot \Phi_e$$

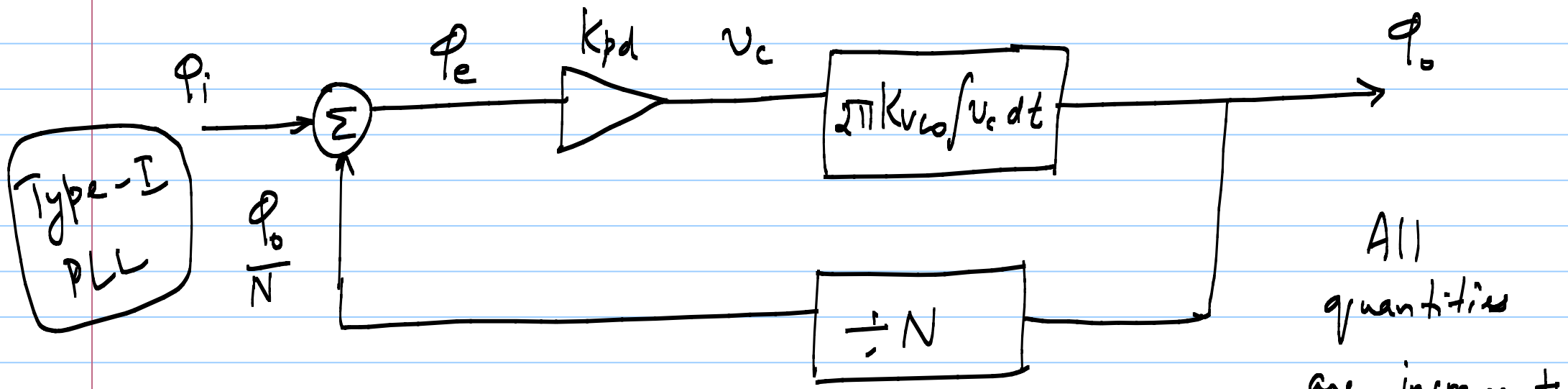
$$= K_{pd} \left[\left(2\pi f_i t + \Phi_i \right) - \underbrace{\left(\frac{2\pi f_o t}{N} + \frac{\Phi_o}{N} \right)}_{\phi_{fb}} \right]$$

$$= K_{pd} \left[\Phi_i - \frac{\Phi_o}{N} \right]$$

$$f_o = f_{free} + K_{vco} \cdot V_c = N f_i$$

$$V_c = \frac{N f_i - f_{free}}{K_{vco}}$$

Incremental PLL model in phase domain



* Loop gain in s-domain
* Closed loop gain in s-domain
* step response in t-domain
* BW — -3 dB
  — unity gain
* order of system
* Stability

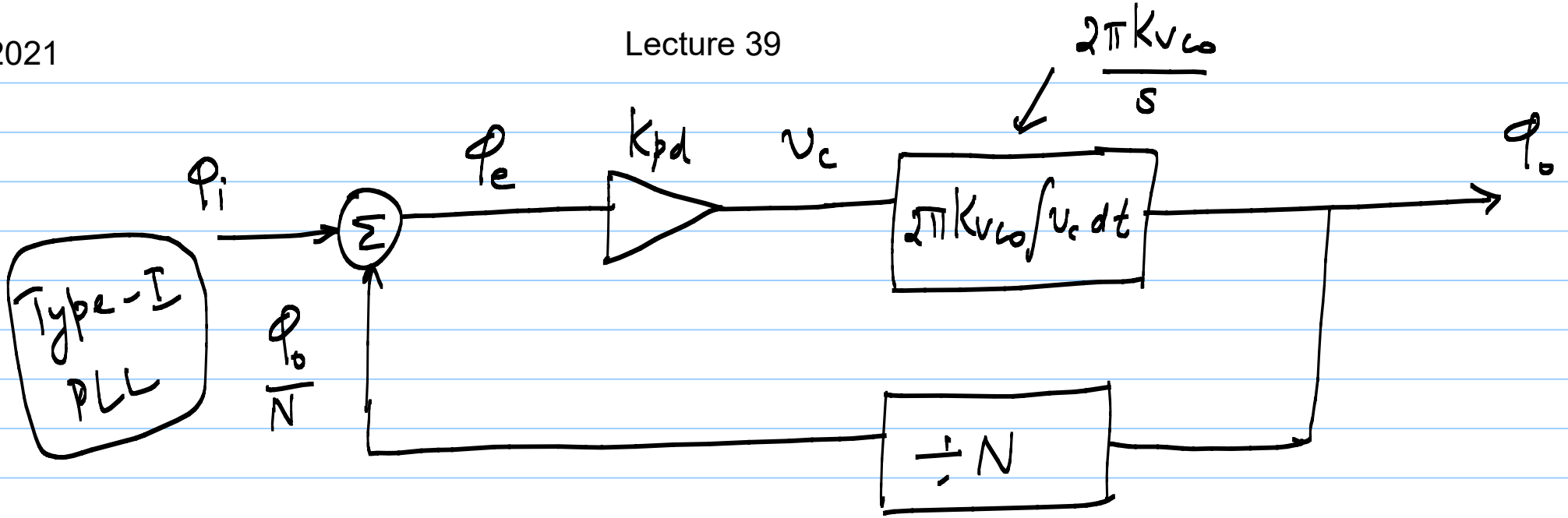
HW: 1) $LG(s) = ?$

2) $CLG(s) = \frac{\varphi_o}{\varphi_i}(s) = ?$

3) $UGF = ?$

4) $f_{-3dB} = ?$

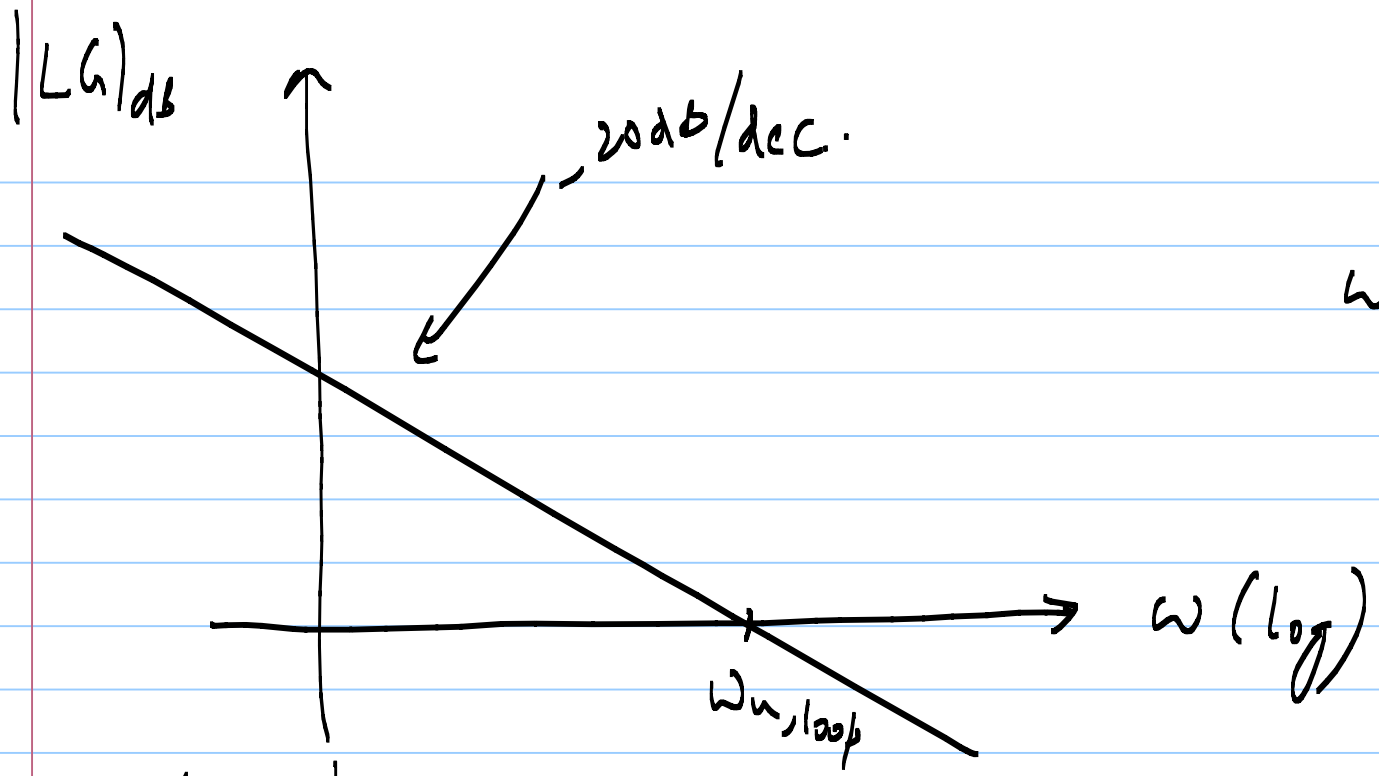
5) Bode plots of $LG(s)$ & $CLG(s)$



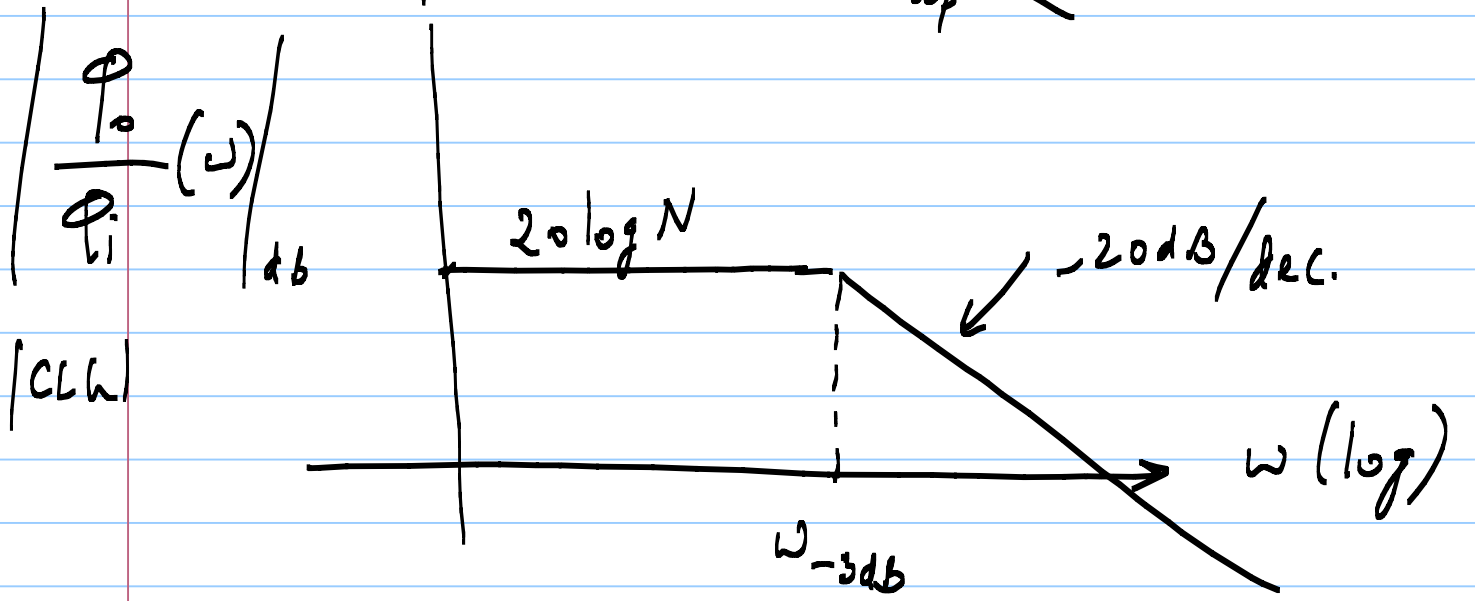
1) Loop gain $LG(s) = \dfrac{2\pi K_{pd} K_{VCO}}{Ns}$

2) Closed loop gain $\dfrac{\varphi_o}{\varphi_i}(s) = N \dfrac{1}{1 + \dfrac{Ns}{2\pi K_{pd} K_{VCO}}}$

3) Closed loop BW $\omega_{-3dB} = \dfrac{2\pi K_{pd} K_{VCO}}{N}$; $f_{-3dB} = \dfrac{K_{pd} K_{VCO}}{N}$



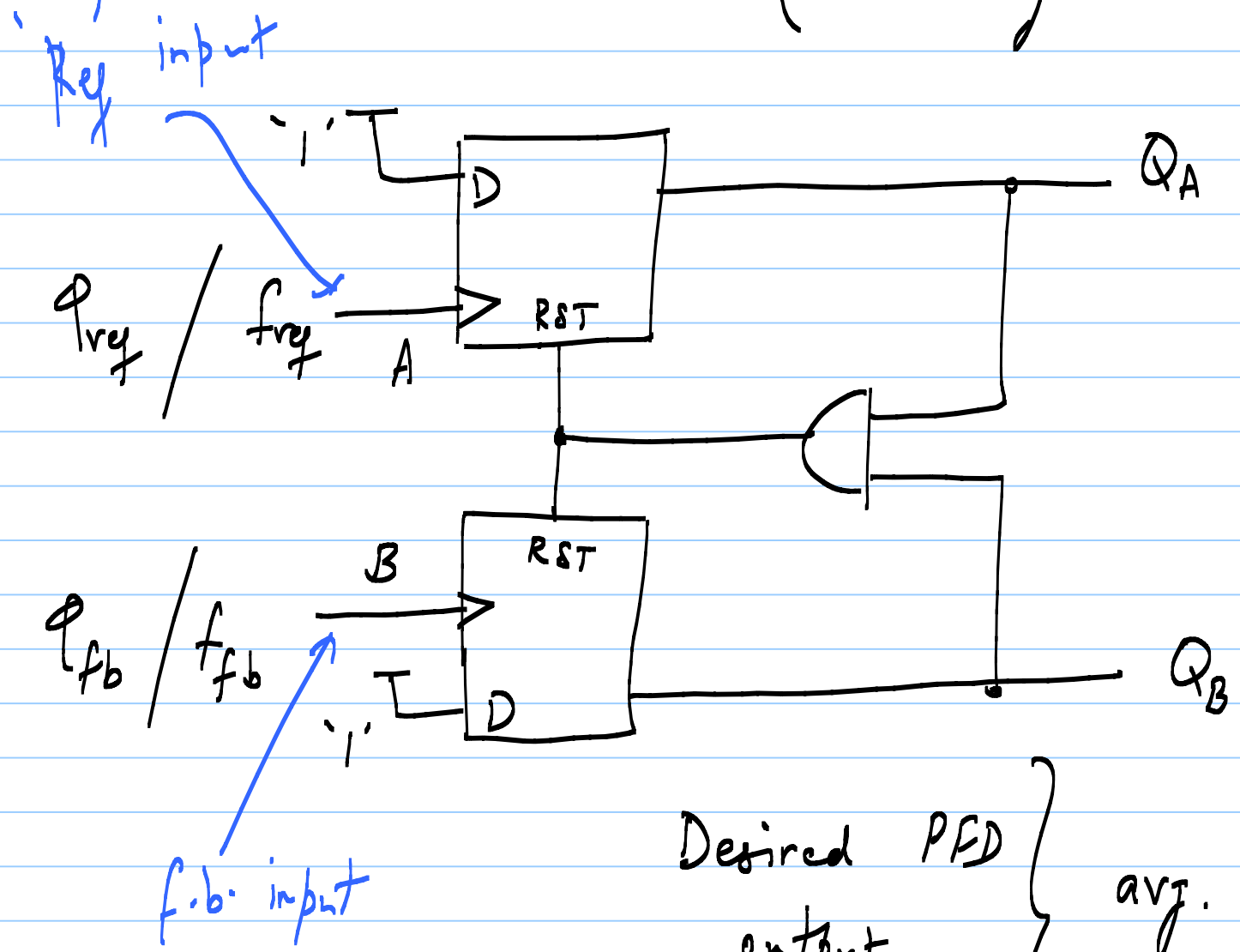
$$\omega_{u,loop} = \frac{2\pi K_{pd} K_{vc}}{N}$$



HW:
step response

Phase detector (Tristate PFD or Phase-frequency Detector)

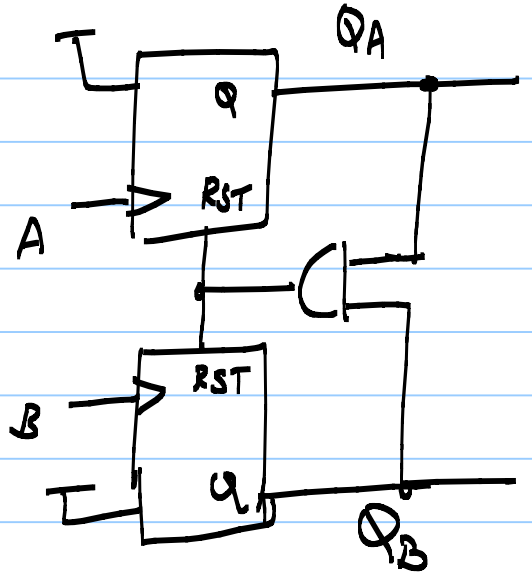
(Commonly used PD)



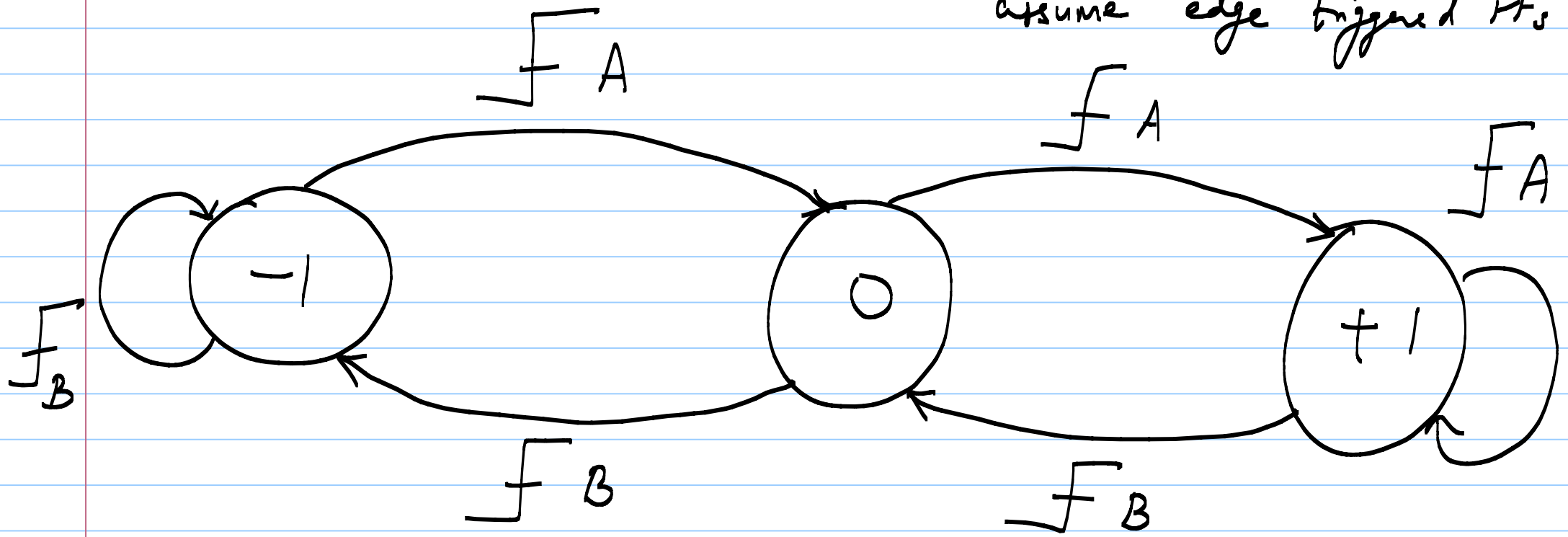
Desired PFD output

avg. $(Q_A - Q_B)$ in Volts

Q_A	Q_B	State
0	0	0
0	1	−1
1	0	+1
1	1	Reset (prohibited)

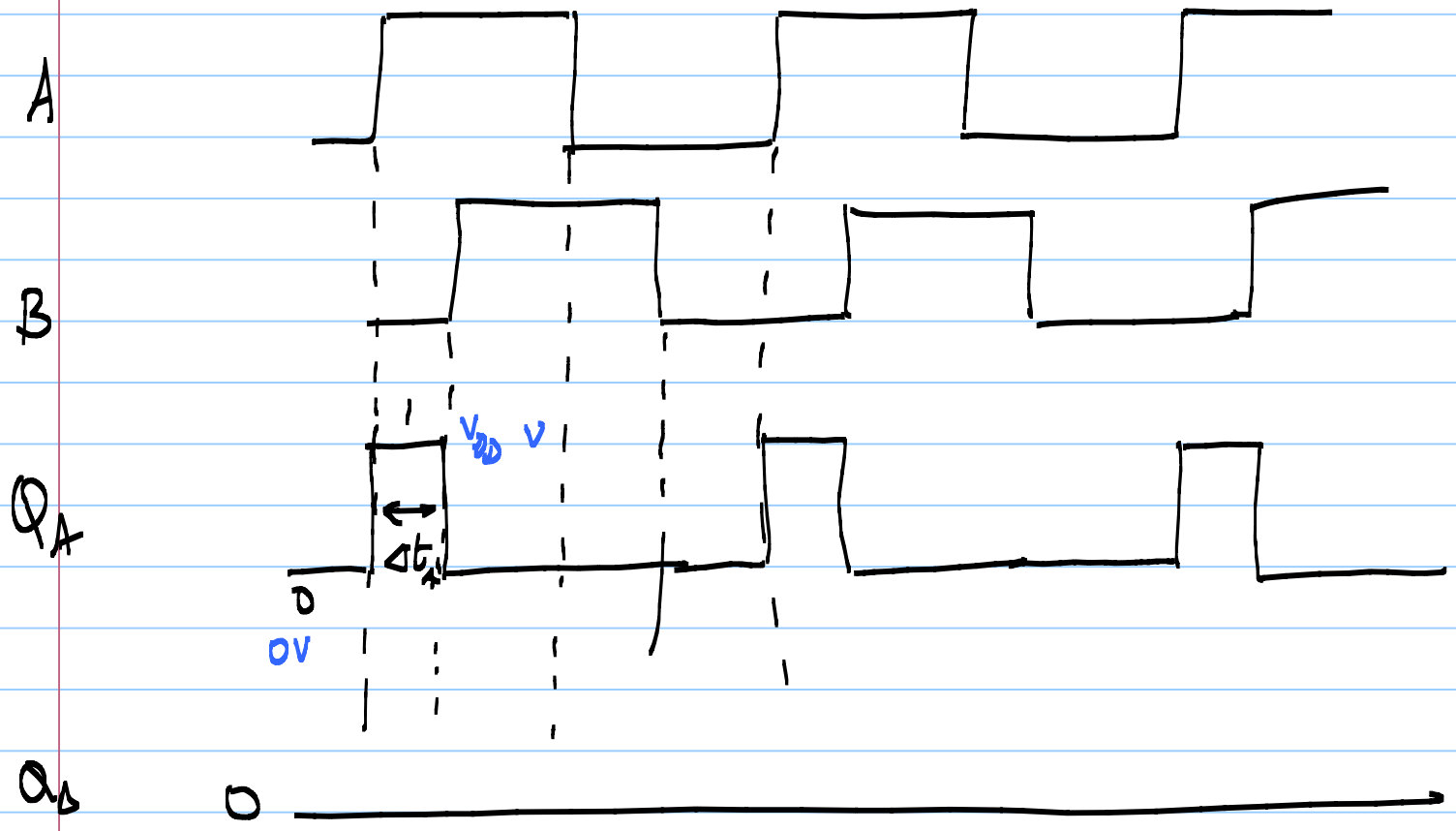


assume edge triggered FFs



* A & B - square wave inputs ; same freq. - diff. phase
  +ve edge triggered DFFs

1) A leading B

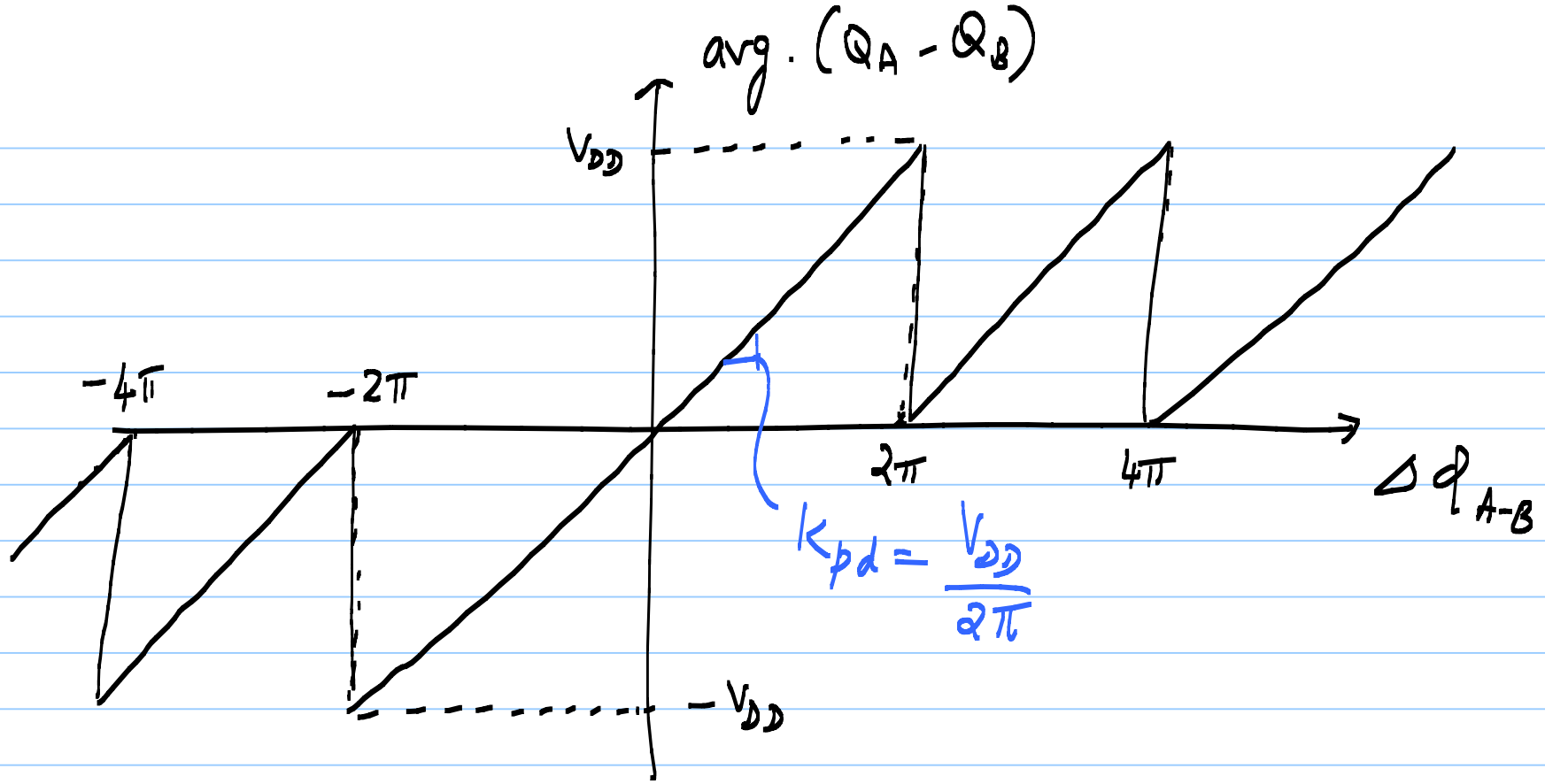


$\Delta t_A \propto \Delta \varphi_{A-B}$

If B were leading A :

$\Delta t_B \propto \Delta \varphi_{B-A}$

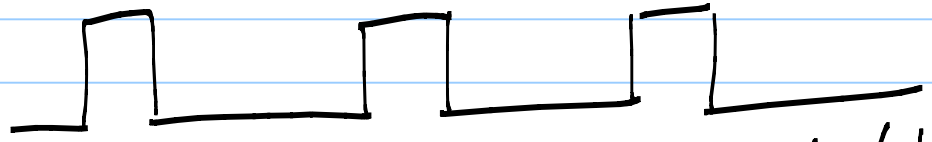
avg. $(Q_A - Q_B)$ is $\propto \Delta \varphi_{A-B}$



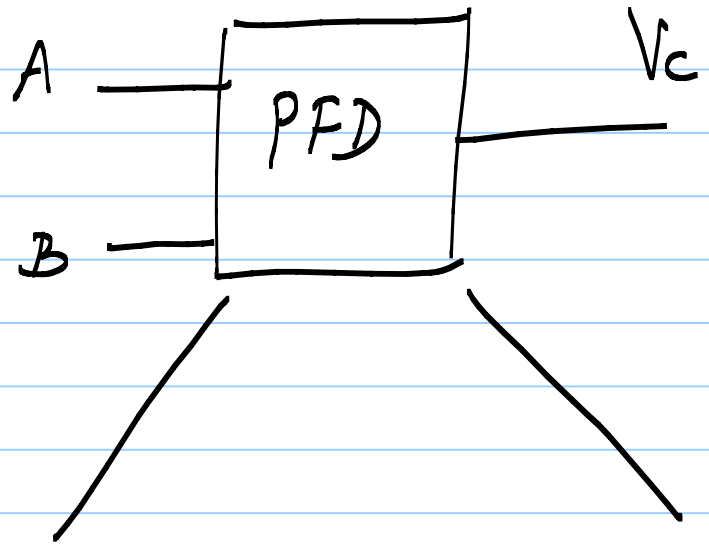
* If $\phi_A = \phi_B \Rightarrow \Delta t = 0 \Rightarrow$ avg. $(Q_A - Q_B) = 0$

* If $\phi_A = \phi_B + 2\pi$ (almost) $\Rightarrow \Delta t = T_{period} \Rightarrow$ avg. $(Q_A - Q_B) \rightarrow V_{DD}$

