3/1/20

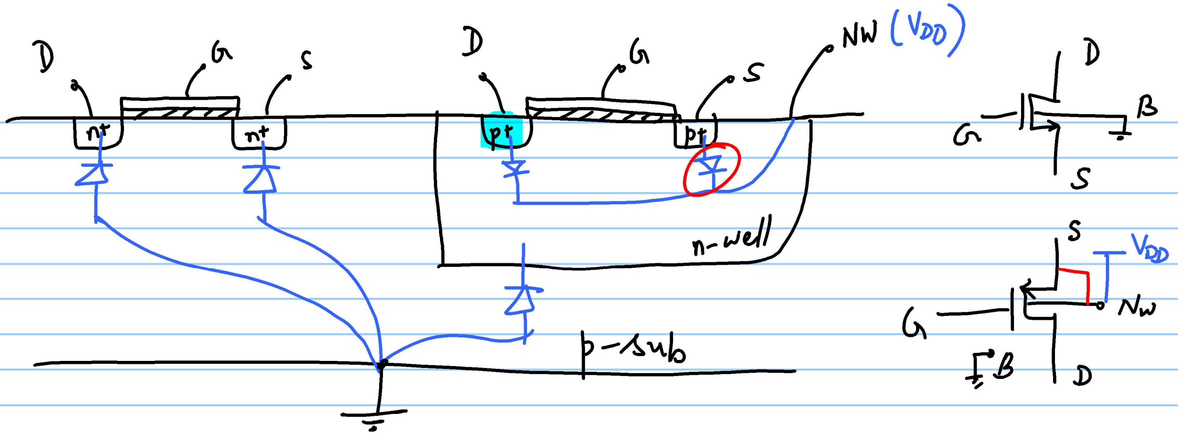
Note Title 03-01-2020

Lec 1

2 'similar'

IC - devices can be assumed to be nominally identical

Assume no $\underset{(\text{ind})}{L}$ for this course


D
G
S
n+
n+
D
G
S
p+
p+
NW (VDD)
n-well
p-sub
D
G
B
S
S
VDD
G
NW
B
D

Resistors → mΩs to 100s of kΩ

polysilicon

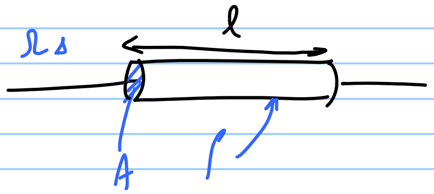
metal — mΩs to Ωs

n-well

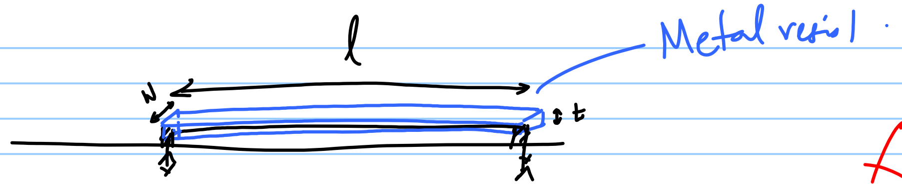
diffusion

larger resistors (10s to 100s of kΩ)

medium



$$R = \frac{\rho l}{A}$$



$$R = \frac{\rho l}{A} = \frac{\rho \cdot l}{w \cdot t} = \left(\frac{\rho}{t}\right) \cdot \frac{l}{w}$$

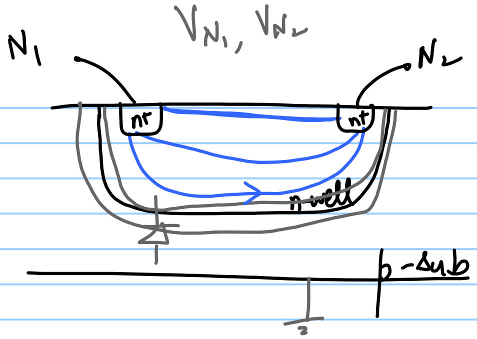
$\equiv \# \text{ of } \square s$

set by process

"sheet resistance"

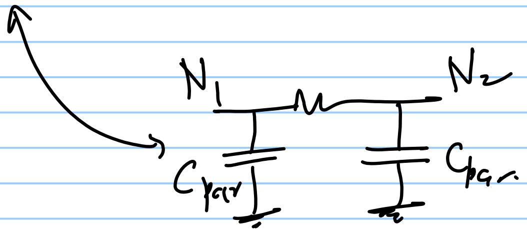
$\Omega / \square$

Value
Power
Accuracy / Tolerance
Temperature Coefficient ✓
Size
Linearity ✓

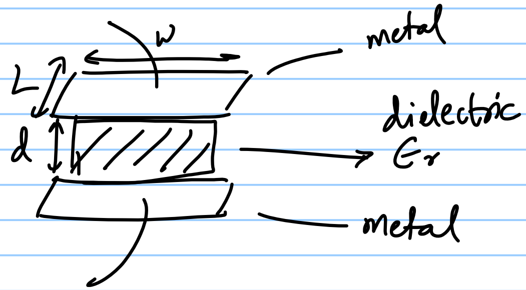


$$V = I \cdot R$$

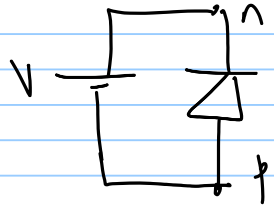
$$I = \frac{V}{R} + (\ )V^2 + (\ )V^3 + \dots$$



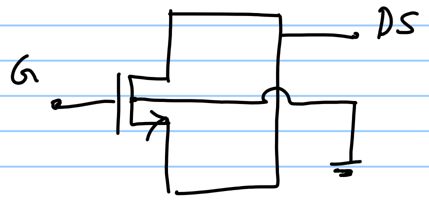
Capacitors

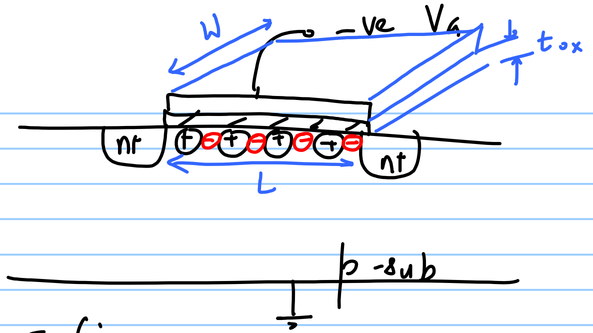


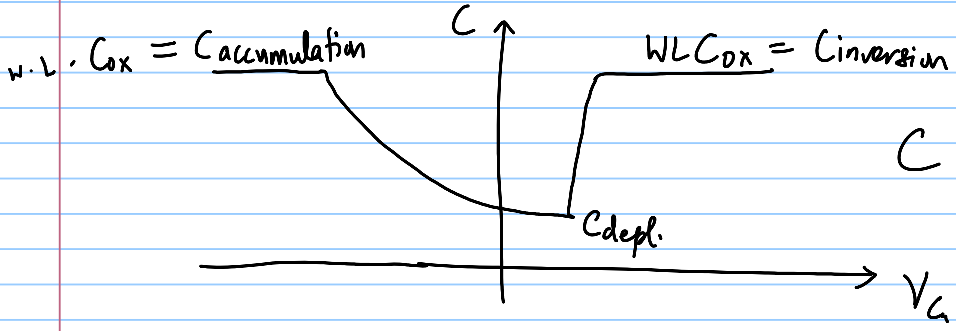
$$C = \frac{\overset{\epsilon_r \cdot \epsilon_0}{\overset{\parallel}{\epsilon}}\,\overset{w \cdot L}{A}}{d}$$



$\equiv C_{diode}$



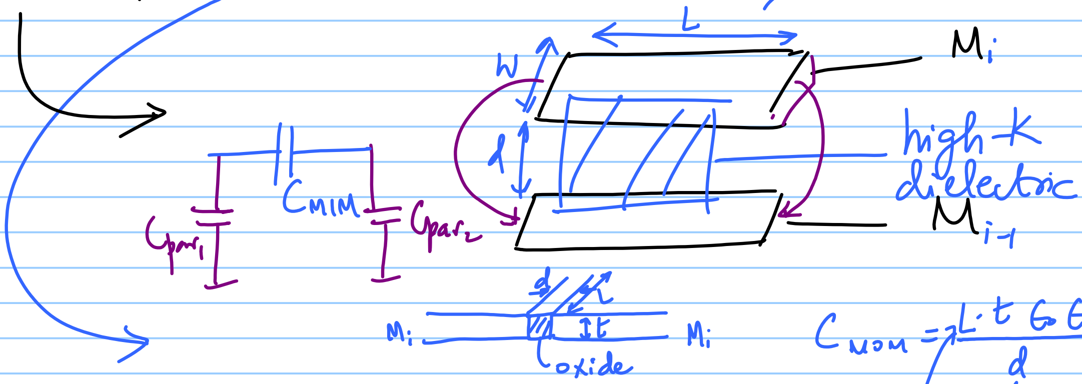




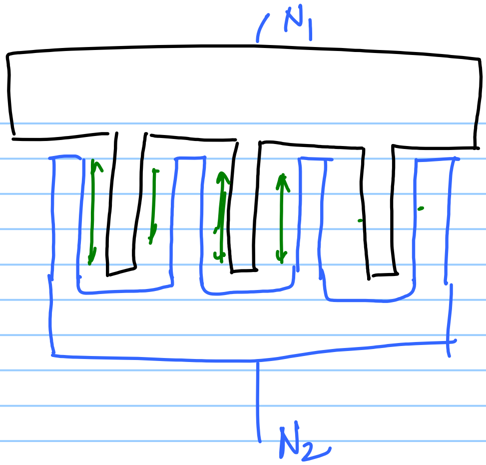
$$C = \frac{\varepsilon_0 \varepsilon_r \cdot W \cdot L}{t_{ox}} \equiv C_{ox}$$

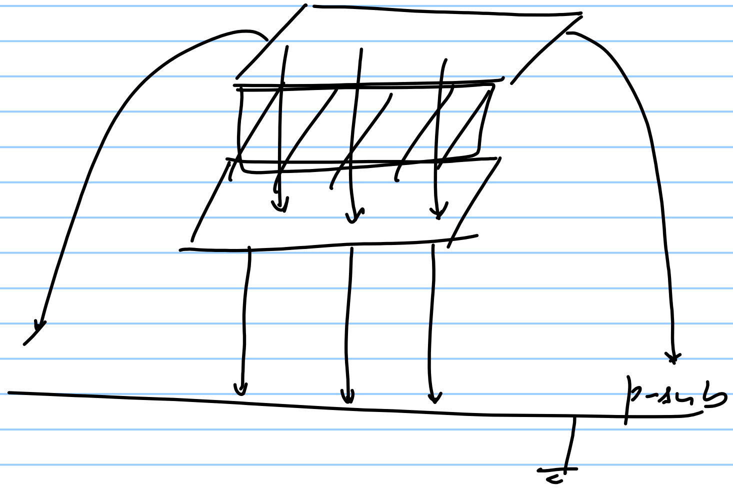
MIM & MOM — linear

$$Cap = \frac{W \cdot L \cdot \epsilon_0 k}{d}$$



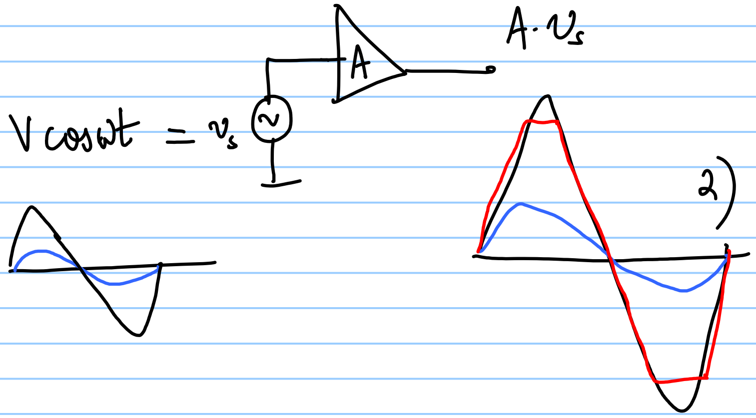
$$C_{MOM} = \frac{L \cdot t \ \epsilon_0 \epsilon_r}{\underset{d_{min.}}{\overset{\parallel}{d}}}$$


N1
N2


p-sub

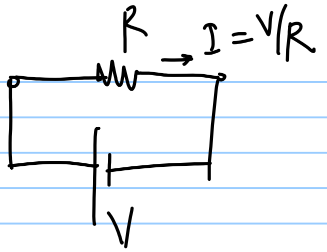
6/1/20

Lec 2



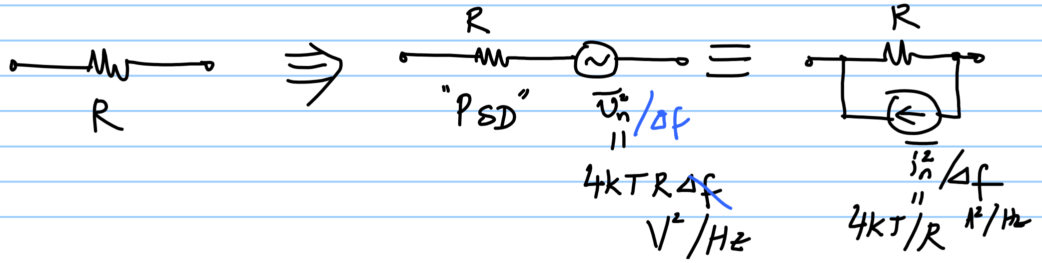
1) As v_s amplitude ↑ : (V↑)
clipping of sinusoid after violation of swing limits

2) As V ↓ : reach noise limits



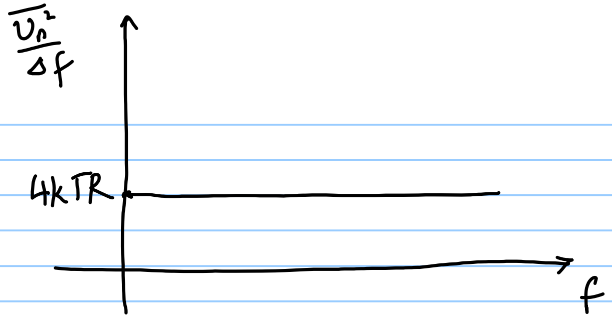
Random motion of charges becomes comparable to deterministic motion

1) Thermal noise of R



* "White" noise — constant PSD

* Either $\overline{v_n^2}$ or $\overline{i_n^2}$ should be considered

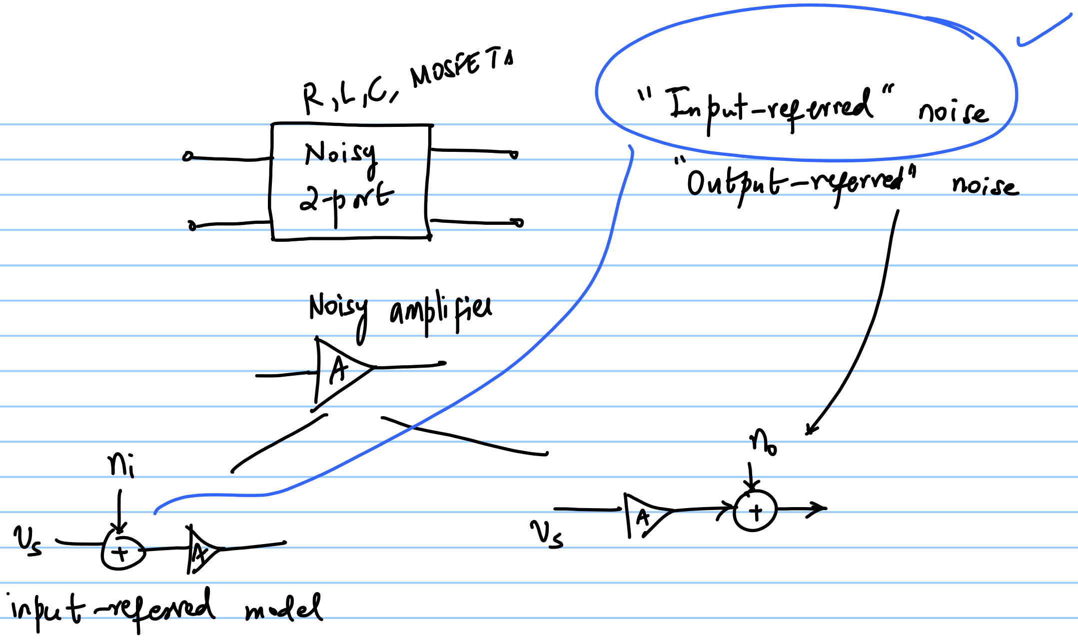


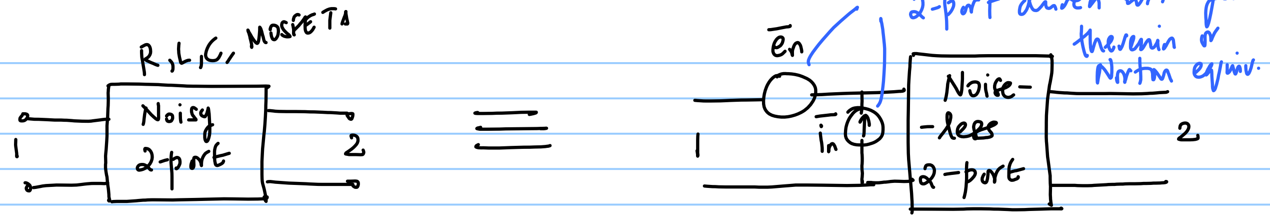
2) pure C, L, M → No noise

e.g. 1kΩ Res:

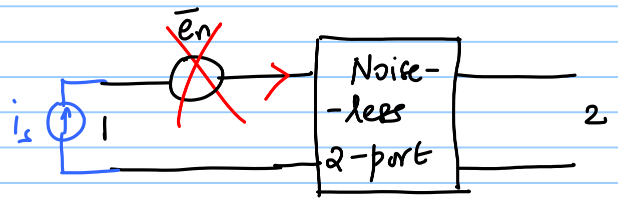
$\overline{v_n^2}/\Delta f = 16\times10^{-18}\ V^2/Hz$ $\quad \overline{v_n}/\sqrt{\Delta f} = 4nV/\sqrt{Hz}$

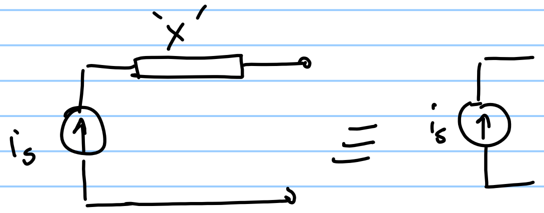
$\overline{i_n^2}/\Delta f = 16\times10^{-24}\ A^2/Hz$ $\quad \overline{i_n}/\sqrt{\Delta f} = 4pA/\sqrt{Hz}$

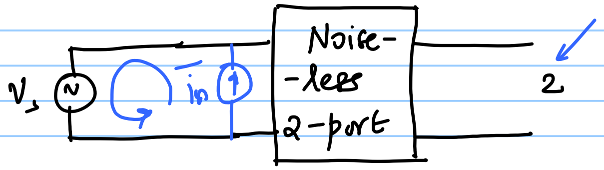

R, L, C, MOSFETs
Noisy 2-port
"Input-referred" noise
"Output-referred" noise
Noisy amplifier
A
n_i
v_s
+
A
input-referred model
n_o
v_s
A
+



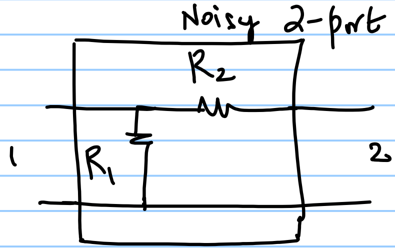
Need both to represent 2-port driven with general thevenin or Norton equiv.





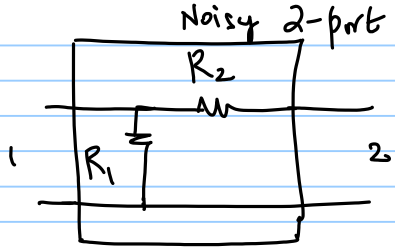


E.g.

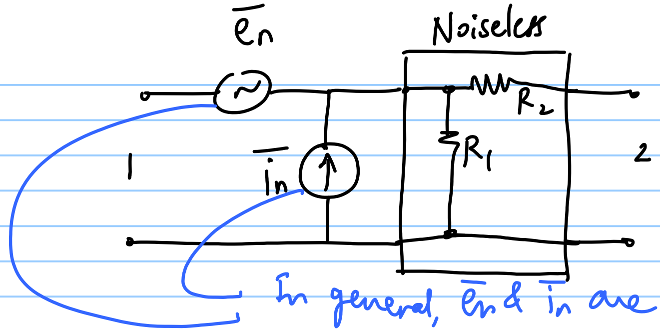


$\overline{e_n} = ?$ $\overline{i_n} = ?$

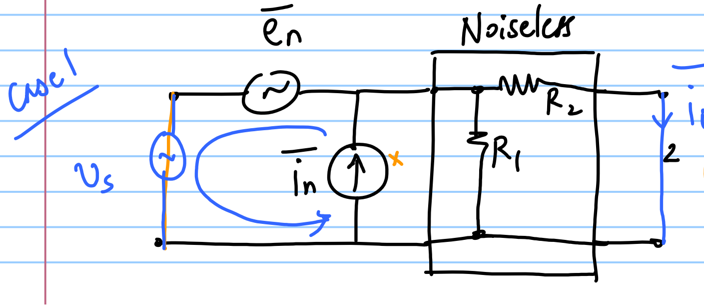
* You need $\overline{e_n}$ & $\overline{i_n}$ separately



$\equiv$

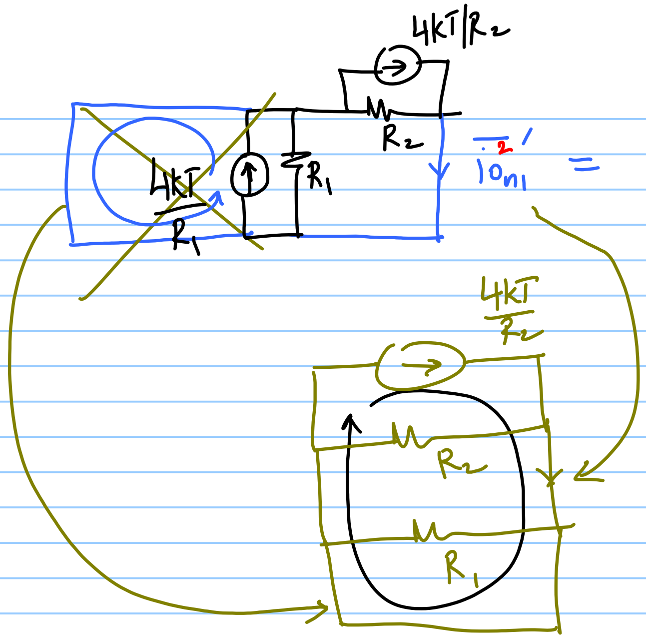


In general, $\overline{e_n}$ & $\overline{i_n}$ are partially correlated only



$\overline{i_{on_1}} = f(\overline{e_n})$

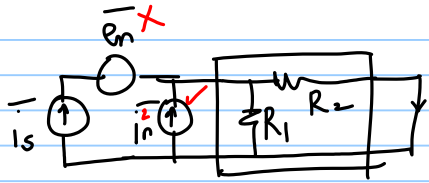
$= \frac{\overline{e_n}}{R_2}$



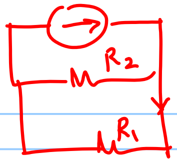
$$\overline{i_{on1}^2}' = \frac{4kT}{R_2}$$

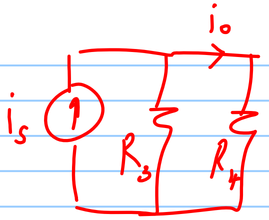
$$\frac{\overline{e_n^2}/\Delta f}{R_2^2} = \frac{4kT}{R_2}$$

$$\overline{e_n^2}/\Delta f = 4kTR_2$$

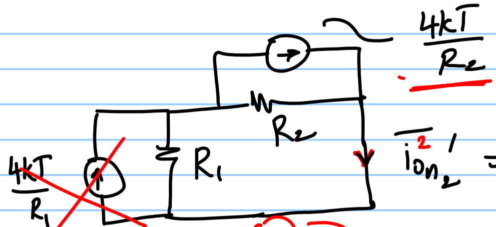


$$\overline{i_{on_2}^2} = \frac{\overline{i_n^2}\cdot R_1^2}{(R_1+R_2)^2}$$

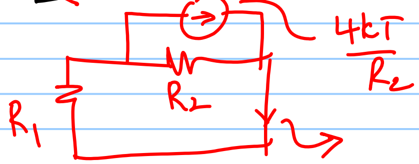




$$i_o = \frac{R_3}{R_3+R_4}\cdot i_s$$



$$\overline{i_{on_2}^2}' = \frac{4kT}{R_1}\cdot\left(\frac{R_1}{R_1+R_2}\right)^2 + \frac{4kT}{R_2}\cdot\left(\frac{R_2}{R_1+R_2}\right)^2$$ ✓



$$\overline{i_n^2} \cdot \left(\frac{R_1}{R_1+R_2}\right)^2 = \frac{4kT}{R_1} \cdot \left(\frac{R_1}{R_1+R_2}\right)^2 + \frac{4kT}{R_2} \cdot \left(\frac{R_2}{R_1+R_2}\right)^2$$