* $(Q_A - Q_B)$

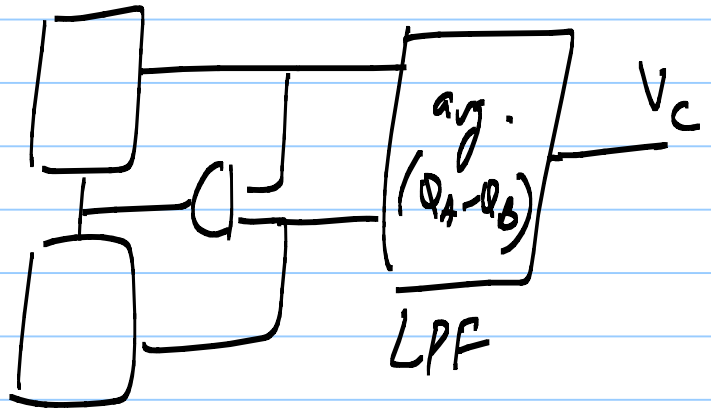


→ dc value + fundamental $\left(\frac{1}{T}\right)$ + harmonics of $\frac{1}{T}$



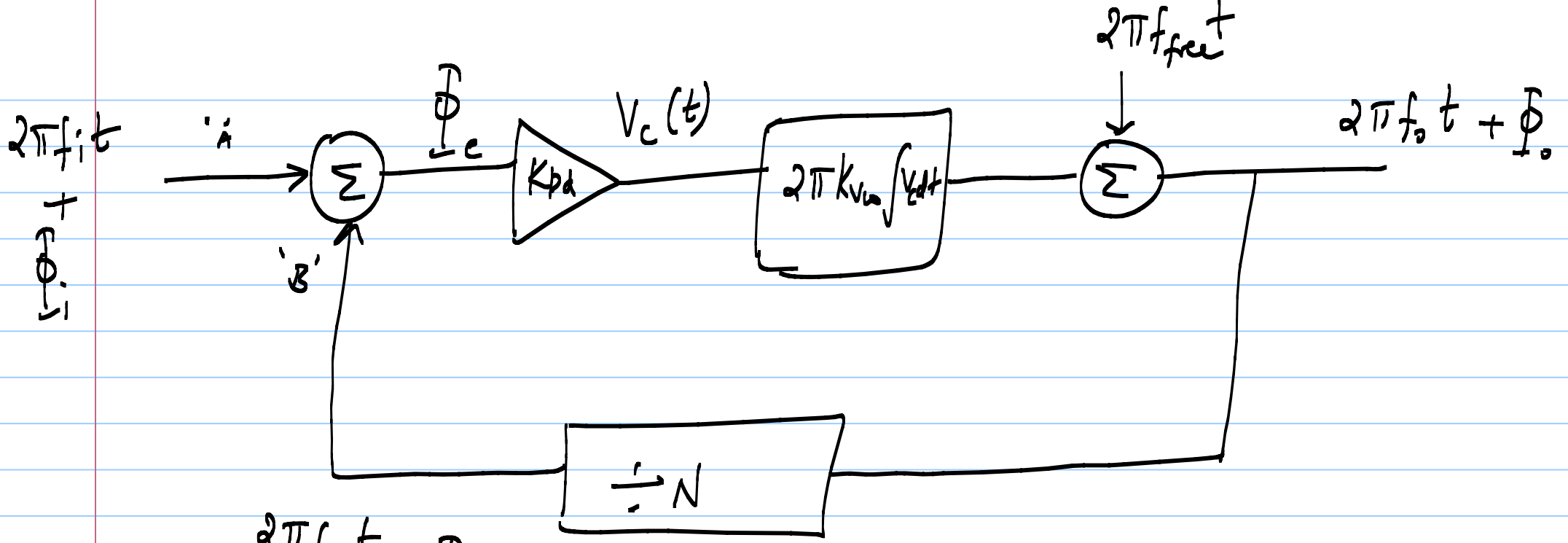
$$K_{pd} = \frac{V_c}{\Delta\Phi_{A-B}} = \frac{V_{DD}}{2\pi}$$

* operates in phase & freq. simultaneously



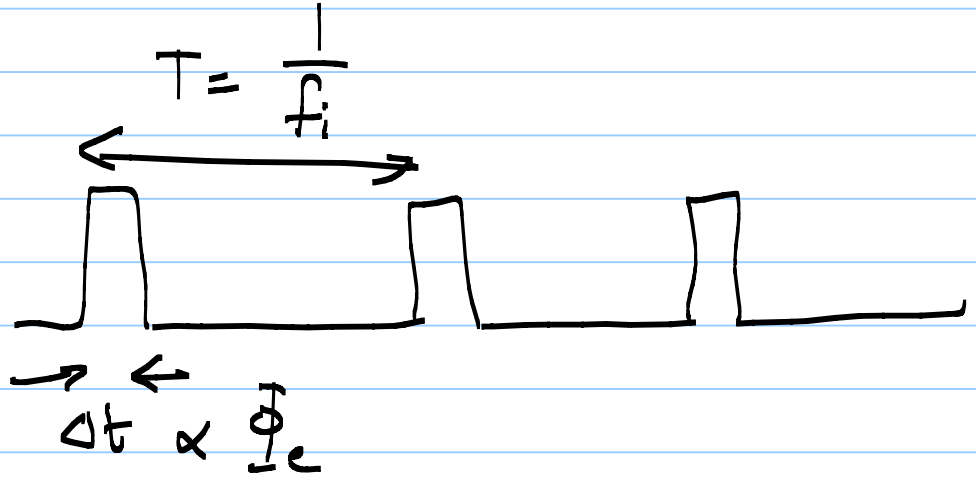
* settles to operating point values of V_c, Φ_e, Φ_o, f_o

@ $f_o = N f_i$



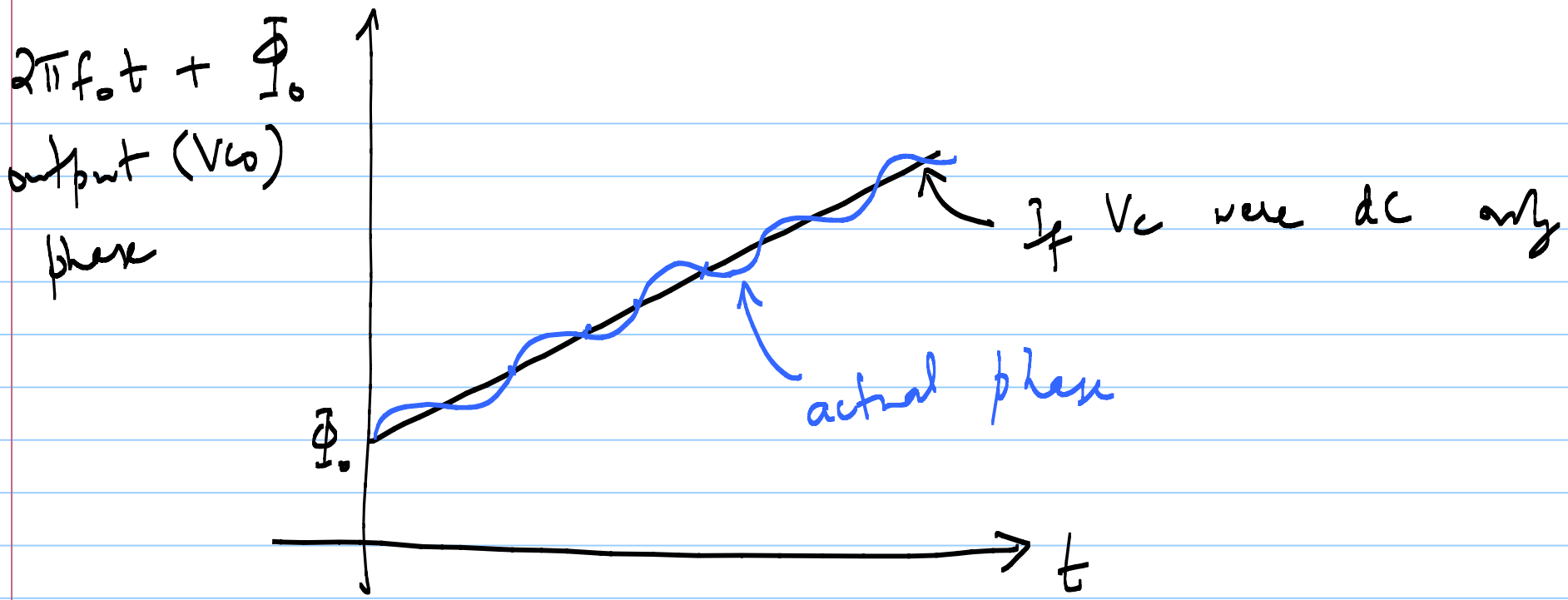
In steady state:

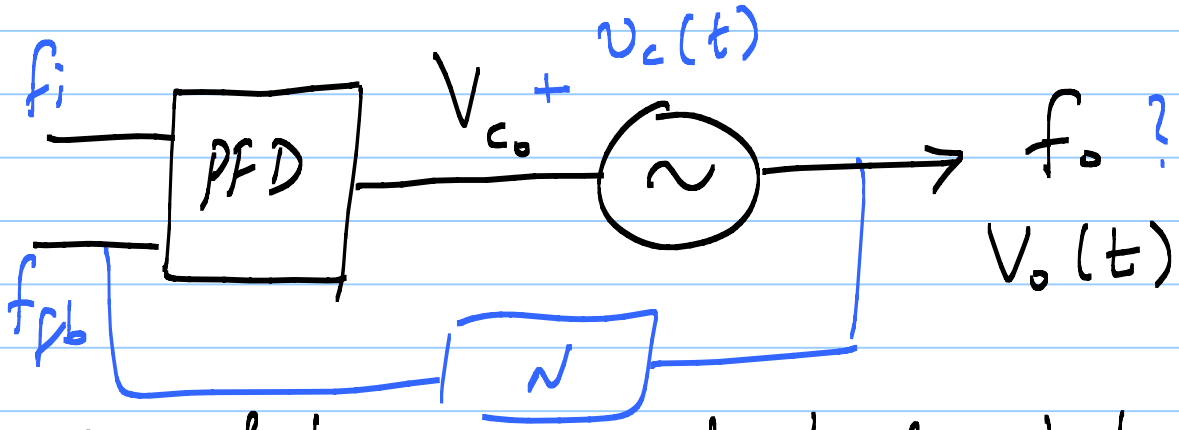
$Q_A - Q_B$ looks like:



$$V_c(t) = \underbrace{V_{c0}}_{(dc)} + \text{fund. @ } f_i + \text{harmonics of } f_i$$

$v_{c_1}(t)$ $\quad$ $v_{c_i}(t)$


$2\pi f_o t + \Phi_o$
output (V_{co})
phase
Φ_o
t
If V_c were dc only
actual phase



* let us understand behaviour of VCO with only fundamental of $v_c(t)$

$$v_c(t) = b_0 \cos(2\pi f_i t)$$

(b_0 small)

$$f_o = f_{free} + K_{vco} V_c(t)$$

$$= f_{free} + K_{vco} \cdot V_{c_0} + K_{vco} v_c(t)$$

Frequency modulation (FM)

output voltage signal

$$V_o(t) = \cos\left[2\pi N f_i\, t + b_1 \sin(2\pi f_i\, t)\right]$$

$$= \cos(2\pi N f_i\, t)\cos\left(b_1 \sin(2\pi f_i\, t)\right)$$

$$\cos\left(b_1 \sin(2\pi f_i\, t)\right) \approx 1$$

$$- \sin(2\pi N f_i\, t)\sin\left(b_1 \sin(2\pi f_i\, t)\right)$$

If b_1 is small

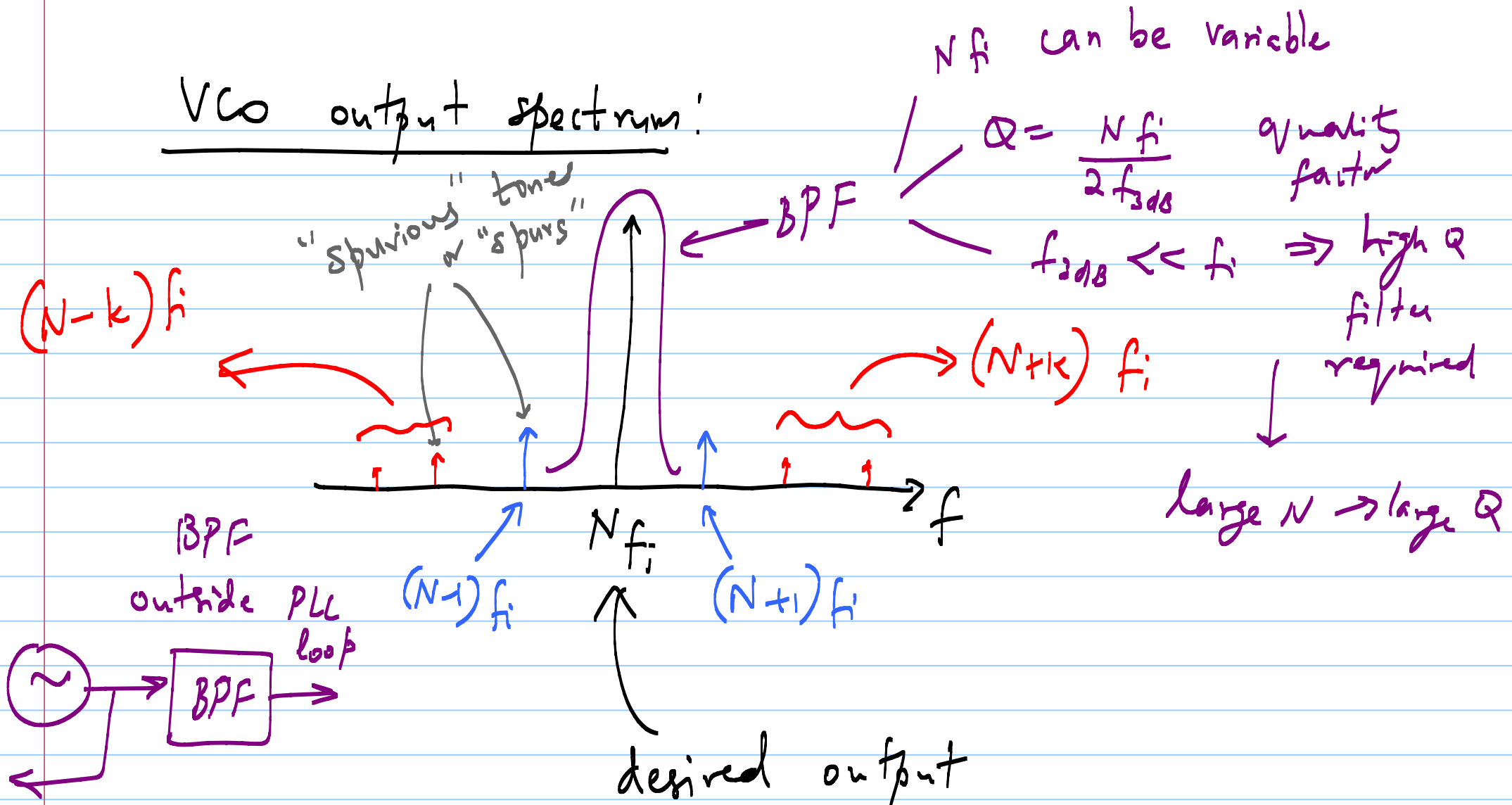
$$\sin\left(b_1 \sin(2\pi f_i\, t)\right) \approx b_1 \sin(2\pi f_i\, t)$$

$$V_o(t) = \cos(2\pi N f_i\, t) - b_1 \sin(2\pi f_i\, t)\sin(2\pi N f_i\, t)$$

problem in wireless comm. systems

$Nf_i + f_i$ $\quad$ $Nf_i - f_i$

VCO output spectrum:



* Reducing $(N \pm k) f_i$ requires an LPF with very low cutoff freq.

→ if BW of LPF < freq ⇒ it affects loop gain etc.

* ↑ order of system affects UGF, stability etc. of the loop.

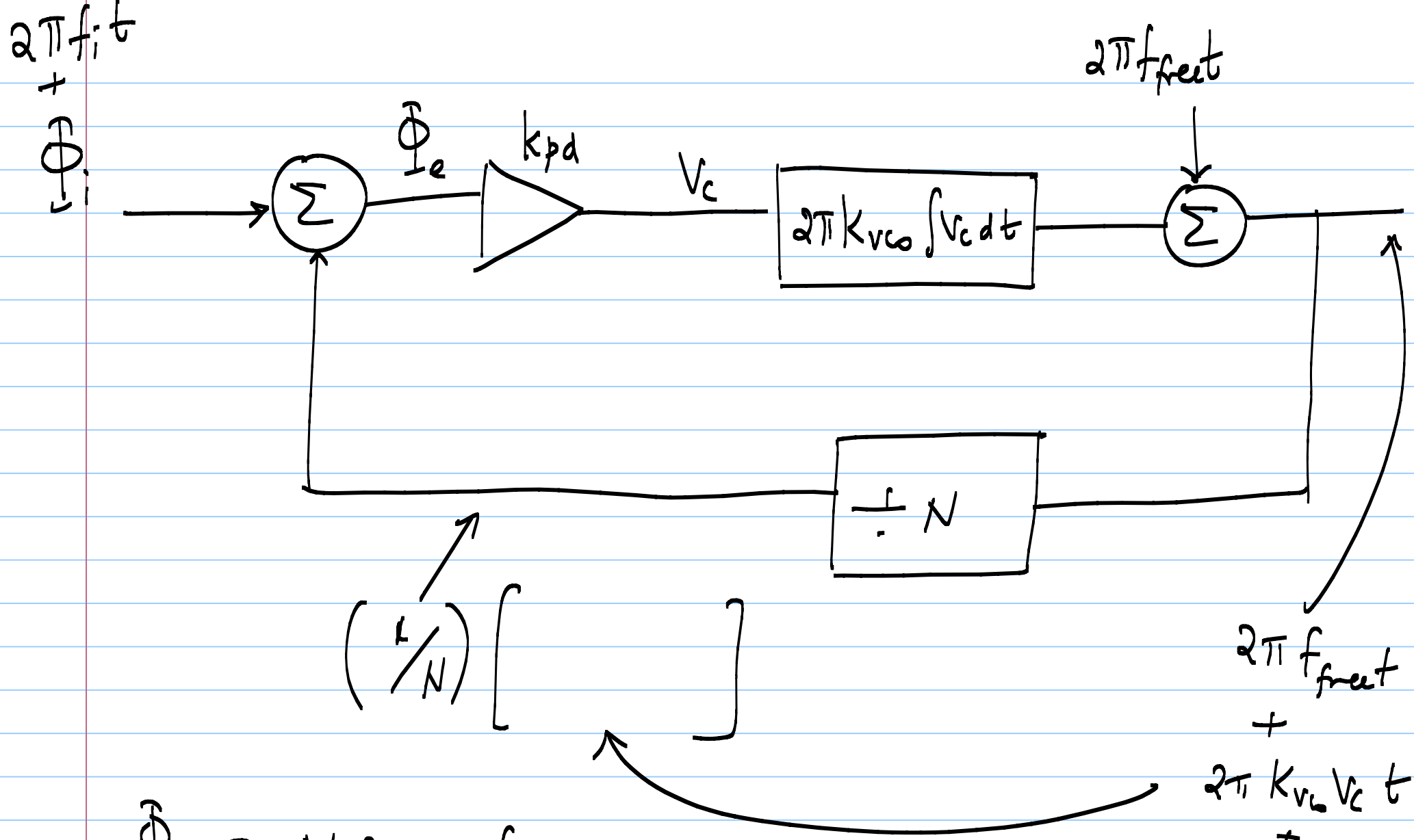
* Type-1 PLL has issues with "reference feedthrough" — ve

  ↳ tradeoff f_i, f_{3dB} & ref. feedthrough

* Type-1 PLL is simple, stable + ve

* Use Type-1 PLL in cases where ref. feedthrough is not an issue.

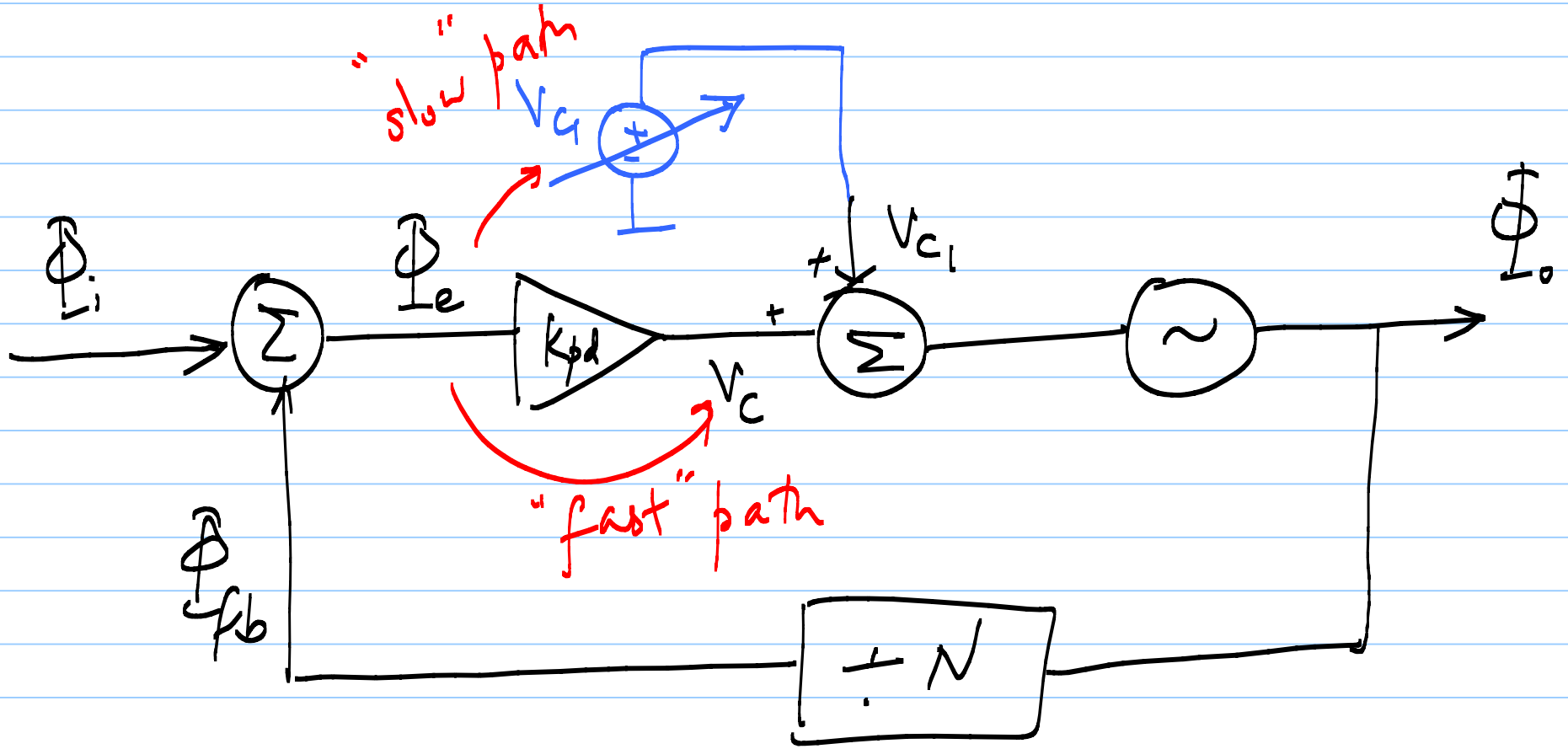
  e.g. clocks for digital systems



$$\Phi_e = \frac{N f_i - f_{free}}{K_{pd} \; K_{vco}}$$

* $\Phi_e = 0$ only when $N f_i = f_{free}$

$\Rightarrow V_c = 0$



* Vary V_{c_1} so that $V_c = 0$ in steady state

* $V_c = 0 \Rightarrow \Phi_e = 0$

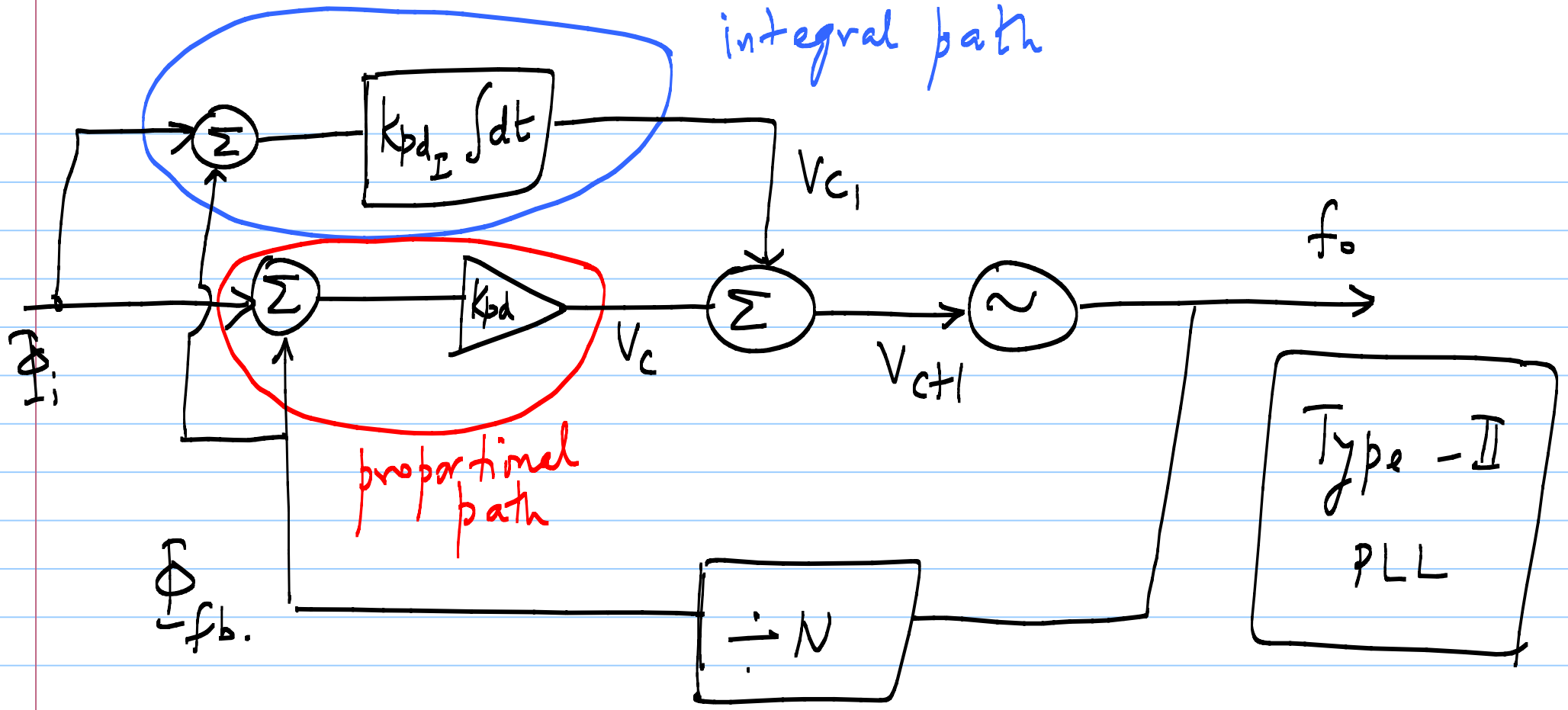
* $V_{c_1} = f(\Phi_e)$



steady state $\Phi_e = 0$

needs to have ∞ dc gain

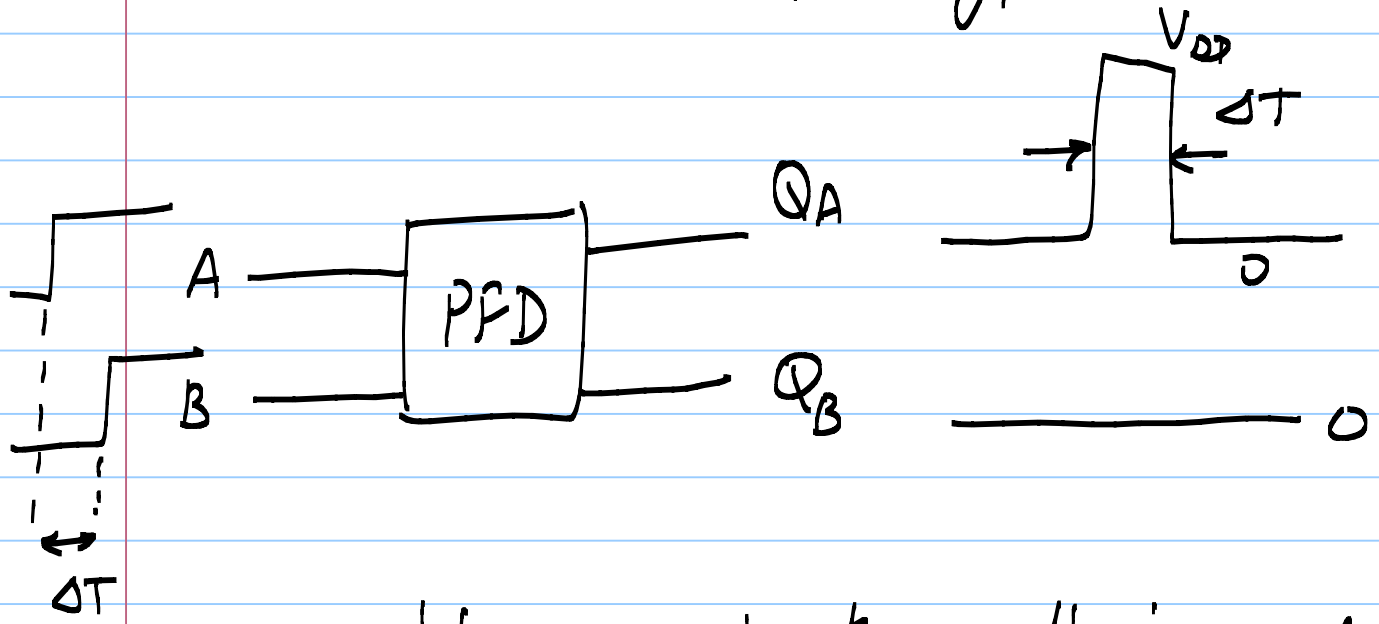
$\Rightarrow$ integrator



In steady state : $V_{C_1} = \dfrac{f_o - f_{free}}{K_{VCO}}$

because : $f_o = \underbrace{(f_{free} + K_{VCO} V_{C_1})}_{f_{free}} + K_{VCO} V_C$

Implementation of Type-II PLL (building blocks)



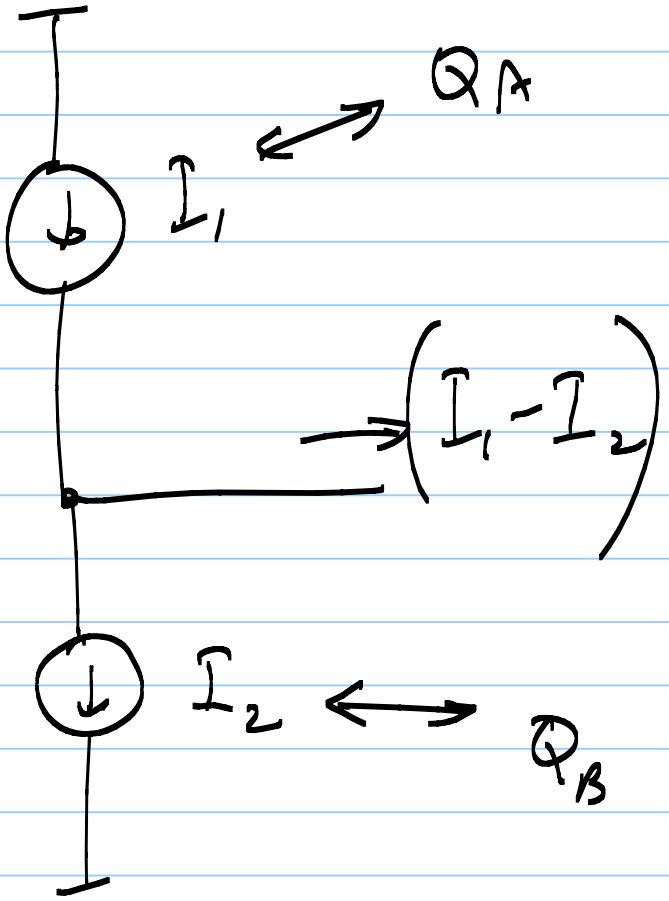
We want the following functionalities:

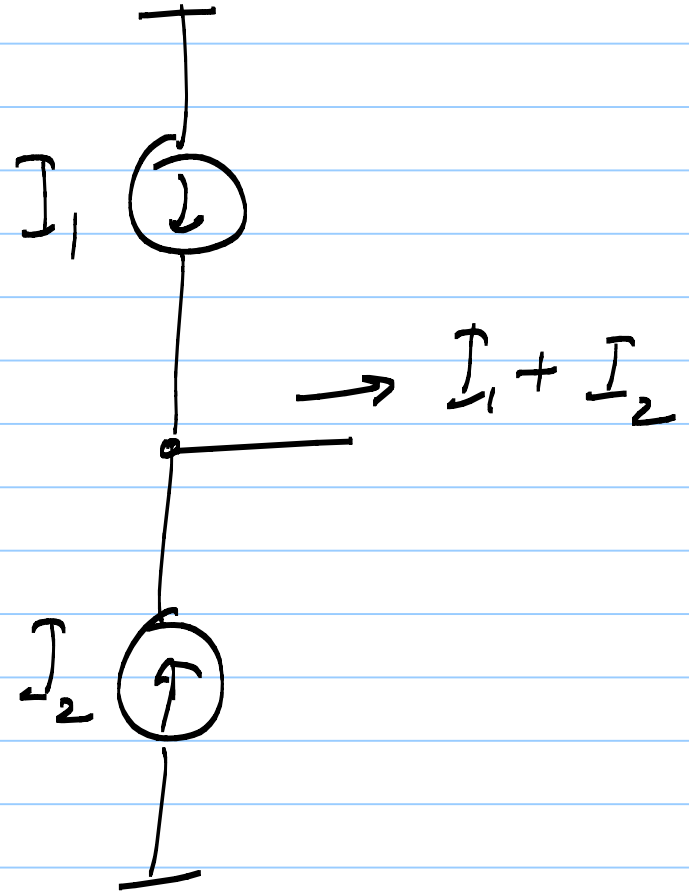
1) $(Q_A - Q_B)$ subtraction ✓

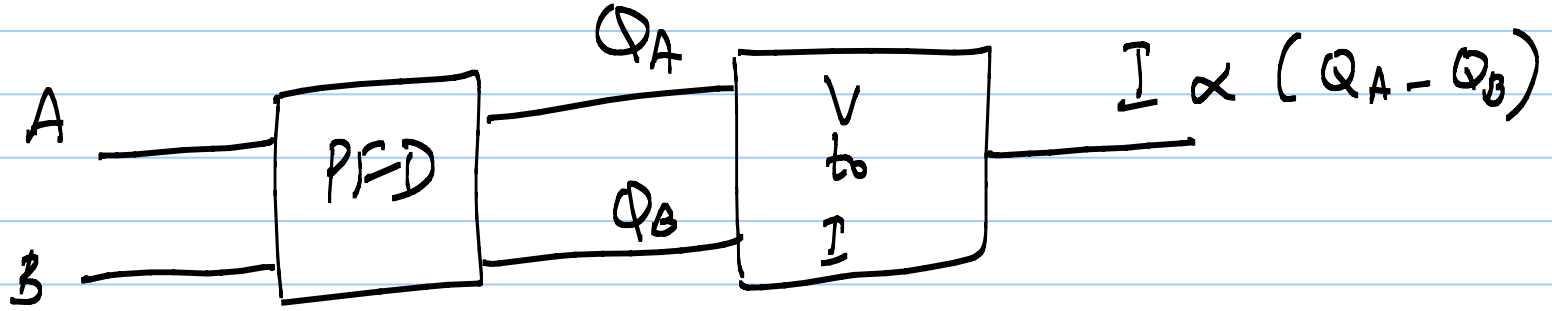
2) K_{pd} & K_{pd_I} paths ✓

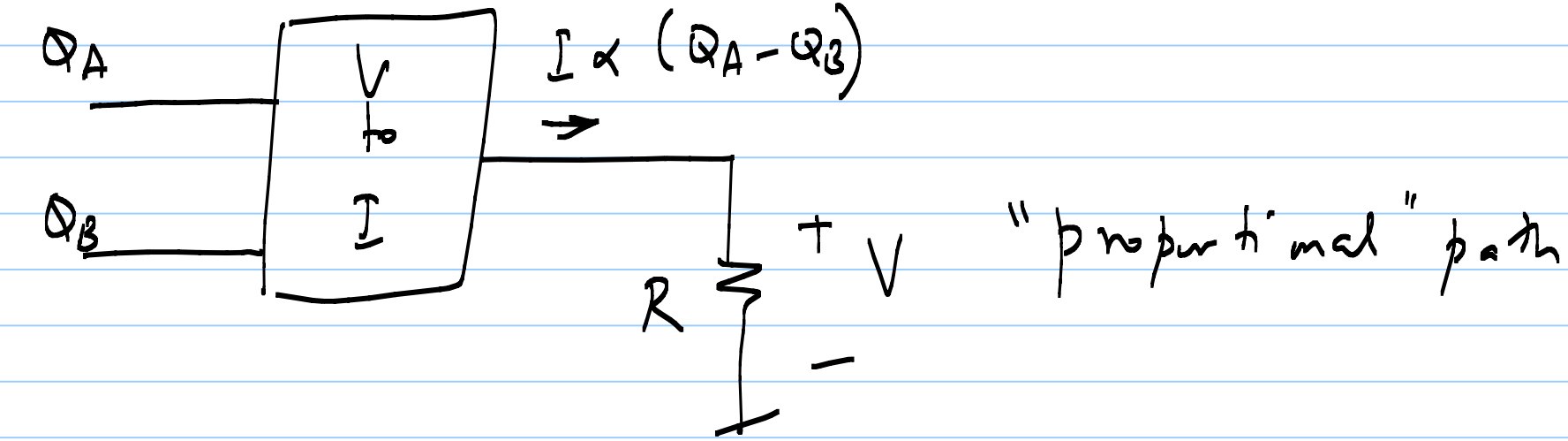
3) We want to add outputs of P & I paths

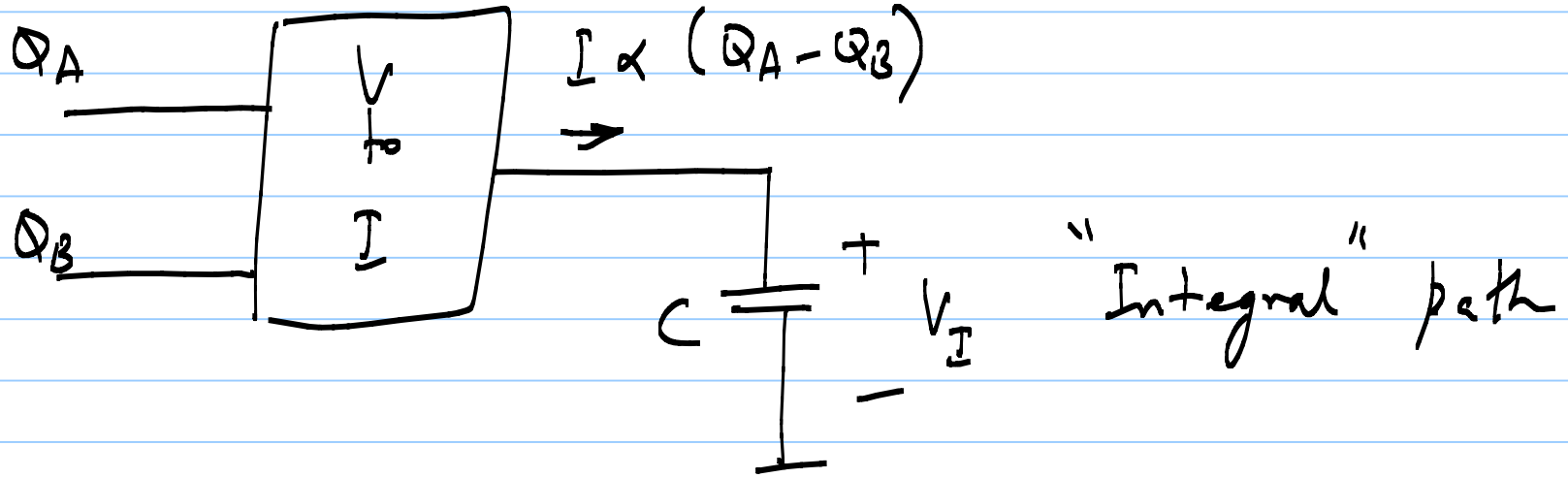
1) $(Q_A - Q_B)$: Easier to add/subtract currents



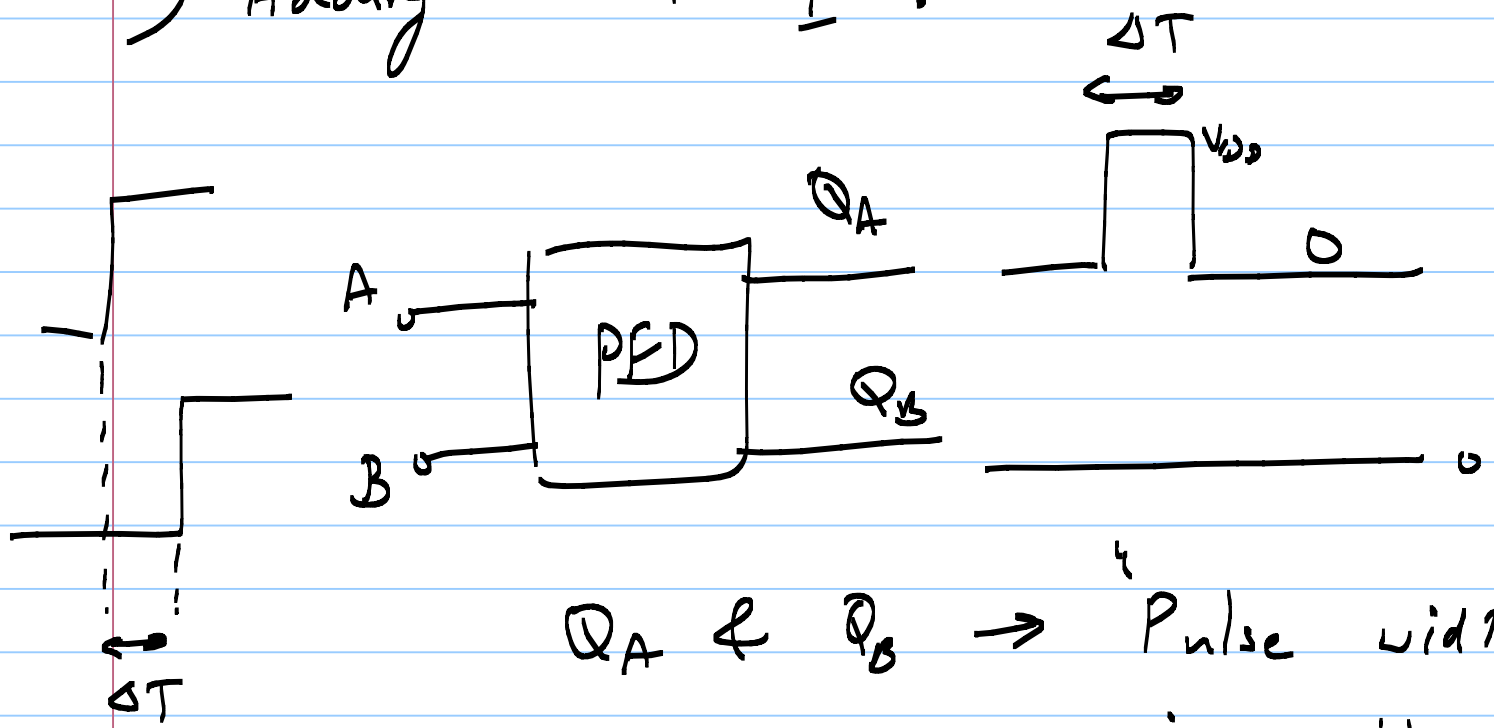





Q_A
Q_B
V
to
I
$I \propto (Q_A - Q_B)$
R
+
V
−
"proportional" path


Q_A
Q_B
V
to
I
$I \propto (Q_A - Q_B)$
C
+
V_I
−
"Integral" path

3) Adding V & V_I :



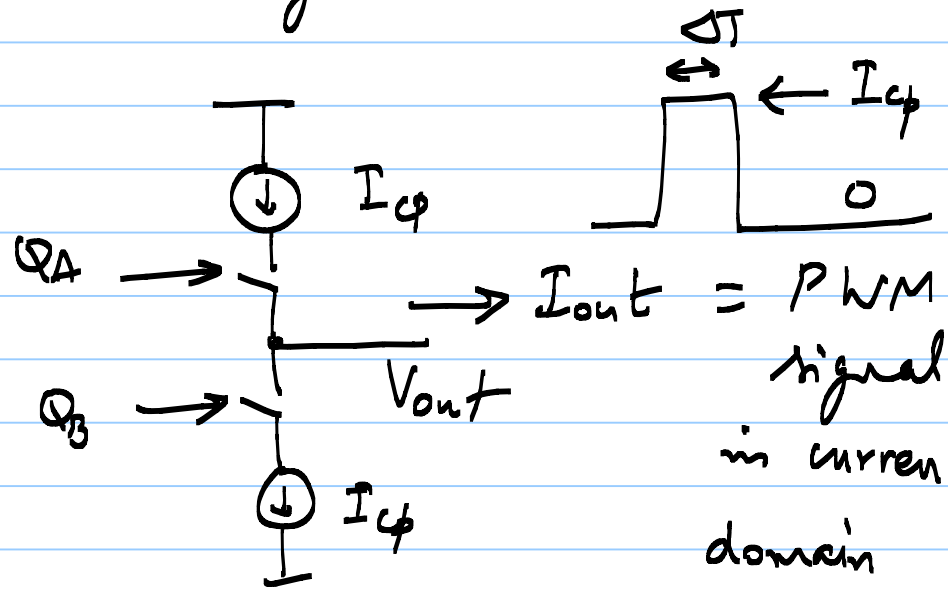
$\Delta T \propto \Delta \varphi_{A-B}$

Q_A & Q_B → "Pulse width modulated" signals in voltage mode

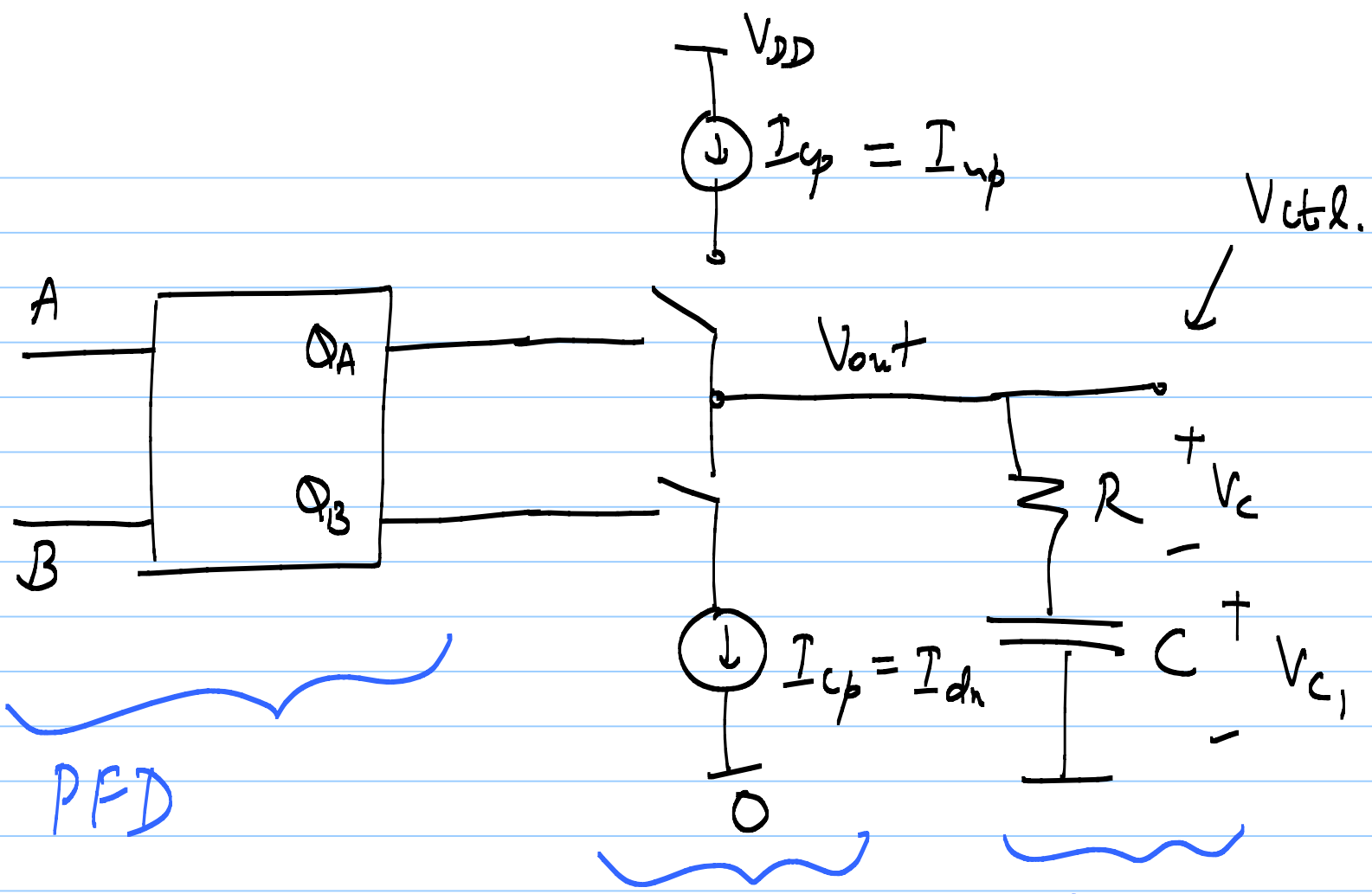
V ⇆ I converter :

"Charge Pump"

Charge is pushed into or pulled from V_{out} ($Q = I_{cp} \cdot \Delta T$)



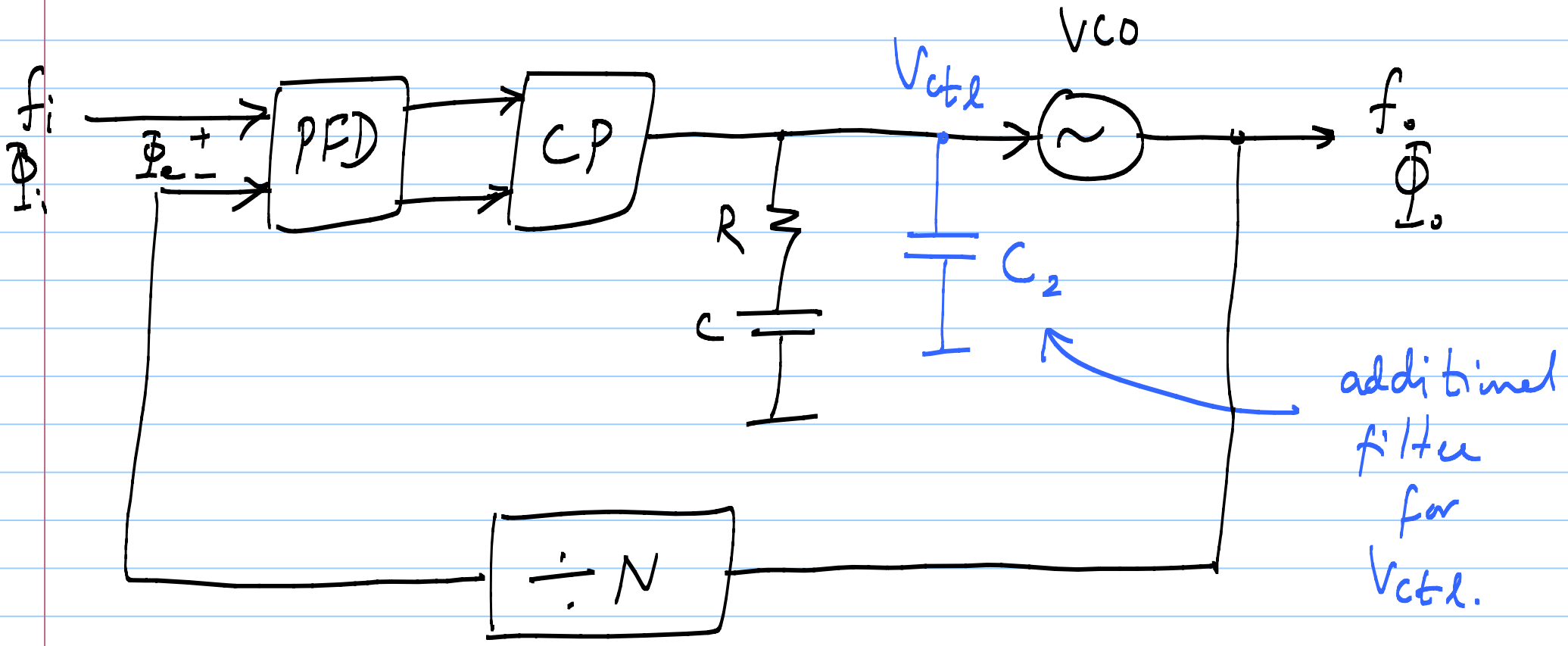
→ I_{out} = PWM signal in current domain



$$K_{pd} = \frac{V_{DD}}{2\pi} \cdot \frac{I_{cp}}{V_{DD}} \cdot R = \frac{I_{cp} \cdot R}{2\pi}$$

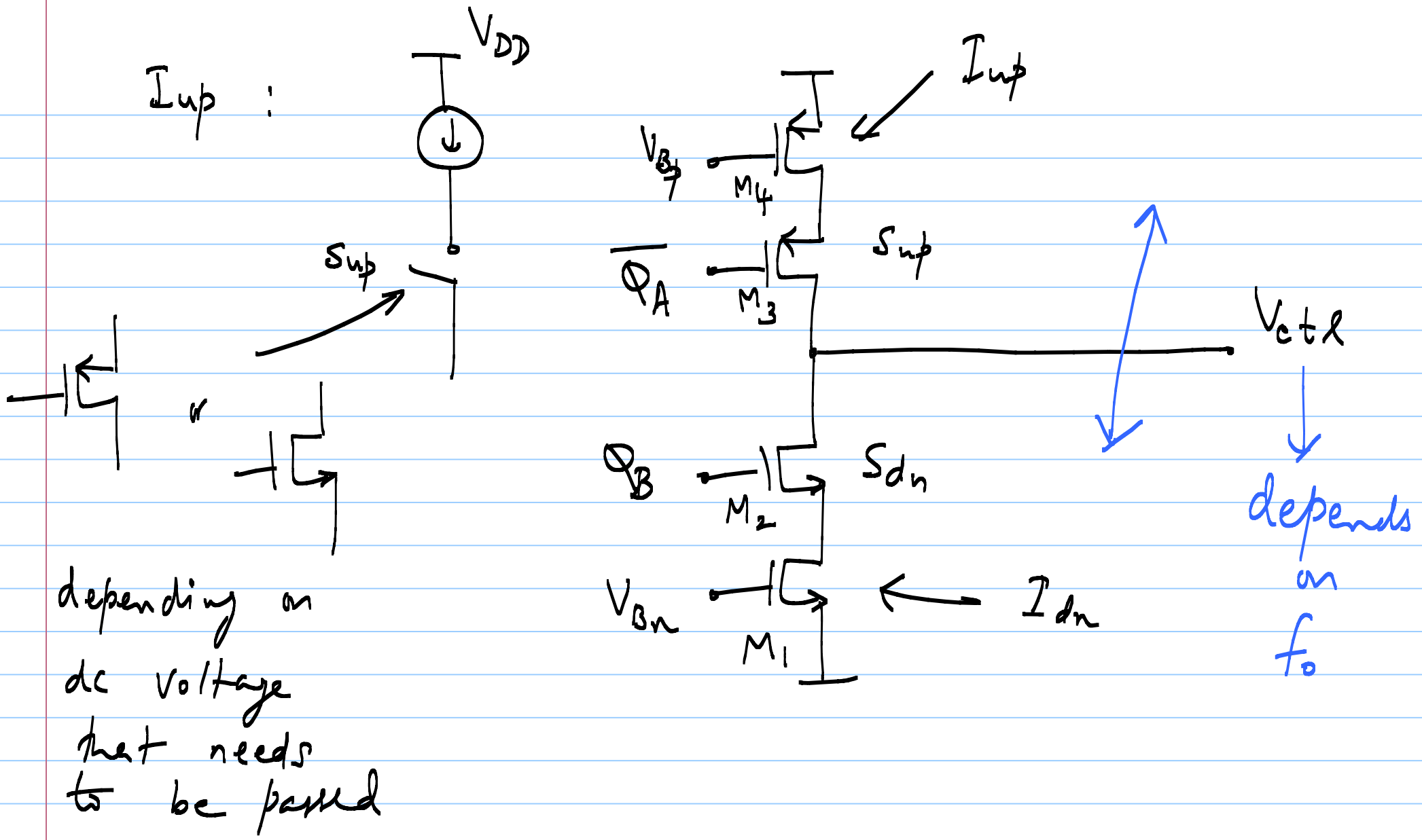
$$K_{pd_I} = \frac{I_{cp}}{2\pi C}$$

Type-II PLL block diagram



$f_o = N f_i$ and $\Phi_o = N \Phi_i$

both f_e & $\Phi_e = 0$


Iup :
VDD
Sup
or
depending on
dc voltage
that needs
to be passed
Iup
VBp
M4
Sup
QA
M3
Vctl
QB
M2
Sdn
VBn
M1
Idn
depends
on
fo

* I_{cp} decides K_{pd} & K_{pd_I} $\Rightarrow$ Loop gain

* I_{cp} should be constant

* M_1 & M_4 should have high r_{out}

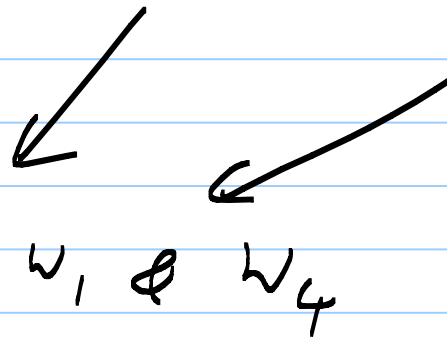
  (V_{ctl} changes with f_o)

  $\hookrightarrow$ L_1 & L_4 may be large

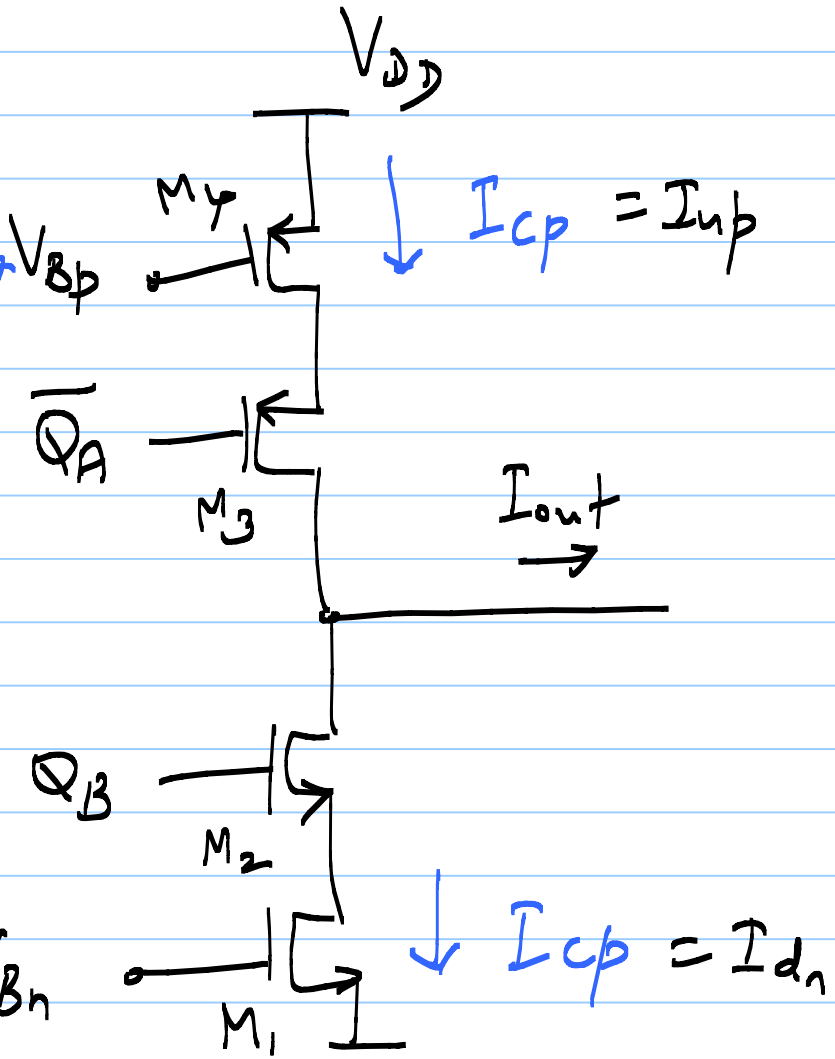
* Cascode current sources are possible

* Large V_{ctl} range $\Rightarrow$ sets V_{Dsat_1} & V_{Dsat_4}

* I_{cp} & V_{Dsat} → sets $\left(\frac{W}{L}\right)_1$ & $\left(\frac{W}{L}\right)_4$



sets W_1 & W_4



should not be generated independently

↓

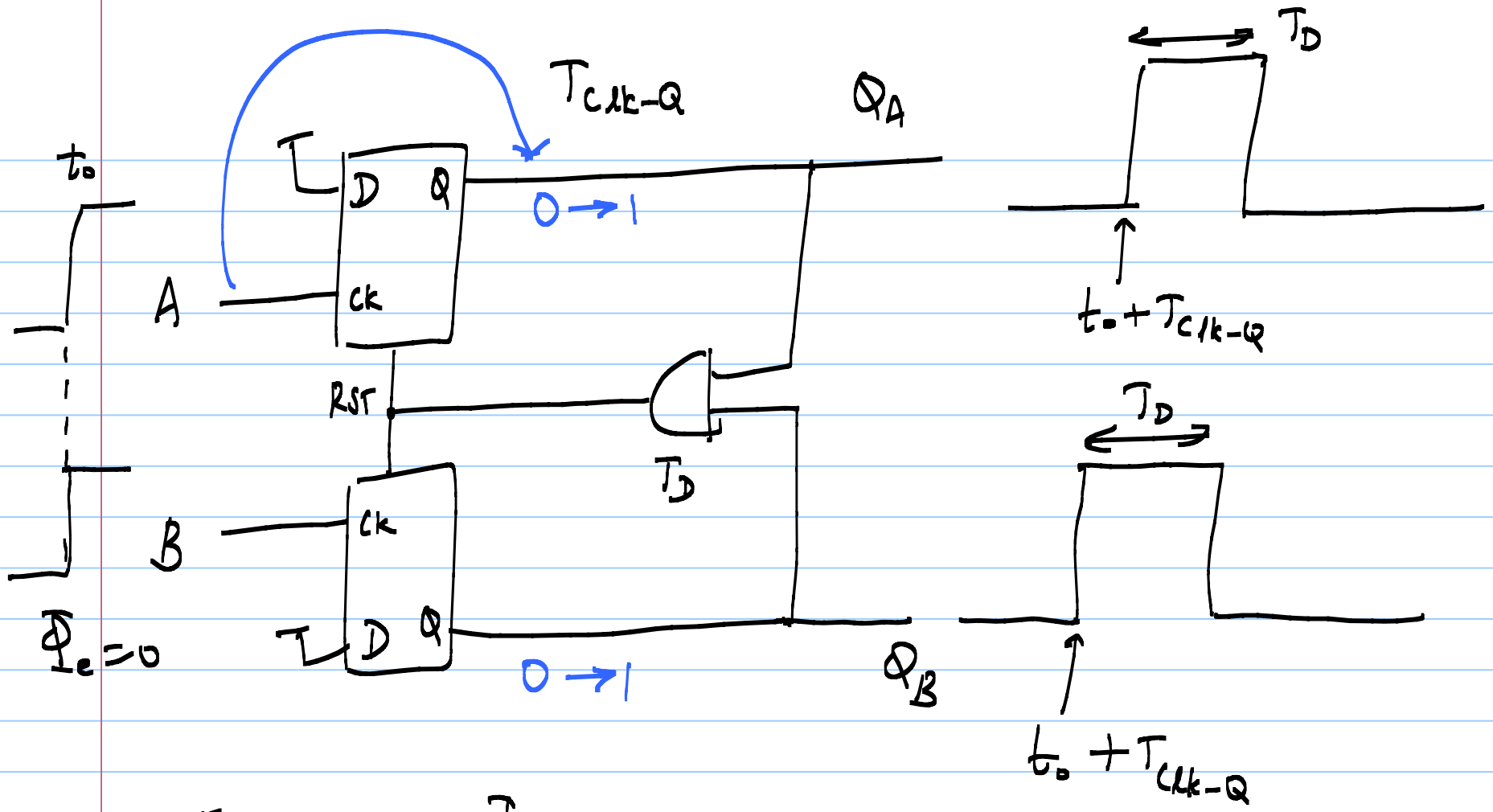
CMFB loop is often used.