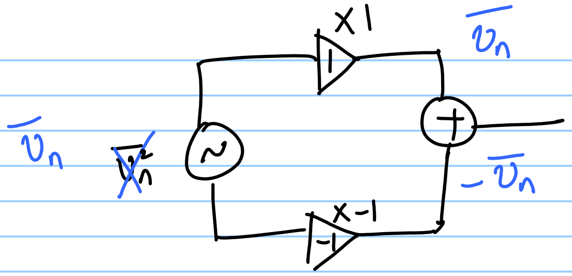
$$\overline{i_n^2} = \frac{4kT}{R_1} + \frac{4kT}{R_2} \cdot \left(\frac{R_2}{R_1}\right)^2$$

mean squared noise

$$\overline{i_n^2} = \frac{4kT}{R_1} + \frac{4kT}{R_2}\left(\frac{R_2}{R_1}\right)^2$$

always add noise powers not I/V

except in cases of noise correlation



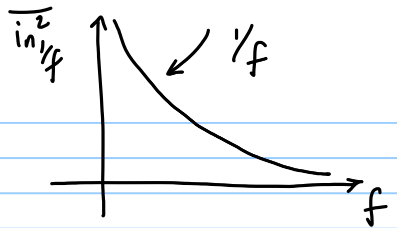
$$\underline{\underline{\overline{v_{on}^2} = 0}}$$

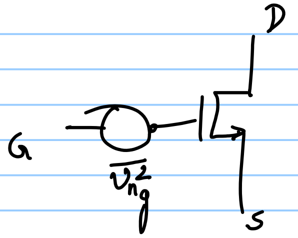
8/1/20

Lec 3

Noise in MOSFETs (saturation)

1) Flicker noise or 1/f noise



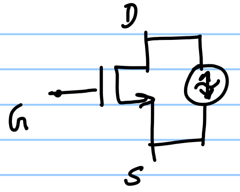


$$\frac{\overline{v_{ng}^2}}{\Delta f} = \frac{K_{1/f}}{W \cdot L \cdot C_{ox}^2} \cdot \frac{1}{f}$$

← empirical parameter

different for nmos & pmos

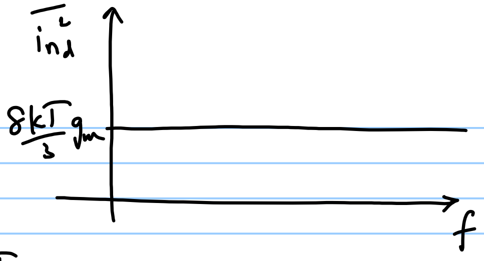
$\equiv$

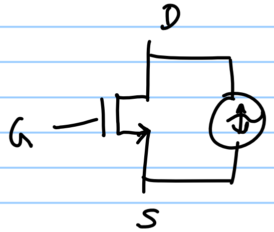


$$\frac{\overline{i_{n_{1/f}}^2}}{\Delta f} = \frac{K_{1/f} \cdot g_m^2}{W L C_{ox}^2} \cdot \frac{1}{f}$$

large devices ↔ low 1/f noise

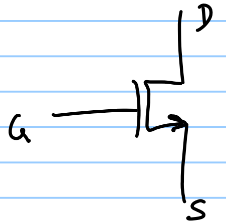
2) Drain Thermal noise ("White")



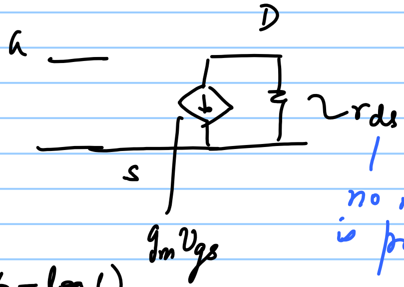


$$\frac{\overline{i_{nd}^2}}{\Delta f} = \frac{8kT}{3} g_m$$

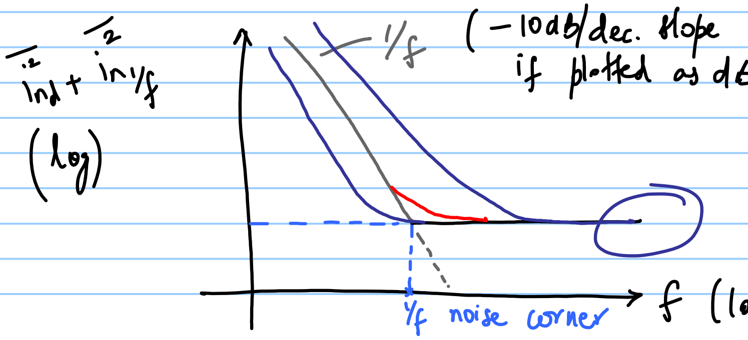
3) MOSFET in triode: $\frac{\overline{i_{nd}^2}}{\Delta f} = 4kT g_{ds}$ X not used in this course



$\equiv$



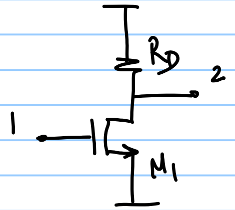
no noise is produced



(−10dB/dec. slope if plotted as dB−log f)

Common Source Amplifier

E.g. 1)

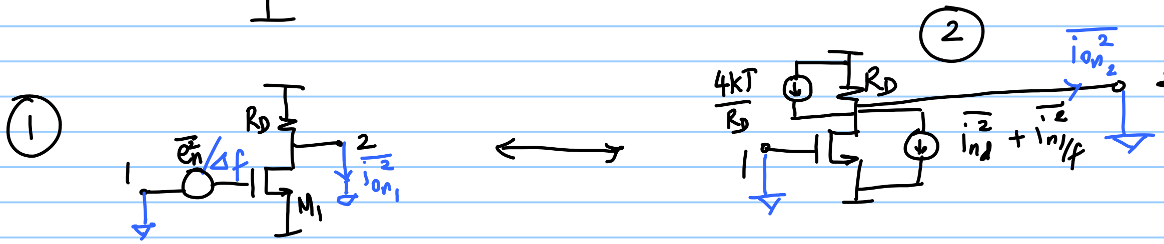


@ low freq. only

$$\frac{\overline{e_n^2}}{\Delta f} = ? \qquad \frac{\overline{i_n^2}}{\Delta f} = 0$$

port 1: choose to drive with ideal v_s

port 2: choose to measure s.c. current



① : $\overline{i_{on_1}^2} = g_{m_1}^2 \cdot \overline{e_n^2}$

② : $\overline{i_{on_2}^2} = \overline{i_{nd}^2} + \overline{i_{n_{1/f}}^2} + \overline{i_{n_{R_D}}^2}$

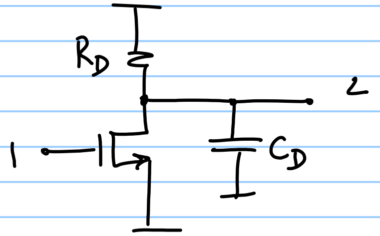
$$\overline{i_{on_1}^2} = \overline{i_{on_2}^2}$$

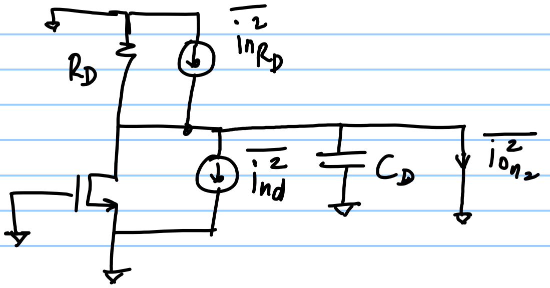
$$\overline{e_n^2} = \frac{\overline{i_{nd}^2} + \overline{i_{n_{1/f}}^2} + \overline{i_{n_{R_D}}^2}}{g_{m_1}^2}$$

$$\frac{\overline{e_n^2}}{\Delta f} = \frac{8kT}{3g_{m_1}} + \frac{K_{1/f}}{W_1L_1C_{ox}^2} \cdot \frac{1}{f} + \boxed{\frac{4kT}{g_{m_1}^2 R_D}}$$

$$\frac{4kT}{g_{m_1}} \cdot \frac{1}{\underbrace{g_{m_1}R_D}_{\text{gain}}}$$

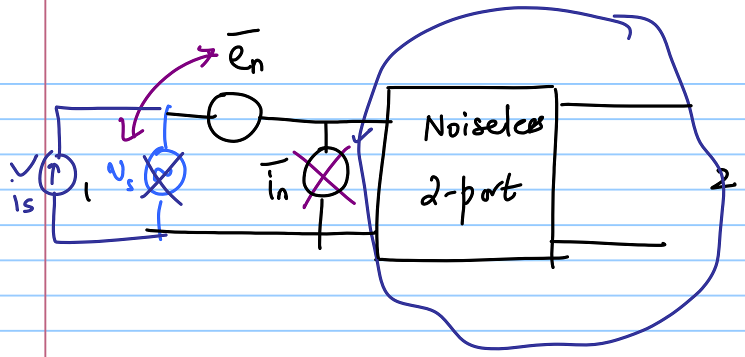
Eg. 2)





$\overline{e_n}$ = same as before ✓

HW: Calculate $\overline{e_n}$ for this circuit using open circuit output voltage



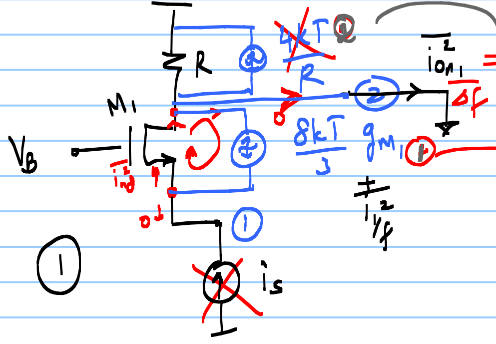
$\overline{e_n}$
i_s
1
v_s
$\overline{i_n}$
Noiseless
2-port
2

10/1/20

Lec 4

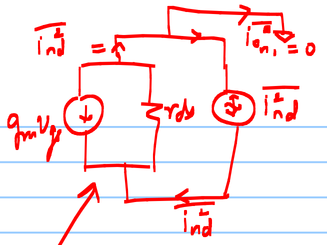
HW

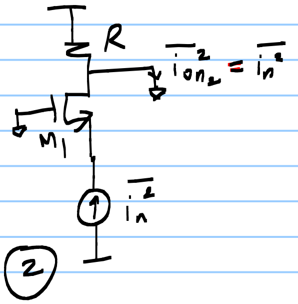
CS, CG, CD amplifiers



$\frac{\overline{i_{on_1}^2}}{\Delta f} = \frac{4kT}{R}$

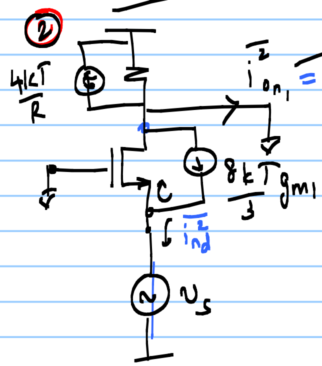
$\overline{e_n^2}$, $\frac{\overline{i_n^2}}{\Delta f} = \frac{4kT}{R}$





case 1

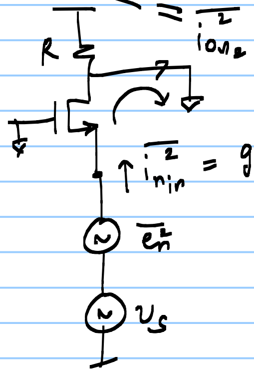
②



$$\overline{i_{on_1}^2} = \overline{i_{nd}^2} + \overline{i_{np}^2} = \frac{4kT}{R} + \frac{8kT}{3} g_{m_1}$$

①

case 2



$$= \overline{i_{on_2}^2} = g_{m_1}^2 \cdot \overline{e_n^2}$$

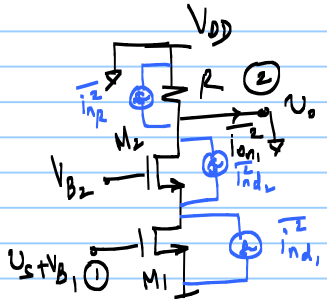
$$\uparrow \overline{i_{n_{in}}^2} = g_{m_1}^2 \cdot \overline{e_n^2}$$

$$\overline{e_n^2} = \frac{4kT}{g_{m_1}^2 \cdot R} + \frac{8kT}{3 g_{m_1}}$$

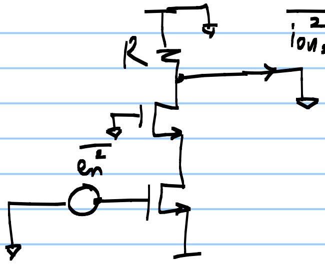
Cascode Amplifier

$\overline{i_n^2} = 0$; $\overline{e_n^2} = ?$

①



②

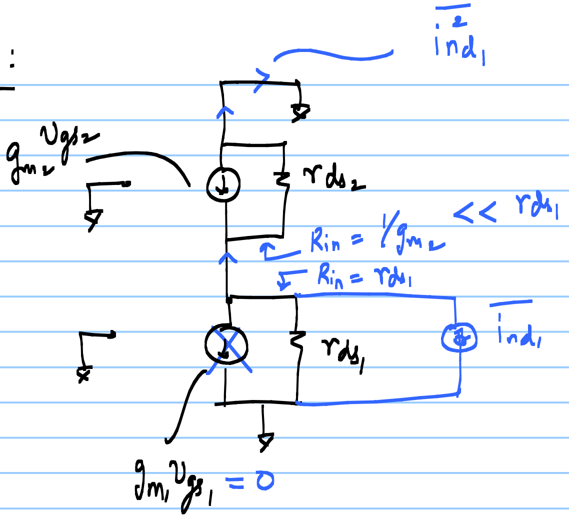


$\overline{i_{on_2}^2} = g_{m_1}^2 \cdot \overline{e_n^2}$

$\overline{i_{on_1}^2} = (R) + (M_1) + (M_2)$

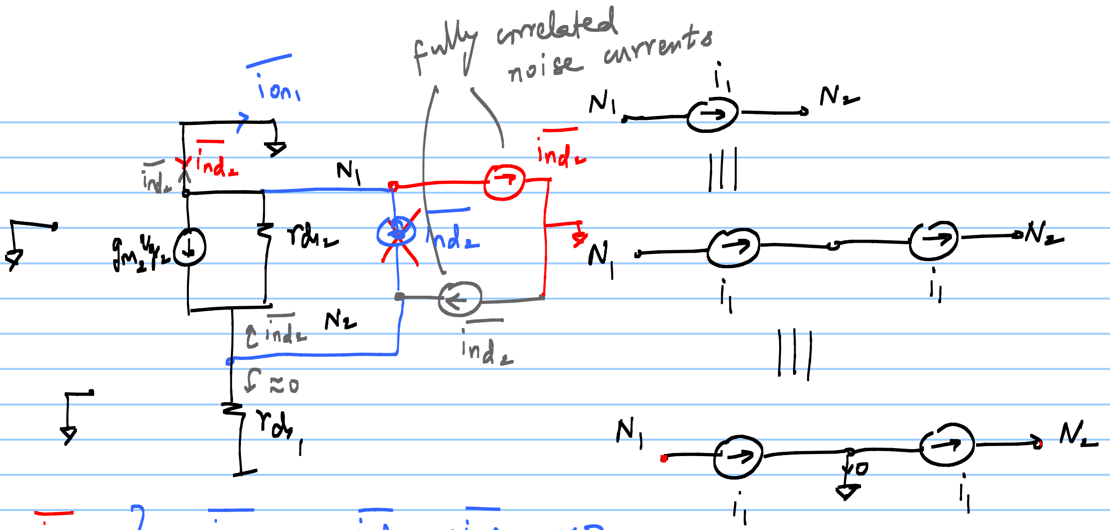
R: $4kT/R_1$; $\frac{8kT}{3} g_{m_1}$; $\simeq 0$

M_1 :



$\overline{i_{nd_1}}$ — flows into cascode

$\underline{M_2}$:



$\overline{i_{on_1}} = \overline{i_{on_1}} + \overline{i_{on_1}}$

$\overline{i_{on_1}} = -\overline{i_{nd_2}}$

$\overline{i_{on_1}} \simeq \overline{i_{nd_2}}$

$\Bigg\}$ $\overline{i_{on_1}} \simeq \overline{i_{nd_2}} - \overline{i_{nd_2}} \simeq 0$

total due to M_2

$$g_{m_1}^2 \cdot \overline{e_n^2} = \frac{4kT}{R} + \frac{8kT}{3} g_{m_1}$$

$$\overline{e_n^2} = \frac{4kT}{g_{m_1}^2 \cdot R} + \frac{8kT}{3 g_{m_1}} \quad \left\{ \begin{array}{l} \text{same as} \\ \text{regular CSA} \end{array} \right\}$$

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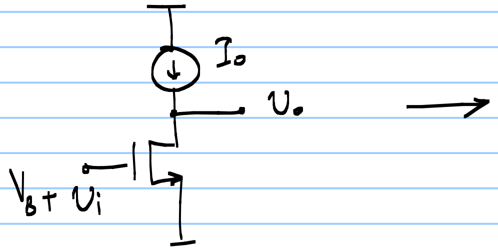
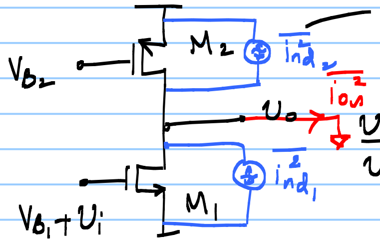
Lec 5



Case 1

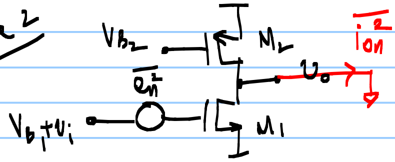


active load

$$\frac{v_o}{v_i} = \frac{g_{m_1}}{g_{ds_1} + g_{ds_2}}$$

$$\overline{i_{on}^2} = \frac{8kT}{3} g_{m_1} + \frac{8kT}{3} g_{m_2}$$

Case 2



$$\overline{i_{on}^2} = g_{m_1}^2 \cdot \overline{e_n^2}$$

$$\overline{e_n^2} = \frac{8kT}{3 g_{m_1}} + \frac{8kT}{3} \frac{g_{m_2}}{g_{m_1}^2}$$

For low noise

* high g_{m_1} $\Longrightarrow$ gain etc — sets I_{DS_1} & $\left(\frac{W}{L}\right)_1$

high g_{m_1} — $\uparrow \left(\frac{W}{L}\right)_1 \Longrightarrow \uparrow C_{par} \Downarrow \downarrow BW$

— $\uparrow I_{DS_1} \Downarrow$ Power $\uparrow$, $r_{ds} \downarrow$

* low g_{m_2}

$\Downarrow$

$\downarrow \left(\frac{W}{L}\right)_2$ only $\left\{ \begin{array}{l} I_{DS_2} = I_{DS_1} \\ = \text{fixed} \end{array} \right\}$ $\longrightarrow$ Power $\uparrow$

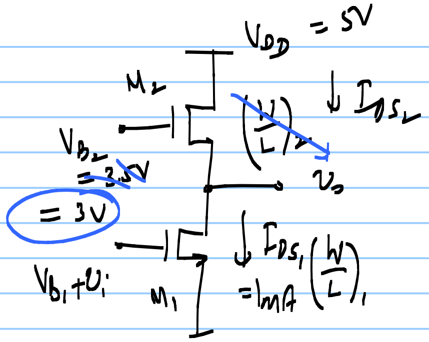
$\Downarrow$

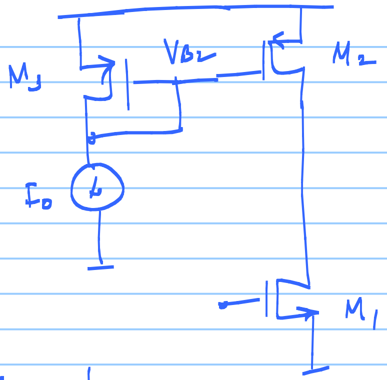
$\downarrow V_{b2}$, $\uparrow V_{SG_2} = V_{T_2} + \nearrow V_{SD\,sat_2}$

$$V_{SD\,sat} = V_{SG} - V_T$$

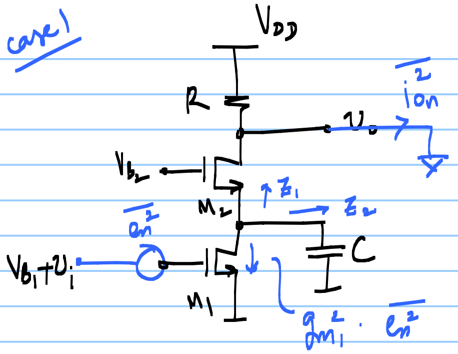
$$= \sqrt{\frac{2 I_D}{\mu_p C_{ox} \left(\frac{W}{L}\right)}}$$

$V_{TA} = 1V$





If I_{DS} is fixed ⇒ V_{B2} can be changed only if $\left(\frac{W}{L}\right)_2$ is changed case 1



$$\overline{e_n^2} = ?$$

$$\frac{Z_2}{Z_1+Z_2} = \frac{1/j\omega C}{1/g_{m2} + \frac{1}{j\omega C}}$$

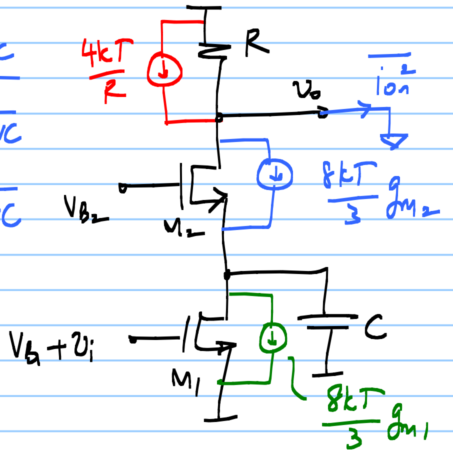
$$= \frac{g_{m2}}{g_{m2}+j\omega C}$$

Case 1

$$\overline{i_{on}^2} = g_{m1}^2 \cdot \overline{e_n^2} \cdot \left|\frac{Z_2}{Z_1+Z_2}\right|^2$$

$$= g_{m1}^2 \cdot \overline{e_n^2} \cdot \frac{g_{m2}^2}{g_{m2}^2+\omega^2C^2}$$ ✓

case 2



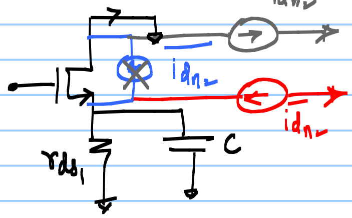
case 2 :

$$\overline{i_{on_3}^2} = \frac{4kT}{R}$$

$$\overline{i_{on_1}^2} = \frac{8kT}{3} g_{m_1} \cdot \left(\frac{r_{ds_1}}{r_{ds_1} + 1/g_{m_2}} \right)^2 \cdot \frac{g_{m_2}^2}{g_{m_2}^2 + \omega^2 C^2}$$

$$\overline{i_{on_2}^2} = ?$$

$$\overline{i_{on_2}} = ?$$



$$\overline{i_{dn_2}} \Longrightarrow \overline{i_{on_2}} = -\overline{i_{dn_2}}$$

$$\overline{i_{on_2}} = \overline{i_{dn_2}} \cdot \left(\frac{r_{ds_1} \| 1/j\omega C}{r_{ds_1} \| 1/j\omega C + 1/g_{m_2}} \right)$$

$$\overline{i_{on_2}} = \overline{i_{on_2}} + \overline{i_{on_2}}$$

$$\overline{i_{on_2}} = \frac{-1/g_{m_2}}{1/g_{m_2} + \left(r_{ds_1} \| 1/j\omega C\right)} \cdot \overline{i_{dn_2}}$$

$$\left(r_{ds_1} \| 1/j\omega C\right) \longrightarrow \frac{r_{ds_1} \cdot 1/j\omega C}{r_{ds_1} + 1/j\omega C}$$

$$= \frac{r_{ds_1}}{1 + j\omega C r_{ds_1}}$$

$$= \frac{-\overline{i_{dn_2}}}{1 + \left(\frac{g_{m_2} r_{ds_1}}{1 + j\omega C r_{ds_1}}\right)}$$

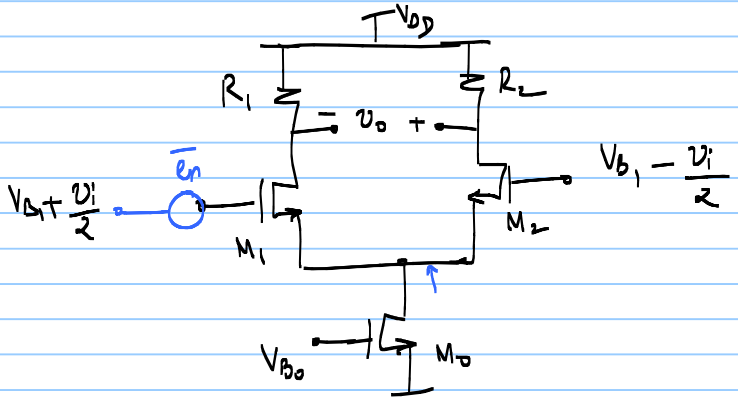
$$= -\overline{i_{dn_2}} \cdot \frac{1 + j\omega C r_{ds_1}}{\left(1 + g_{m_2} r_{ds_1}\right) + j\omega C r_{ds_1}}$$

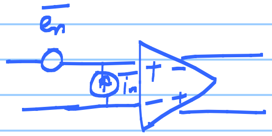
$$\overline{i^2_{on_2}} = \overset{\frac{8kTg_{m_2}}{3}}{\overline{i^2_{dn_2}}} \cdot \frac{1 + \omega^2 C^2 r_{ds_1}^2}{\left(1 + g_{m_2} r_{ds_1}\right)^2 + \omega^2 C^2 r_{ds_1}^2}$$

$$\overline{i_{on}^2} = \underbrace{\overline{i_{on_1}^2}}_{M_1} + \underbrace{\overline{i_{on_2}^2}}_{M_2} + \underbrace{\overline{i_{on_3}^2}}_{R} = g_{m_1}^2 \cdot \overline{e_n^2} \cdot \frac{g_{m_2}^2}{g_{m_2}^2 + w^2 C^2}$$

$$\overline{e_n^2} = \frac{\overline{i_{on_1}^2} + \overline{i_{on_2}^2} + \overline{i_{on_3}^2}}{g_{m_1}^2} \cdot \left(\frac{g_{m_2}^2 + w^2 C^2}{g_{m_2}^2}\right)$$

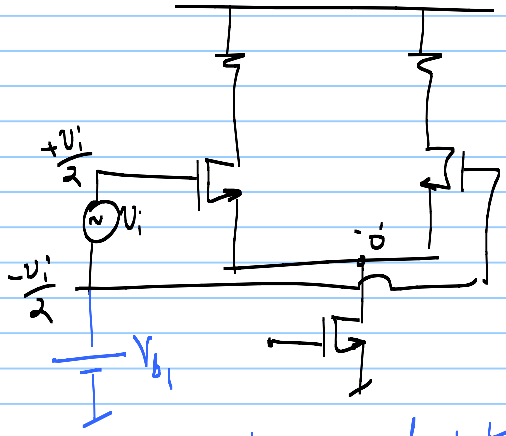
Noise in differential amplifiers





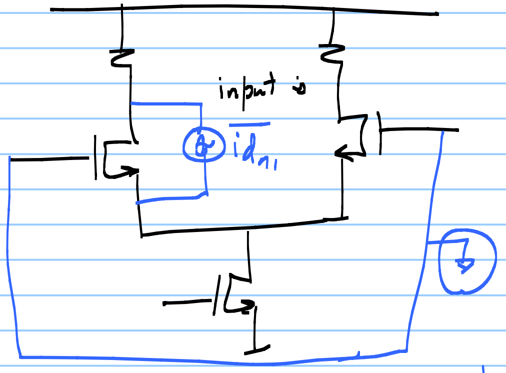
$\overline{i_n^2} = 0$ for this diff. amp.

Signal



H.C. analysis ✓

Noise analysis



Noise sources are not symmetrically applied!

No H.C. analysis!

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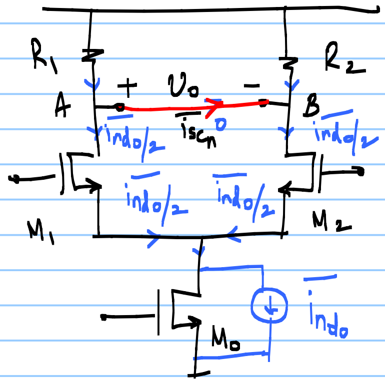
Lec 6



$\overline{V_{n_a}^2}$, $\overline{V_{n,b}^2}$

$R_1 = R_2 = R$

1) noise of M_o :

$\overline{v_{na}} = -\frac{\overline{ind_o}}{2} \cdot R_1$

$\overline{v_{n,b}} = -\frac{\overline{ind_o}}{2} \cdot R_2$

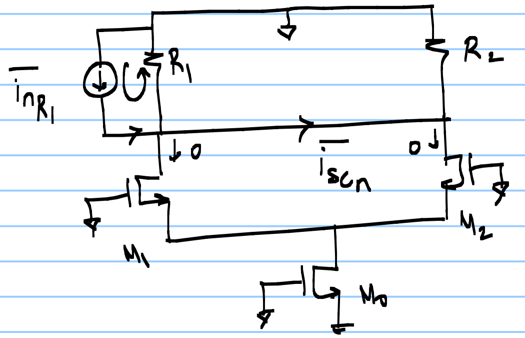
$\overline{v_{na}} = \overline{v_{n,b}}$ common mode noise

diff. output noise voltage

$\overline{v_{o,n}} = \overline{v_{na}} - \overline{v_{n,b}} = 0$

$\overline{i_{scn}}$ from this point onwards

2) Noise from R_1:



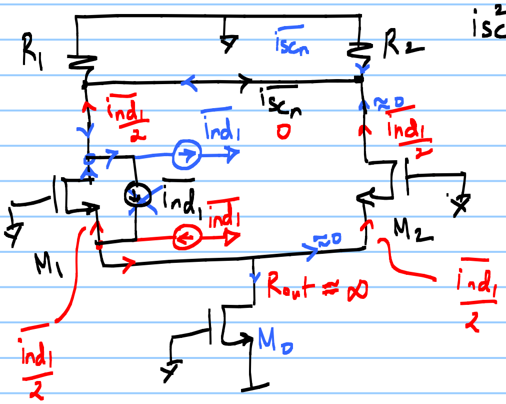
$$\overline{i_{scn}} = \frac{1}{2}\overline{i_{nR_1}}$$

$$\frac{\overline{i^2_{scn}}}{\Delta f} = \frac{1}{4}\overline{i^2_{nR_1}} = \frac{kT}{R_1}$$

↳ Total noise PSD from resistors = $\frac{2kT}{R}$

Noise from R_2: $\frac{\overline{i^2_{scn}}}{\Delta f} = \frac{kT}{R_2}$

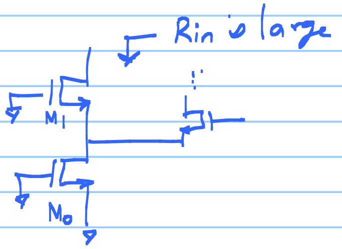
3) Noise from M_1:

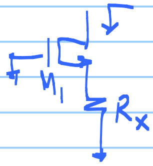


$$\overline{i_{scn}} = -\frac{\overline{i_{nd_1}}}{2} + 0$$

$$\overline{i_{scn}^2} = \frac{\overline{i_{nd_1}^2}}{4}$$

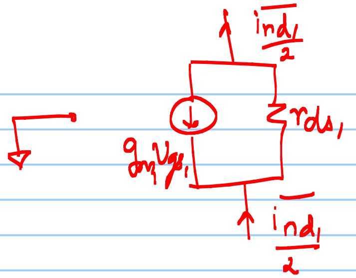
$$= \frac{2kT}{3} g_{m1}$$



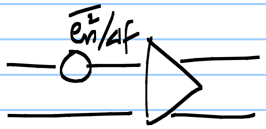


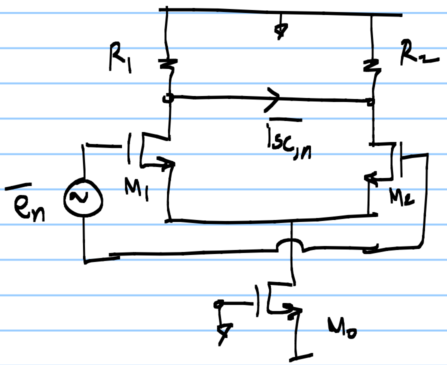
$$R_{in} = g_{m_1} r_{ds_1} \cdot R_X + r_{ds} + R_X$$

Noise of M_2: $\overline{i^2_{sc,n}} = \frac{\overline{i^2_{nd_2}}}{4} = \frac{2kT}{3} g_{m_2}$



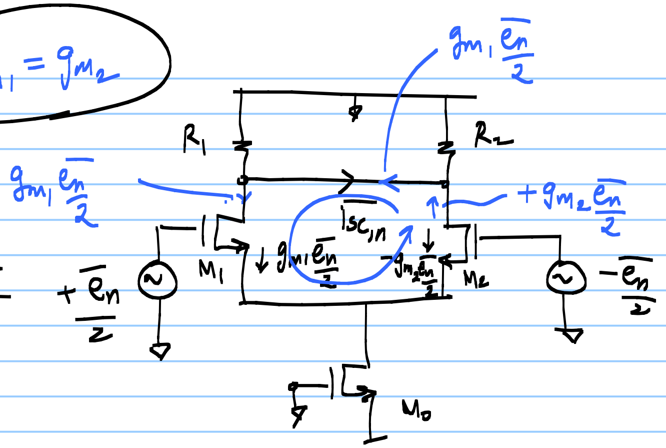
$$\frac{\overline{i^2_{scn,tot.}}}{\Delta f} = \frac{2kT}{R} + \frac{4kT}{3} g_m$$





$$\boxed{g_{m1} = g_{m2}}$$

$\equiv$



$\overline{id_1} = g_{m1}\frac{\overline{e_n}}{2}$; $\overline{id_2} = -g_{m2}\frac{\overline{e_n}}{2}$

$\overline{isc_n} = -g_{m1}\frac{\overline{e_n}}{2}$; $\overline{i^2_{sc_n}} = \left(\frac{g_{m1}^2}{4}\right)\cdot \overline{e_n^2}/\Delta f$

Equate $\overline{i_{scn}^2}$ in the two cases:

$$\frac{g_m^2}{4}\cdot\frac{\overline{e_n^2}}{\Delta f} = \frac{2kT}{R} + \frac{4kT}{3}g_m$$

$$\boxed{\frac{\overline{e_n^2}}{\Delta f} = \frac{8kT}{g_m^2 R} + \frac{16kT}{3g_m}} \longleftrightarrow$$

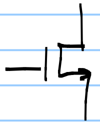
compare with $\overline{e_n}$ of CSA:

$$\frac{4kT}{g_m^2 R} + \frac{8kT}{3g_m}$$

low noise: ↑ g_m, ↑ $g_m R$

HW: calculate $\overline{v_{on}^2}$ instead of $\overline{i_{scn}^2}$

Mismatch

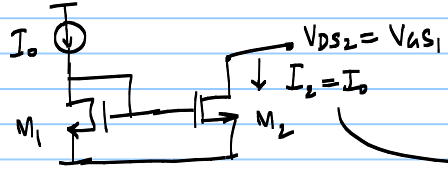


μ_n , C_{ox} , V_T . . .

process variations

temperature var^ns etc.

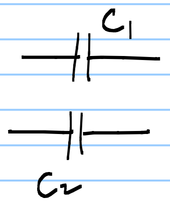
2 quantities are expected to be identical, but are not.

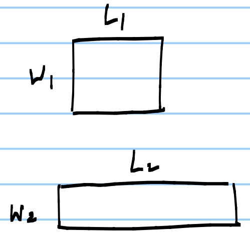


$M_1 \equiv M_2$: same V_T, $\boxed{W, L,}$ μ_n, C_{ox}

W, L → design parameters {expected to be equal}

μ_n, C_{ox}, V_T → process parameters

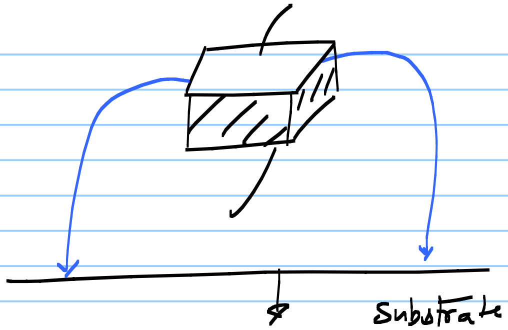


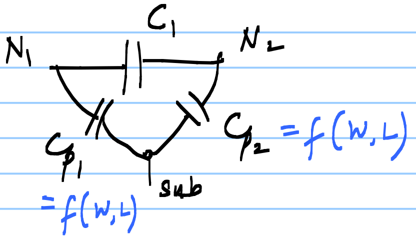


$W_1 L_1 = W_2 L_2$

$C_1 = C_2$

$$C = \frac{\varepsilon_0 k A}{d}$$





Mismatch → Systematic

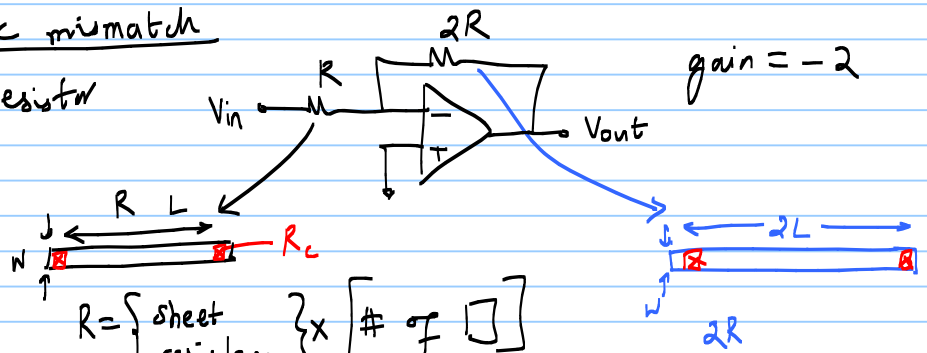
→ Random

20/1/20

Lec 7

Systematic mismatch

e.g. Resistor



gain = −2

$$R = \begin{Bmatrix} \text{sheet} \\ \text{resistance} \end{Bmatrix} \times \left[\# \text{ of } \square \right]$$

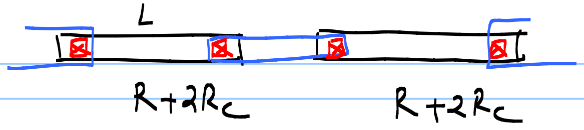
actual Resistance = $R + 2R_c$

Actual res.

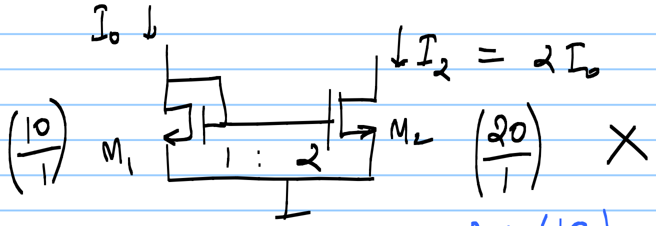
= $2R + 2R_c$

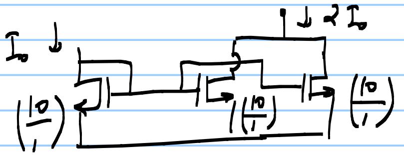
≠ 2x resistance

Build $2R$:

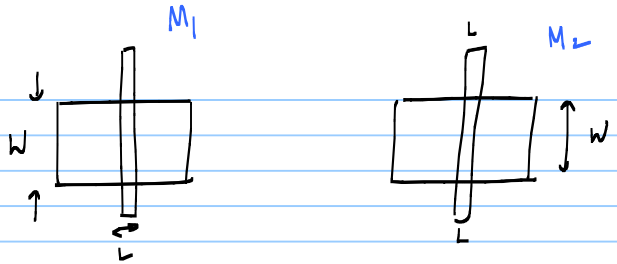


total res. $= 2R + 4R_c = 2(R+2R_c)$



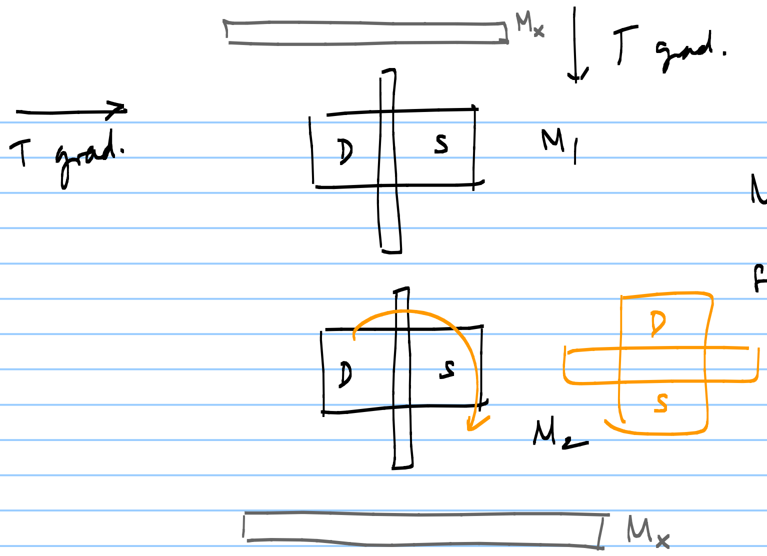


$2\times\left(\frac{10}{1}\right)$ ✓



$\xrightarrow{\quad}$ T gradient ⇒ properties of M_1 & M_2 will be different

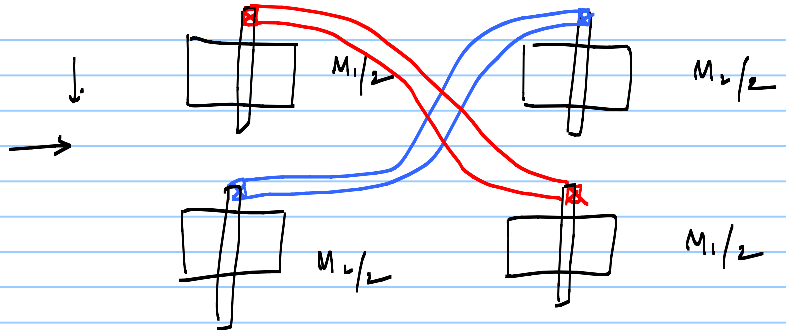
* place devices as close as possible to each other



Make M_1 & M_2

feel same

environment (identically)

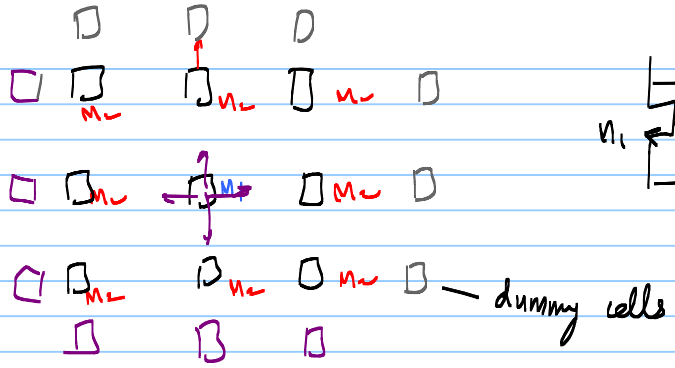


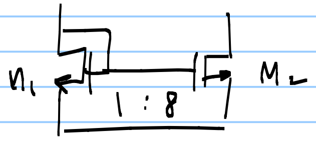
Common

centroid

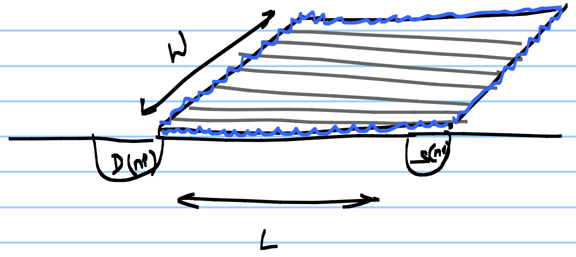
layout

W_f	width / finger
N_f	# of fingers
⋮	


M2
M2
M2
M2
M1
M2
M2
M2
M2
dummy cells


M1
1 : 8
M2

Random mismatch



ΔW

ΔL) for the device

$\frac{\Delta L}{L}$, $\frac{\Delta W}{W}$

Similar effects for R, C, V_T, ...

* Mean = 0 , σ describes statistical effects

empirical parameter

$$\sigma_{V_T} = \frac{A_{V_T}}{\sqrt{W \cdot L}}$$

V_T = threshold voltage

$$\sigma_{\frac{\Delta W/L}{W/L}} = \frac{A_{W/L}}{\sqrt{W \cdot L}}$$

$$\sigma_{\beta} = \frac{A_{\beta}}{\sqrt{W \cdot L}}$$

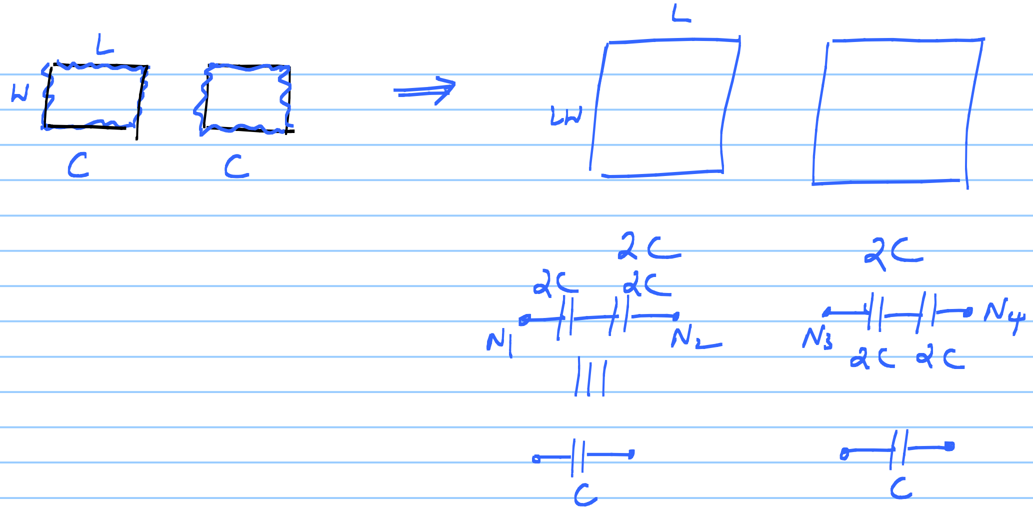
$\beta = \mu C_{ox}$

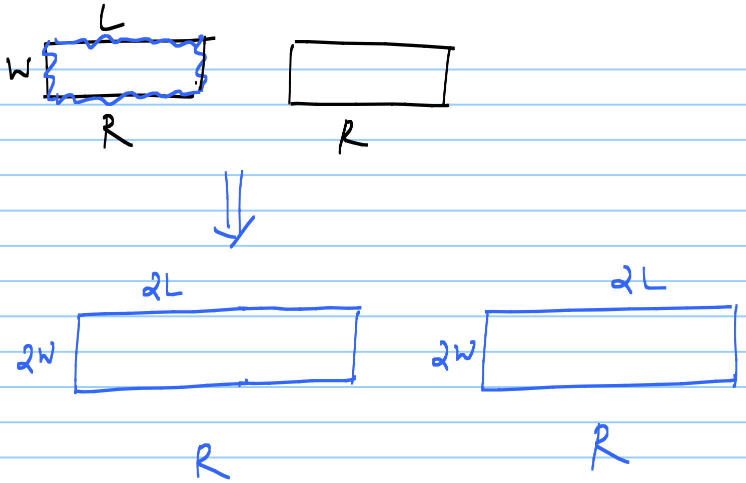
$$\sigma_{\frac{\Delta R}{R}} = \frac{A_R}{\sqrt{W_R \cdot L_R}}$$

Low mismatch (low σ)