* If there is mismatch between I_{up} & I_{dn} (due to CLM etc.)

↓

?

Need a CMFB loop that tracks V_{cm}.



* Even if $\Phi_e = 0$, Q_A & Q_B go high for a short period of time = RST path delay

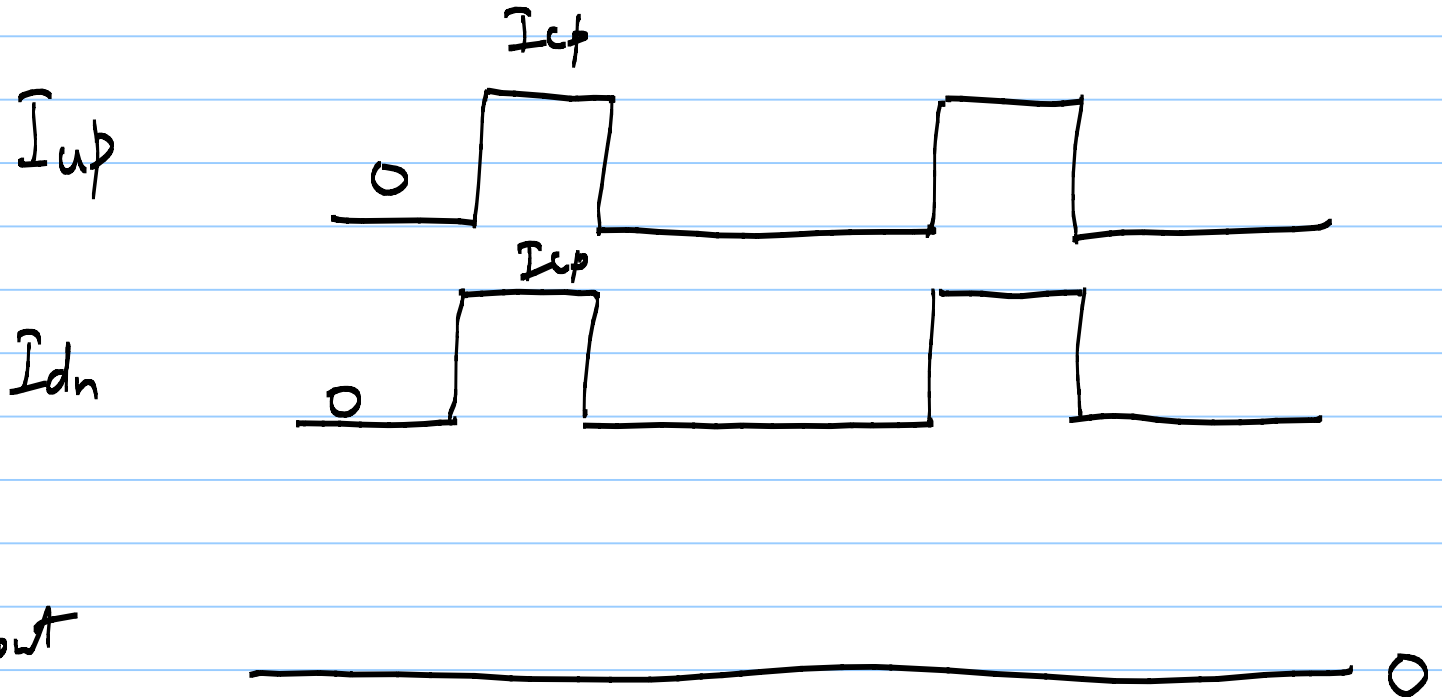
* As long as the UP & DN paths are symmetric $(Q_A - Q_B) = 0 \Rightarrow$ CP output does not change

* In case of mismatch (due to CLM or random mismatches) → $I_{up} \neq I_{dn}$

→ for T_D period, $Q_{up} \neq Q_{dn}$

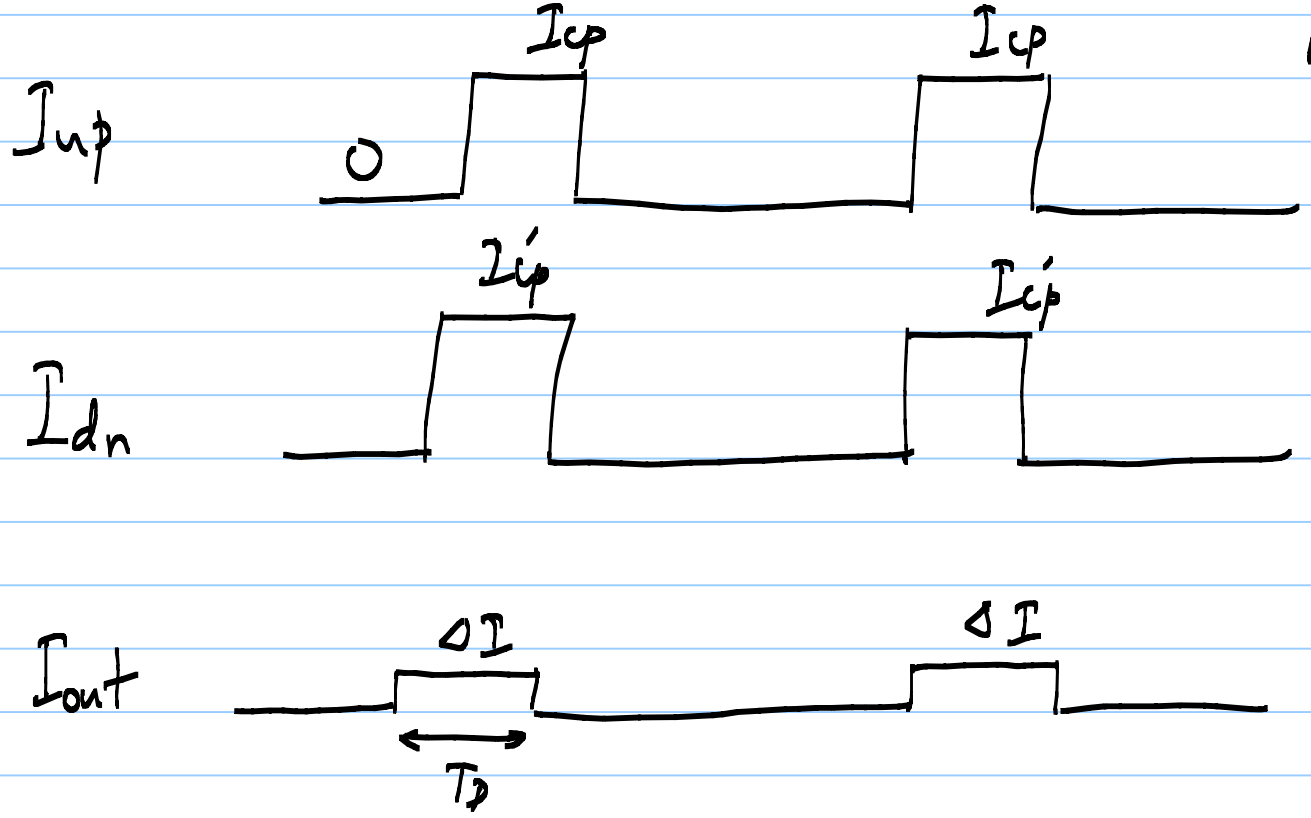
i.e. charge deposited onto V_{ctl} ≠ charge removed

Symmetric case:



Mismatched I_{cp}'s

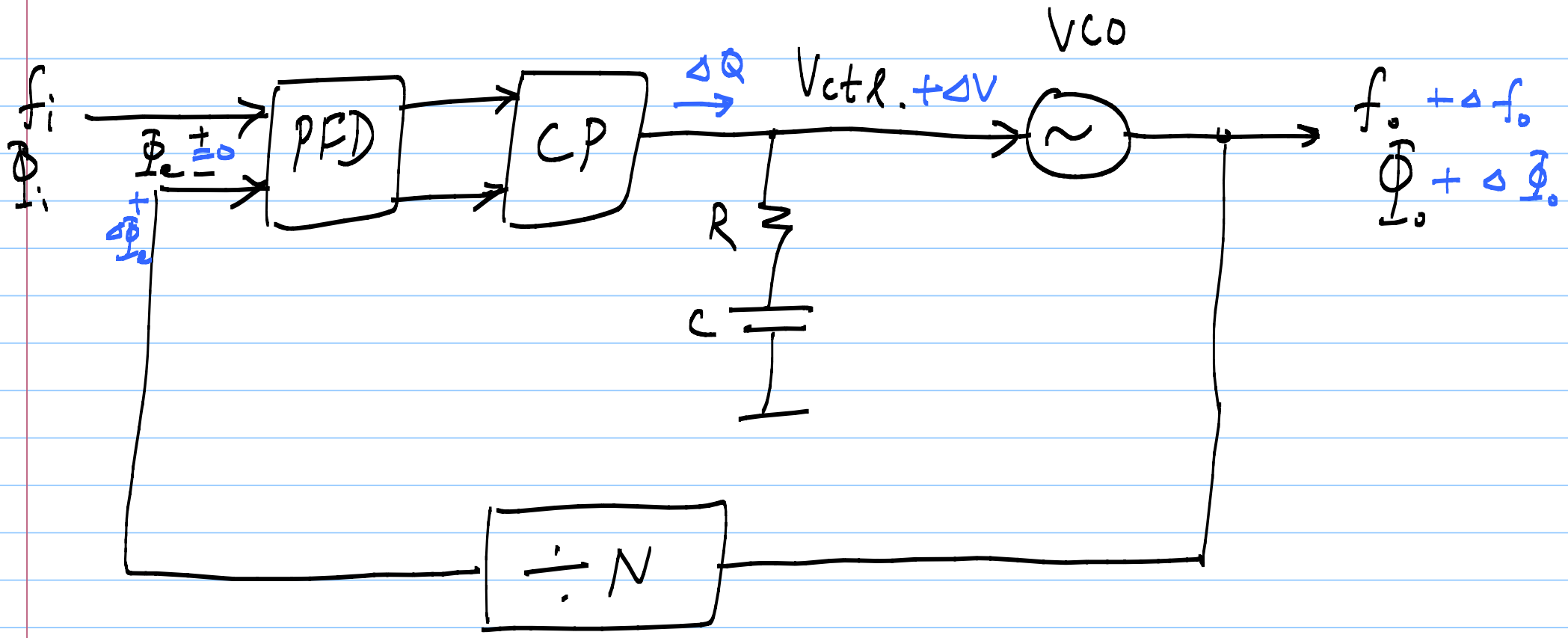
e.g. $I_{cp} = I_{cp}' + \Delta I$



In open loop: $\Delta Q = \Delta I \cdot T_D$ is deposited onto V_{ctl}.

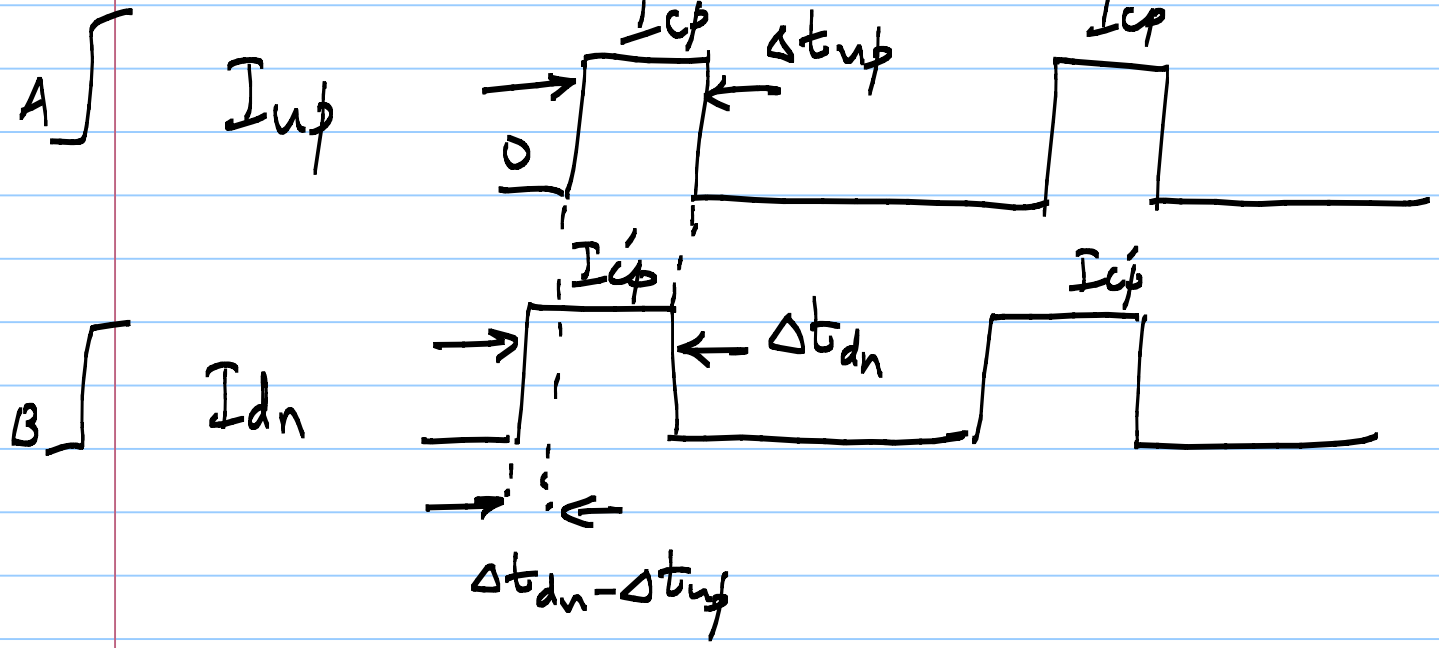
In closed loop: negative f.b. will compensate for this to maintain $f_o = N f_i$

& $\Phi_o = N \Phi_i + \Delta\Phi$



* Loop settles to a new steady state with $\Phi_e \neq 0$

In closed loop, with I_{cp} mismatch:



$I_{cp} > I_{cp}'$

$\Delta I > 0$

$\Downarrow$

for $\Delta Q = 0$,

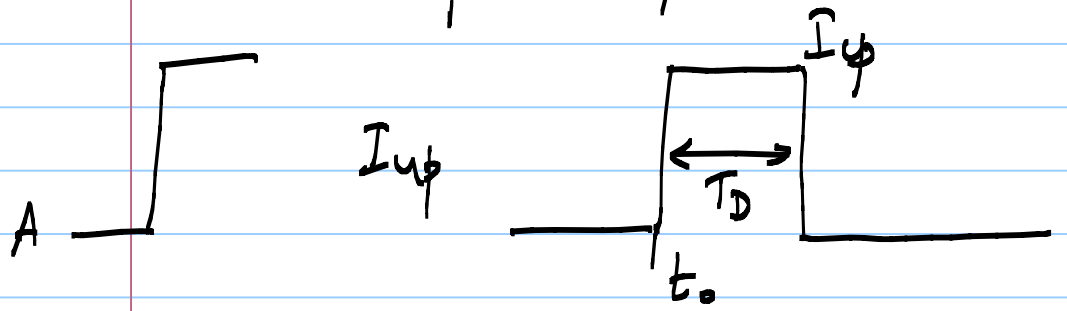
$\Delta t_{dn} > \Delta t_{up}$

Mismatch in path delays

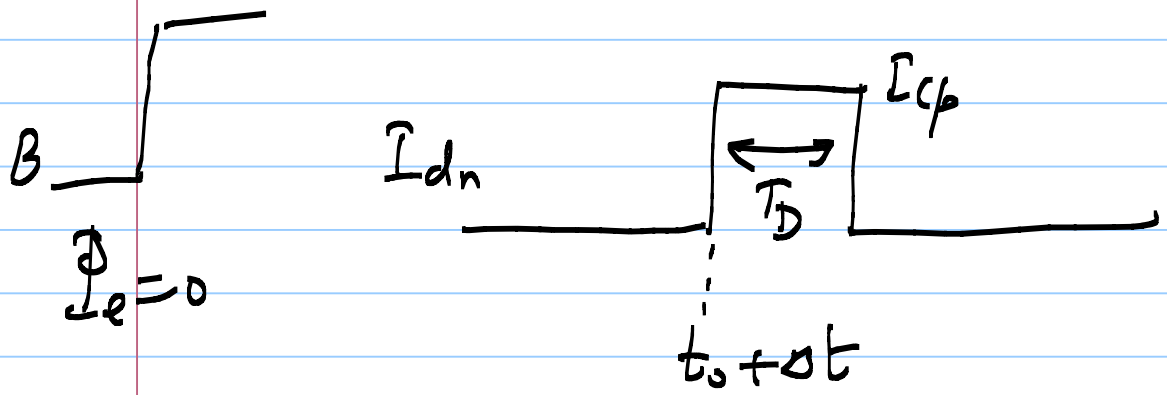
$I_{up} = I_{dn} = I_{cp}$

but delays of two paths are different

In open loop:



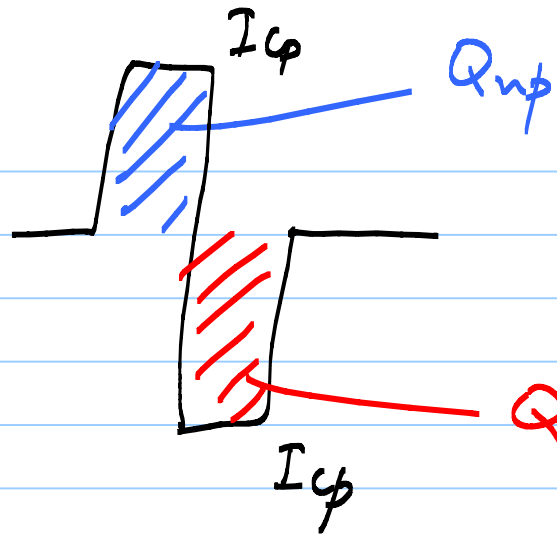
$\Delta Q_{up} = I_{cp} \cdot T_D$

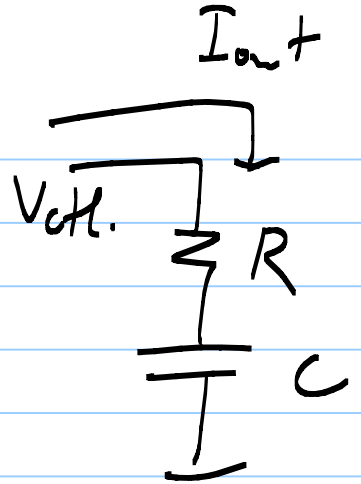


$\Delta Q_{dn} = I_{cp} \cdot T_D$

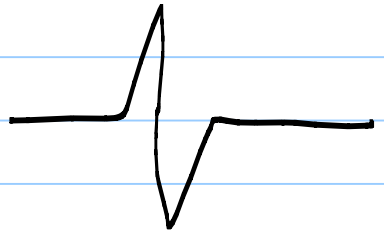
$\Delta Q = 0$ over f_{ref} period

$I_{out} = I_{up} - I_{dn}$



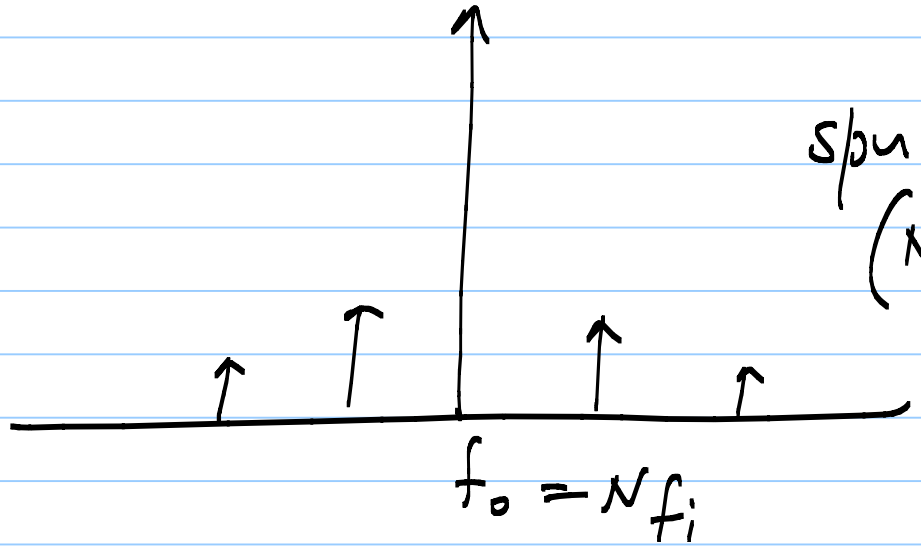


V_{ctl}

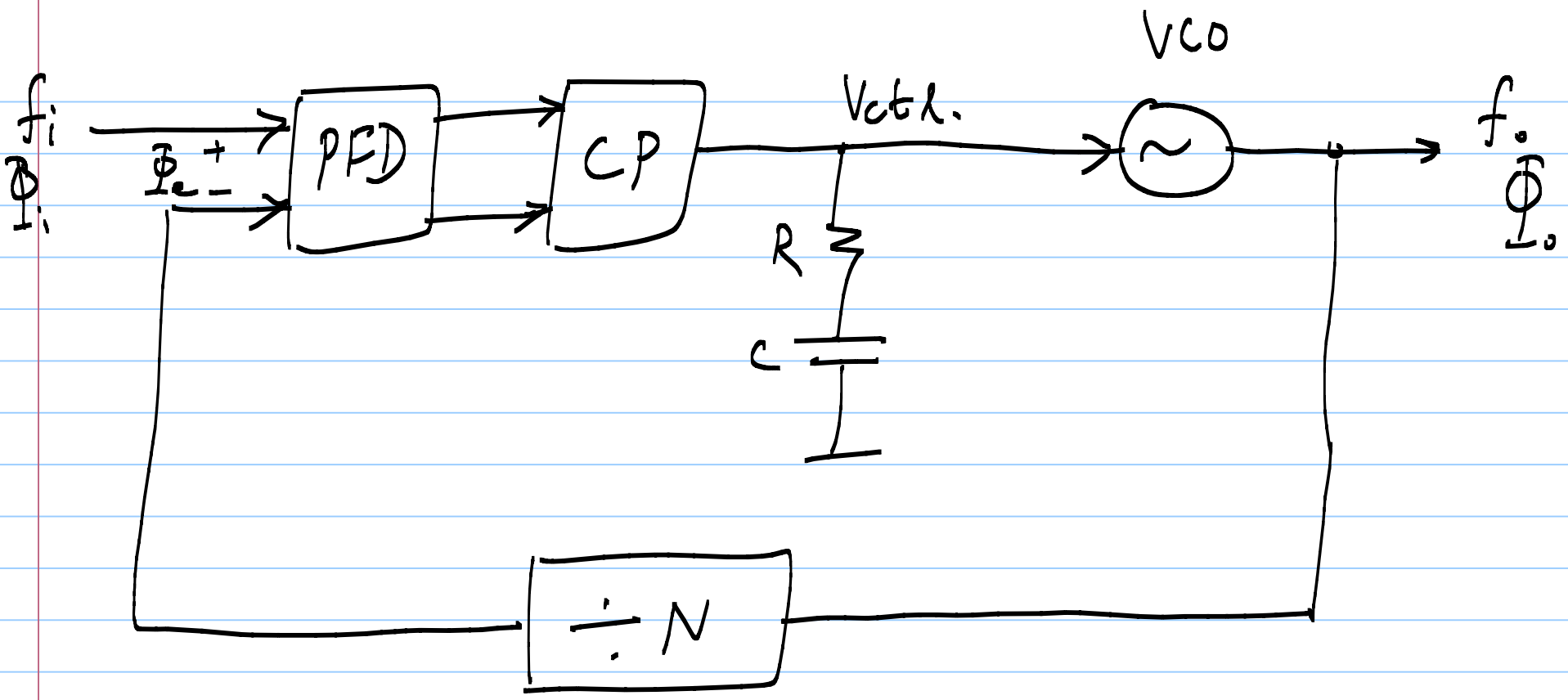


← periodic disturbance of $V_{ctl.}$ even in closed loop

VCO output



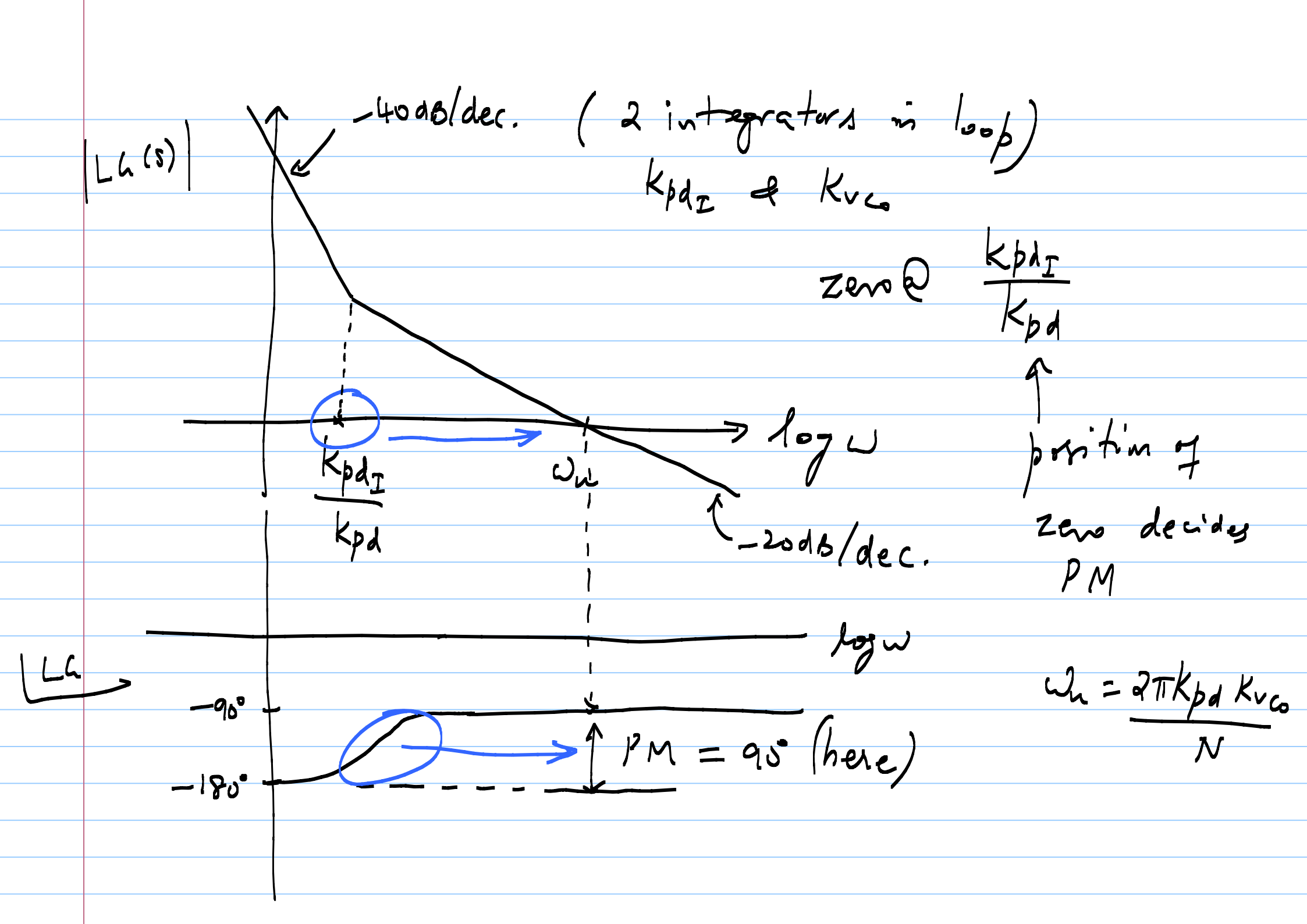
spurs @ $(N \pm k) f_i$

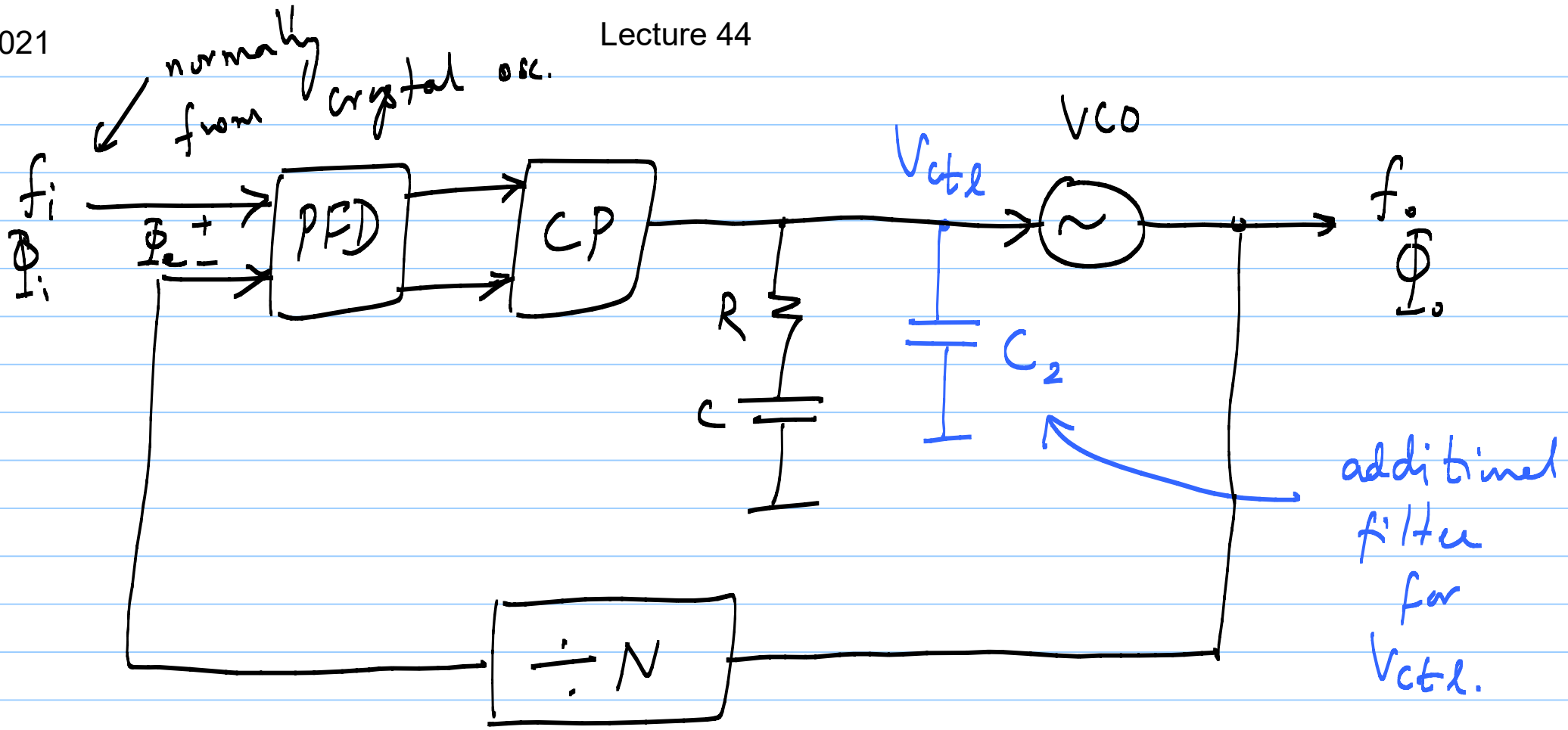


Bode plots of $L_G(s)$ for stability, etc.

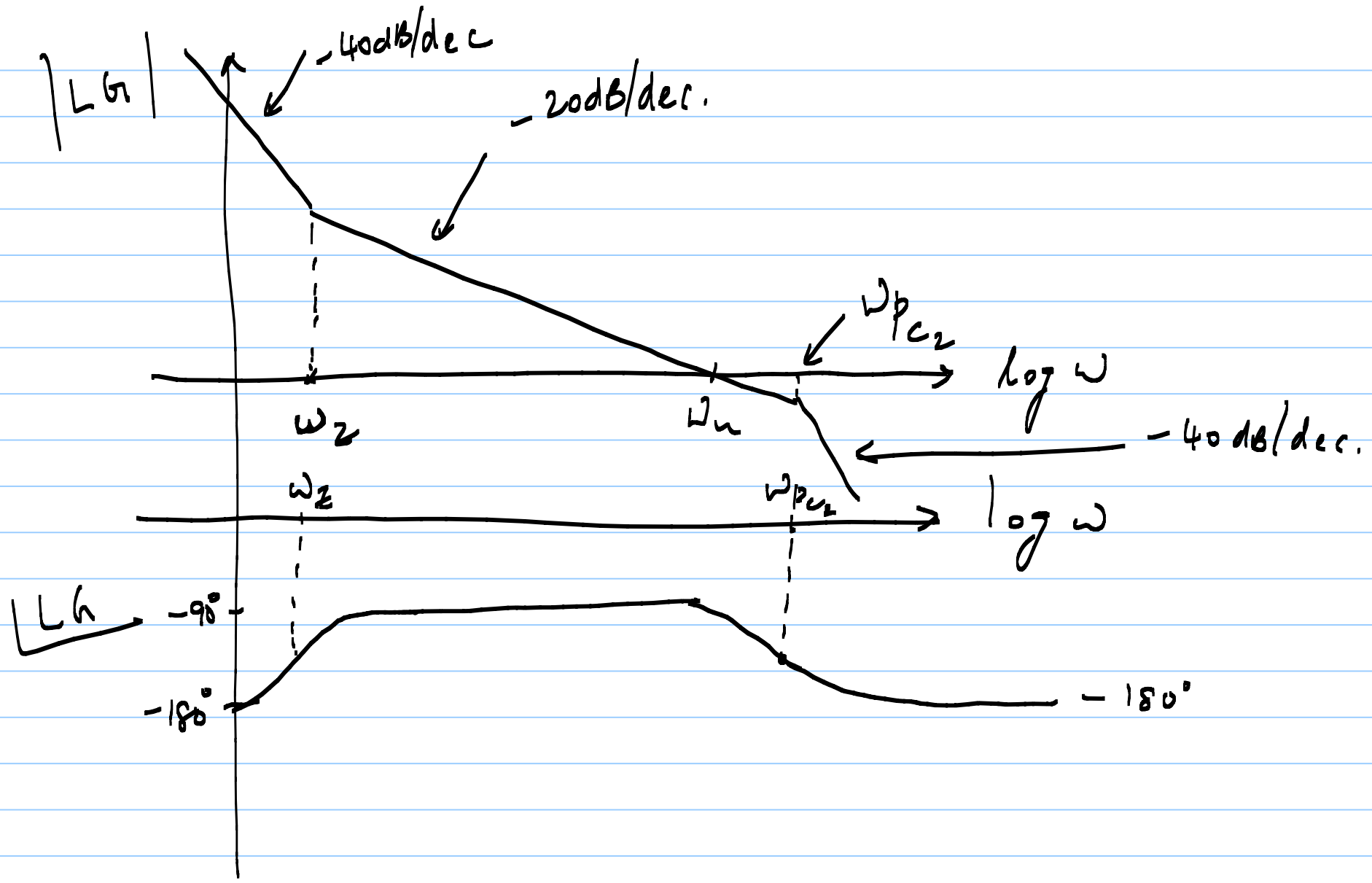
for phase signals

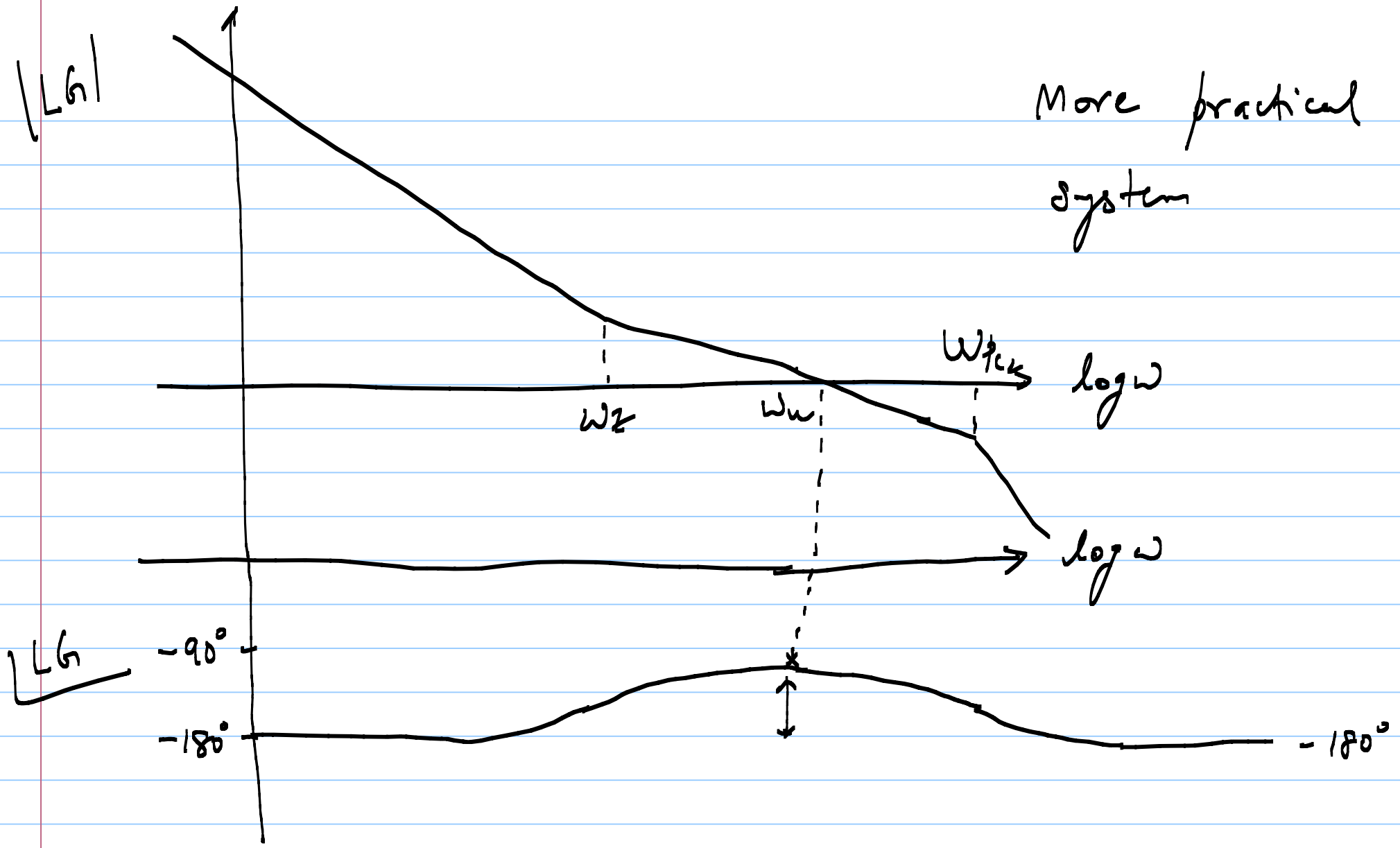
ω_n


$|L_h(s)|$
-40 dB/dec.
(2 integrators in loop)
K_{pd_I} & K_{vco}
zero @ $\frac{K_{pd_I}}{K_{pd}}$
position of zero decides PM
$\log \omega$
K_{pd_I}
K_{pd}
ω_u
-20 dB/dec.
$\log \omega$
$\angle L_h$
-90°
-180°
PM = 95° (here)
$\omega_u = \frac{2\pi K_{pd} K_{vco}}{N}$



* Addition of C_2 changes Bode Plot


|LG|
-40dB/dec
-20dB/dec.
$\omega_{p_{c_2}}$
log ω
ω_z
ω_n
-40 dB/dec.
ω_z
$\omega_{p_{c_2}}$
log ω
∠G
-90°
-180°
-180°



f_i = Quartz crystal-based osc → electrical resonance

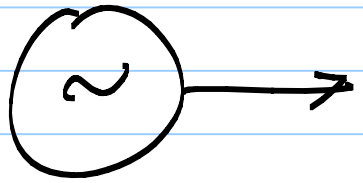
Voltage Controlled Oscillator (VCO)

Relaxation Osc ; Ring Osc., Opamp RC osc.,

LC Osc. → Colpitts osc.

→ Hartley Osc

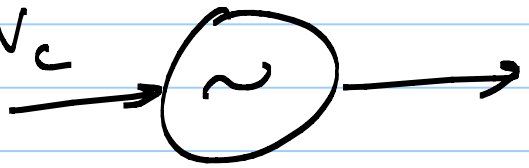
→ Cross-coupled osc



const → Osc.

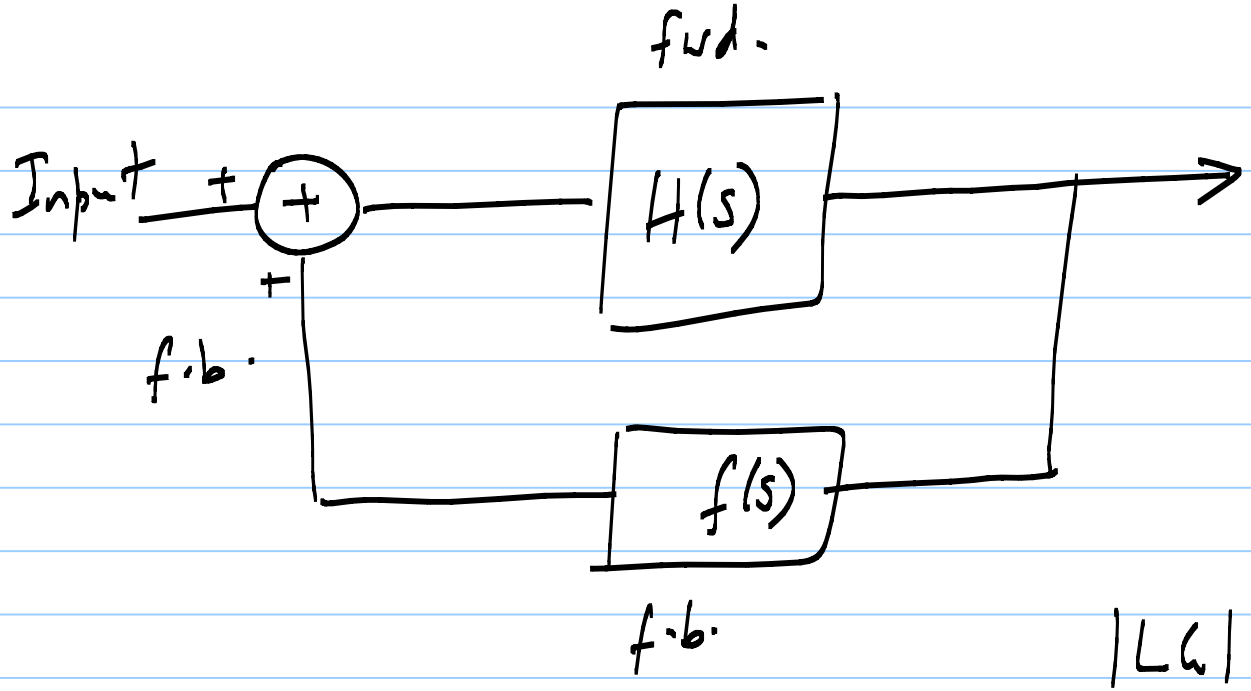
incorporates +ve f.b.

Output without input



V_c → cos ωt → VCO

avoided in negative f.b. systems



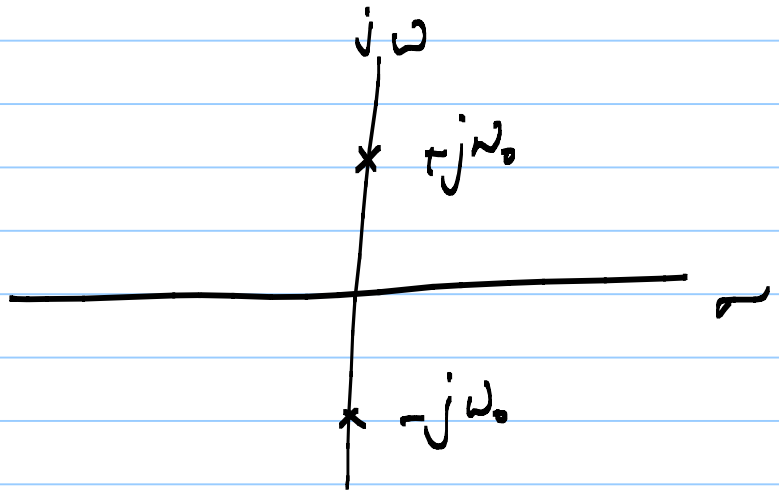
$$Out = \frac{1}{f(s)} \cdot \frac{f(s)H(s)}{1 - f(s)H(s)} \cdot In$$

$f(s)H(s) = |\angle 0$

Output without input

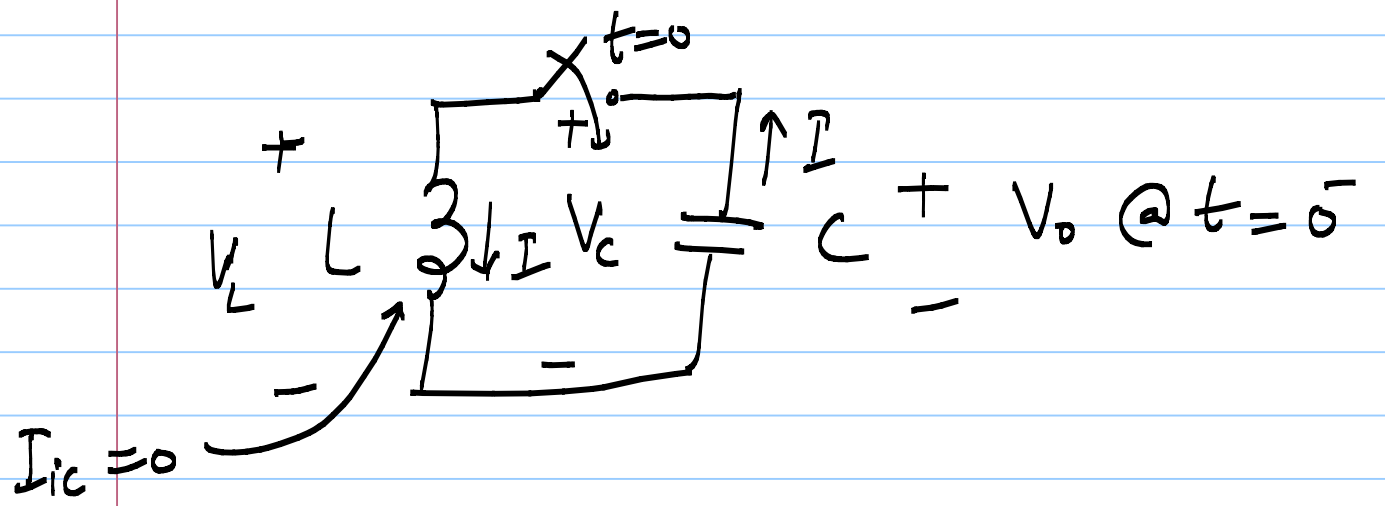
$|LG| = 1 \quad \underline{\&} \quad \angle LG = 0^\circ / 360^\circ$

$\frac{Out}{In}(j\omega)$



poles @ $\pm j\omega_0$

Cross-coupled LC osc.



$$V_L = V_c = L\frac{dI}{dt}$$

$$I = -C\frac{dV_c}{dt}$$

HW: Solve system of equations

to find $V_c(t)$ & $I(t)$

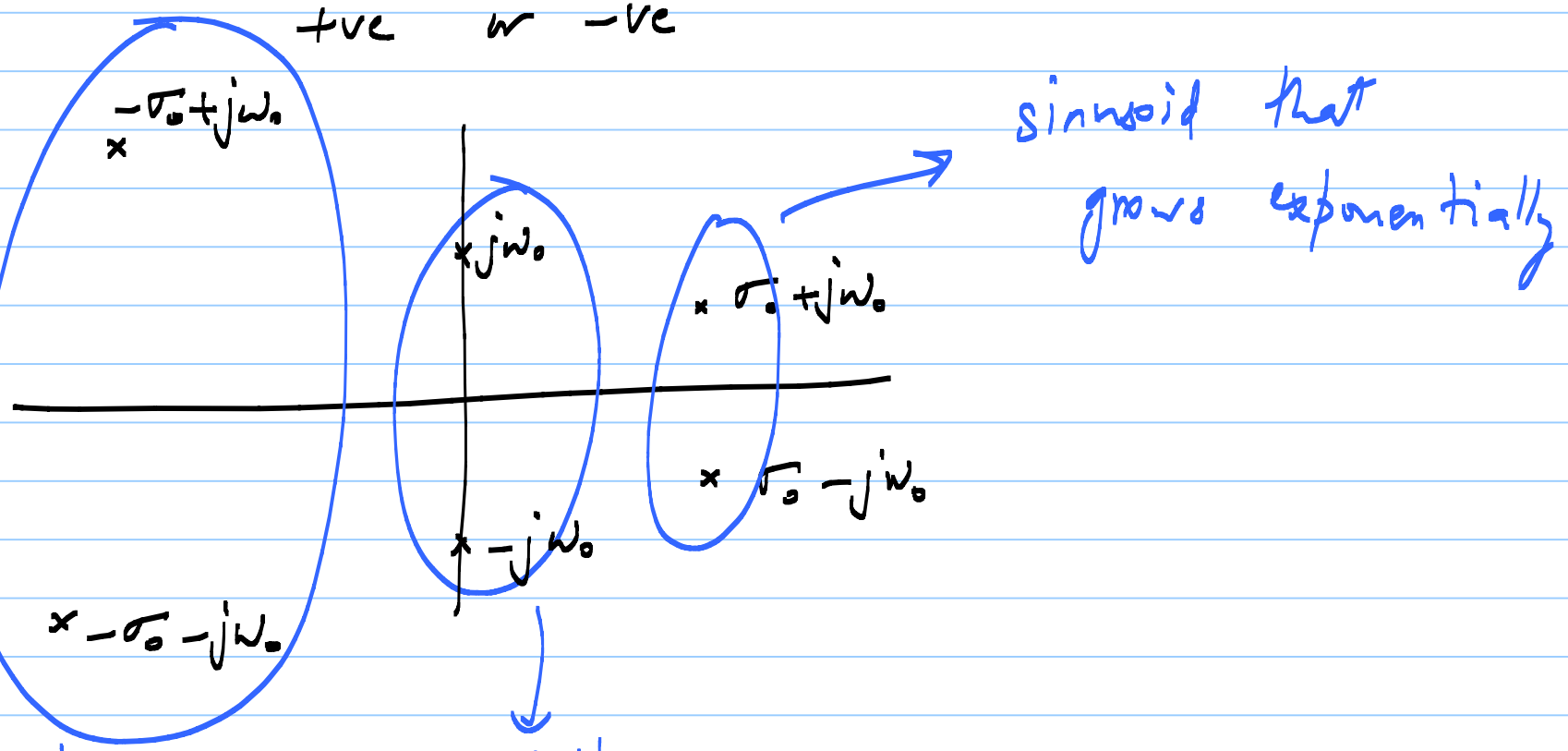
* Second order differential equation without the first order term

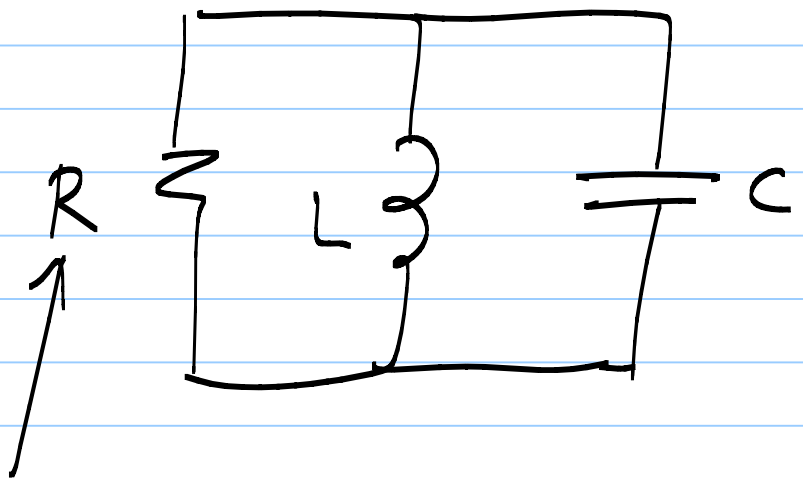
⇒ purely sinusoidal solution → poles @ $\pm j\omega_0$

* if you had first order term

↳ $\sigma_0 \pm j\omega_0$ (poles)

+ve or -ve





gives first order term

Real system

$R>0 \Rightarrow \sigma_0 < 0$ (LHP)

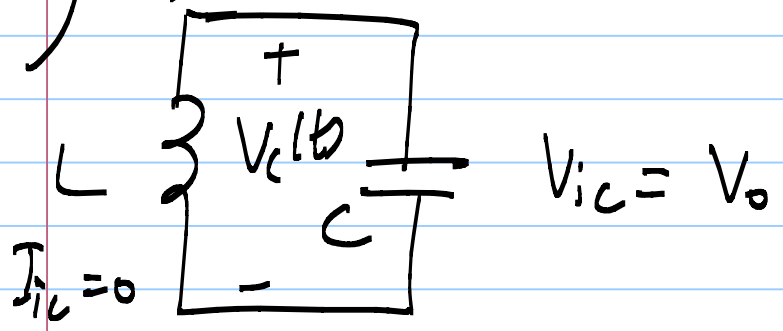
$R<0 \Rightarrow \sigma_0 > 0$ (RHP)

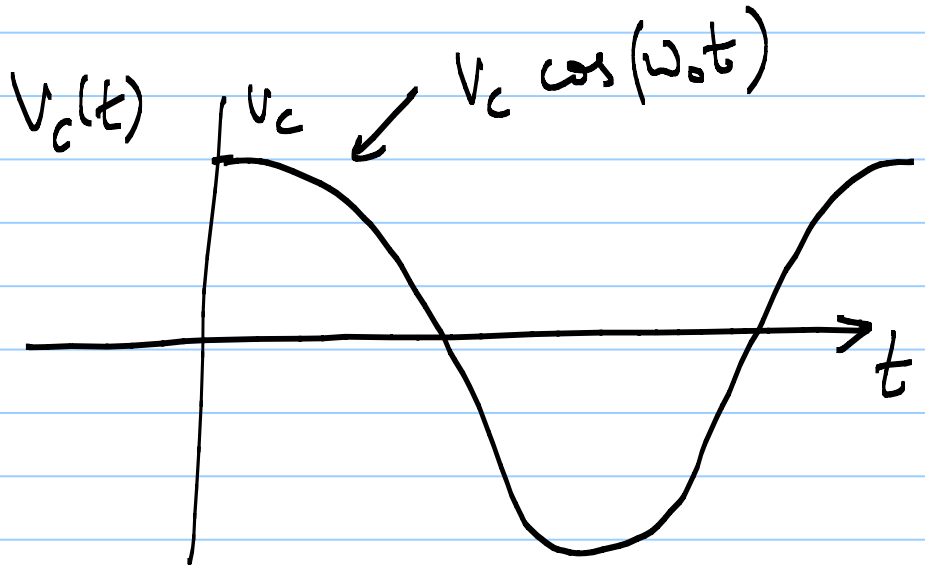
?

$R=0 \Rightarrow \sigma_0 = 0$ (poles on jω axis)

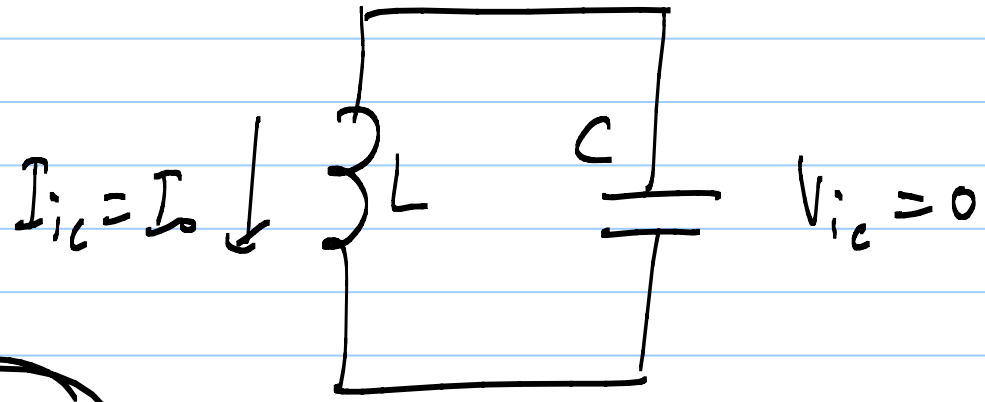
Osc.