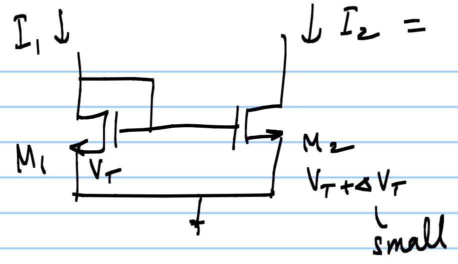
- ↑ W, L ⇒ ↓ σ
- Area is traded ff with mismatch
- as Area ↑, distance between devices ↑

$$\sigma_{\frac{\Delta C}{C}} = \frac{A_C}{\sqrt{W_C \cdot L_C}}$$


L
W
C
C
L
LW
2C
2C
2C
N1
N2
2C
N3
N4
2C
2C
C
C



of □ is the same



$I_2 = I_1 + \Delta I$

$\left(\frac{\Delta I}{I}\right)$

$$\frac{\partial I_D}{\partial V_{GS}} = g_m$$

$$\frac{\partial I_D}{\partial V_T} = -\frac{\partial I_D}{\partial V_{GS}} = -g_m$$

$$I_D = \frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_T)^2$$

Consider only V_T mismatch

$$\Delta I = \frac{\partial I_D}{\partial V_T}\cdot\Delta V_T = -g_m\cdot\Delta V_T \checkmark$$

$$\sigma_{\Delta I}^2 = g_m^2\cdot\sigma_{V_T}^2$$

nominal

$$\sigma^2_{\frac{\Delta I}{I}} = \frac{g_m^2}{I_1^2} \cdot \sigma_{V_T}^2$$

$$g_m = \mu_n C_{ox} \left(\frac{W}{L}\right) \left(V_{GS} - V_T\right)$$

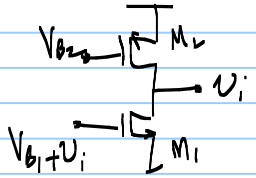
$$= \frac{2 I_D}{V_{GS} - V_T} \quad \longleftarrow$$

$$= \sqrt{2 \mu C_{ox} \left(\frac{W}{L}\right) \cdot I_D}$$

$$\frac{g_m^2}{I_1^2} = \frac{4}{V_{Dsat}^2}$$

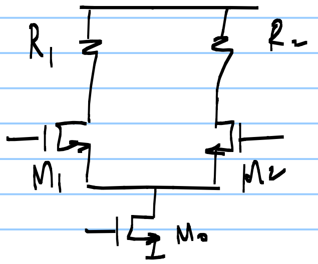
$$\sigma^2_{\frac{\Delta I}{I}} = \frac{4}{\underbrace{V_{Dsat}^2}} \cdot \frac{A^2_{V_T}}{W \cdot L}$$

larger V_{Dsat} ⟷ smaller $\frac{\Delta I}{I}$



No matching considerations

Diff. Amplifier

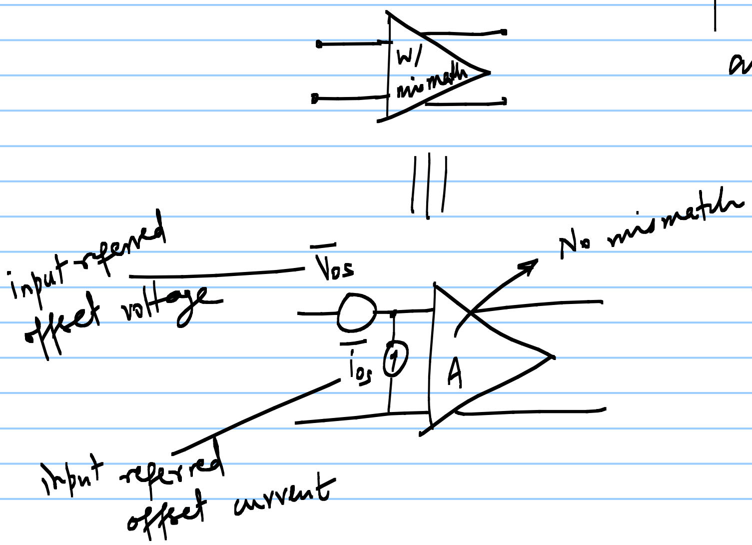


$M_1 \equiv M_2$

$R_1 \equiv R_2$

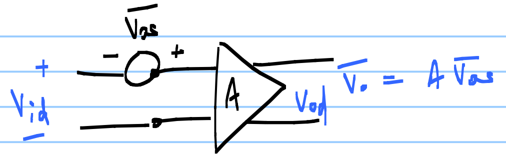
$I_{os} = 0$

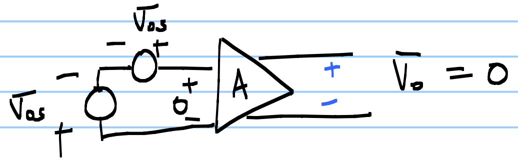
Diff. circuit



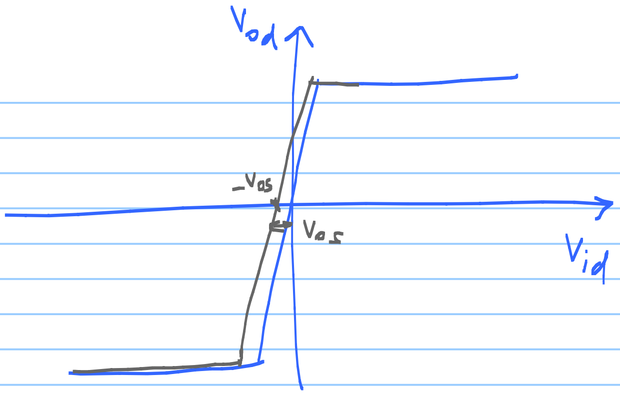
pull out all mismatches as input-referred quantities

$\overline{V_{os}}$ & $\overline{i_{os}}$ → statistical quantities





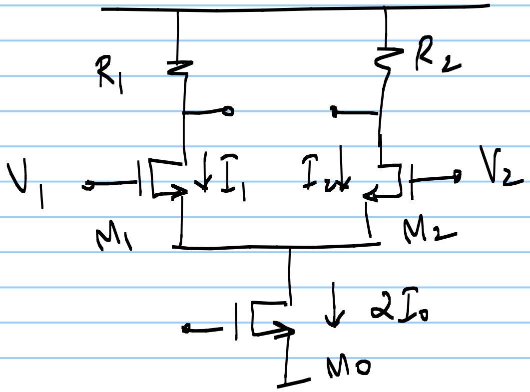
input-referred offset voltage $\overline{V_{os}}$ = voltage that is applied at input so that output voltage offset is zero



23/01/2020

Lec 8

E.g. 1



$R_1 \neq R_2$, $V_{T_1} \neq V_{T_2}$, $\left(\frac{W}{L}\right)_1 \neq \left(\frac{W}{L}\right)_2$

$V_{os} = ?$

$I_1 \neq I_2$

$V_1 - V_2 = V_{GS_1} - V_{GS_2}$ —— ①

$I_1 R_1 = I_2 R_2$ —— ② {output offset = 0}

$$R = \frac{R_1 + R_2}{2} \; ; \quad \Delta R = \frac{R_1 - R_2}{2}$$

$$R_1 = R + \Delta R \; ; \quad R_2 = R - \Delta R$$

$$I = \frac{I_1 + I_2}{2} \; ; \quad \Delta I = \frac{I_1 - I_2}{2}$$

$$I_1 = I + \Delta I \; ; \quad I_2 = I - \Delta I$$

$$\left(\frac{W}{L}\right) = \frac{\left(\frac{W}{L}\right)_1 + \left(\frac{W}{L}\right)_2}{2}$$

$$\Delta\left(\frac{W}{L}\right) = \frac{\left(\frac{W}{L}\right)_1 - \left(\frac{W}{L}\right)_2}{2}$$

$$I_1 R_1 = I_2 R_2$$

$$(I + \Delta I)(R + \Delta R) = (I - \Delta I)(R - \Delta R)$$

$$\left(1 + \frac{\Delta I}{I}\right)\left(1 + \frac{\Delta R}{R}\right) = \left(1 - \frac{\Delta I}{I}\right)\left(1 - \frac{\Delta R}{R}\right)$$

$$1 + \frac{\Delta I}{I} + \frac{\Delta R}{R} + \frac{\Delta I \Delta R}{IR} = 1 - \frac{\Delta I}{I} - \frac{\Delta R}{R} + \frac{\Delta I \Delta R}{IR}$$

$$\boxed{\frac{\Delta I}{I} = \frac{-\Delta R}{R}}$$

$$I_D = \frac{\beta}{2}\left(V_{GS} - V_T\right)^2$$

$$\beta = \mu C_{ox}$$

$$V_1 - V_2 = V_{GS_1} - V_{GS_2}$$

$$= V_{T_1} + \sqrt{\frac{2I_1}{\beta\left(\frac{W}{L}\right)_1}} - \left[V_{T_2} + \sqrt{\frac{2I_2}{\beta\left(\frac{W}{L}\right)_2}}\right]$$

$$V_1 - V_2 = \Delta V_T + \sqrt{\frac{2}{\beta}} \left[\sqrt{\frac{I + \Delta I}{\left(\frac{W}{L}\right) + \Delta\left(\frac{W}{L}\right)}} - \sqrt{\frac{I - \Delta I}{\left(\frac{W}{L}\right) - \Delta\left(\frac{W}{L}\right)}} \right]$$

$$= \Delta V_T + \sqrt{\frac{2I}{\beta\left(\frac{W}{L}\right)}} \cdot \left[\sqrt{\frac{1 + \Delta I/I}{1 + \frac{\Delta(W/L)}{W/L}}} - \sqrt{\frac{1 - \Delta I/I}{1 - \frac{\Delta(W/L)}{W/L}}} \right]$$

$$= \Delta V_T + V_{Dsat} \left[\left(1 + \frac{\Delta I}{2I}\right)\left(1 - \frac{\Delta(W/L)}{2\left(\frac{W}{L}\right)}\right) - \left(1 - \frac{\Delta I}{2I}\right)\left(1 + \frac{\Delta(W/L)}{2(W/L)}\right) \right]$$

$$V_1 - V_2 = \Delta V_T + V_{Dsat}\left[\left(\frac{\Delta I}{I}\right) - \frac{\Delta(W/L)}{(W/L)}\right] \longrightarrow$$

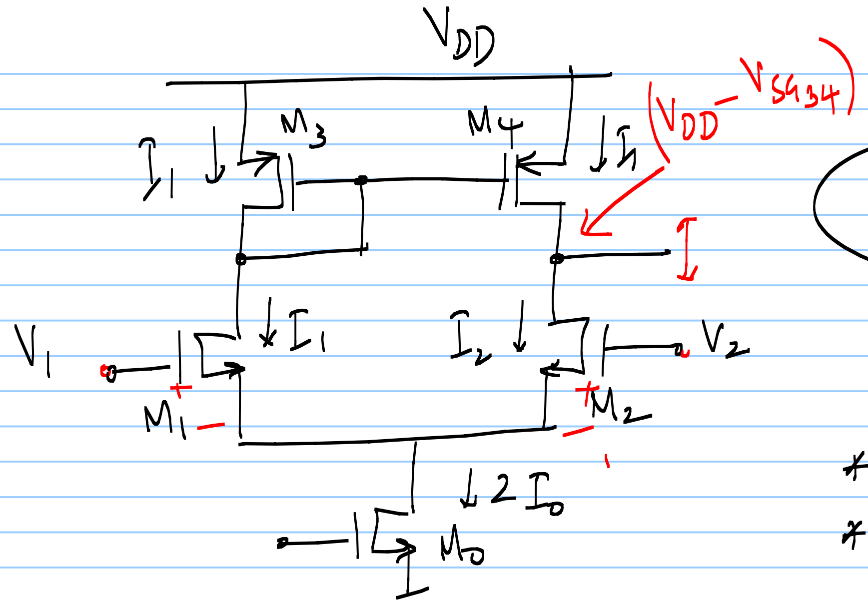
true for all V_1, V_2

$$= \Delta V_T + V_{Dsat}\left[\frac{-\Delta R}{R} - \frac{\Delta(W/L)}{(W/L)}\right]$$

$V_1 - V_2 = V_{os}$ if output offset $= 0$

$$V_{os} = \Delta V_T - V_{Dsat}\left[\frac{\Delta R}{R} + \frac{\Delta(W/L)}{(W/L)}\right] \longleftarrow \sigma_{os}^2$$

E.g.2



$V_{T_1} - V_{T_2} = \Delta V_{T_{12}}$

$\cancel{V_{T_3} - V_{T_4} = \Delta V_{T_{34}}}$ ✓

$V_{os} = ?$

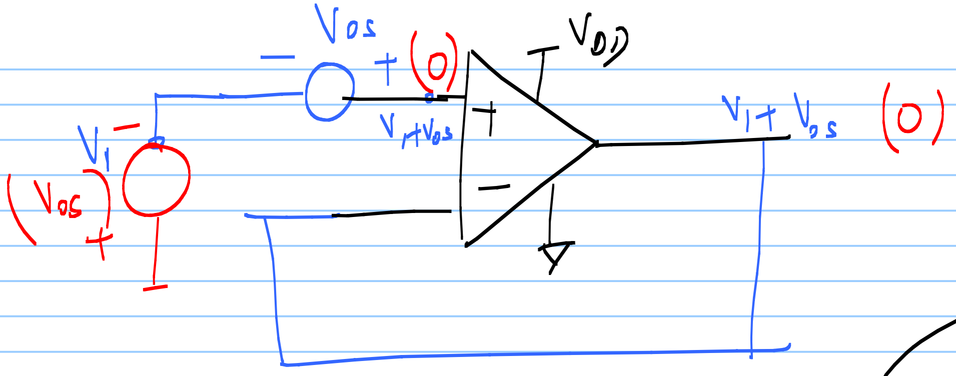
* $V_1 - V_2 = 0$

* $V_{GS_1} = V_{GS_2}$

* $I_1 = I_0 + \Delta I$

$= I_0 - \frac{g_m \Delta V_{T_{12}}}{2}$

$I_2 = I_0 - \Delta I$



$2\Delta I$ flows into r_{out}

$$r_{out} = r_{ds_3} \| r_{ds_4}$$

$$\Delta V_{out} = 2\Delta I \cdot r_{out} \checkmark$$

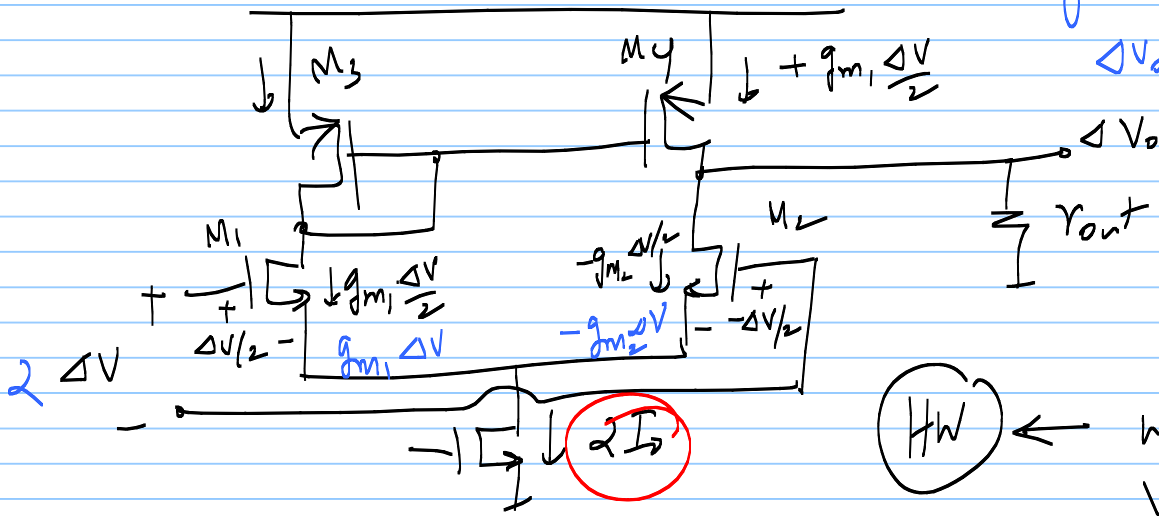
$$V_{os} = \Delta V_{in} = -\frac{2\Delta I \cdot r_{out}}{g_{m_1} r_{out}}$$

$$= -\frac{2\Delta I}{g_m} = \Delta V_{T/2}$$

apply V_{os} so that $\Delta V_{out} = 0$

$$gain = \frac{\Delta V_o}{2\Delta V} = g_m r_{out}$$

$$\Delta V_o = 2 g_m r_{out} \Delta V$$



$$\Delta V_o = g_m r_{out} \Delta V$$

$$gain = \frac{\Delta V_o}{\Delta V}$$

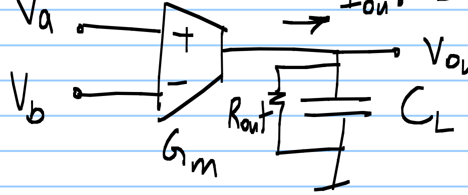
$$= g_m r_{out}$$

HW ← with $\Delta V_{T3,4}$, $V_{os} = ?$

24/1/20

Lec 9

One-stage Opamp

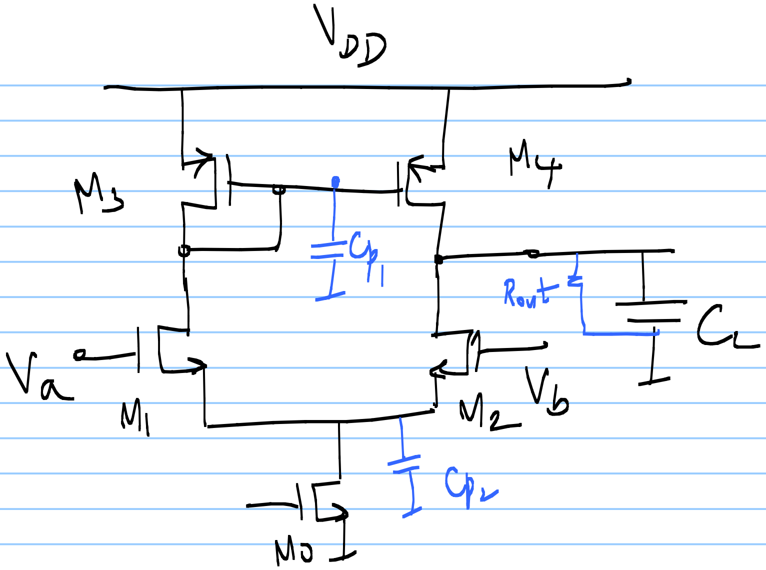


$I_{out} = G_m (V_a - V_b)$

$V_{out} = \frac{G_m}{s C_L} \cdot (V_a - V_b)$

$\frac{V_{out}}{V_{id}}(s) = \frac{\omega_u}{s}$

$\omega_u = \frac{G_m}{C_L}$



Data sheet

1) DC gain $= g_{m_1} \overbrace{(r_{ds_2} || r_{ds_4})}^{R_{out}}$

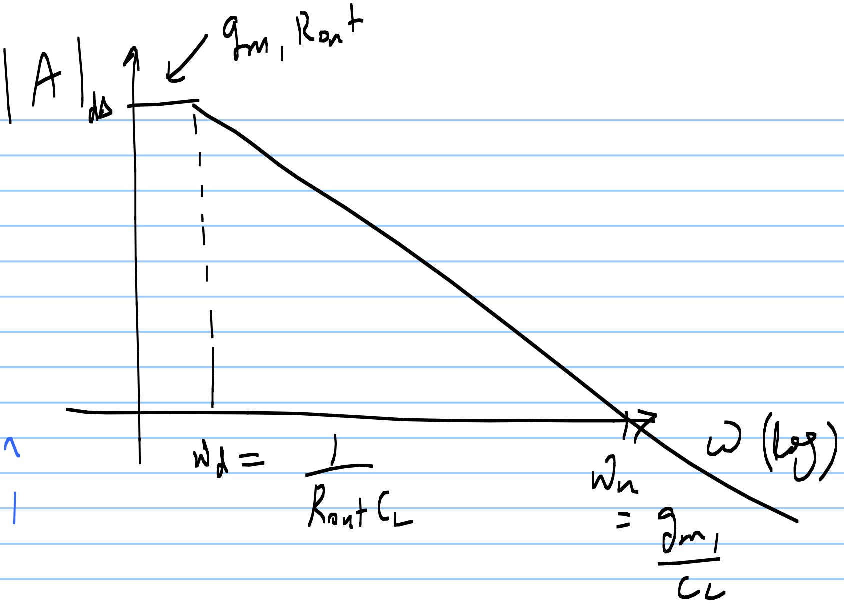
2) $G_m = g_{m_1}$

3) $\omega_u = \frac{g_{m_1}}{C_L}$;

Dominant pole @ $\omega_d = \frac{1}{R_{out} C_L}$

open loop gain of opamp

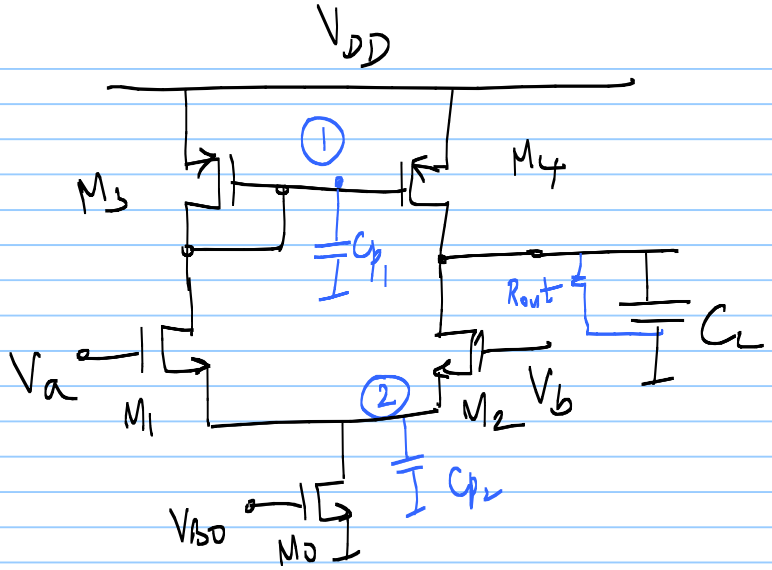
= Loop gain if $f = 1$



4) Non-dominant poles & zeroes?

$$\omega_{p_2} = \frac{-g_{m_3}}{C_{p_1}}$$

$$\omega_z = \frac{-2 g_{m_3}}{C_{p_1}} = 2\omega_{p_2}$$

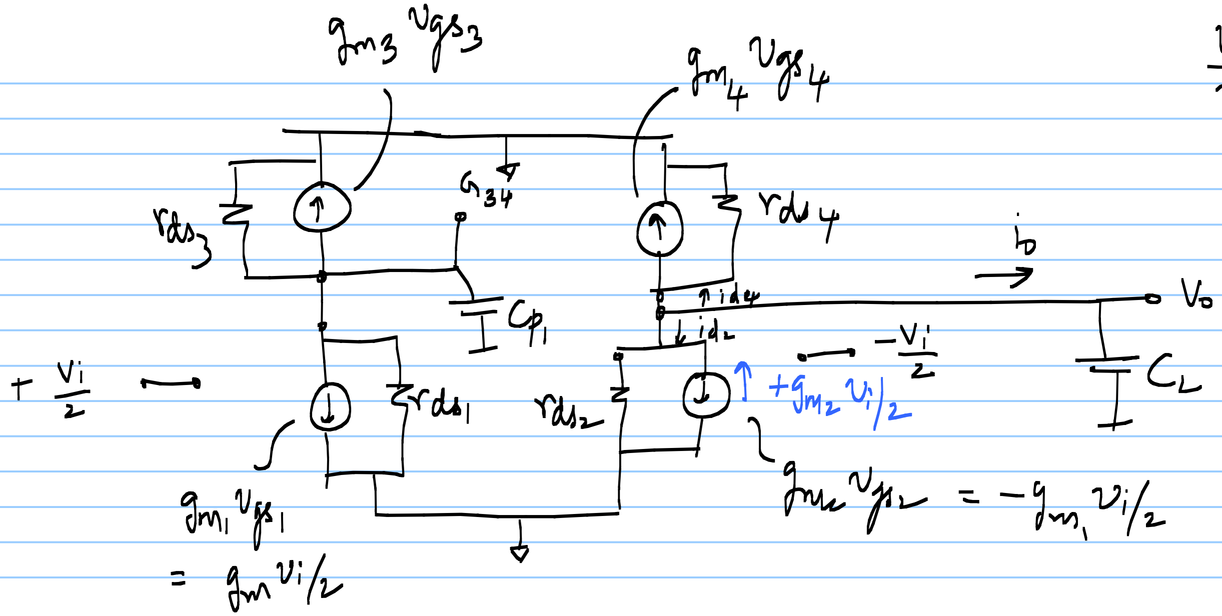


* Node ② = Virtual ground, C_{p_2} does not affect differential mode operation

C_{p_2} will affect CM gain & CMRR

$$CMRR = \left|\frac{A_{dm}}{A_{cm}}\right|$$

* Node 1 : C_{p_1} could affect A_{dm}



$$\frac{v_o}{v_i}(s) = ?$$

$$i_o = -(i_{d2} + i_{d4})$$

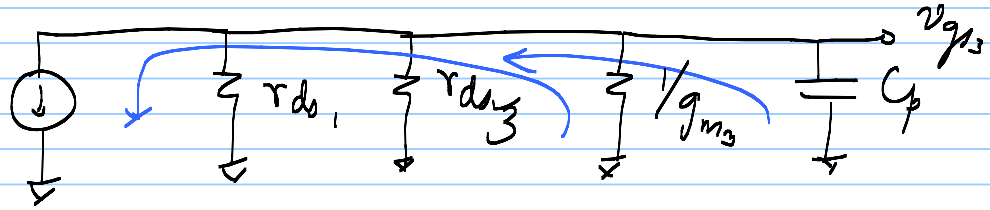
$$= \frac{-(g_{m2} v_{gs2} + g_{m4} v_{gs4}) \cdot (r_{ds2} \| r_{ds4})}{(r_{ds2} \| r_{ds4}) + \frac{1}{sC_L}}$$

$$i_o = \left[-\left(-g_{m_2}\frac{v_i}{2}\right) - \left(g_{m_4}\, v_{gs_4}\right)\right] \cdot \frac{R_{out}}{R_{out} + \frac{1}{sC_L}}$$

$v_{gs_4} = v_{gs_3}$

assume $r_{ds_1}, r_{ds_3} \gg 1/g_{m_3}$

$g_{m_1} v_{gs_1}$



$$v_{gs_3} = -g_{m_1}\frac{v_i}{2} \times \left({}^{1}\!/g_{m_3} \parallel \frac{1}{sC_{p_1}} \right)$$

$$= -g_{m_1}\frac{v_i}{2} \cdot \frac{{}^{1}\!/g_{m_3} \times \frac{1}{sC_{p_1}}}{\frac{1}{g_{m_3}} + \frac{1}{sC_{p_1}}} = -g_{m_1}\frac{v_i}{2} \frac{1}{g_{m_3} + sC_{p_1}} = v_{gs_4}$$

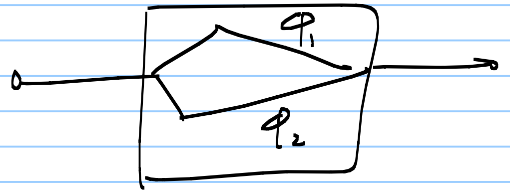
$$g_{m_4} \; v_{gs_4} = -g_{m_1}\frac{v_i}{2} \cdot \left(\frac{g_{m4}}{g_{m3} + sC_{p_1}} \right)$$

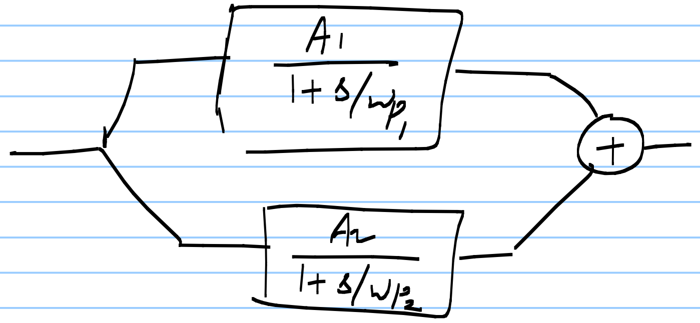
$$i_o = \left[g_{m_2}\frac{v_i}{2} + g_{m_1}\frac{v_i}{2}\cdot\left(\frac{g_{m_4}}{g_{m_3}+sC_{p_1}}\right)\right]\frac{R_{out}}{R_{out}+\frac{1}{sC_L}}$$

$$g_{m_1} = g_{m_2} \; ; \; g_{m_3} = g_{m_4}$$

$$i_o = \frac{g_{m_1} v_i}{2}\left[1 + \frac{g_{m_3}}{g_{m_3}+sC_{p_1}}\right]\cdot\frac{sC_L R_{out}}{1+sC_L R_{out}}$$

$$v_o = i_o \cdot \frac{1}{sC_L} = \frac{g_{m_1} v_i}{2} \cdot \frac{R_{out}}{1 + sC_L R_{out}} \cdot \frac{2 g_{m_3} + sC_{p_1}}{g_{m_3} + sC_{p_1}}$$





$$\frac{v_o}{v_i}(s) = (DC\,gain)\,\frac{\left(1+s/\omega_z\right)}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)} \qquad \omega_z = 2\,\omega_{p2}$$

$$= g_{m1} R_{out} \cdot \frac{\left(1 + \frac{s}{(2g_{m3}/C_{p1})}\right)}{\left(1+\frac{s}{(1/C_L R_{out})}\right)\left(1+\frac{s}{g_{m3}/C_{p1}}\right)}$$

$$\frac{1}{\cancel{2}} \cdot \frac{\cancel{2} g_{m3} + \frac{1}{2} s C_{p1}}{g_{m3} + s C_{p1}} = \frac{1 + \frac{sC_{p1}}{2g_{m3}}}{1 + sC_{p1}/g_{m3}}$$

$$\omega_z = -\frac{2g_{m3}}{C_{p1}} \;;\; \omega_{p1} = \frac{-1}{R_{out}C_L} \;;\; \omega_{p2} = \frac{-g_{m3}}{C_{p1}}$$

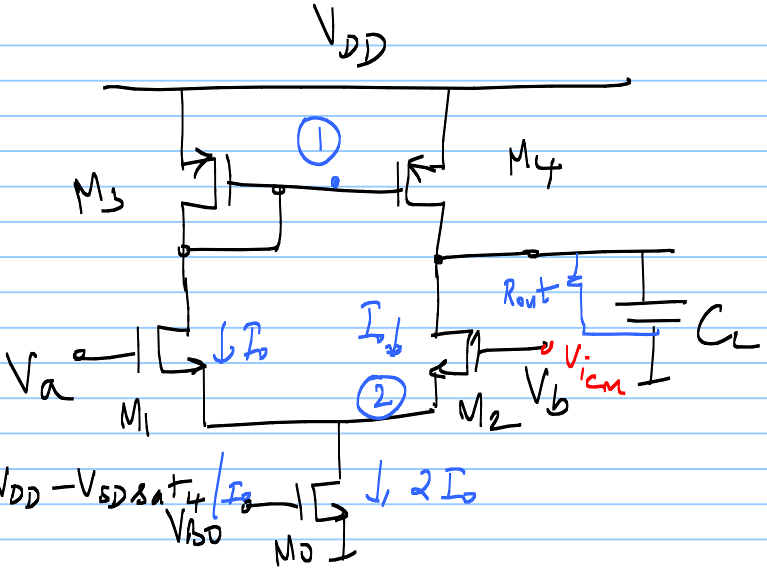
5) Input CM range ;

$$V_{CM,in}\,(min) = V_{GS_1}\Big|_{I_0} + V_{Dsat_0}\Big|_{2I_0}$$

$$V_{cm,in}\,(max) = V_{T_1} + V_{DD} - V_{sa_3}\Big|_{I_0}$$

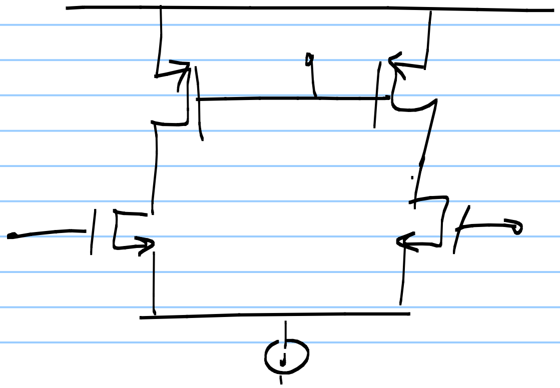
6) Output CM range ;

$$V_{CM,out}\,(min) = V_{icM} - V_{T_2}\ ;\ V_{CMout}(max) = V_{DD} - V_{SDsat_4}\Big|_{I_0}$$

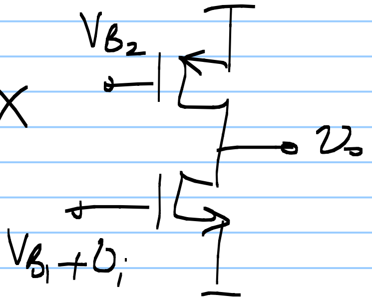


7) <u>Noise</u> : $\overline{i_n} = 0$

$\overline{e_n^2} = \boxed{HW}$



$\Rightarrow \quad \overline{e_n^2} = 2 \times$



8) Offset

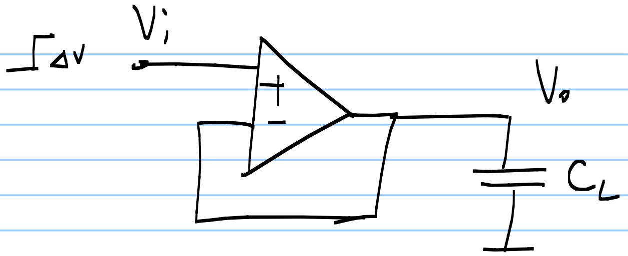
$$V_{os} = \Delta V_{T_{1,2}} + \frac{g_{m_3}}{g_{m_1}} \cdot \Delta V_{T_{3,4}}$$

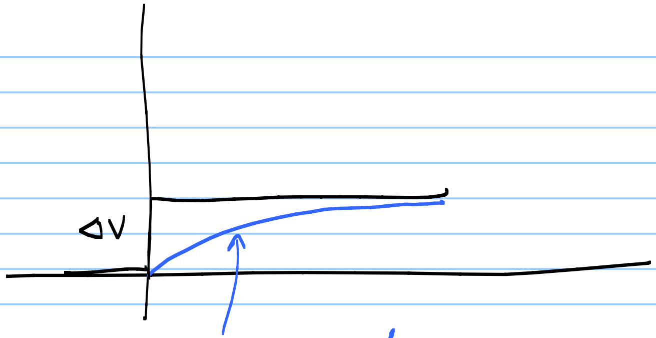
$$\sigma_{os}^2 = \underline{\underline{\sigma_{V_{T_{1,2}}}^2}} + \left(\frac{g_{m_3}}{\underbrace{g_{m_1}}}\right)^2 \sigma_{V_{T_{3,4}}}^2$$

large W_1, L_1, W_2, L_2

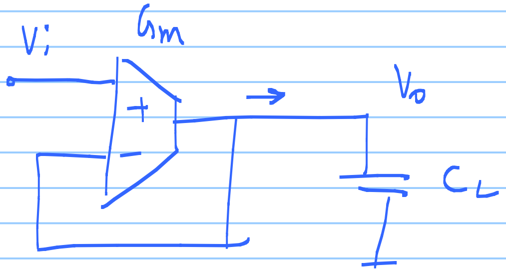
also reduces flicker noise

9) Slew Rate :

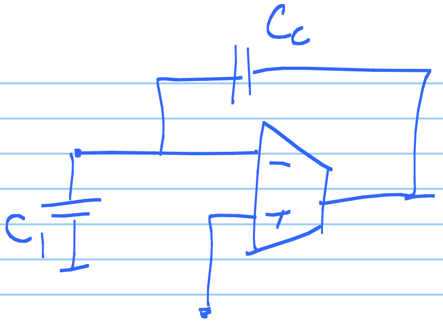




exponential ; $\tau \approx \frac{C_L}{g_{m1}}$

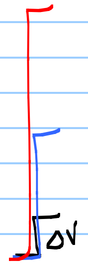


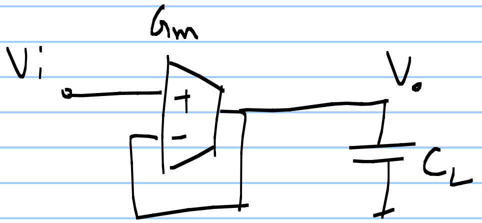
$\Rightarrow \frac{V_o}{V_i}(s) =$



$R_{out} = ?$

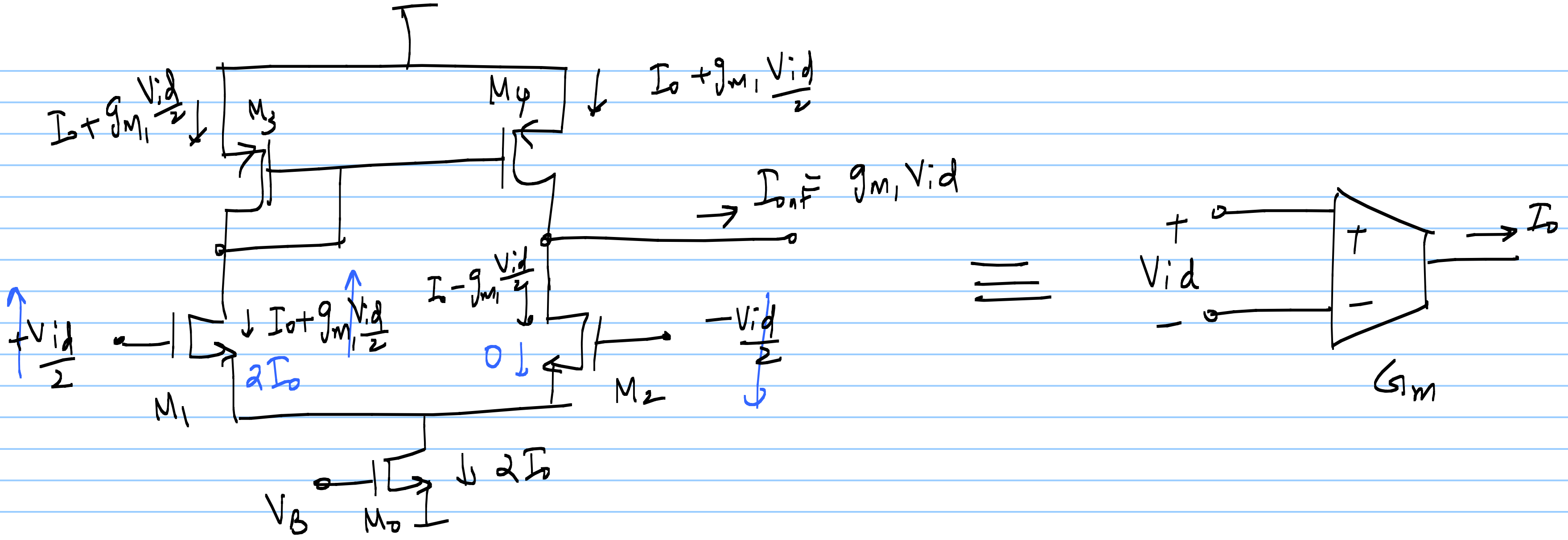
$\boxed{HW}$



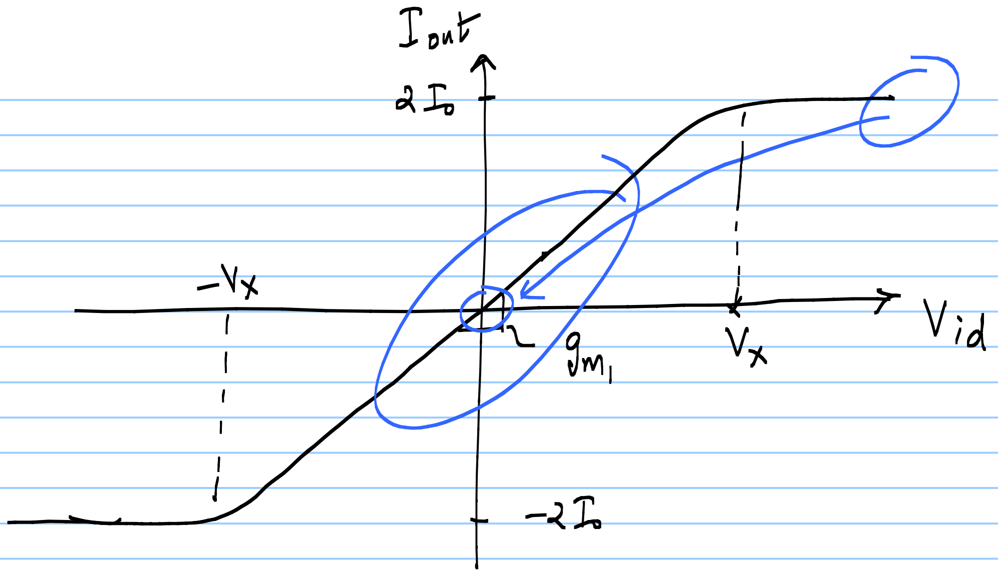


$$(V_i - V_o) \cdot G_m = V_o \cdot sC_L$$

$$\frac{V_o}{V_i}(s) = \frac{1}{1 + sC_L/G_m} \nearrow \tau = \frac{C_L}{G_m}$$



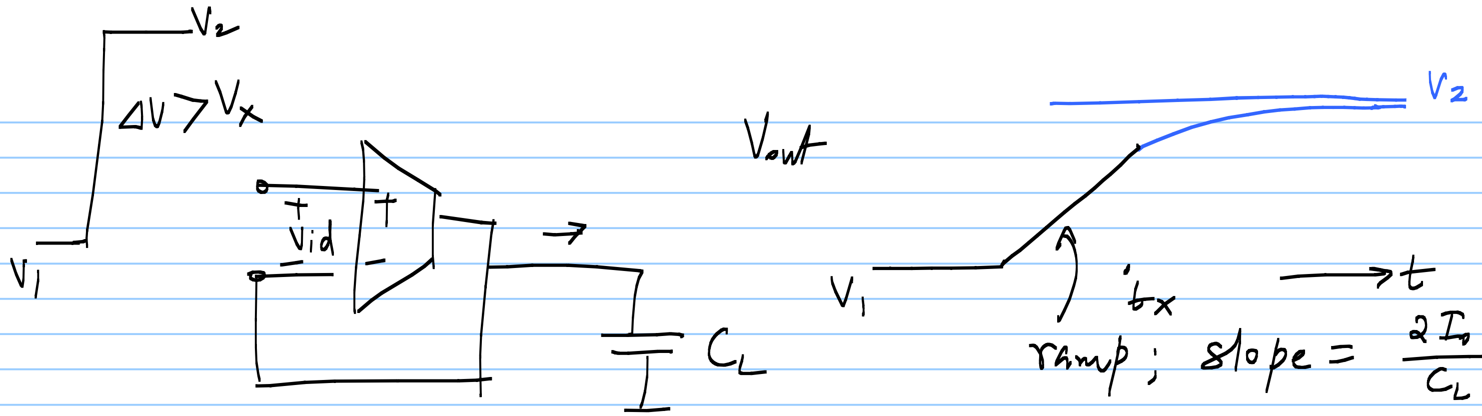
$I_o + g_{m_1}\frac{V_{id}}{2}$
M_3
M_4
$I_o + g_{m_1}\frac{V_{id}}{2}$
$I_{out} = g_{m_1}V_{id}$
$+\frac{V_{id}}{2}$
$I_o + g_{m_1}\frac{V_{id}}{2}$
$2I_o$
$I_o - g_{m_1}\frac{V_{id}}{2}$
0
$-\frac{V_{id}}{2}$
M_1
M_2
V_B
M_0
$2I_o$
$\equiv$
$+$
V_{id}
$-$
$+$
$-$
I_o
G_m



If ΔV (or V_{id}) $> V_x$,

$I_{out} = 2I_o$

slope $= 0$ $\{G_m = 0\}$



ramp ; slope = $\frac{2I_o}{C_L}$ = "slew rate"

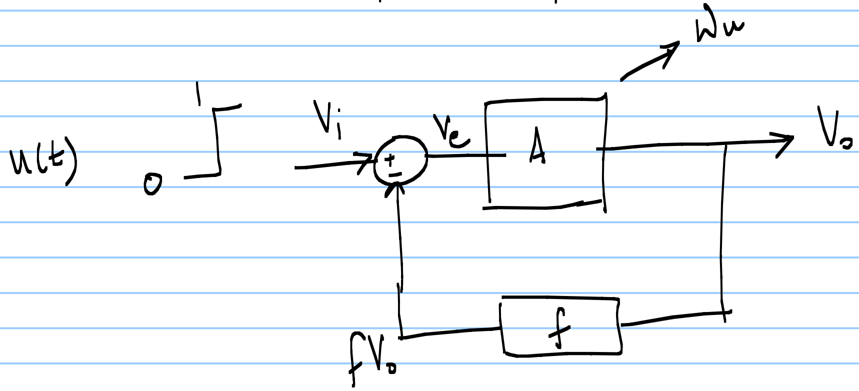
@ $t=0^+$, $V_{id} = \Delta V$; $I_{out} = 2I_o$; V_{out} = ramp @ $\frac{2I_o}{C_2}$

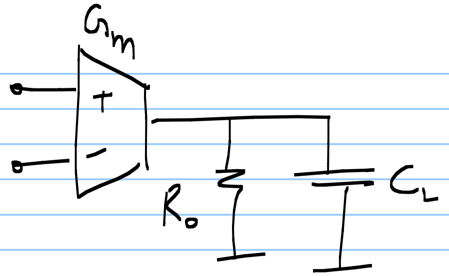
at $t = t_x$, $\Delta V < V_x$

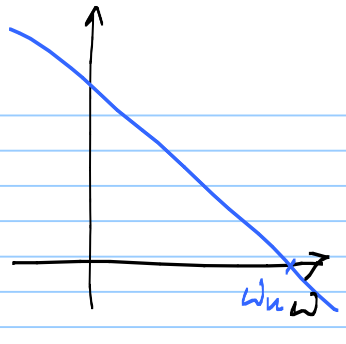
28/1/20

Lec 10

Telescopic Opamp

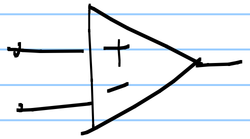






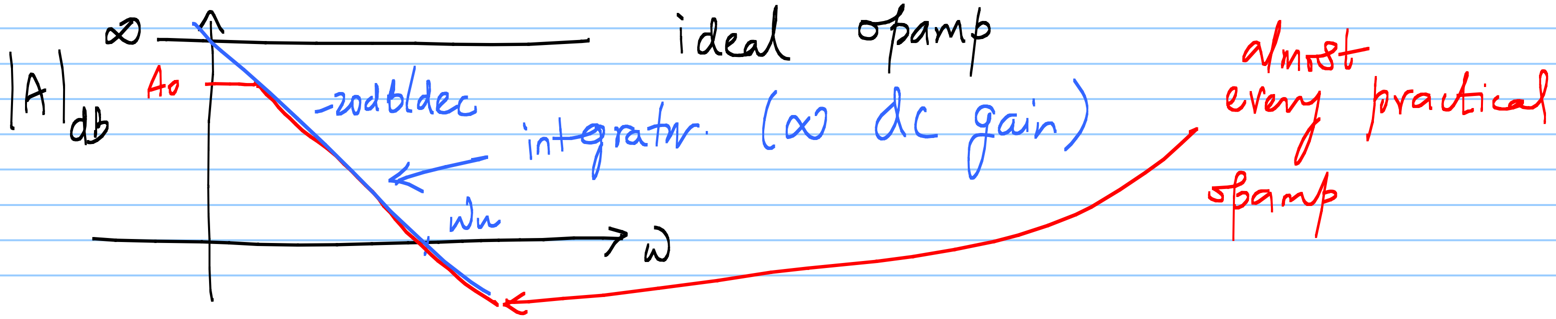
$$V_e(\infty) = 0$$

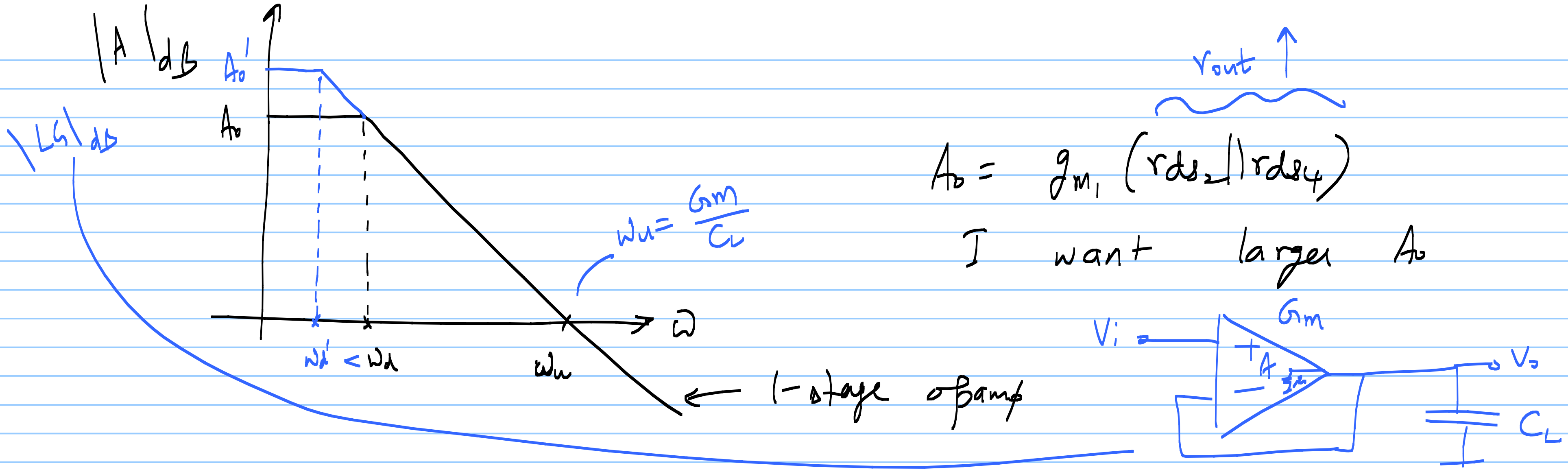
$$\Rightarrow V_o(\infty) = V_i(\infty)/f = \frac{1}{f} \; V$$



⇒ 1) Closed Loop gain = $\frac{1}{f}$ in steady state

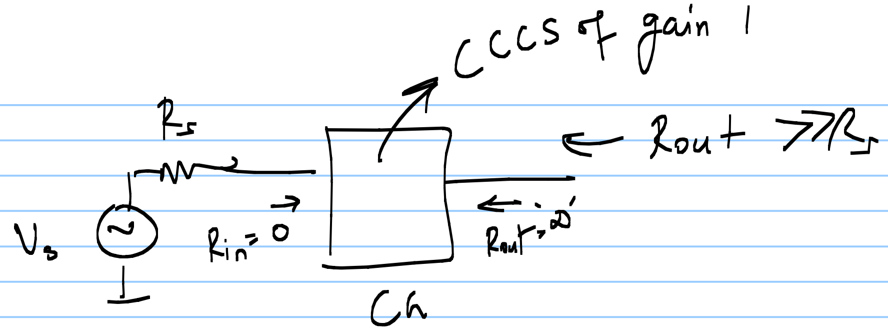
2) $V_e(\infty) = 0$ ⇒ A should have ∞ DC gain