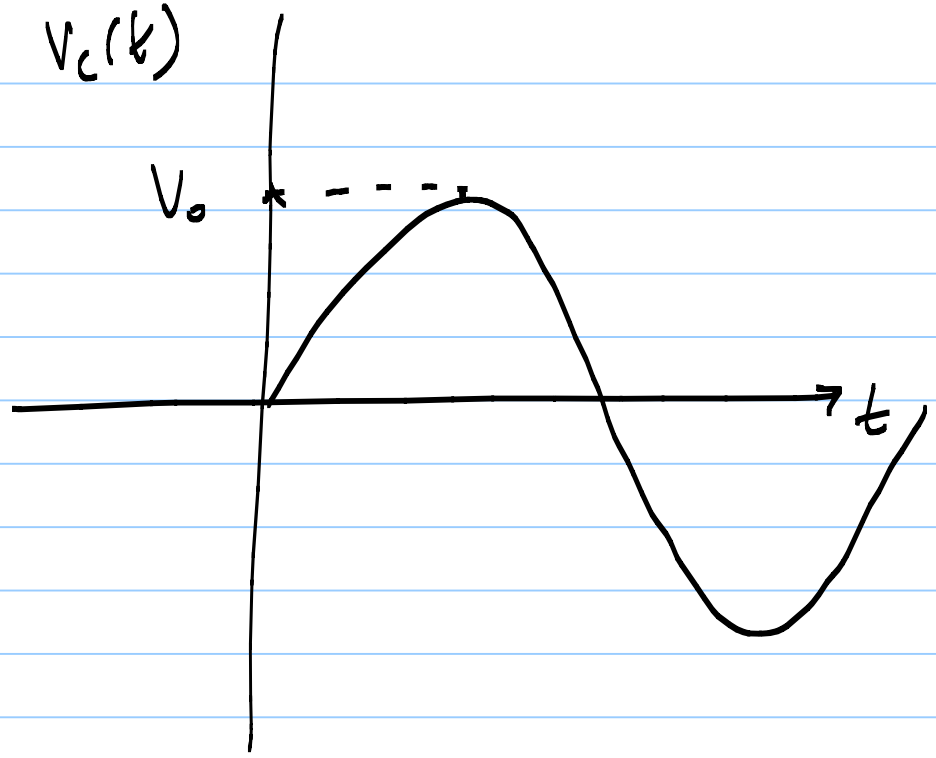
1) $V_{ic} = V_0$





2) $I_{ic} = I_o$

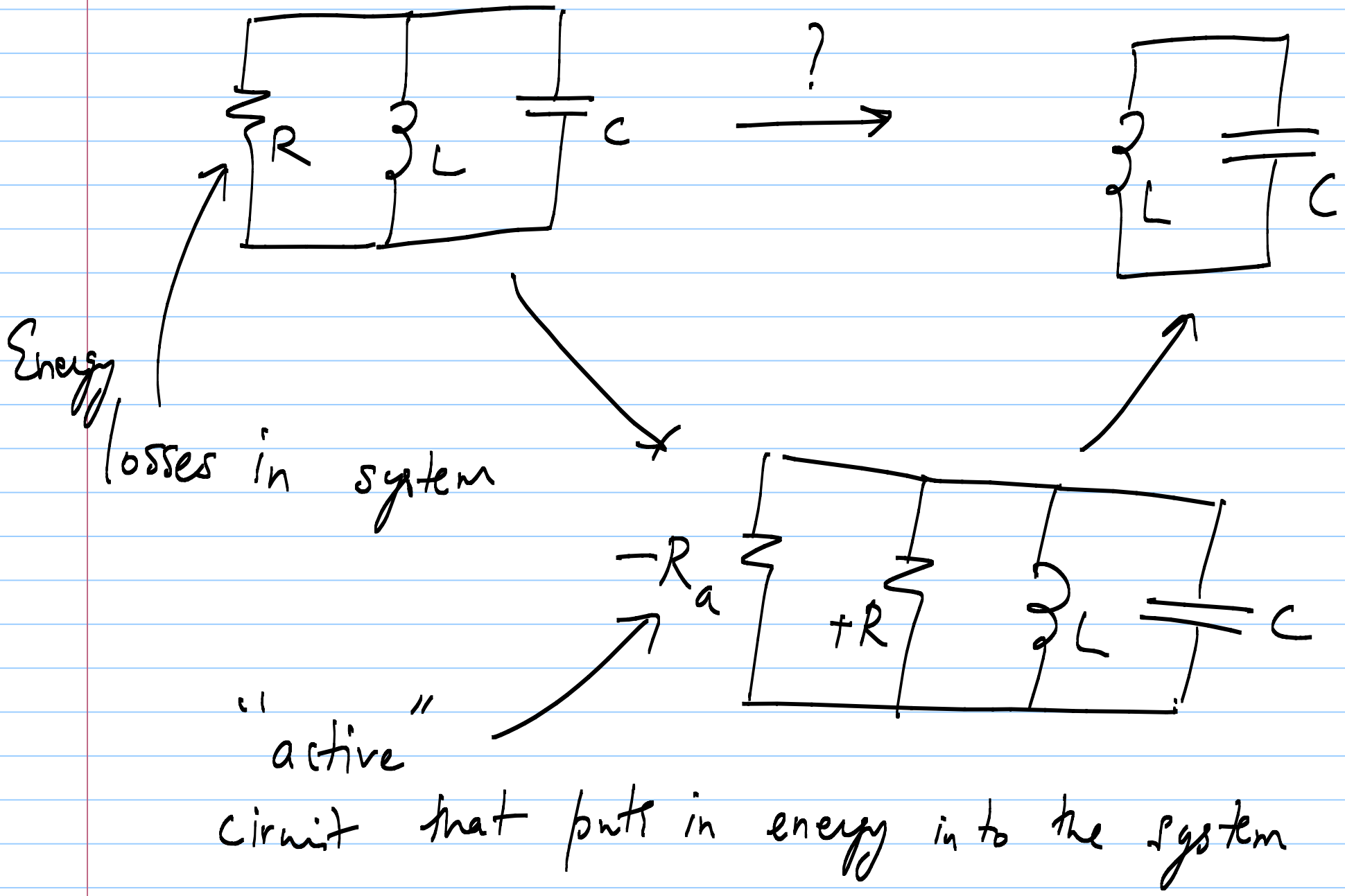




HW $V_o = ?$ here

3) $I_{ic} = I_o$; $V_{ic} = V_o$

$I = I_{pk} \sin(\omega t + \varphi)$; $V_c = V_{pk} \cos(\omega_o t + \varphi)$


R
L
C
?
L
C
Energy
losses in system
$-R_a$
+R
L
C
"active"
circuit that puts in energy into the system

e.g. $I_{D_0} = 50\mu A$ through $\left(\frac{W}{L}\right)_0 = \frac{2\,\mu m}{0.5\,\mu m}$

$\Downarrow$

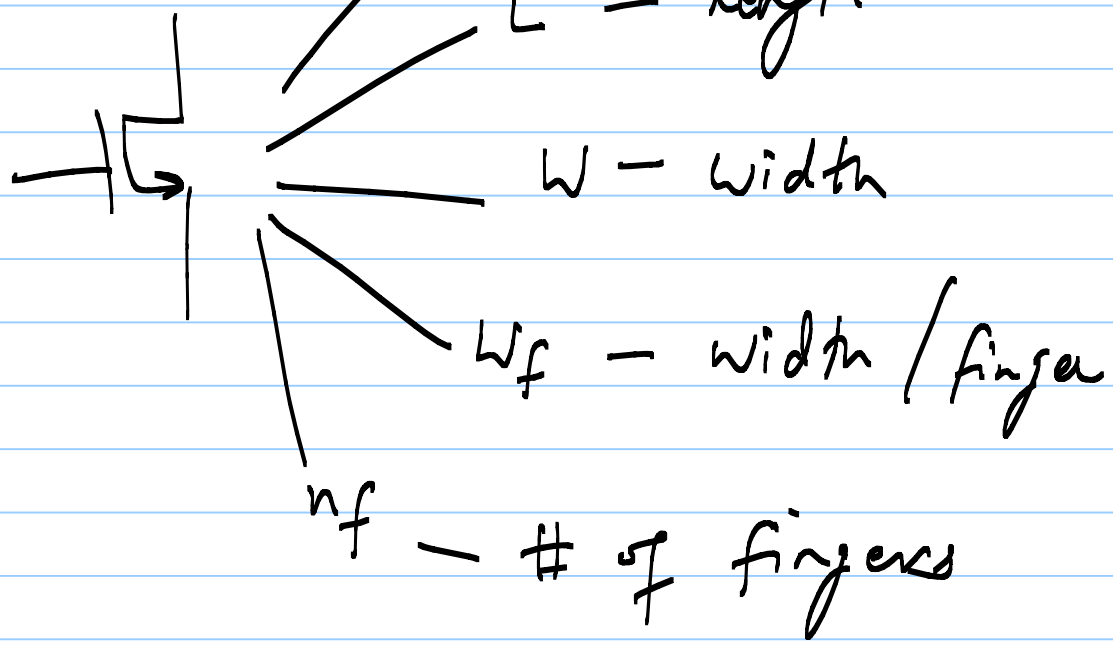
$g_{m_0} = 880\,\mu S$

required $g_{m_1} = 88\,mS \Rightarrow \left(\frac{W}{L}\right)_1 = \frac{200\,\mu}{0.5\,\mu m}$

$= 100\, g_{m_0}$

$I_{D_1} = 5mA$

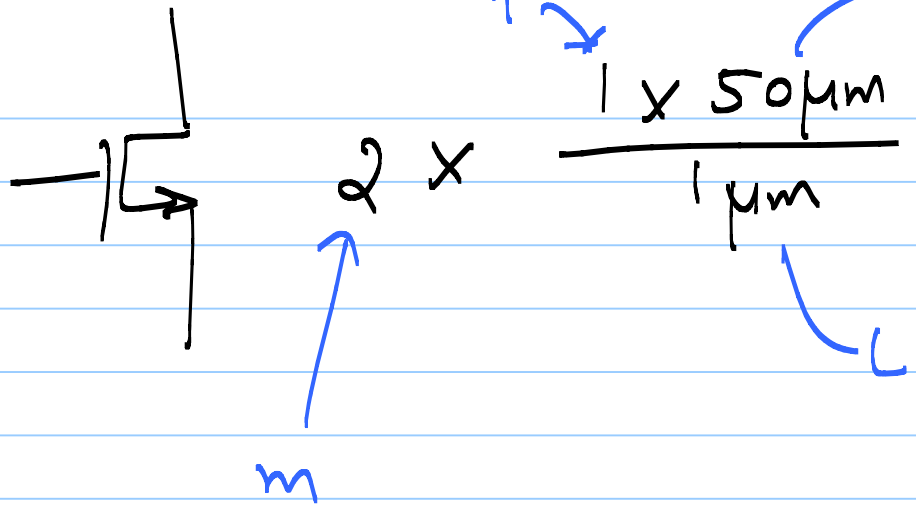
If $r_{ds} \downarrow \Rightarrow L \uparrow \Rightarrow W \uparrow \Rightarrow$ choose new current density



1) $\frac{100\mu}{1\mu m} = \frac{W}{L}$; $W_f = 100\mu m$; $W = 100\mu$
$n_f = 1$
$m = 1$

2) $\frac{2 \times 50\mu m}{1\mu m} = \frac{W}{L}$; $W_f = 50\mu m$; $W = 100\mu$
$n_f = 2$; $m = 1$

3)



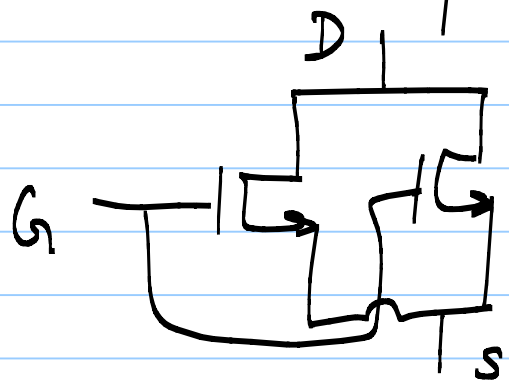
$W_f = 50\mu m$; $W = 100\mu m$

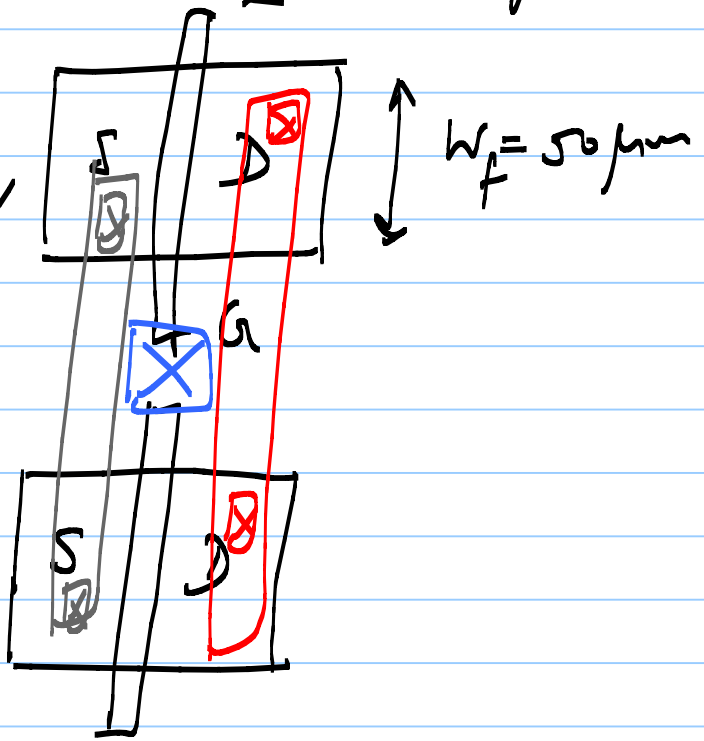
$nf = 1$; $m = 2$

total $W = m \times W_f \times nf$

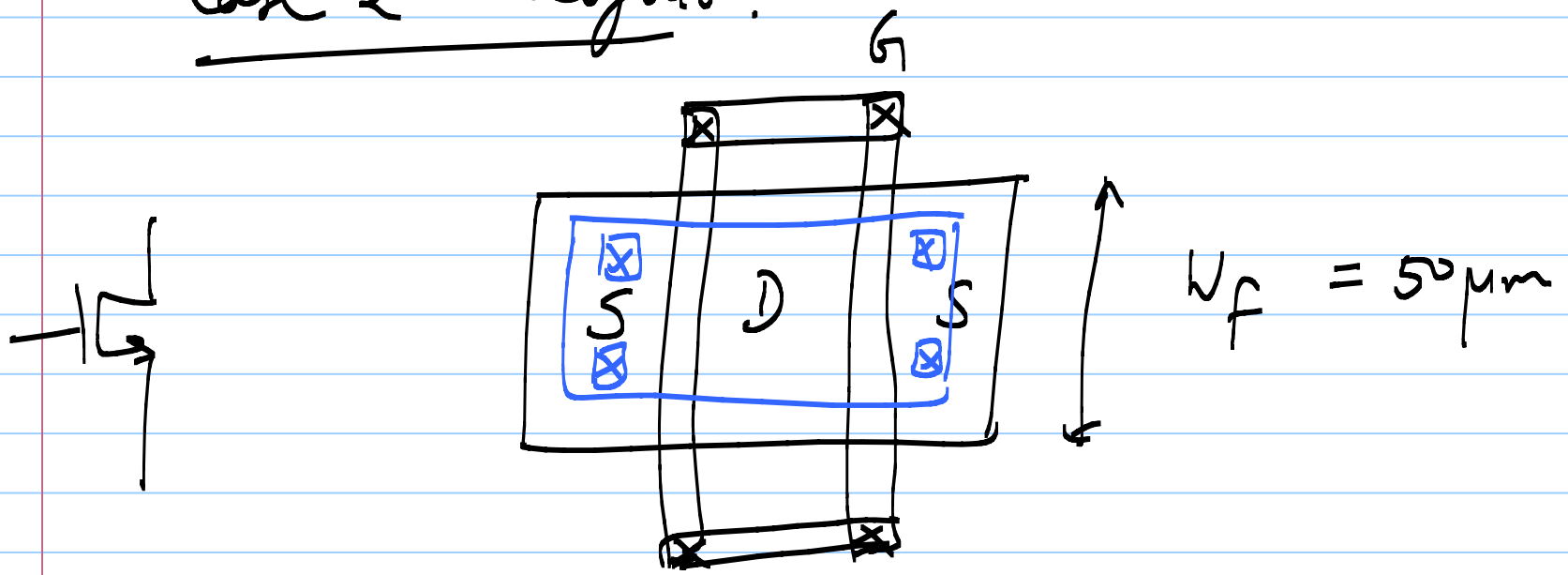
m → # of devices in parallel

nf → 1 finger

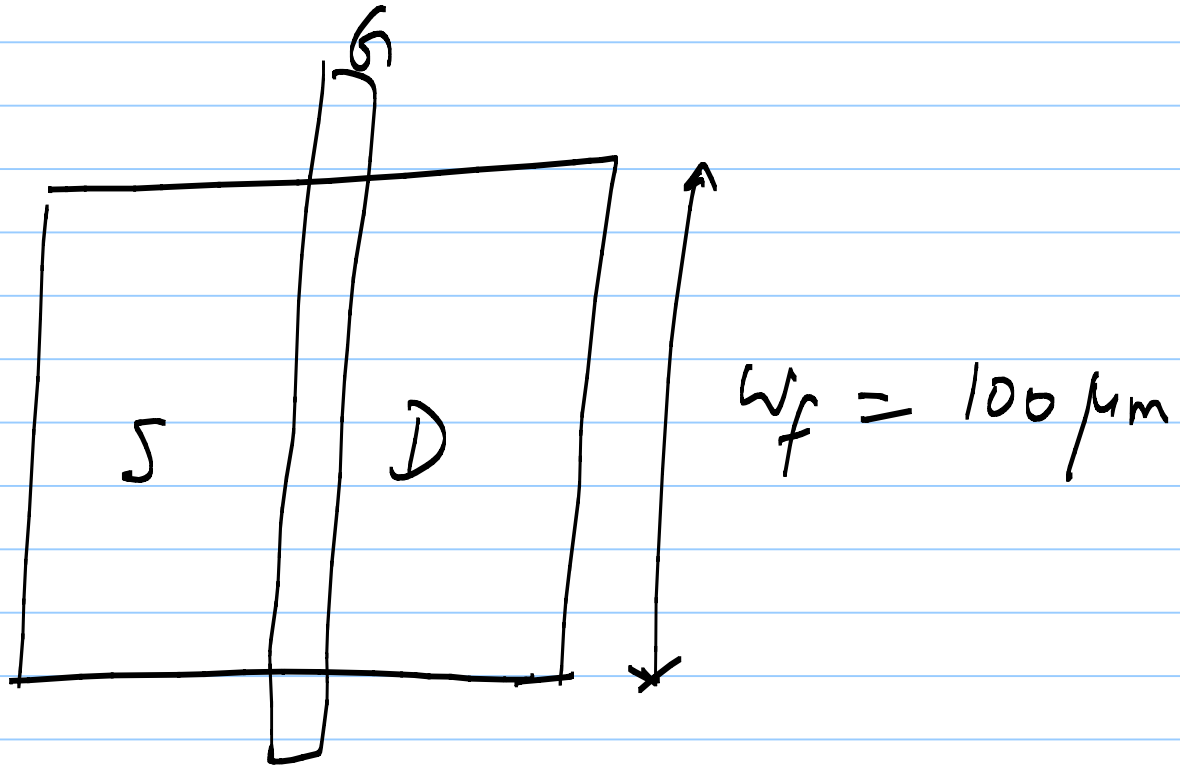




Case 2 layout:



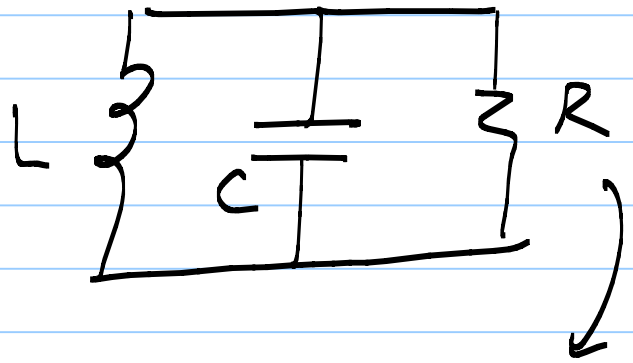
Case 1 layout



scale using m (multiplier)

↳ gives best matching

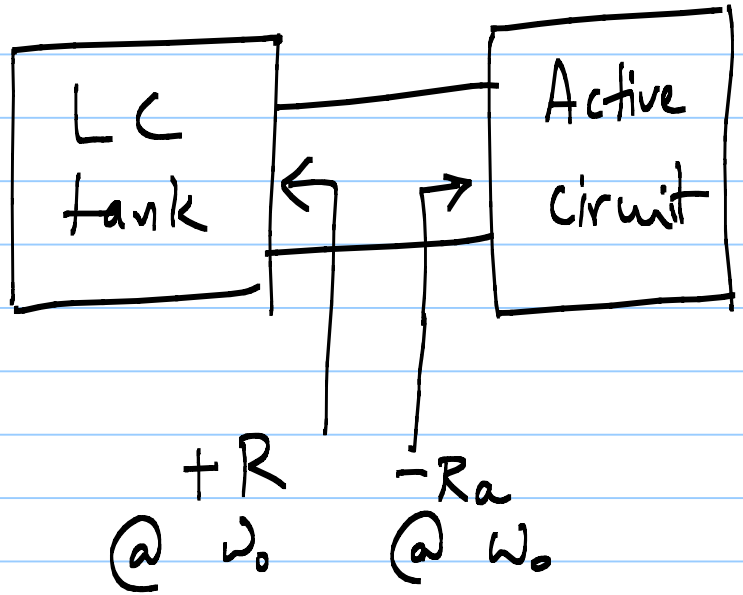
LC tank:

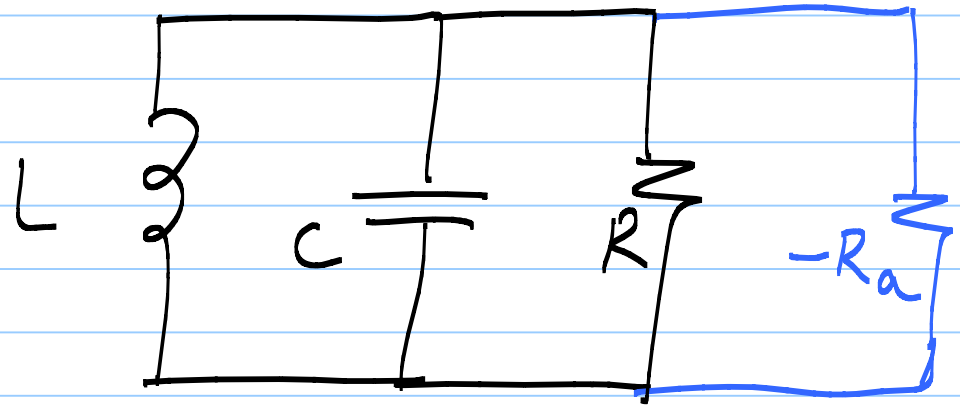


$$\omega_0 = \frac{1}{\sqrt{LC}}$$

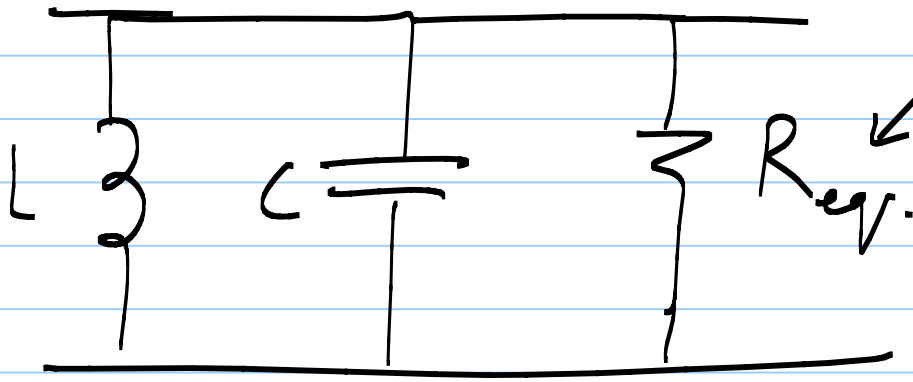
$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

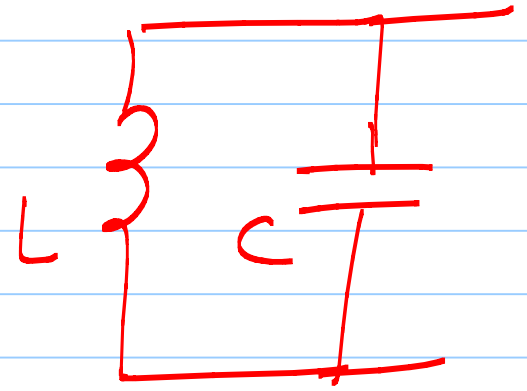
LHP complex conjugate poles





$$\frac{1}{Re\{Y_{in}\}}$$





tank osc.

$$R_{eq.} = \frac{R \cdot (-R_a)}{R + (-R_a)} = \frac{R \cdot R_a}{R_a - R}$$

* For stable osc → complex conjugate poles on jω axis

↳ $R_{eq.} = \infty$ (ideal LC tank)

@ ω_0

↳ $R_a = R$

∴ magnitude of negative resistance should be exactly equal to the equivalent resistance of LC tank.

{ for stable oscillations }

* If $R_a > R \Rightarrow R_{eq}$ is positive & finite

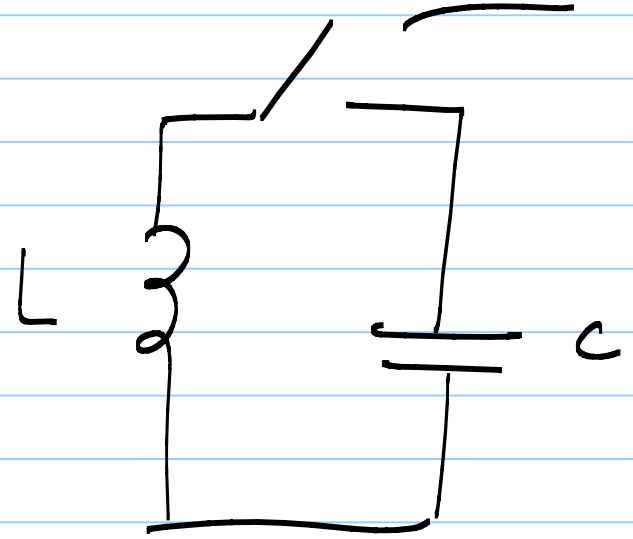
⇒ osc. will eventually die out

* If $R_a < R \Rightarrow R_{eq}$ is negative & finite

$\Rightarrow$ oscillation amplitude will rise

* How do you start up oscillations?