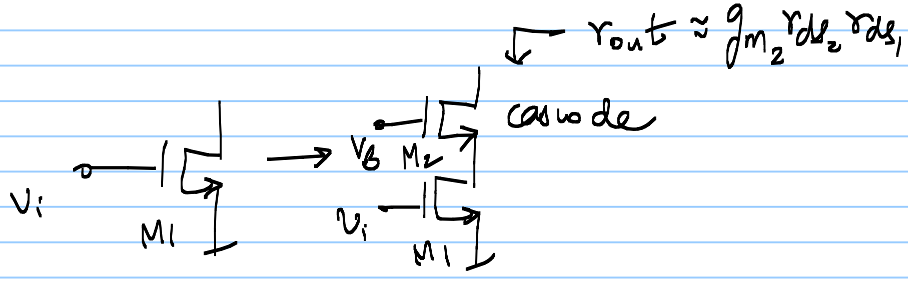




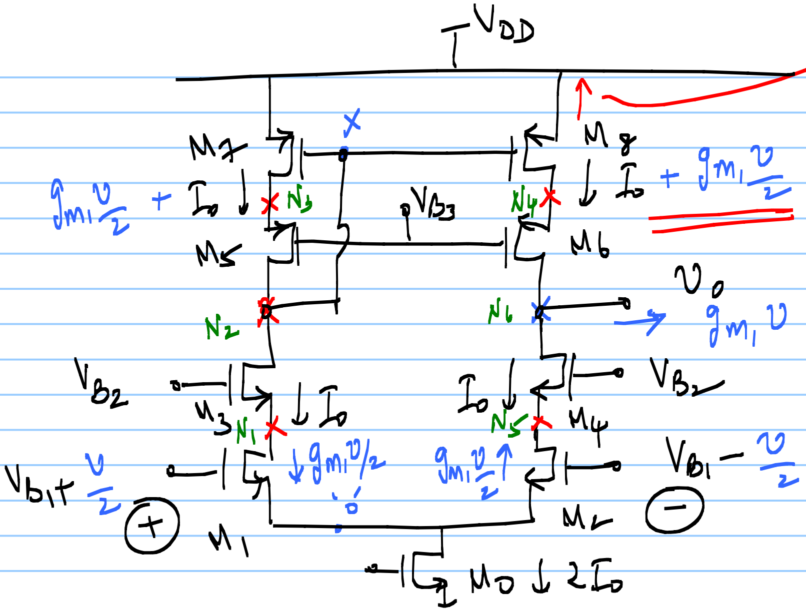
$$A_o = g_{m_1} \overbrace{\left(r_{ds_2} \| r_{ds_4}\right)}^{r_{out} \uparrow}$$

I want larger A_o


CCCS of gain 1
Rs
Rout >> Rs
Vs
Rin = 0
Rout = ∞
CG


rout ≈ gm2 rds2 rds1
cascode
Vi
M1
VB
M2
vi
M1

$id_8 = g_{m_8} \cdot v_x = -g_{m_1}\frac{v}{2}$



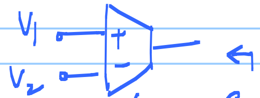
Telescopic opamp

$$V_x\ (DC) = V_{DD} - V_{SG_{7,8}}\Big|_{I_o}$$

$$v_x = -g_{m_1}\frac{v}{2} \times \frac{1}{g_{m_7}}$$

HW

≡



$R_{out} = g_{m_6} r_{ds_6} r_{ds_8} \| g_{m_4} r_{ds_4} r_{ds_2}$

$G_m = g_{m_1}$

Datasheet of Telescopic Opamp

1) DC gain = $g_{m_1} r_{out} = g_{m_1} \left[(g_{m_6} r_{ds_6} r_{ds_8}) \,||\, (g_{m_4} r_{ds_4} r_{ds_2}) \right]$

2) $G_m = g_{m_1}$

3) $\omega_u = \dfrac{g_{m_1}}{C_L}$; dominant pole $\omega_d = \dfrac{1}{r_{out} \; C_L}$

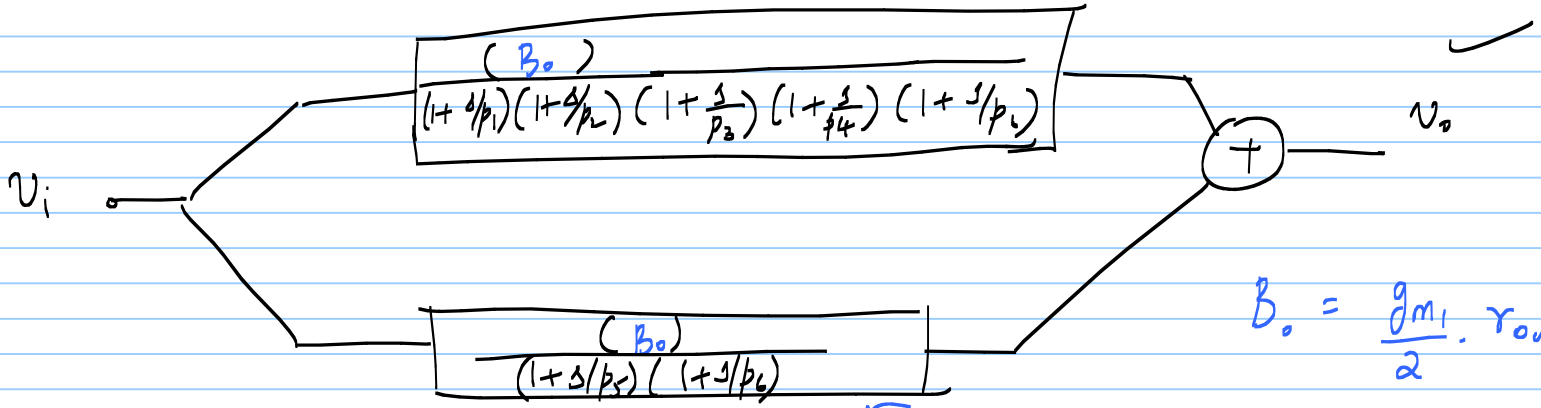
4) ND poles & zeroes : * 5 nodes of interest

assume node $N_k \rightarrow \frac{1}{1 + s/p_k}$; p_k depends of r & c @ N_k

in general p_k s are unique

$+\frac{v}{2}$ → encounters effects of p_1, p_2, p_3, p_4, p_6

$- v/2$ → encounters effects of p_5, p_6



$$B_o = \frac{g_{m1}}{2} \cdot r_{out}$$

@ low freq. $A_o = g_{m1} r_{out}$

$= 2B_o$

HW

$$\frac{v_o}{v_i} = \frac{N(s)}{D(s)}$$

$N(s)$ — degree 3 if N_1 & N_5 are symmetric

degree 4 if N_1 & N_5 are not symmetric

$D(s) \rightarrow$?? 5 poles if N_1 & N_5 symmetric

6 poles if N_1 & N_2 are asymmetric

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Lec 11

$N_1 \Leftrightarrow N_5$ (i.e. $p_1 = p_5$)

$$\frac{v_o}{v_i} = \frac{B_o}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)\left(1+\frac{s}{p_3}\right)\left(1+\frac{s}{p_4}\right)\left(1+\frac{s}{p_6}\right)} + \frac{B_o}{\left(1+\frac{s}{p_5}\right)\left(1+\frac{s}{p_6}\right)}$$

$$\frac{v_o}{v_s} = \frac{B_o + B_o\left(1+s/p_2\right)\left(1+\frac{s}{p_3}\right)\left(1+\frac{s}{p_4}\right)}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)\left(1+\frac{s}{p_3}\right)\left(1+\frac{s}{p_4}\right)\left(1+\frac{s}{p_6}\right)}$$

3 zeroes

5 poles

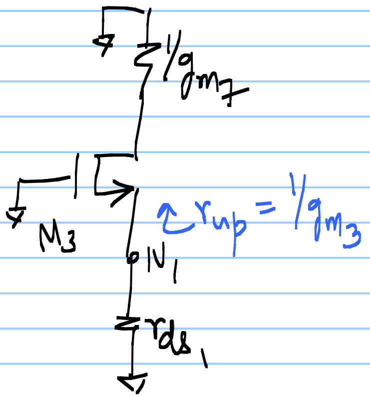
if $p_1 \neq p_5$

$$\frac{v_o}{v_i} = \frac{B_o\left(1+s/p_5\right) + B_o\left(1+s/p_1\right)\left(1+s/p_2\right)\left(1+s/p_3\right)\left(1+s/p_4\right)}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)\left(1+\frac{s}{p_3}\right)\left(1+\frac{s}{p_4}\right)\left(1+\frac{s}{p_5}\right)\left(1+s/p_6\right)}$$

4 zeroes

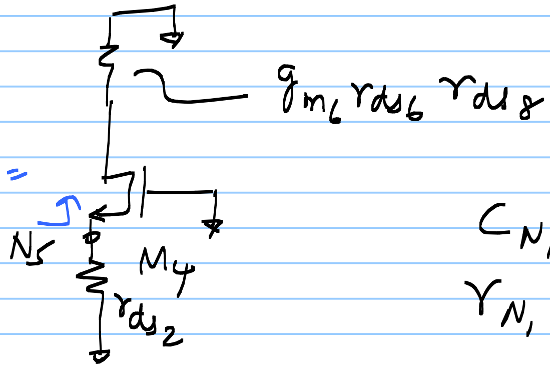
6 poles

$1/g_{m8} << r_{ds3}$



$$r_{up} = \frac{1}{g_{m3}}\left[\frac{1/g_{m7} + r_{ds3}}{r_{ds3}}\right] \approx \frac{1}{g_{m3}}$$

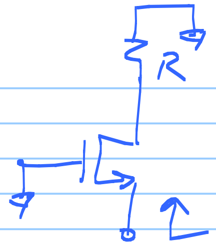
$r_{up}' =$



$$r_{up}' = \frac{1}{g_{m4}}\left[\frac{r_{ds4} + g_{m6} r_{ds6} r_{ds8}}{r_{ds4}}\right]$$

$C_{N1} = C_{N5}$

$r_{N1} \neq r_{N5}$



$\uparrow R_{in} =$

$$\frac{1}{g_m}\left[\frac{R + r_{ds}}{r_{ds}}\right]$$

If $r_{ds} >> R$,

$R_{in} \approx \frac{1}{g_m}$

$$r_{up}' \approx \frac{g_{m6} r_{ds6} r_{ds8}}{g_{m4} r_{ds4}} \neq \frac{1}{g_{m3}}$$

N_1 & N_5 poles are different.

4) ND poles & zeroes → 6 poles & 4 zeroes in total

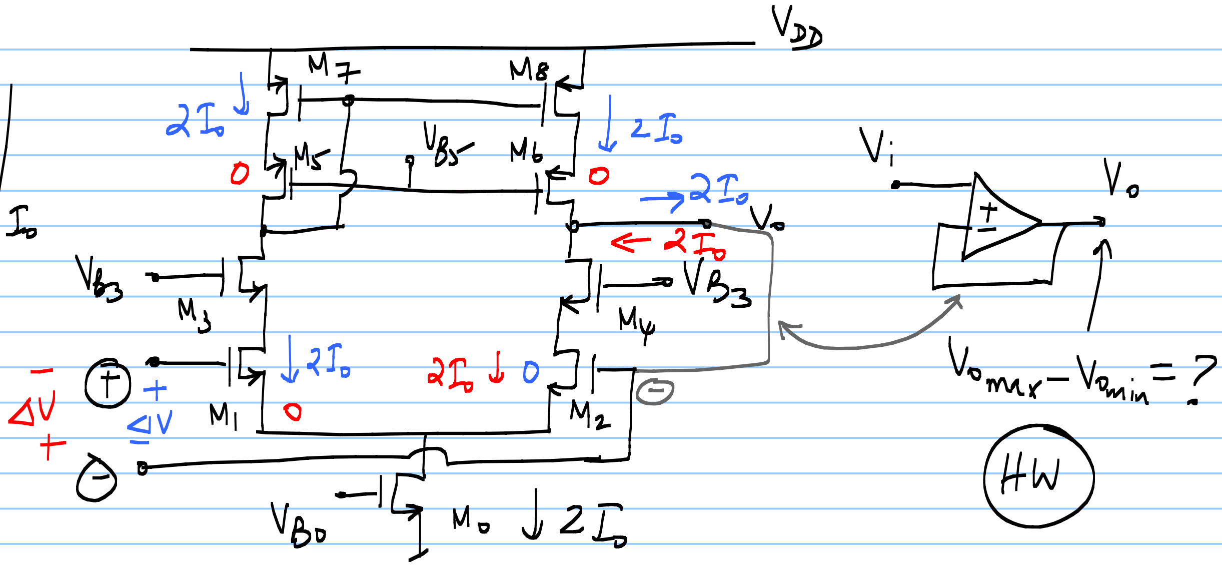
p_6 = Dominant pole

$p_1 - p_5$ = ND poles

5) ICMR

$$V_{CMin}\,(\text{min.}) = V_{Dsat_0}\Big|_{2I_0} + V_{GS_1}\Big|_{I_0}$$

$$V_{CMin}\,(\text{max}) = V_{B_3} - V_{GS_3}\Big|_{I_0} + V_{T_1}$$



$V_{o\max} - V_{o\min} = ?$

HW

6) OCMR

$$V_{CM_{out}}(min) = V_{B_3} - V_T$$

$$V_{B_3}(min) = V_{CM_{in}} - V_{T_2} + V_{GS_4}$$

$$V_{CM_{out}}(max) = V_{B_5} + V_{T6}$$

7) **Noise:** $\overline{i_n} = 0$ @ low freq.

5-transistor

$\overline{e_n}$ = same as 1-stage opamp $= \frac{16kT}{3g_{m_1}} + \frac{16kT \cdot g_{m_7}}{3g_{m_1}^2}$

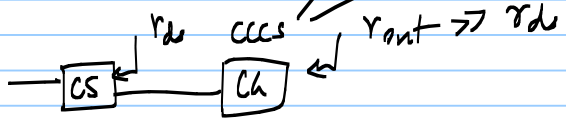
- * Noise from M_0 is Common-mode
- * Noise from M_3 M_4 M_5 M_6 — does not affect $\overline{e_n}$ @ low freq. ; affects $\overline{e_n}$ @ high freq.

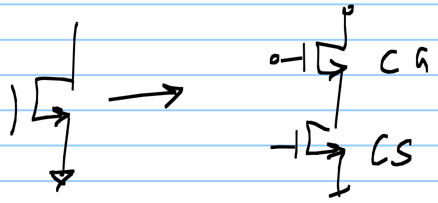
8) Offset : $\sigma_{os}^2 = ?$ HW assume only V_T mismatch

9) Slew rate : $SR = \pm \frac{2I_0}{C_L}$

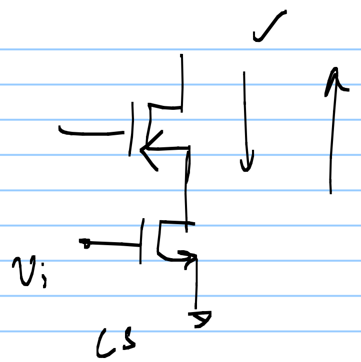
Cacode : NMOS CS + NMOS CG

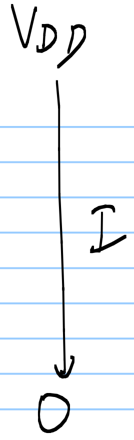
PMOS / NMOS → Cascode

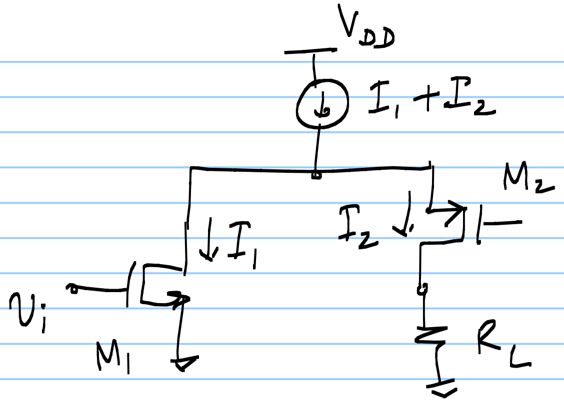




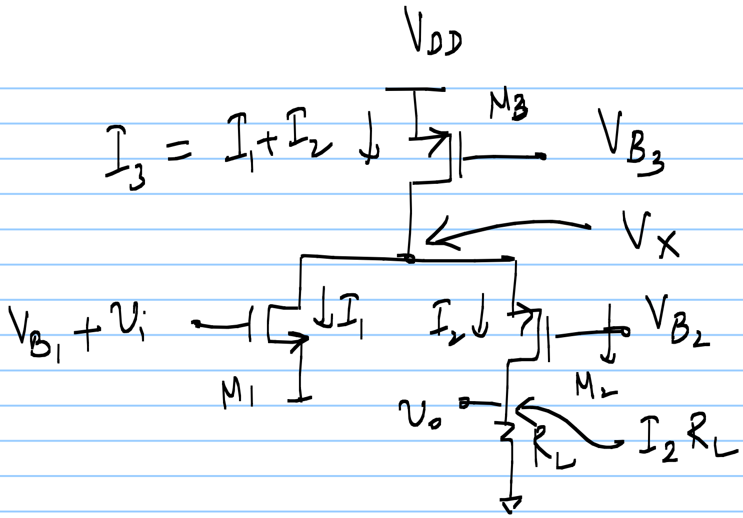
NMOS CS + PMOS CG







"Folded Cascode" amplifier



$V_{SG_3} = V_{DD} - V_{B_3}$ $\longrightarrow$ causes $I_{D_3} = I_1 + I_2$

$V_{B_1} = V_{GS_1}\Big|_{I_1}$; V_X is set by V_{B_2} $\left(V_X = V_{B_2} + V_{SG_2}\right)$

$V_X\,(\max) = V_{DD} - V_{SD\,sat_3}$

M₂ in triode

$V_X\,(\min) = V_{D\,sat_1}$ or $\underbrace{(\quad / \quad)}$

✗ M_2 goes into triode when $V_{B_2} = I_2 R_L - V_{T_2}$

$$\min V_X = I_2 R_L - V_{T_2} + V_{SG_2}$$

$$= I_2 R_L + V_{SDsat_2}$$

min V_X = larger of $\{ V_{Dsat_1} , I_2 R_L + V_{SDsat_2} \}$

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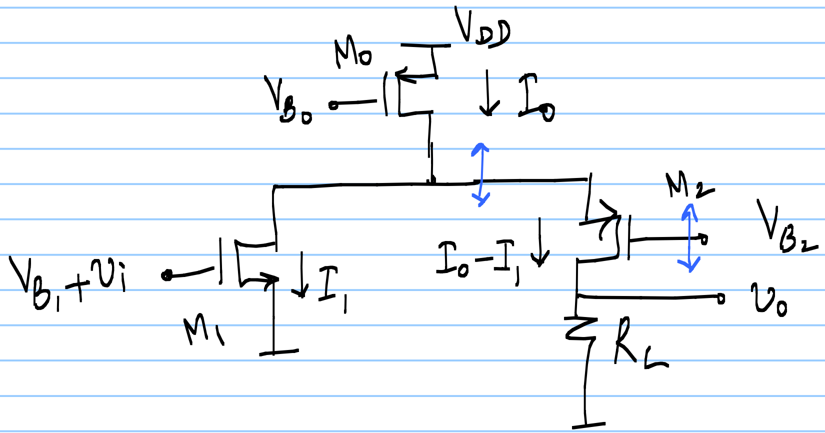
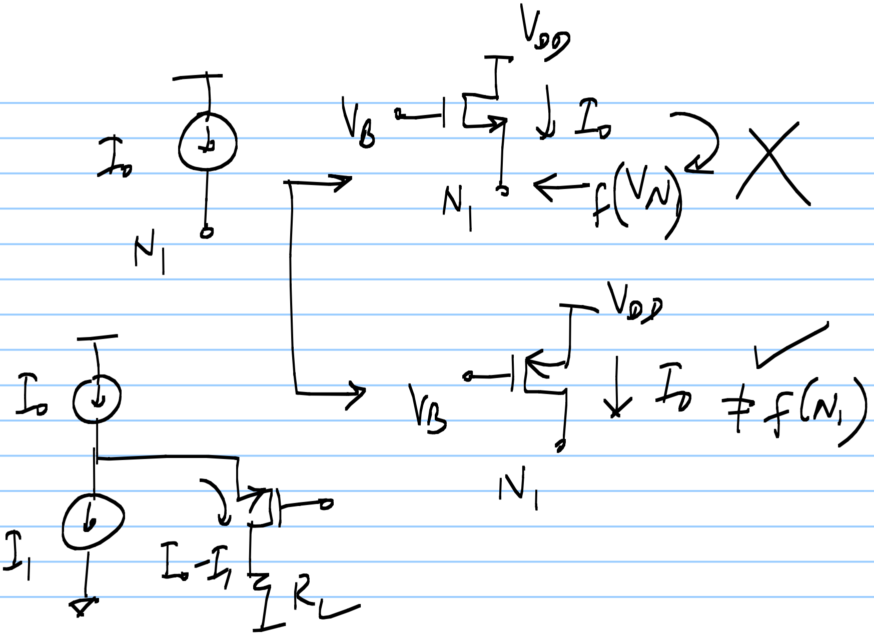
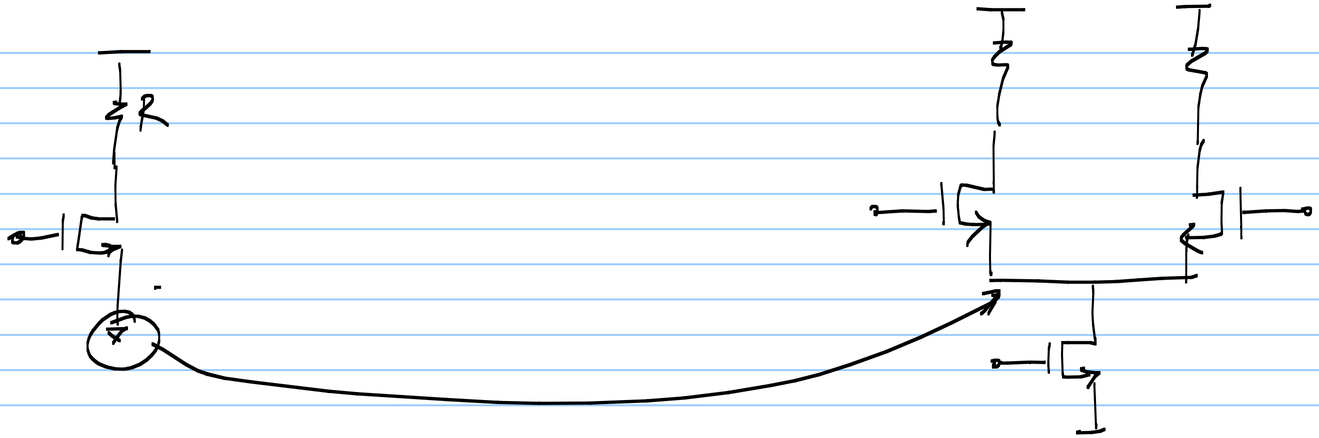
LT Spice — Circuit simulator that runs on windows

- simulation assignments will start soon.
- Read the manual & learn to use the simulator

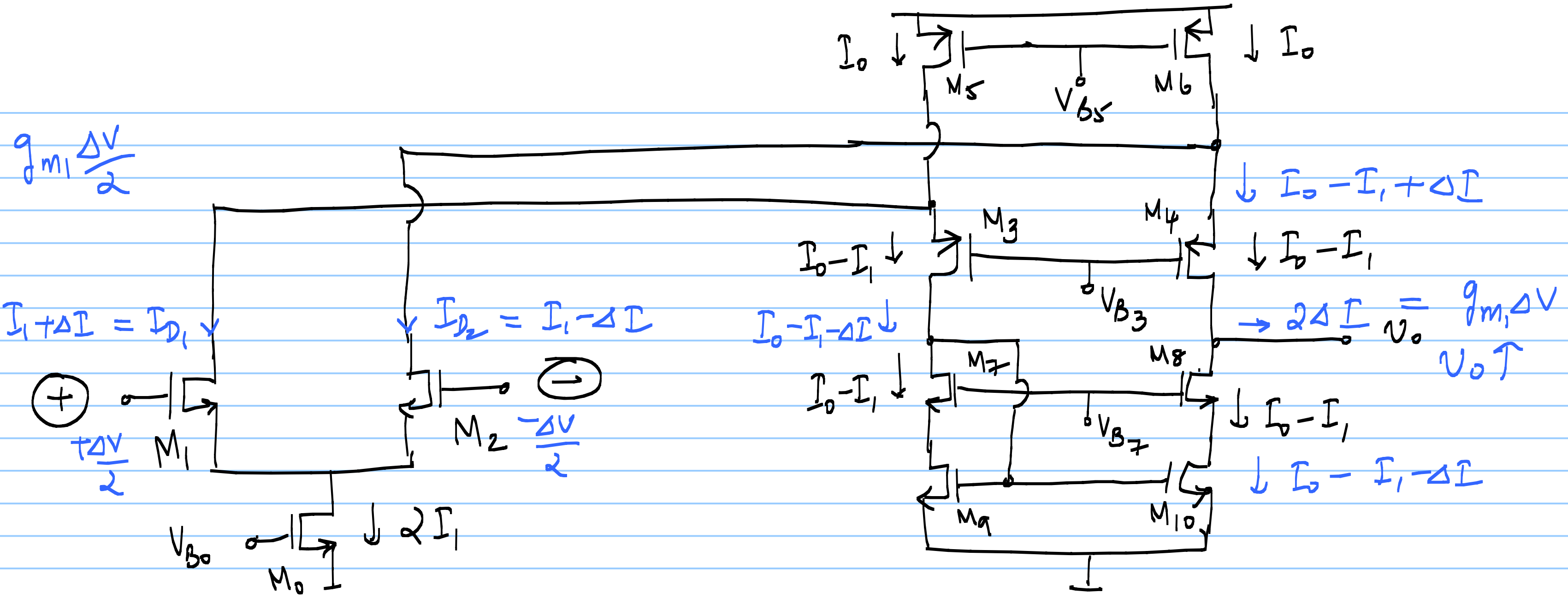
Folded Cascode

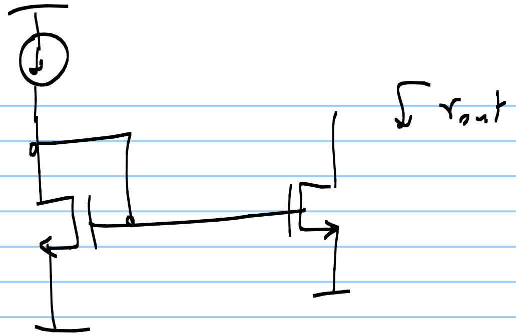


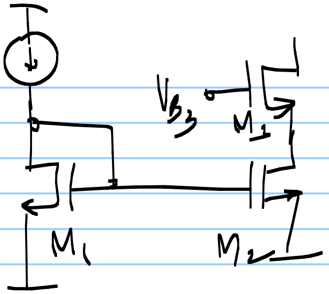


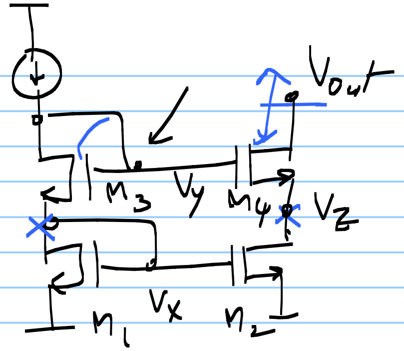

R

$\Delta I = g_{m1}\frac{\Delta V}{2}$









$V_y = V_{GS_1} + V_{GS_3}$; $V_x = V_{GS_1}$

$V_Z = V_{GS_1} + V_{GS_3} - V_{GS_4} = V_{GS_1}$

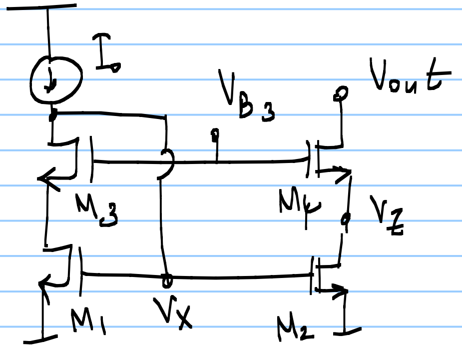
$$\left.\right\} \quad V_{out_{min}} = V_y - V_{T_4}$$

$$= V_{GS_1} + V_{Dsat_4}$$

If $\underbrace{V_Z = V_{Dsat_2}}$, $V_{out_{min}} = V_{Dsat_2} + V_{Dsat_4}$

↳ we need $V_y = V_Z + V_{GS_4} = V_{Dsat_2} + V_{GS_4}$

high swing cascode current mirror



$V_X = V_{GS_1}$ through negative f.b.