1)



resistance will increase tank losses

X

2) Allow osc. to build up naturally

↳ small noise + mismatches get amplified + disturbance

characteristic of active circuit

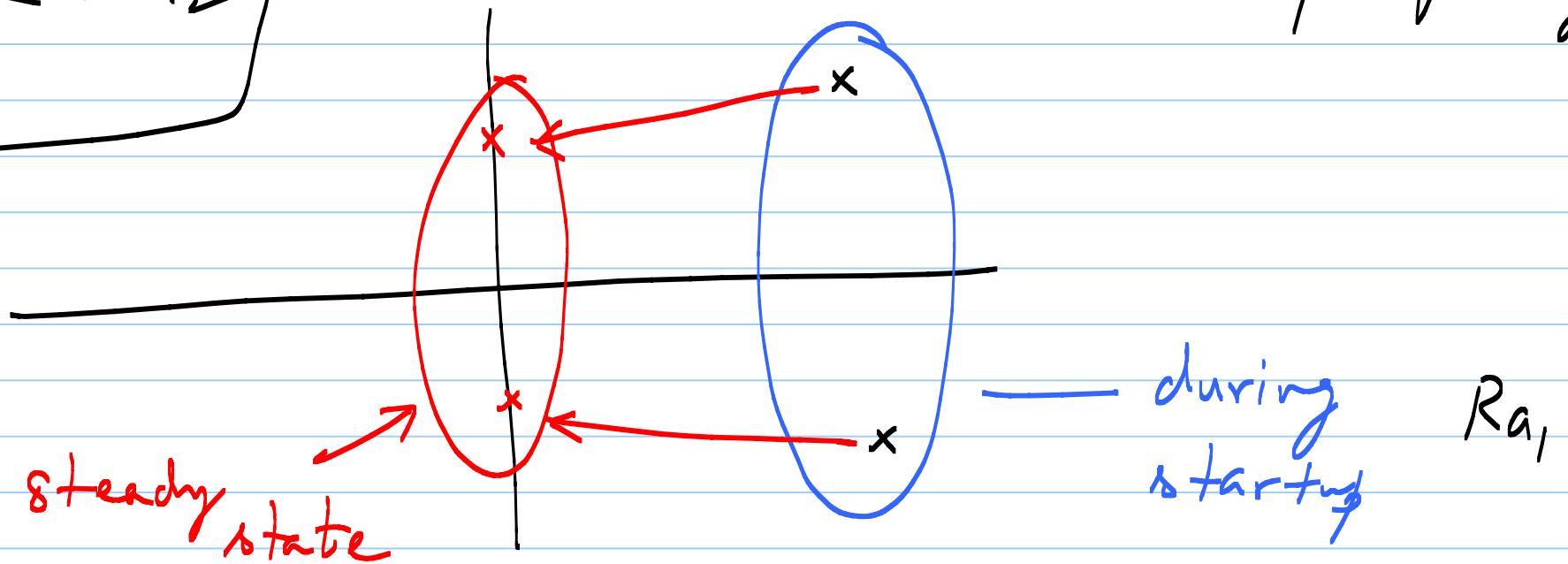
↳ for startup : $R_a \ll R$

autonomous

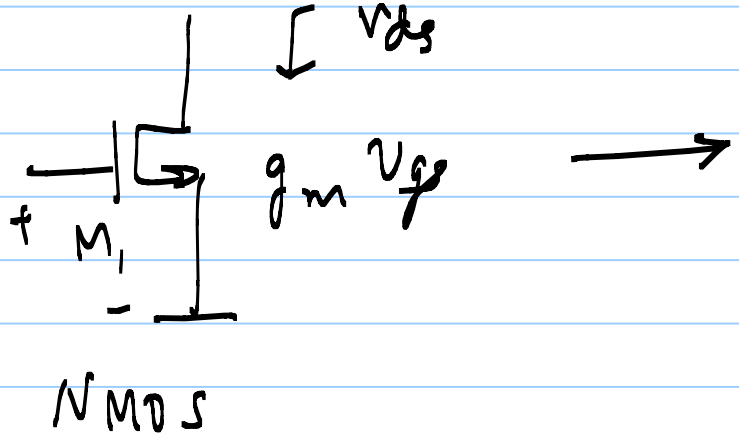
$\Downarrow$

osc. build up quickly

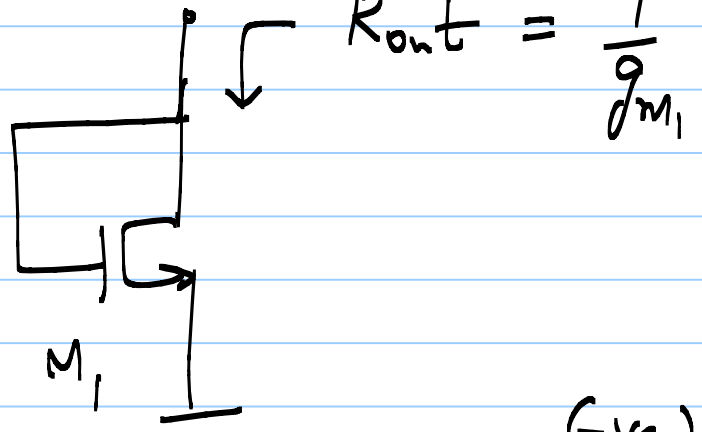
$R_{a_1} \ll R_{a_2}$



* We want simplest possible circuit that gives $-R_a$

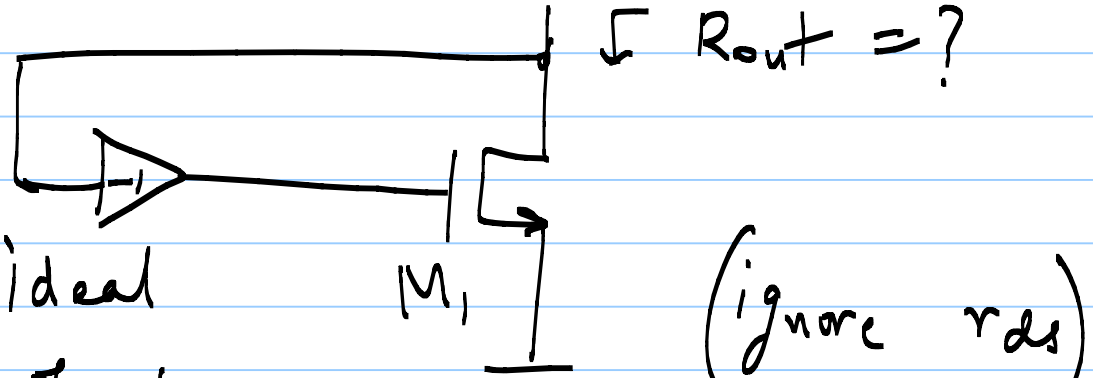


NMOS

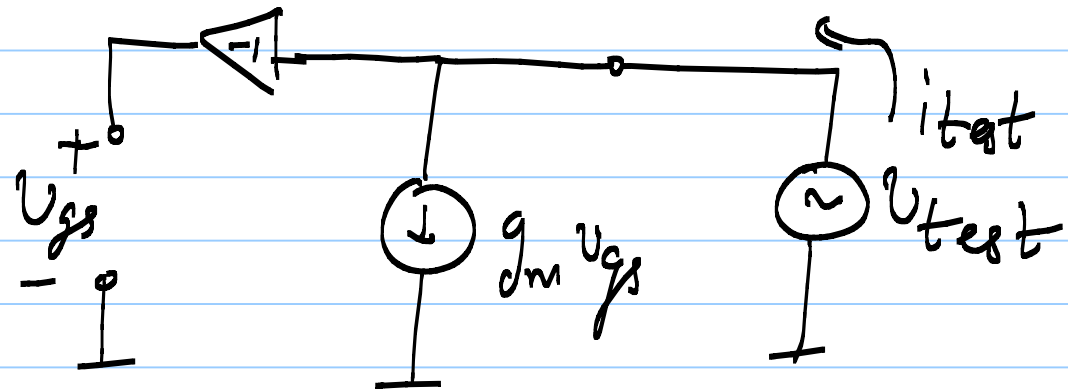


NMOS with D – G feedback (-ve)

* We need +ve f.b. to generate $-R_a$



Convert f.b. to +ve



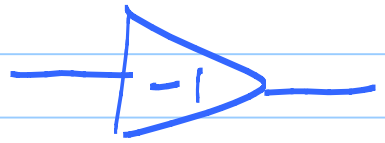
$$v_{gs} = -v_{test}$$

$$g_m v_{gs} = -g_m v_{test}$$

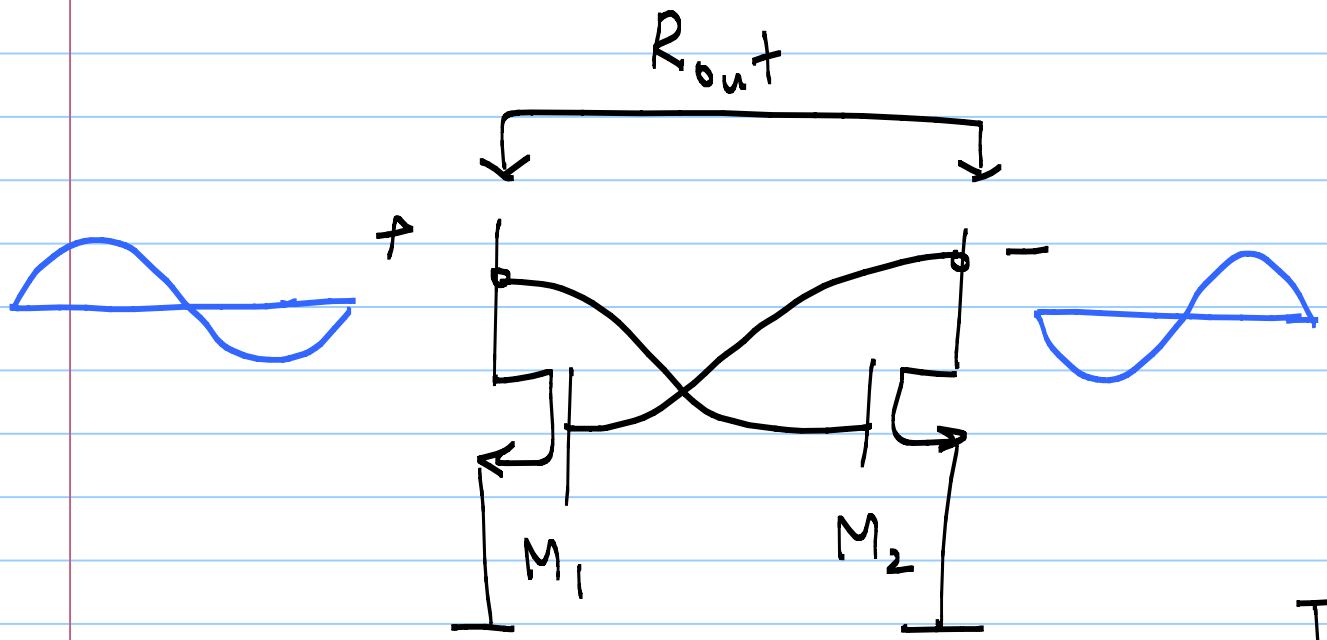
$$= i_{test}$$

$$\boxed{\frac{v_{test}}{i_{test}} = \frac{-1}{g_m}}$$

$$-R_a = \frac{-1}{g_m}$$



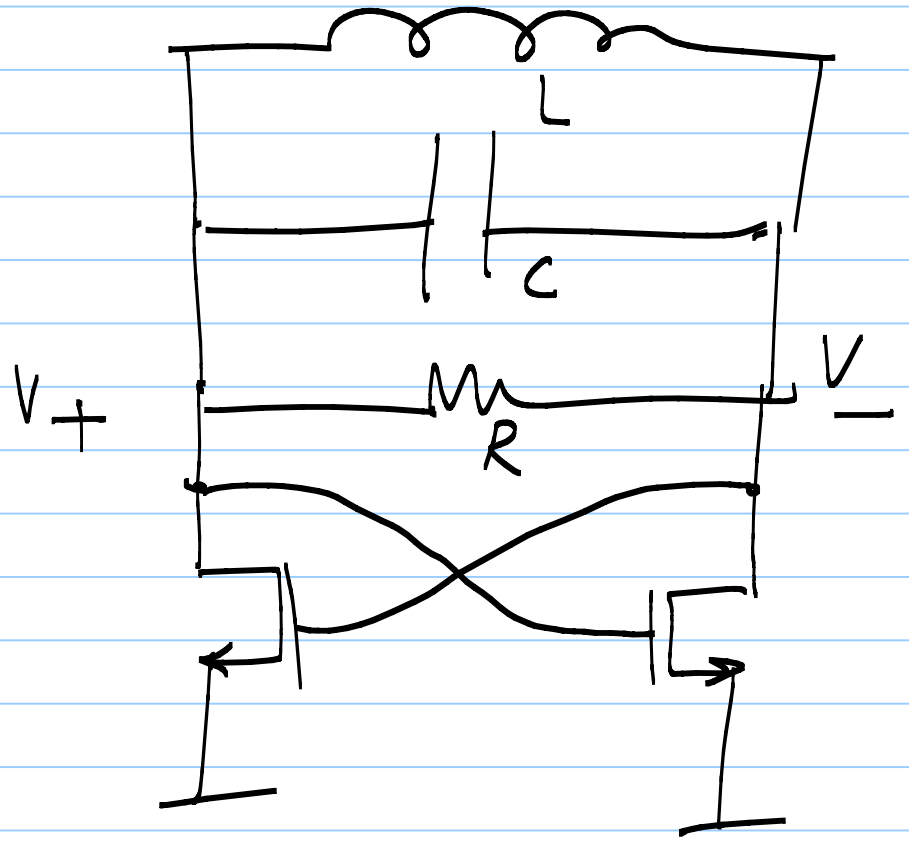
→ get it automatically in a fully differential oscillator

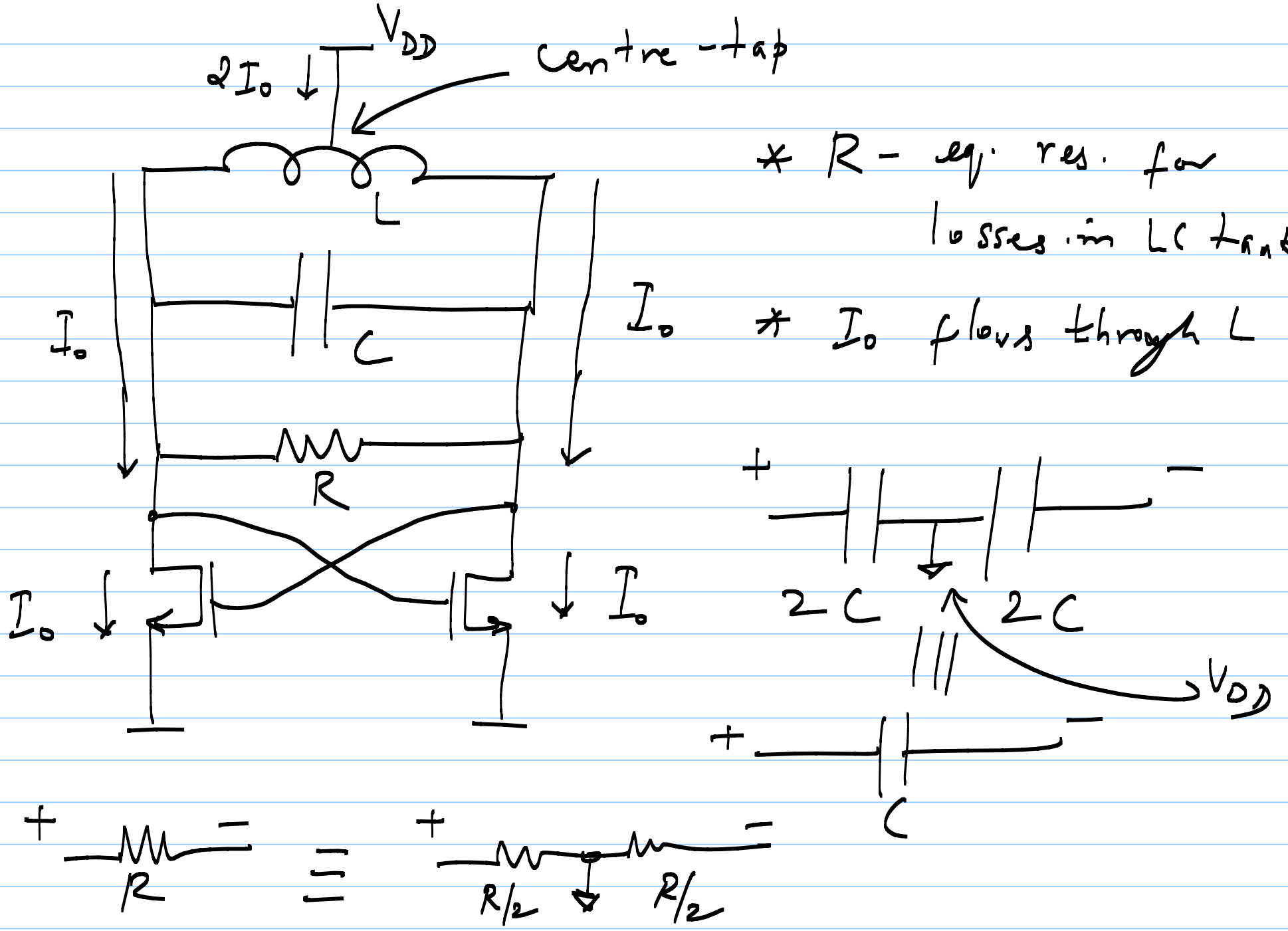


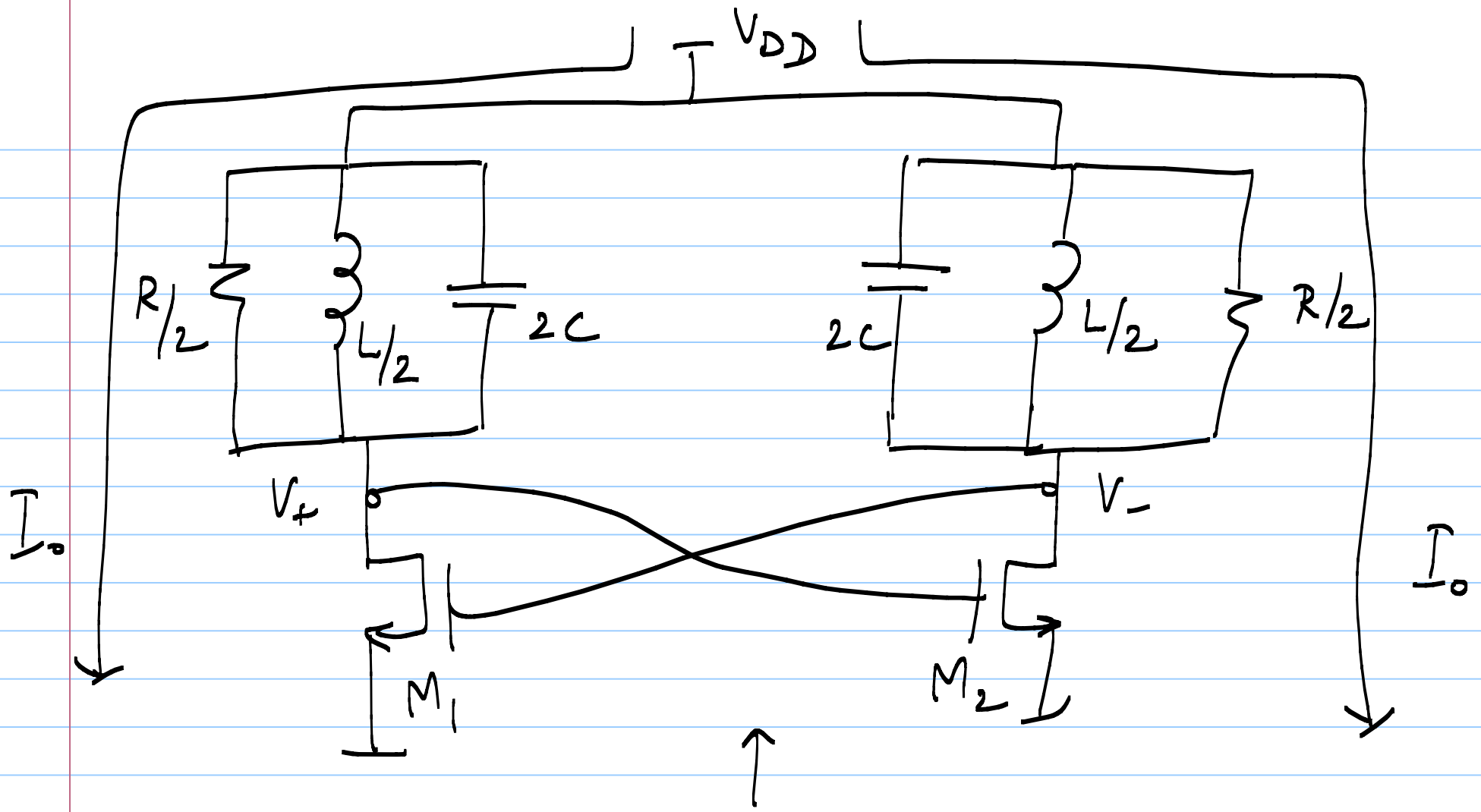
$$R_{out} = -\frac{2}{g_m}$$

$M_1 \equiv M_2$

Fully diff. Osc (small signal only)

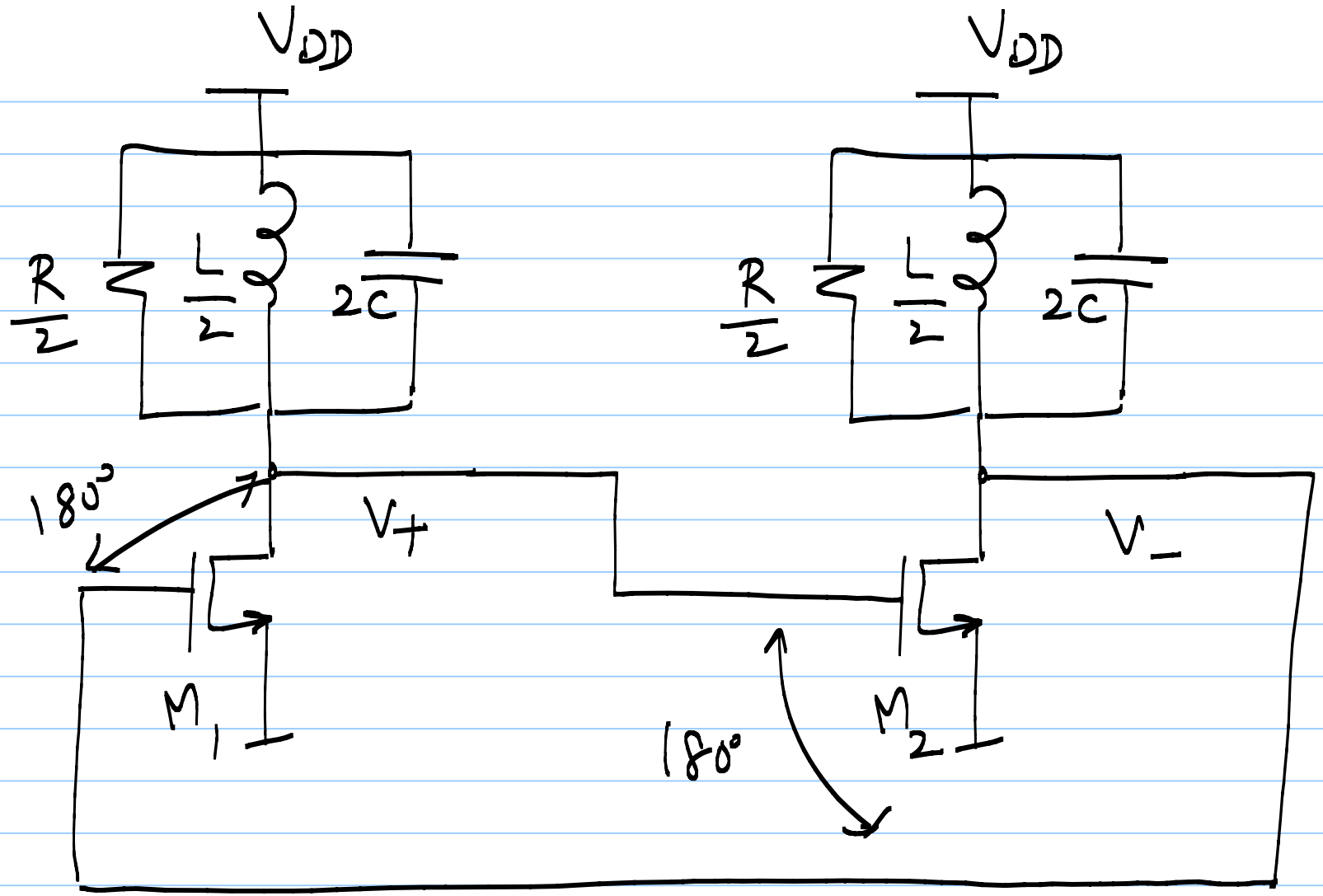



V_{DD}
$2I_o$
centre-tap
L
C
R
I_o
I_o
I_o
I_o
* R - eq. res. for losses in LC tank
* I_o flows through L
+
−
2C
2C
V_{DD}
C
R
=
R/2
R/2



alternative view of circuit

(Half-circuit view)



Feed back view of the osc.

$$|LG| = \left(g_m \cdot \frac{R}{2}\right)^2$$

@ resonance when phase shift $= 360°$

@ $\omega_0 = \frac{1}{\sqrt{LC}}$

$|LG| > 1$ for increasing osc.

$|LG| = 1$ for stable osc.

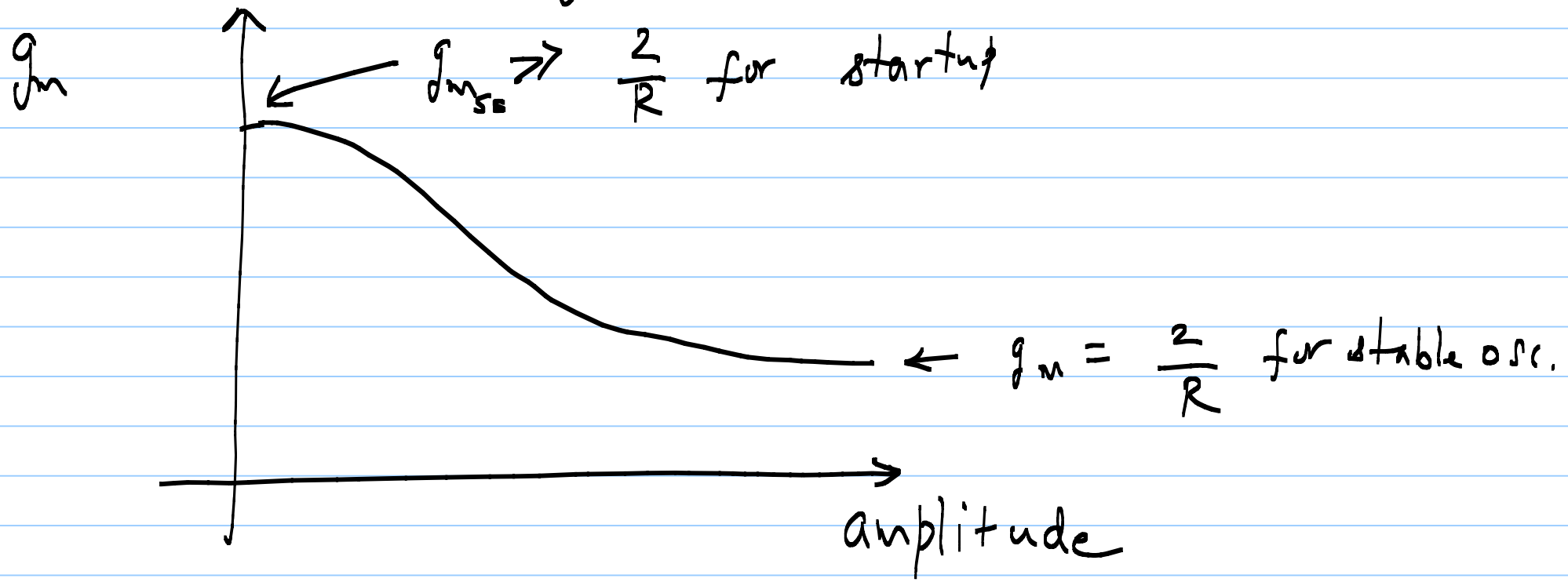
$$\left(g_m \frac{R}{2}\right)^2 = 1 \Rightarrow \boxed{R = \frac{2}{g_m} = R_a}$$

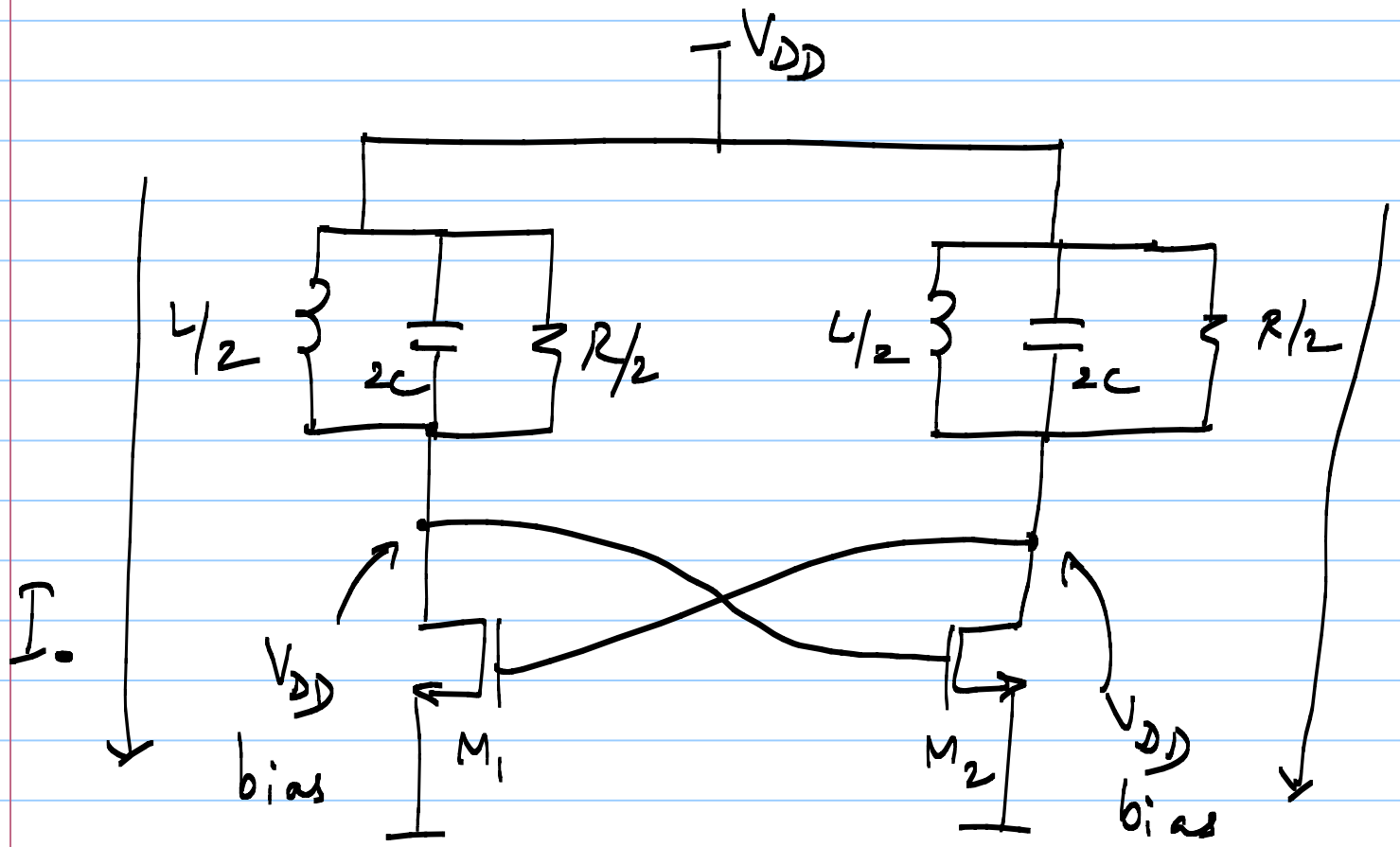
Design condition: $g_m = \frac{2}{R}$ for stable osc

For startup: $g_{m_{small\ signal}} > \frac{2}{R}$

* Make R large by design

↳ high quality L & C



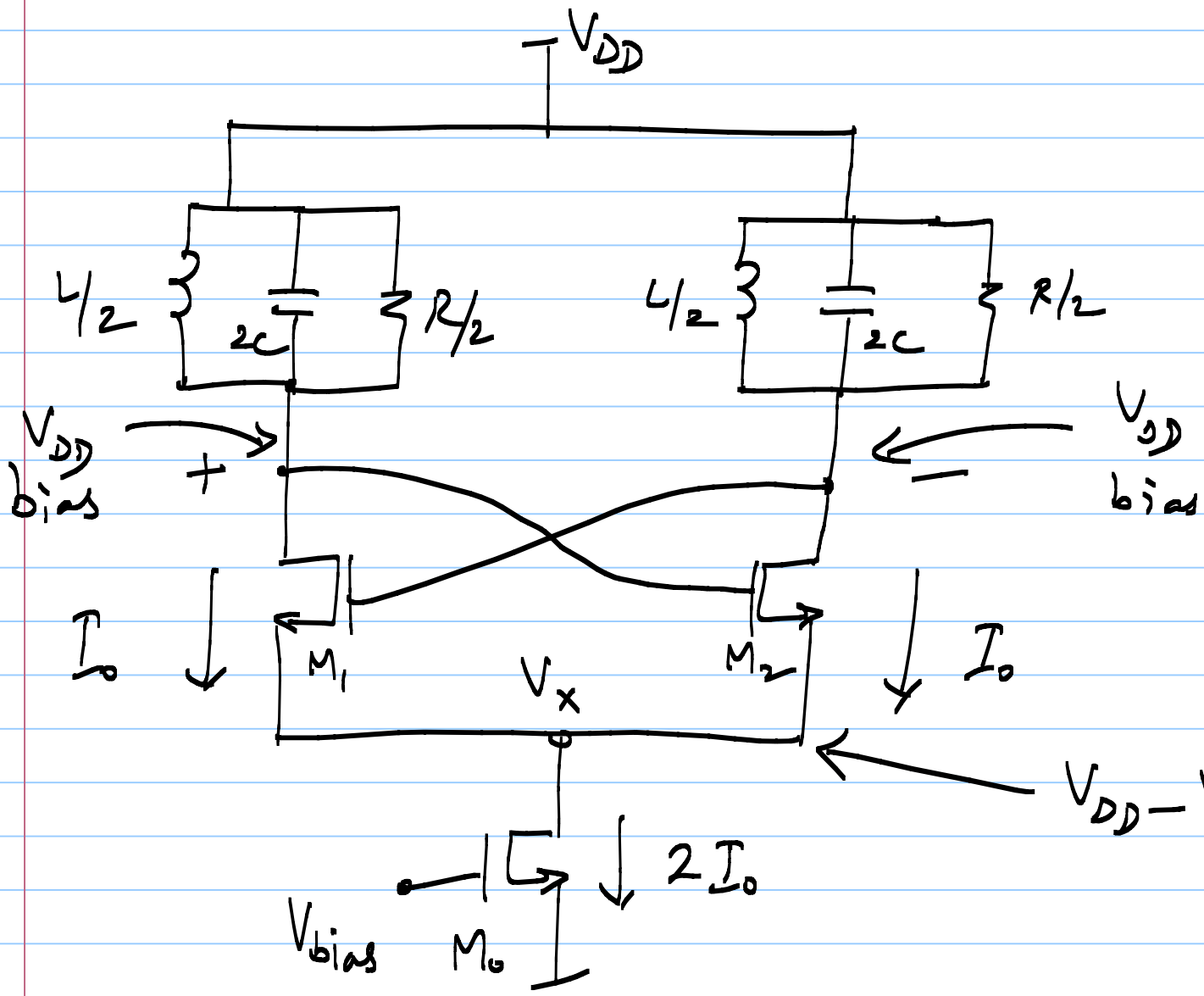


$I_o = ?$

$V_{GS} = V_{DD}$

$$I_o = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\left(V_{DD} - V_{Tn}\right)^2$$