Set $V_{B3} \geq V_{Dsat_1} + V_{GS_3}$

$V_{out_{min}} = V_{B3_{min}} - V_{T_4} = V_{Dsat_1} + V_{Dsat_3}$

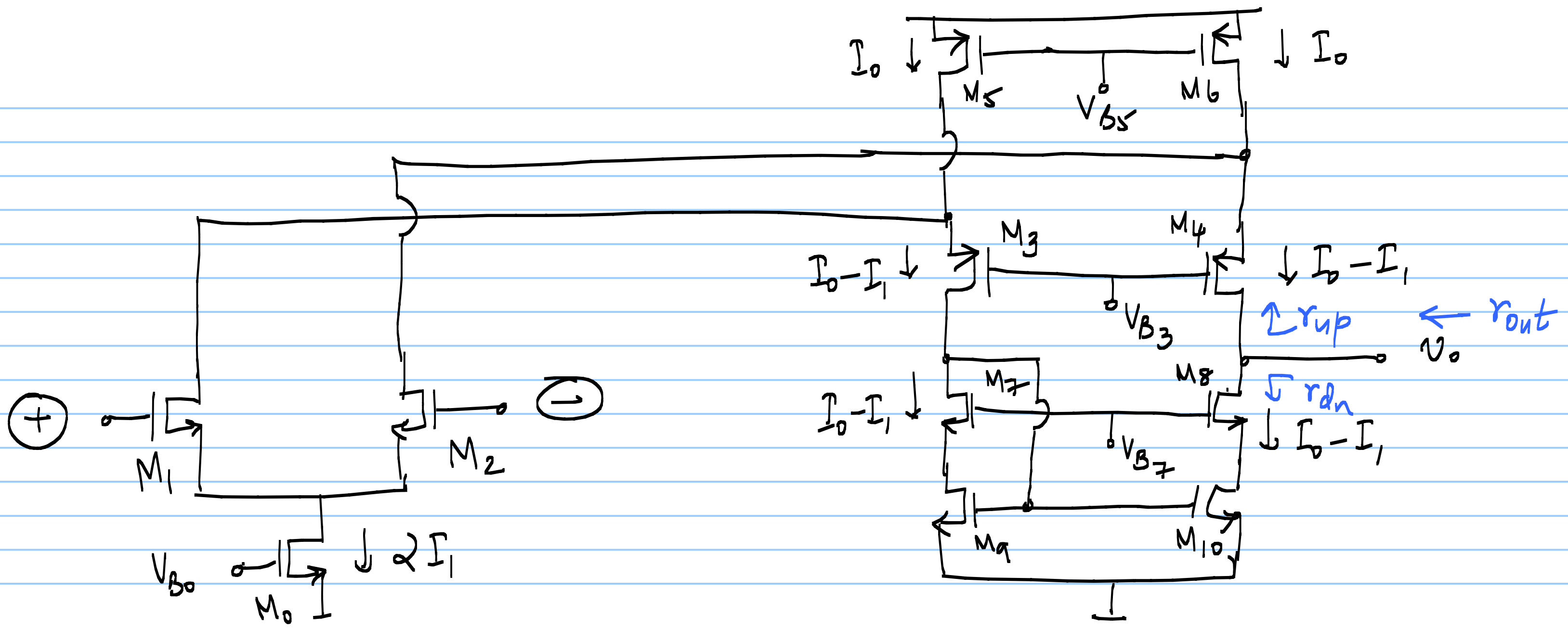
Datasheet

$G_m = g_{m_1}$

$r_{out} = r_{up} \| r_{dn}$

$= \left[(g_{m_4} r_{ds_4})(r_{ds_6} \| r_{ds_2})\right] \| \left[(g_{m_8} r_{ds_8})(r_{ds_{10}})\right]$

1) DC gain = $g_{m_1} r_{out}$ → slightly lower than that of telescopic opamp


I0
M5
VB5
M6
I0
M3
M4
I0-I1
VB3
I0-I1
r_up
r_out
v_o
M7
M8
r_dn
I0-I1
VB7
I0-I1
M9
M10
+
M1
−
M2
VB0
M0
2I1

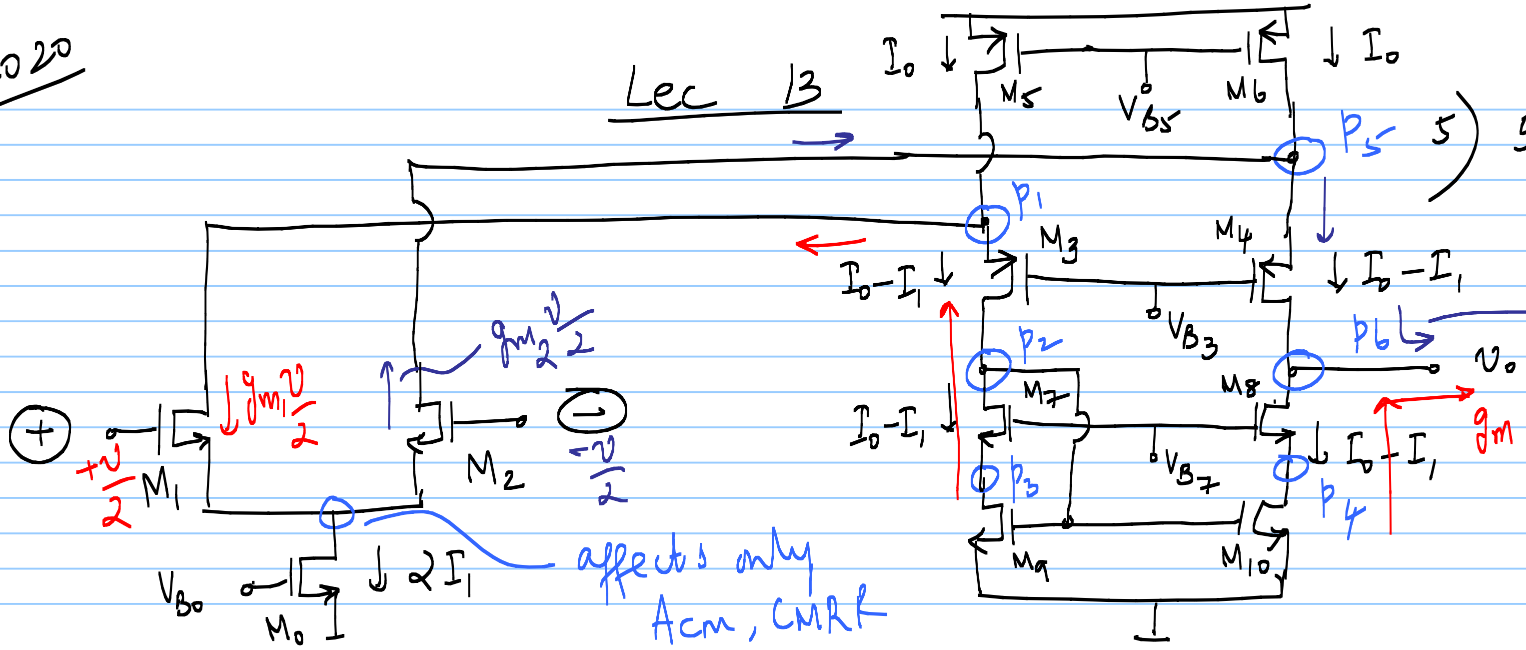
2) $G_m = g_{m1}$

3) $\omega_u = \frac{g_{m1}}{C_L}$; Dominant pole $\omega_d = \frac{1}{r_{out} C_L}$

4) ND poles & zeroes ← next class (HW − # of poles & zeroes)

05/02/2020

Lec 13



5) 5 Non dominant poles @ $p_1 - p_5$

$g_{m_2} v/2 \{ [p_5] + p_6 \}$

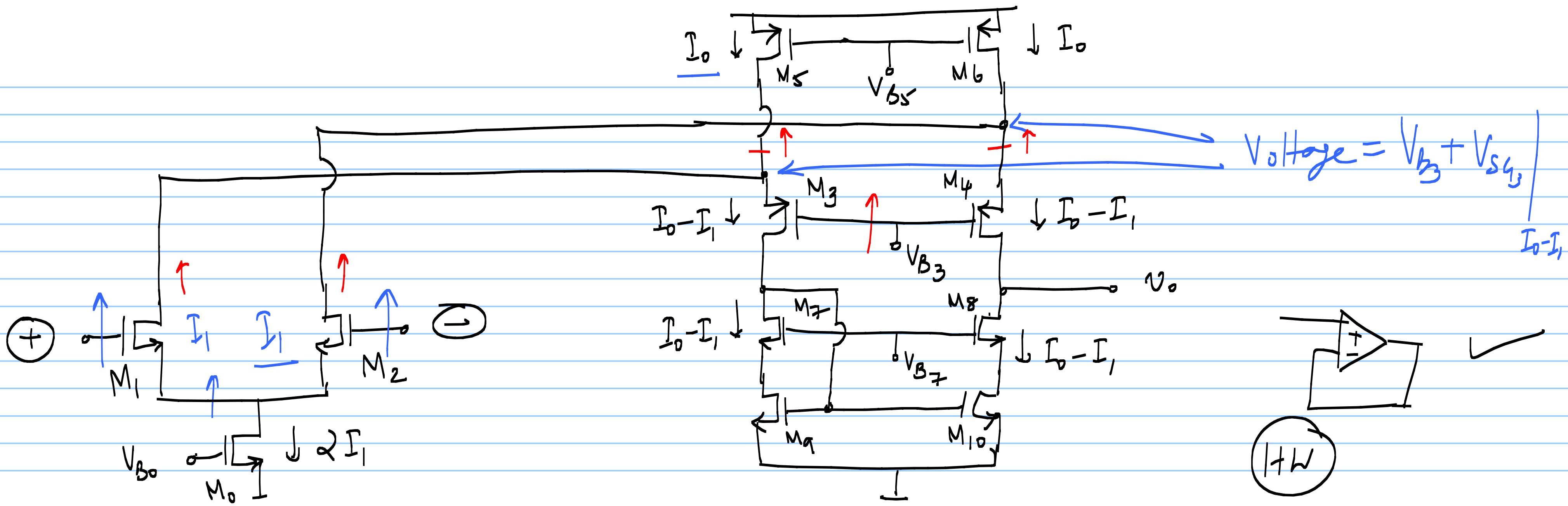
$g_{m_1} \frac{v}{2} \{ [p_1 - p_4] + p_6 \}$

6) <u>ICMR</u> $= \left\{ V_{Dsat_0}\Big|_{2I_1} + V_{GS_1}\Big|_{I_1} \; , \; V_{B_3} \overset{\uparrow}{+} V_{SG_3}\Big|_{I_0-I_1} + V_{T_1} \right\}$

7) <u>OCMR</u> $= \left\{ \underbrace{V_{B_7} - V_{T_8}}_{\text{can be set } \approx 2V_{Dsat}} \; , \; \underbrace{\overset{\uparrow}{V_{B_3}} + V_{T_4}}_{\longrightarrow \; V_{DD} - 2V_{Dsat}} \right\}$

i.e. $V_{B_7} = V_{GS_8}\Big|_{I_0-I_1} + V_{Dsat_{10}}\Big|_{I_0-I_1}$

i.e. $V_{B_3} = V_{DD} - V_{SDsat_5}\Big|_{I_0} - V_{SG_3}\Big|_{I_0-I_1}$


+
−
M1
I1
I1
M2
VB0
M0
2I1
I0
M5
VB5
M6
I0
M3
M4
I0−I1
VB3
I0−I1
M7
M8
I0−I1
VB7
I0−I1
vo
M9
M10
Voltage = VB3 + VSG3
I0−I1
+
−
HW

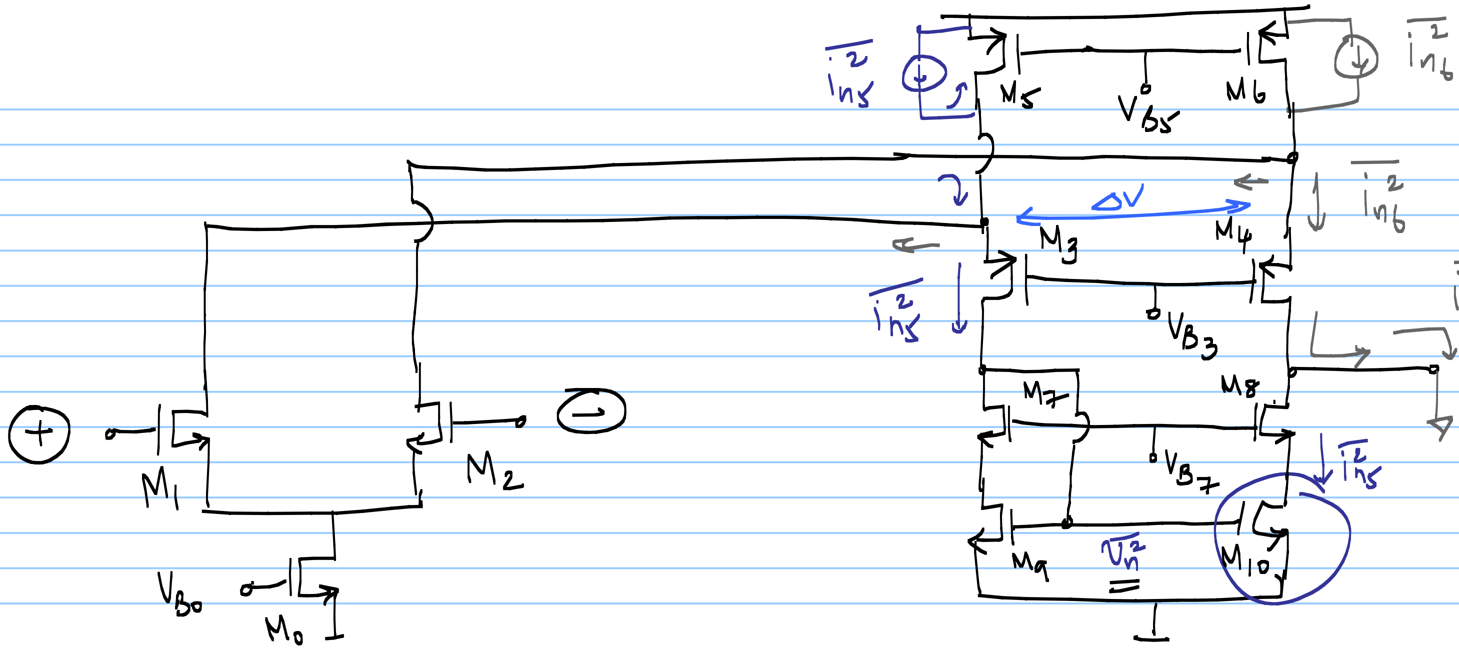
8) $\overline{e_n^2} = ?$ * $M_3 - M_4$, $M_7 - M_8$ → Cascode devices

→ no noise @ low freq.

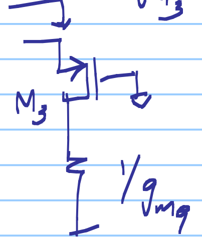
* noise contribution from $M_1 - M_2$, $M_5 - M_6$, $M_9 - M_{10}$

* Expect noise to be worse than 1-stage or telescopic

$$\frac{\overline{e_n^2}}{\Delta f} = \frac{16kT}{3g_{m_1}} + \frac{16kT}{3} \cdot \frac{g_{m_5}}{g_{m_1}^2} + \frac{16kT}{3} \cdot \frac{g_{m9}}{g_{m1}^2}$$



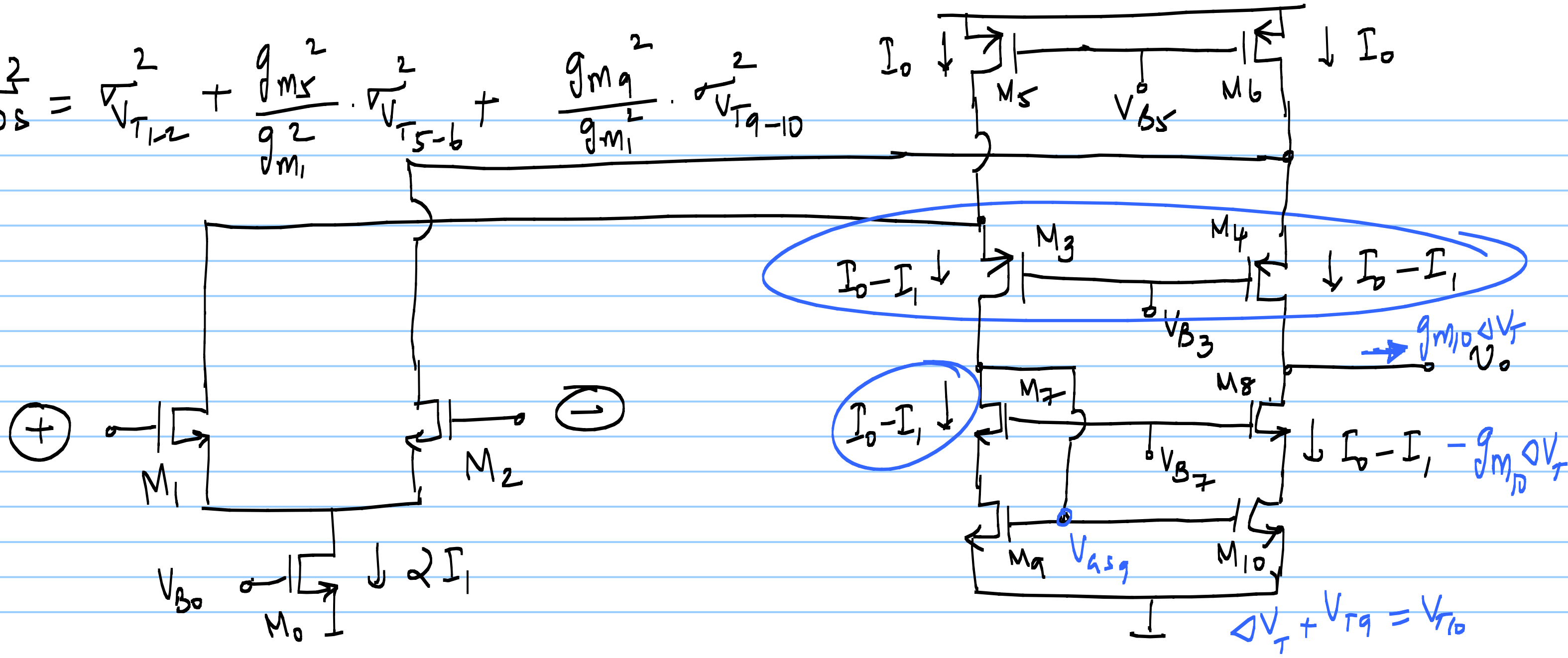
$r_{in} = 1/g_{m_3}$



$\overline{i_{scn}^2} = \overline{i_{n6}^2} + \overline{i_{n5}^2}$

$v_o \quad + \overline{i_{n10}^2} + \overline{i_{n9}^2}$ if $1/g_{m9} << r_{d3}$

9) Offset : $\sigma_{os}^2 = \sigma_{V_{T1-2}}^2 + \frac{g_{m5}^2}{g_{m1}^2} \cdot \sigma_{V_{T5-6}}^2 + \frac{g_{m9}^2}{g_{m1}^2} \cdot \sigma_{V_{T9-10}}^2$