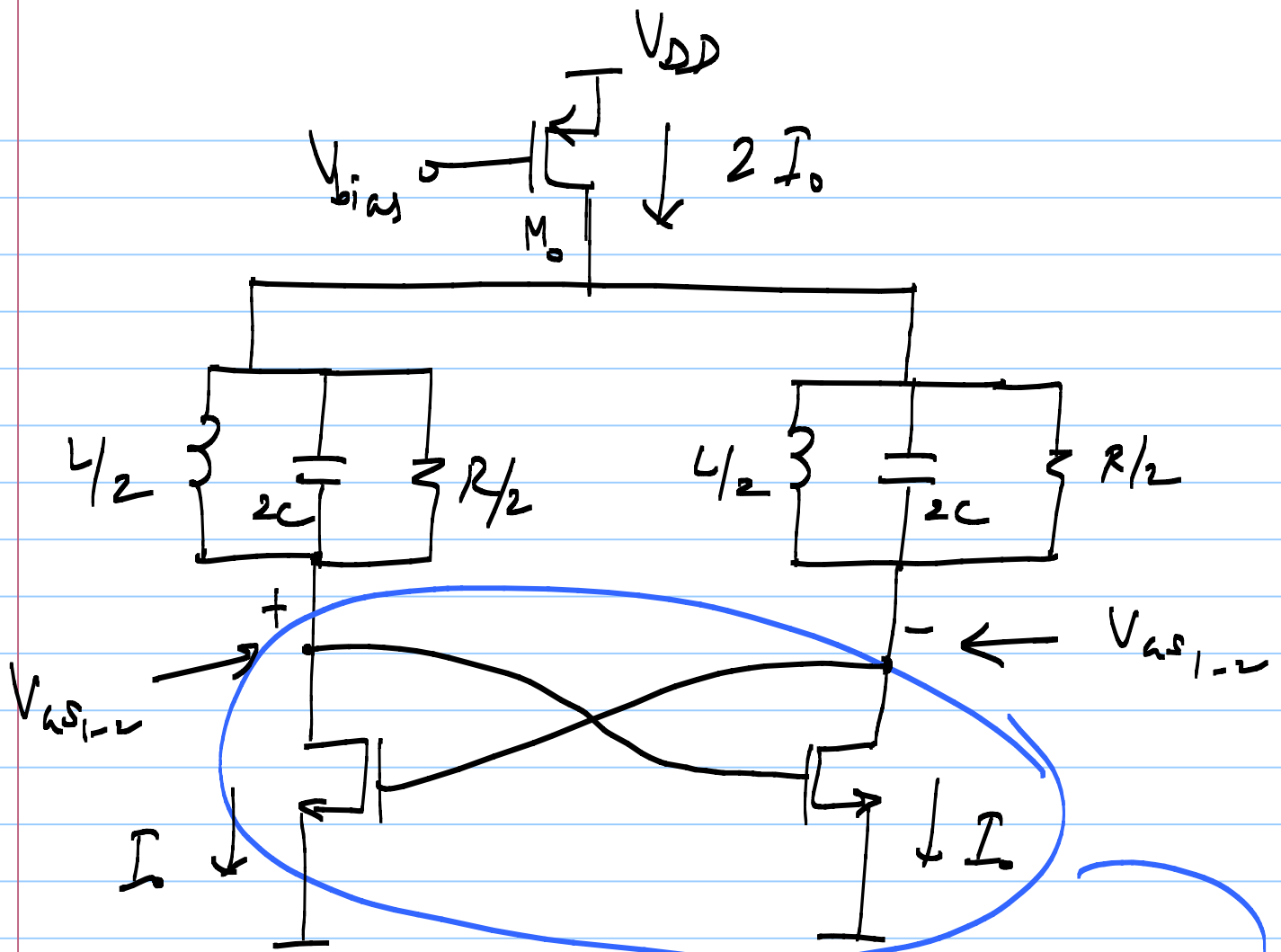
$\left(\frac{W}{L}\right)$ → decides everything



I_o & $\left(\frac{W}{L}\right)_{1-2}$ are independent design parameters

$V_{DD} - V_{GS_{1,2}}\Big|_{I_o} > V_{Dsat_o}$

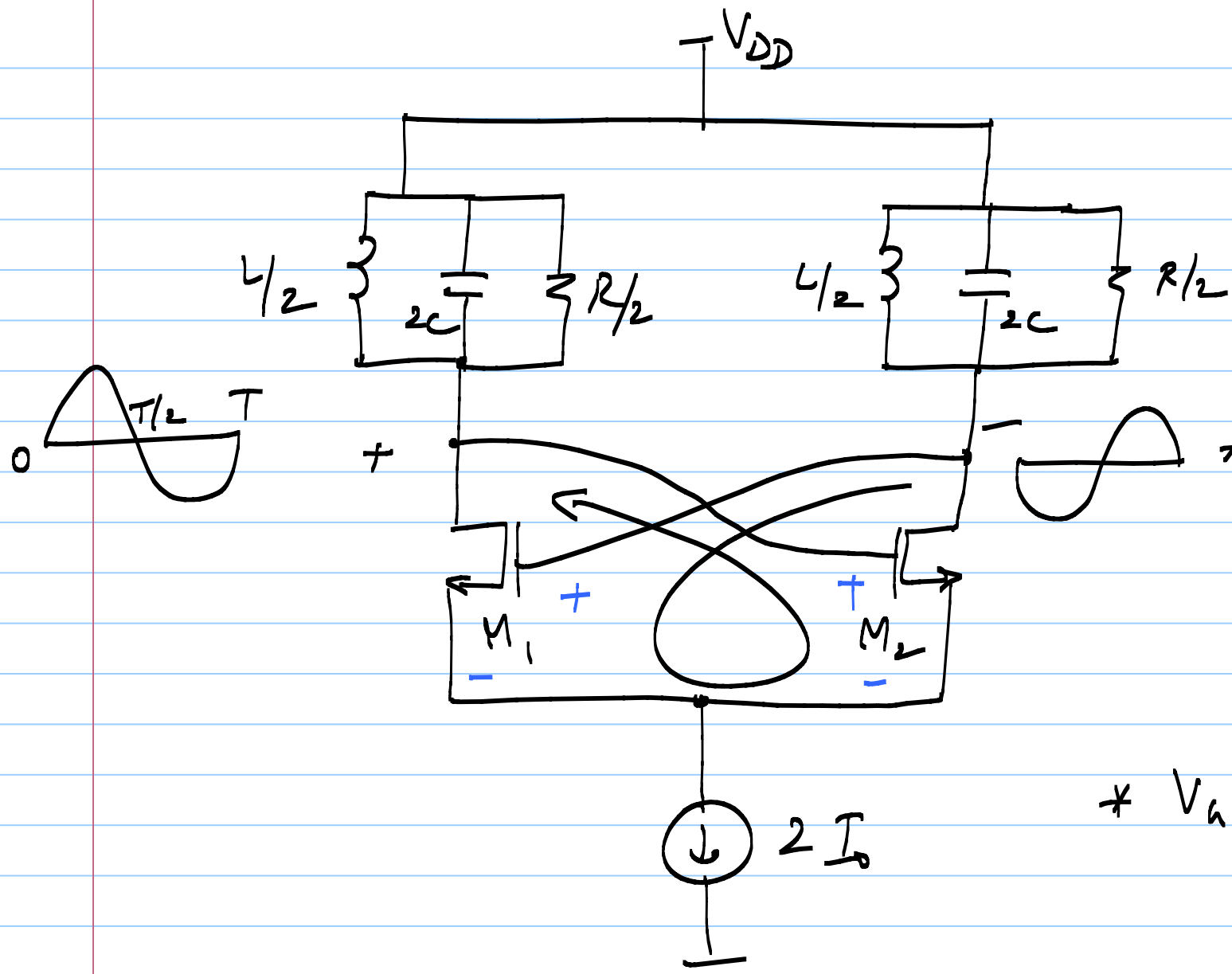
$> V_{bias} - V_T$



* Indep. control on I_0 & $\left(\frac{W}{L}\right)_{1-2}$

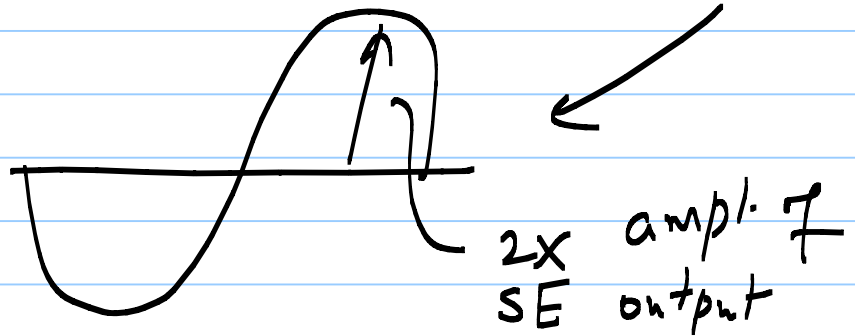
4 variants → even can be cross coupled pair M_1-M_2 PMOS

"Cross-coupled LC osc." or "$-g_m$ LC osc."



* $M_1 - M_2$ are placed in +ve f.b.

* $V_{gs_1} - V_{gs_2} = V_- - V_+$



* Switching of current from M_1 to M_2 happens very quickly due to +ve f-b.

* M_1 & M_2 conduct $2I_o$ in successive half periods

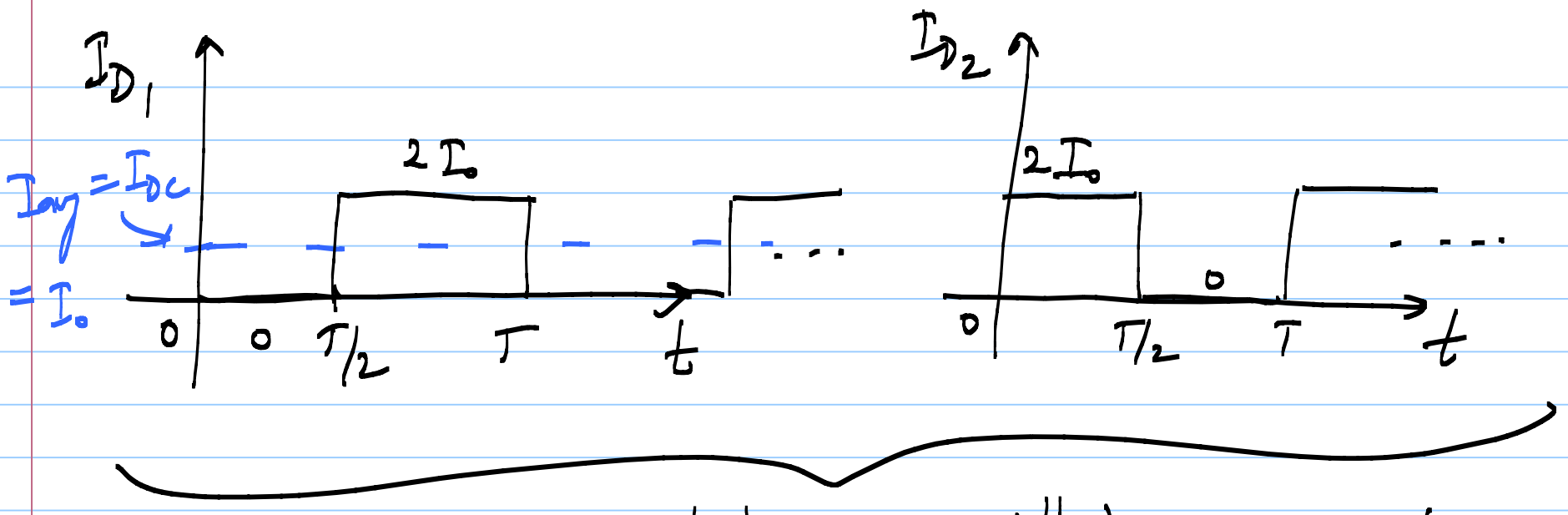
e.g. during 0 to $T/2$ → $V_{G_2} \uparrow$, $V_{G_1} \downarrow$

→ M_2 ON, M_1 OFF

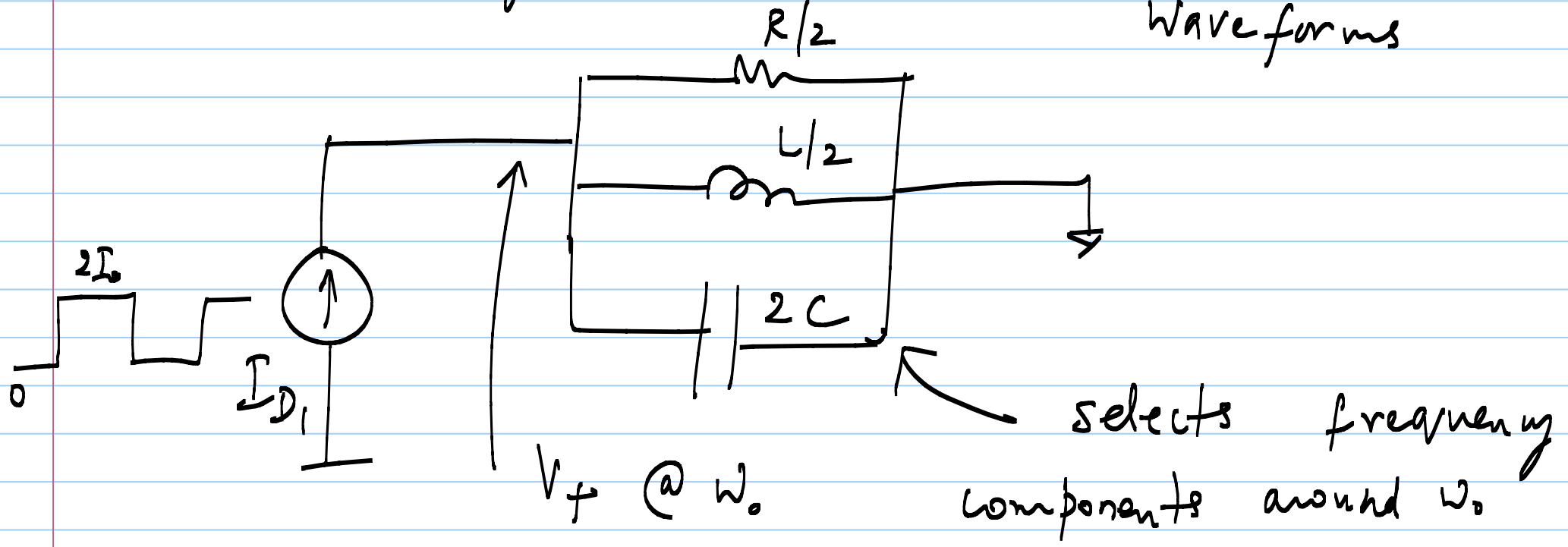
$T/2$ to T → M_2 OFF, M_1 ON

OFF device $I_D = 0$

ON device $I_D = 2I_o$



steady state oscillator current waveforms



$$I_{D_1} = 2I_0 \left[\frac{1}{2} - \frac{1}{\pi} \sin\omega_0 t + \text{odd harmonics} \right]$$

$$V_{+ @ \omega_0} = \frac{2I_0}{\pi} \cdot \frac{R}{2} = \frac{I_0 R}{\pi} \quad (-\sin\omega_0 t)$$

$$V_{- @ \omega_0} = \frac{I_0 R}{\pi} \quad (+\sin\omega_0 t)$$

$$V_{od} = (V_- - V_+) = \frac{2I_0 R}{\pi}$$

fully differential amplitude

Osc $\longrightarrow$ VCO ?

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

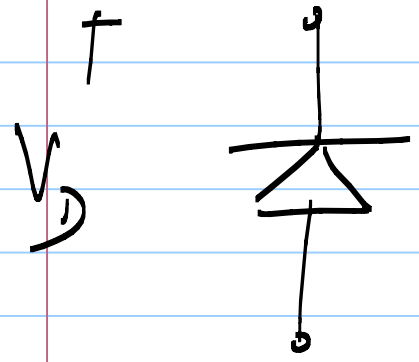
tuning L is difficult

tuning C is easier

$C = f(V)$ → Varactor

$$C = \frac{\epsilon A}{d}$$

1)



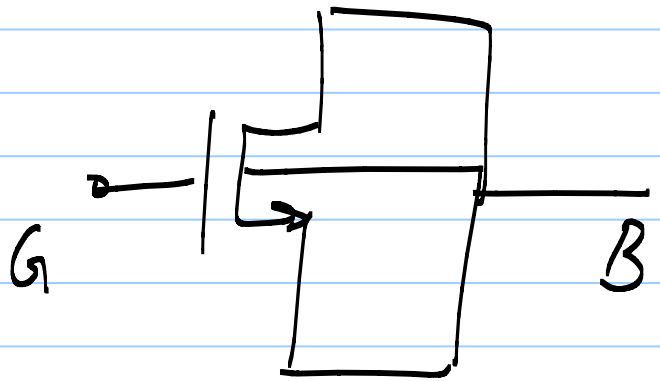
reverse biased diode

↳ depletion region width

depends on V_D

→ $C = f(V_D)$

2)



MOS varactor

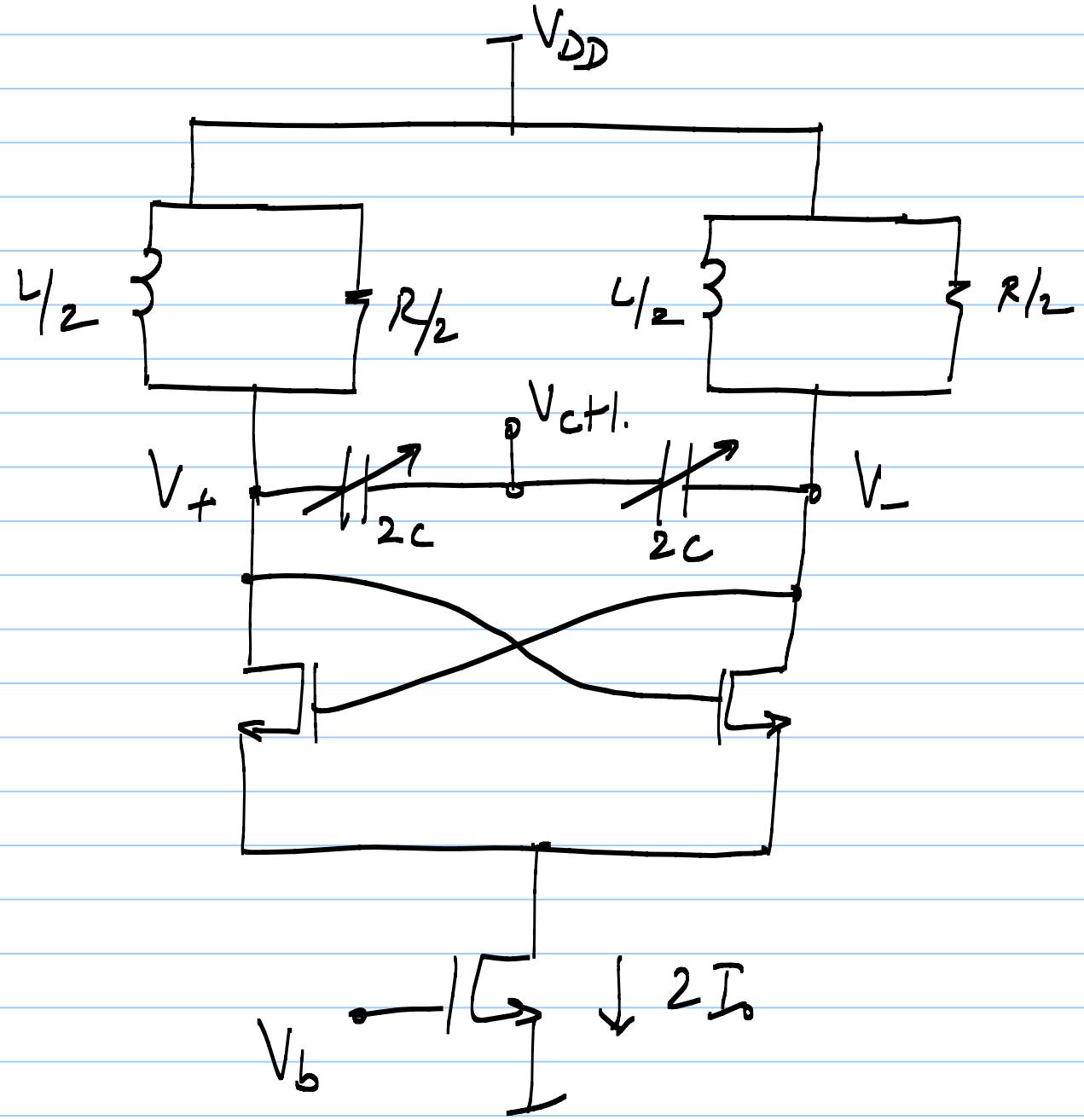
$C_{gb} = f(V_{gb})$

Accumulation → Depletion → Inversion

C_{ox} C_{ox}

$C_{depl.} < C_{ox}$

t_{une} t_{une}



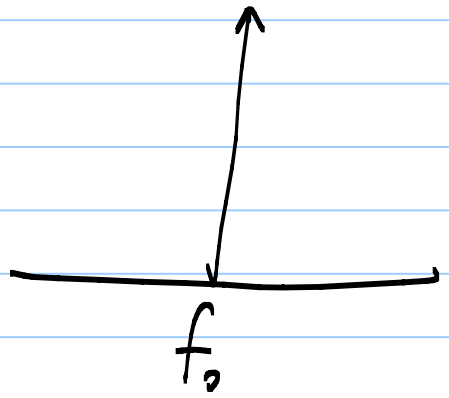
VCO

Voltage

Controlled

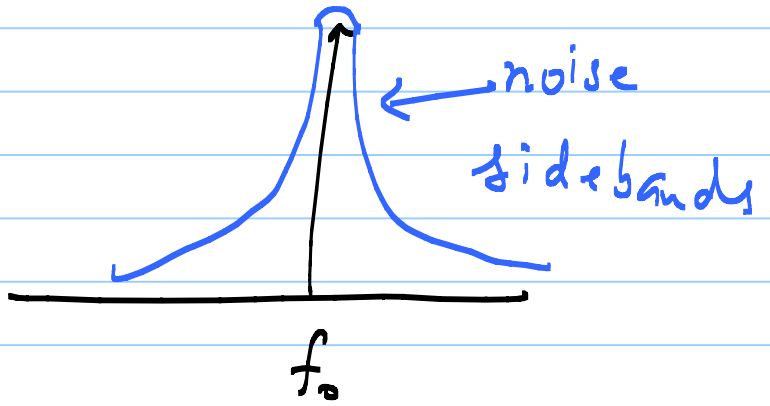
Oscillator

What you want



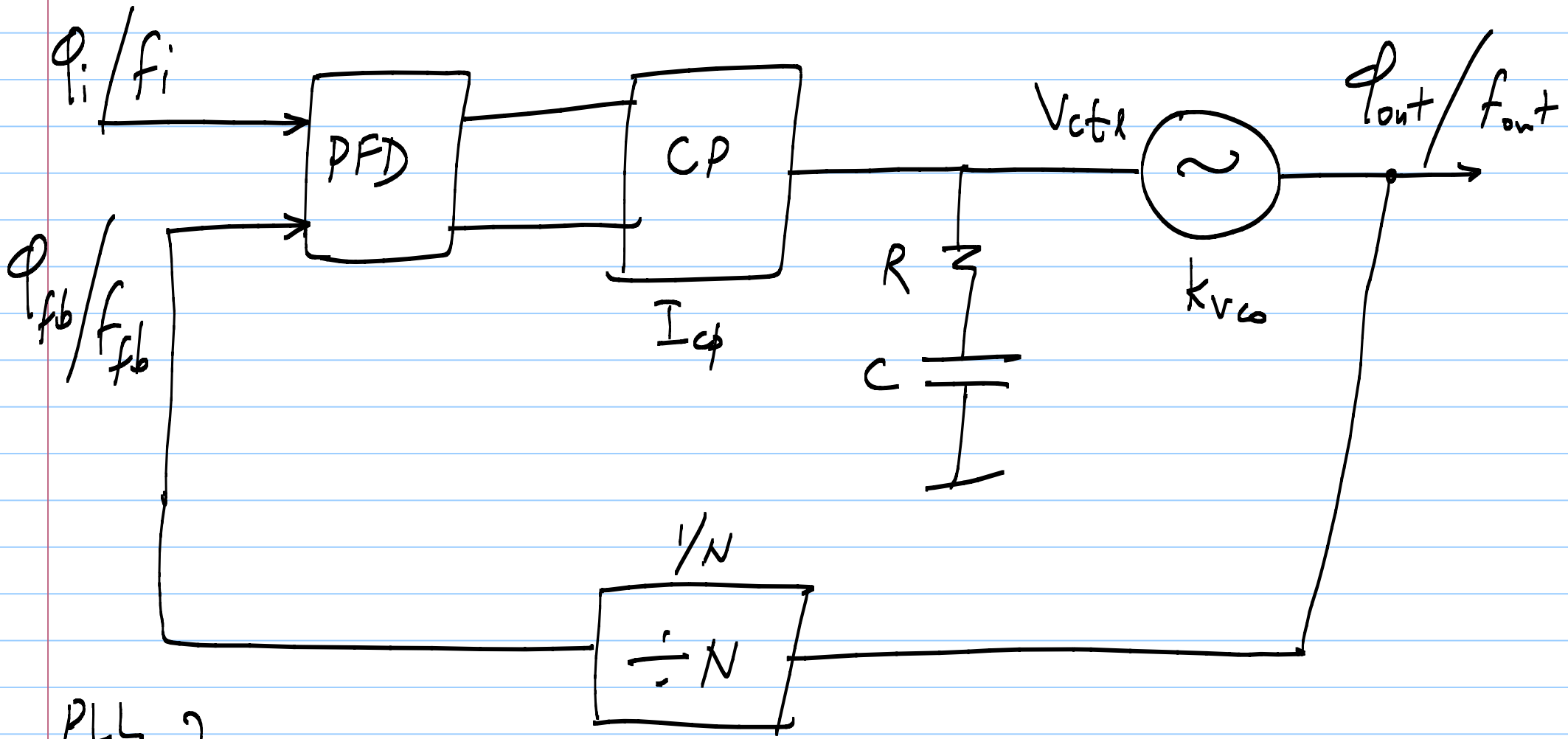
ideal osc. spectrum

What you get



Real osc. spectrum

Type-II PLL Design Example



PLL Specs

$f_i = 10\,kHz$; $f_o = 1\,MHz$; PLL loop $BW = 500\,Hz$;

$\zeta = \frac{1}{\sqrt{2}} = 0.707$; $K_{VCO} = 500\,kHz/V$

* ζ → decides the shape of transient locking of PLL to phase step

$$Q = \frac{1}{2\zeta} = 0.707$$

$\frac{\phi_{out}}{\phi_i}$ → closed loop

LG $\quad L(s) = \frac{I_{cp}}{2\pi}\left[R + \frac{1}{sC}\right] \cdot \frac{2\pi K_{vco}}{s} \cdot \frac{1}{N}$

$$= \frac{I_{cp} \cdot K_{vco}\,(1+sCR)}{s^2 NC}$$

CLG $\quad \dfrac{\phi_{out}}{\phi_i}(s) = N \cdot \dfrac{L(s)}{1 + L(s)}$

$$= N \cdot \frac{I_{cp} K_{vco} (1 + sCR) / s^2 NC}{1 + I_{cp} K_{vco} (1 + sCR) / s^2 NC}$$

$$= N \cdot \frac{I_{cp} K_{vco} (1 + sCR)}{s^2 NC + s (I_{cp} K_{vco} CR) + I_{cp} K_{vco}}$$

$$D(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$\omega_n = \sqrt{\frac{I_{cp} K_{vco}}{NC}}$$

$$\omega_{-3dB} = 2\pi \times \underbrace{f_{-3dB}}_{\text{loop BW}} = 2\pi \times 500$$

ω_{-3dB} & ω_n & ζ relation for a second order system:

$$\omega_{-3dB}^2 = \omega_n^2 \left[(2\zeta^2 + 1) + \sqrt{(2\zeta^2+1)^2 + 1} \right]$$

determine ω_n

$$(3140)^2 = \omega_n^2 \left[2 + \sqrt{5}\right]$$

$$\omega_n = 1525.6 \ \text{rad/s}$$

$$\omega_n = \sqrt{\frac{I_{cp} K_{VCO}}{N C}} \longrightarrow \text{sets } \frac{I_{cp}}{C}$$

I_{cp} ← normally set by noise constraint

here : set $I_{cp} = 10\mu A$

$$N = \frac{f_{out}}{f_i} = 100$$

$$\Rightarrow \quad C = 21.5\,nF$$

$$\zeta = \frac{1}{\sqrt{2}}$$

coeff. of s term in $D(s) = \dfrac{I_{cp} K_{vco} R}{N}$

set this $= 2\zeta\omega_n$

→ gives R

$$R = 43.1\ k\Omega$$

* $K_{pd} = \dfrac{I_{cp} \cdot R}{2\pi}$ * $f_n = \dfrac{K_{pd} \cdot K_{vco}}{N}$;

* $K_{pdI} = \dfrac{I_{cp}}{2\pi C}$ * $\omega_n = \dfrac{2\pi K_{pd} K_{vco}}{N}$