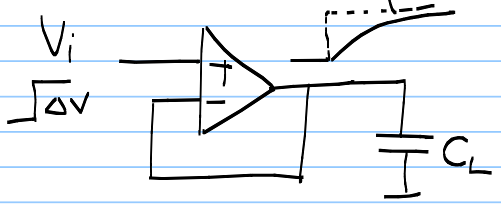
5/2/20

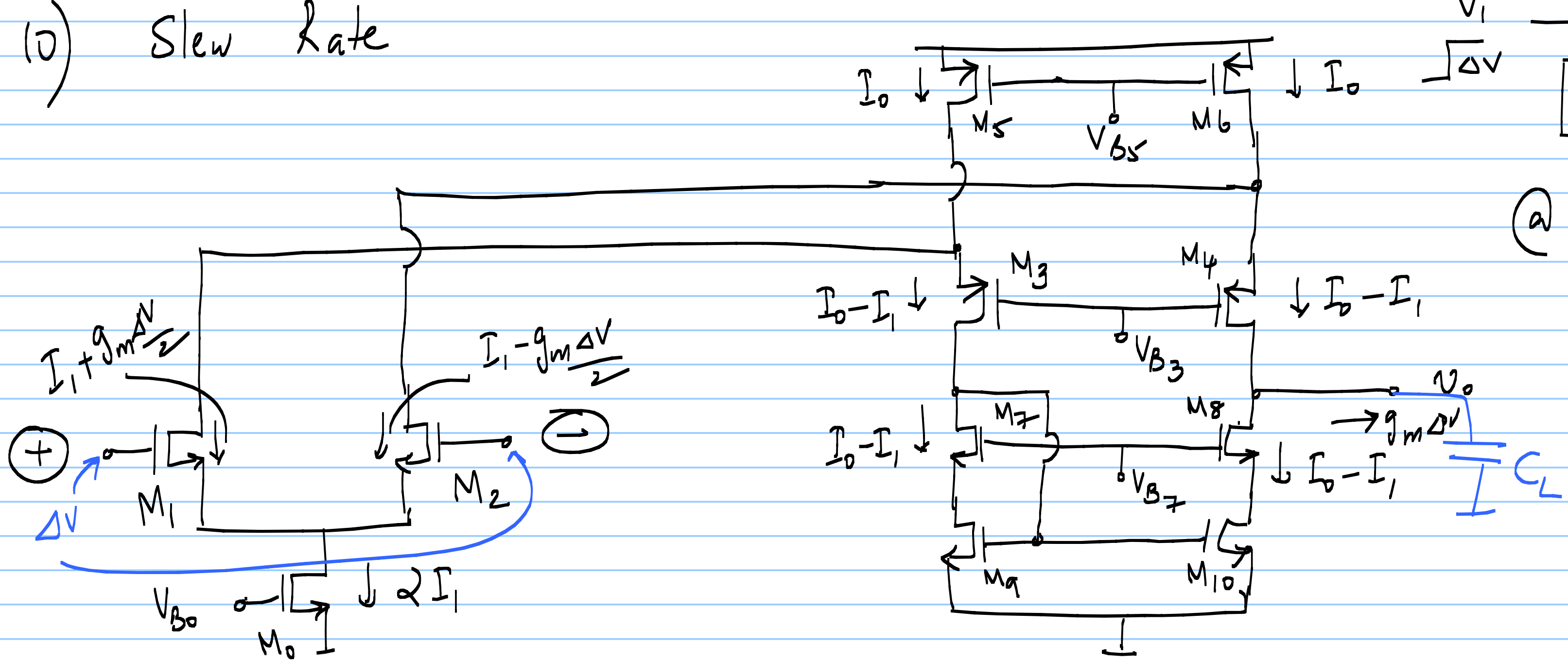
Lec 13

Folded cascode opamp datasheet

(10) Slew Rate

$\tau \approx \frac{1}{\omega_n}$ $\tau = \frac{C_L}{G_m}$

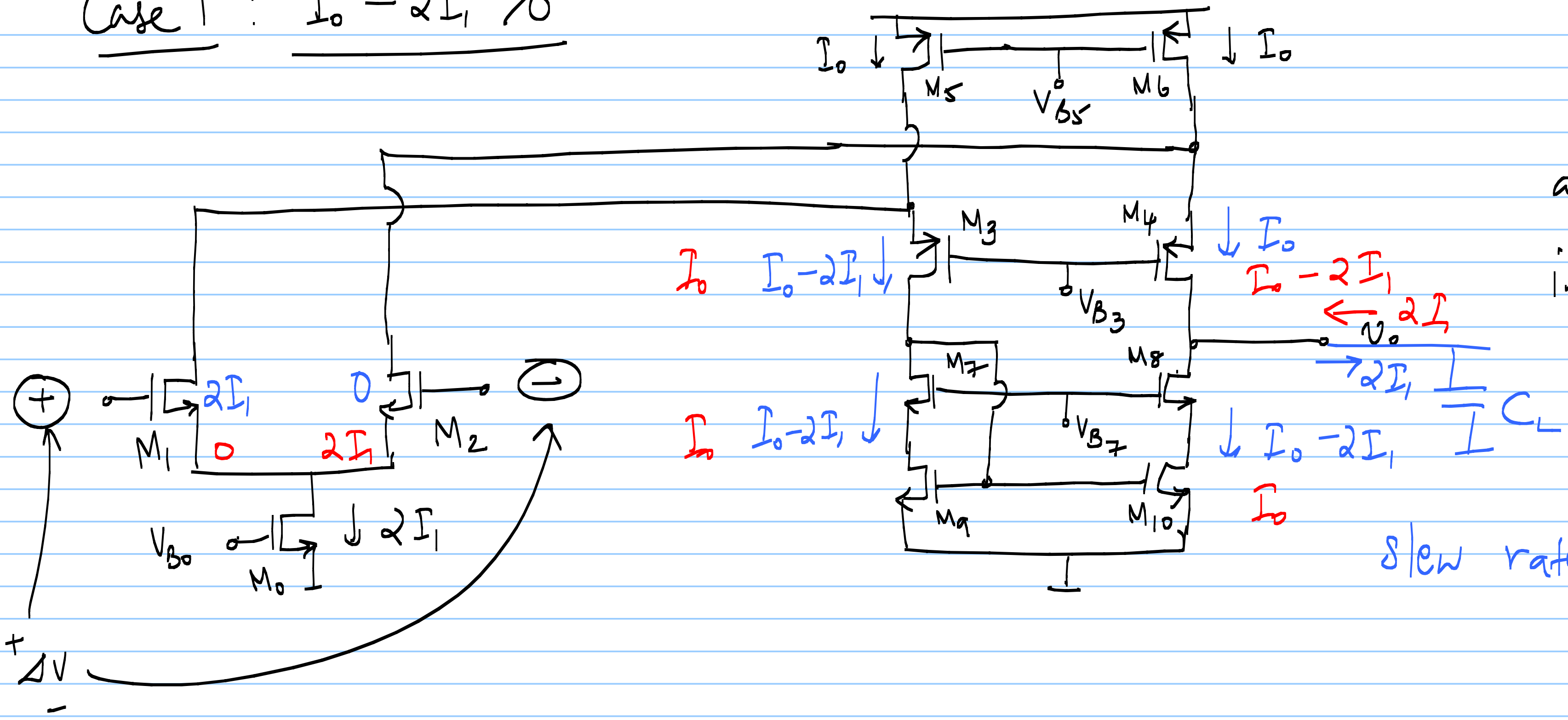




@ $t = 0^+$, ΔV appears as V_{id}

$(V_+ - V_-) = \Delta V$

Case 1 : $I_o - 2I_1 > 0$

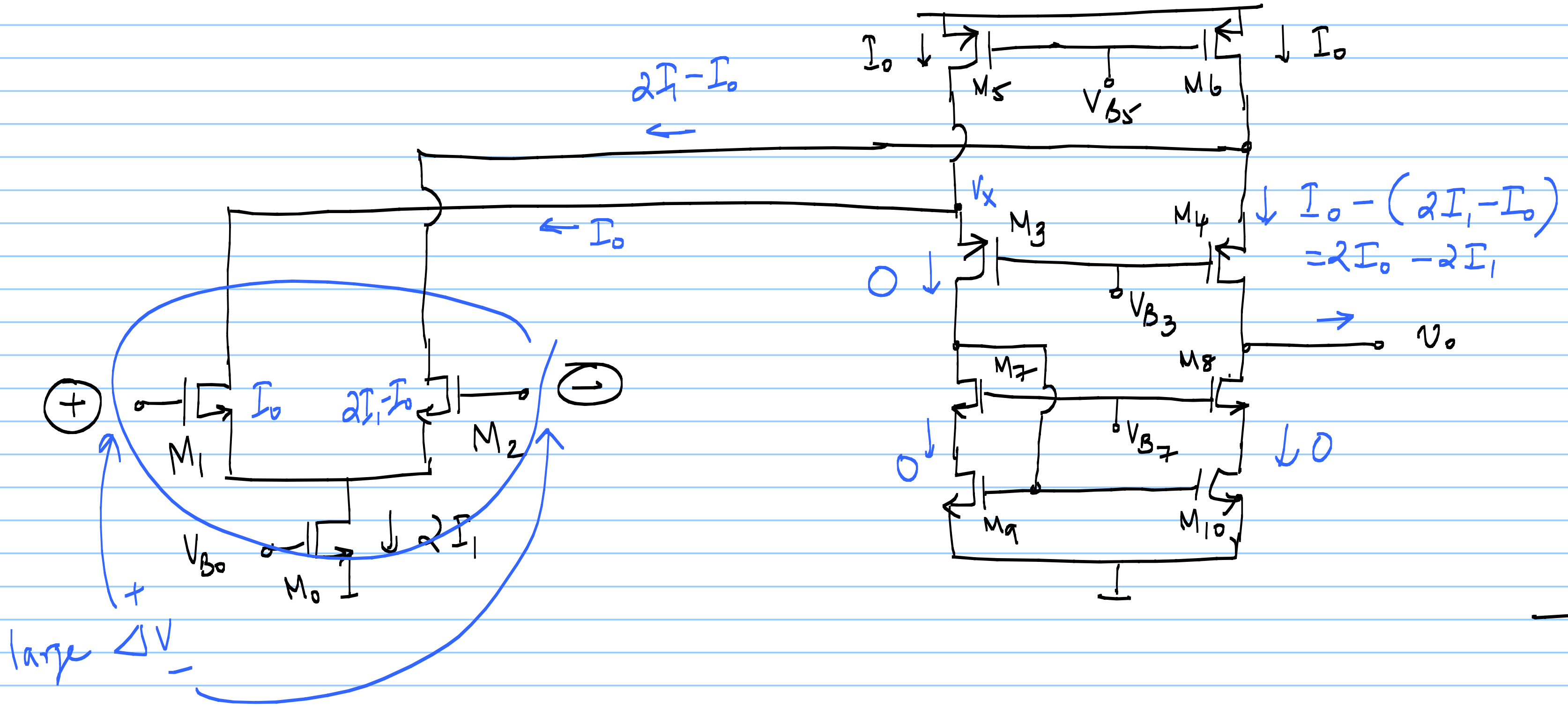


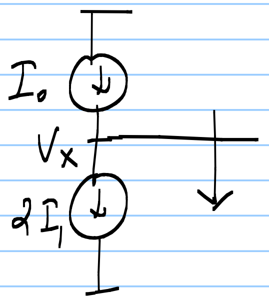
large ΔV
applied @
input

$$\text{slew rate} = +\frac{2I_1}{C_L}$$

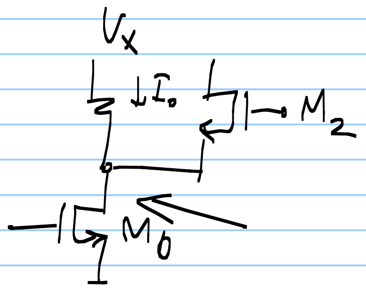
$$-\frac{2I_1}{C_L}$$

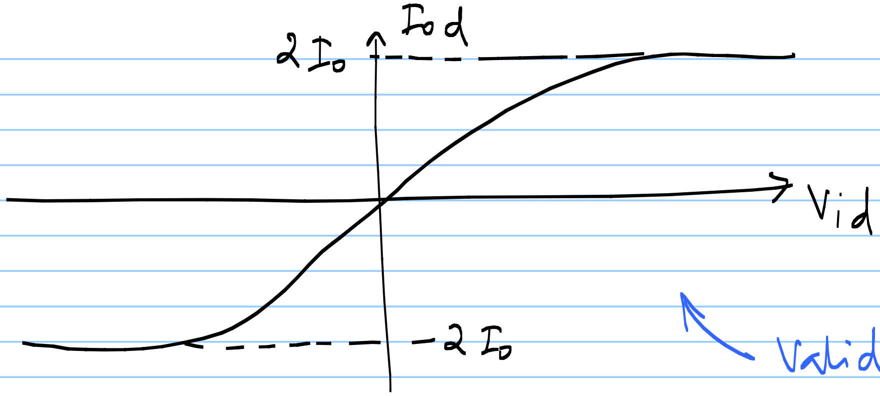
Case 2: $I_o - 2I_1 < 0$ $\boxed{HW}$

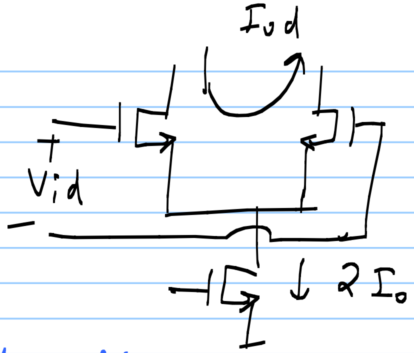




$2I_1 > I_o$







Valid if $M_1 - M_2$ are in saturation

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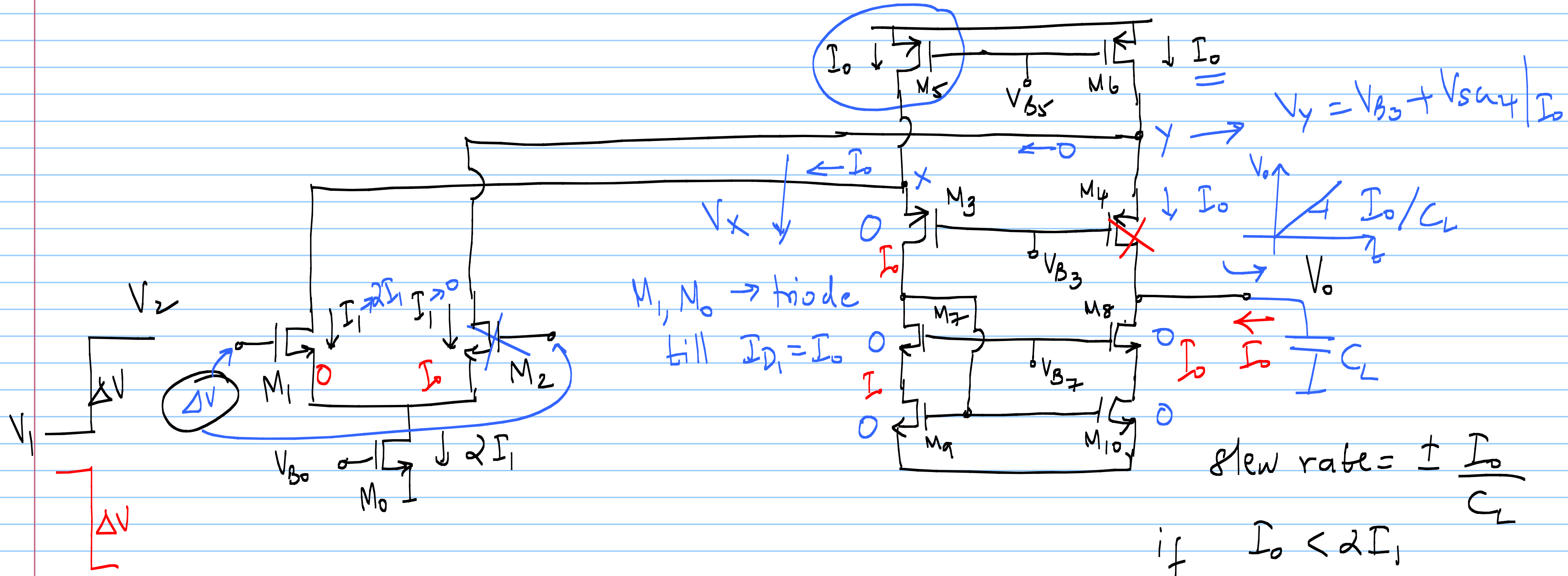
Lec 14

$I_o < 2I_1$



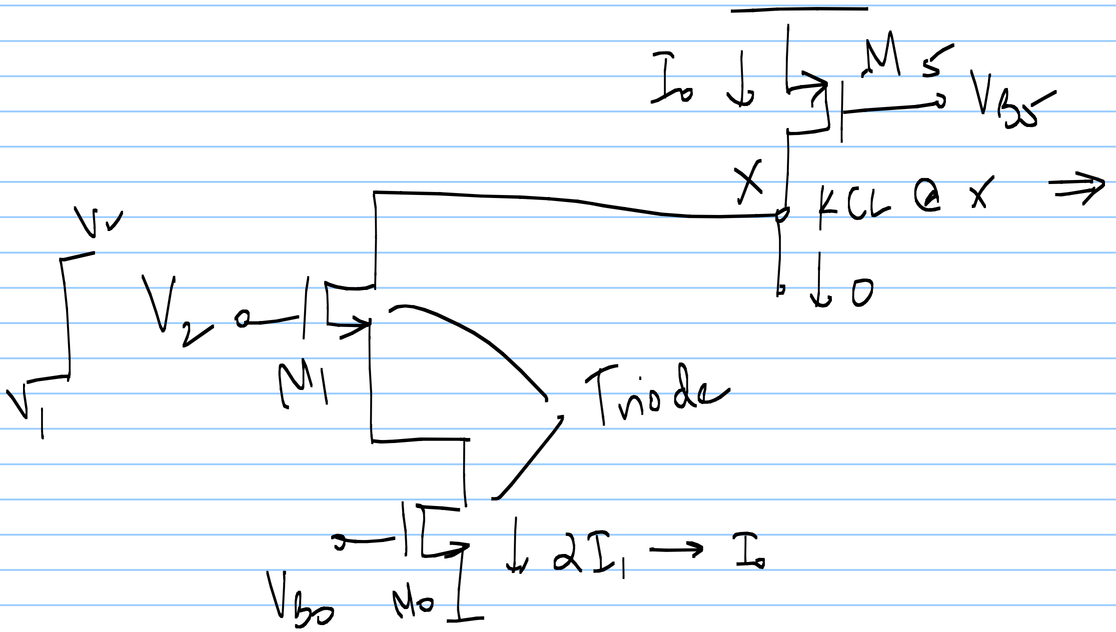
$V_y = V_{B3} + V_{SG4}|_{I_o}$

$M_1, M_0 \rightarrow$ triode till $I_{D_1} = I_o$

Slew rate $= \pm \dfrac{I_o}{C_L}$

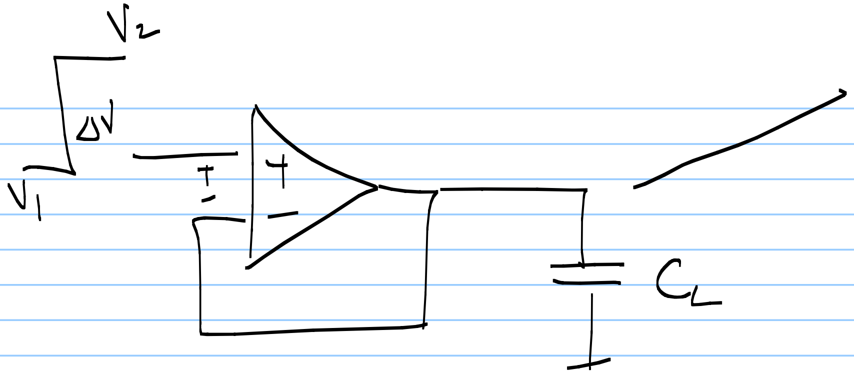
if $I_o < 2I_1$

$I_0 < 2I_1$



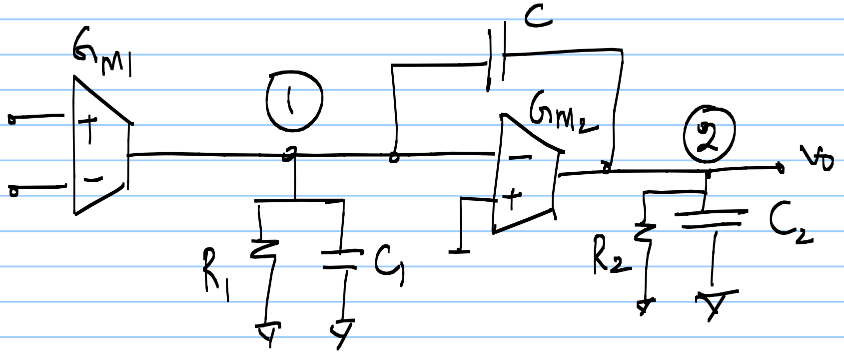
V_1
V_2
V_2
M_1
Triode
I_0
M_5
V_{B5}
X
KCL @ X ⇒
0
$2I_1 \rightarrow I_0$
V_{B0}
M_0

V_X
t
t = 0



2- stage opamp

Block level picture



$R_1 \rightarrow r_{out}$ of G_{m_1}

$R_2 \rightarrow$ " " G_{m_2}

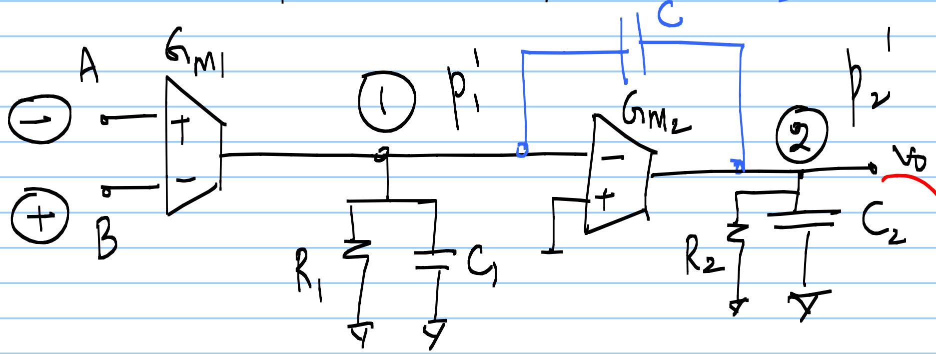
$C_1, C_2 \rightarrow$ total cap @ ① & ② resply

p_1, p_2, z_1

ω_d @ ①

Before compensation

Miller effect : $C_{①} \approx C_1 + G_{m2}R_2 \cdot C$
$\approx G_{m2}R_2C$



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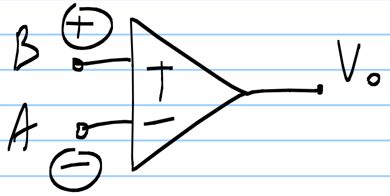


$A_o = G_{m1}R_1\, G_{m2}R_2$

$C_{②}$ ↑ slightly

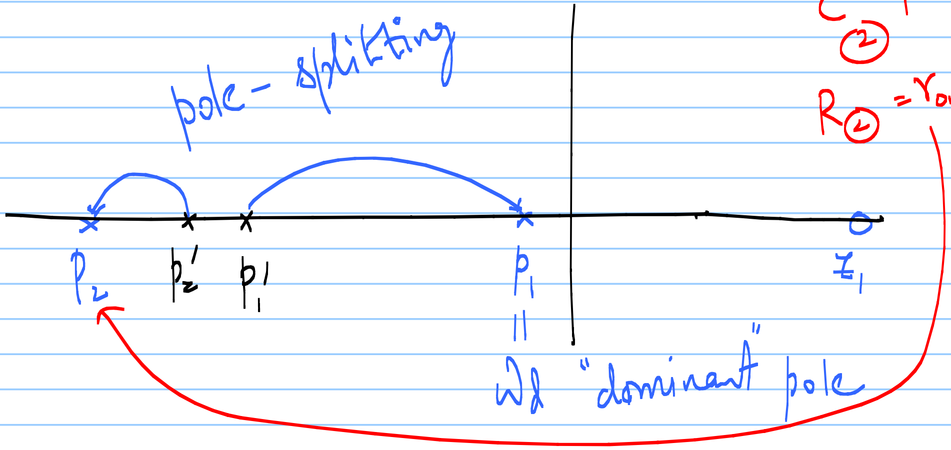
$R_{②} = r_{out}' \| R_2$

$$p_1' = \frac{1}{R_1C_1}$$

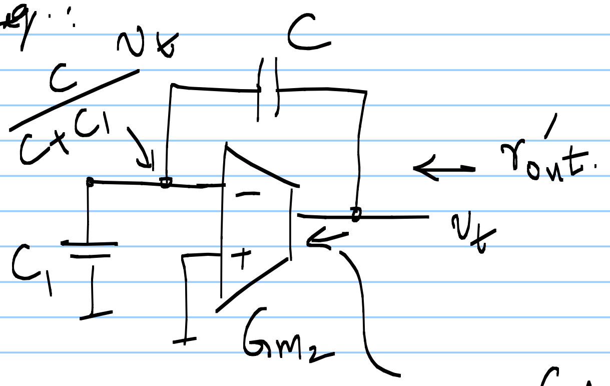
$$p_1 = \frac{1}{R_1 \cdot G_{m2}R_2C} = \omega_d$$

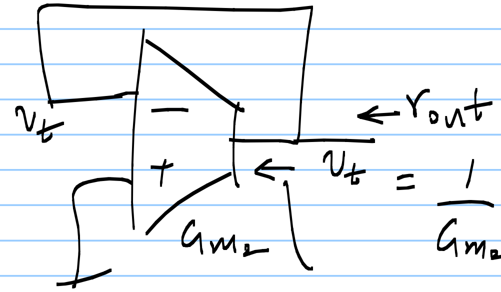
$$p_2' = \frac{1}{R_2C_2}$$

p_2 = — — — .



@ high freq.:





$= \frac{1}{G_{m_2}}$

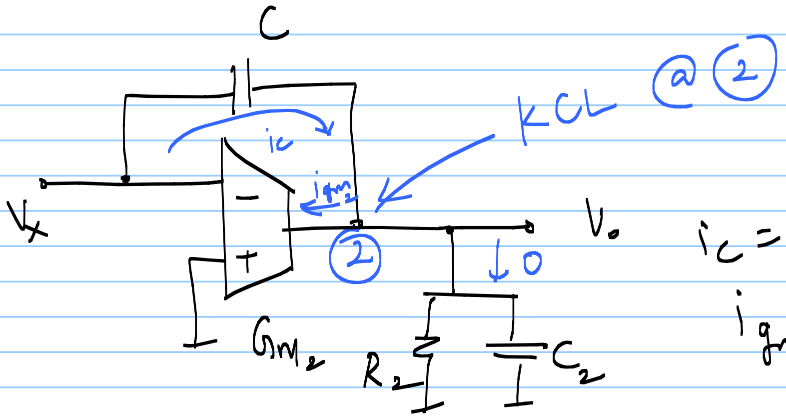
$$i_t = \frac{G_{m_2} \cdot C}{C+C_1} \cdot v_t$$

$$i_t = G_{m_2} \cdot v_t$$

$$r_{out} = \frac{1}{G_{m_2}}$$

$$r'_{out} \approx \frac{v_t}{i_t} = \underline{\left(\frac{C+C_1}{C}\right) \cdot \frac{1}{G_{m_2}}}$$

Zero : $z_1 = +\frac{G_{m2}}{C}$



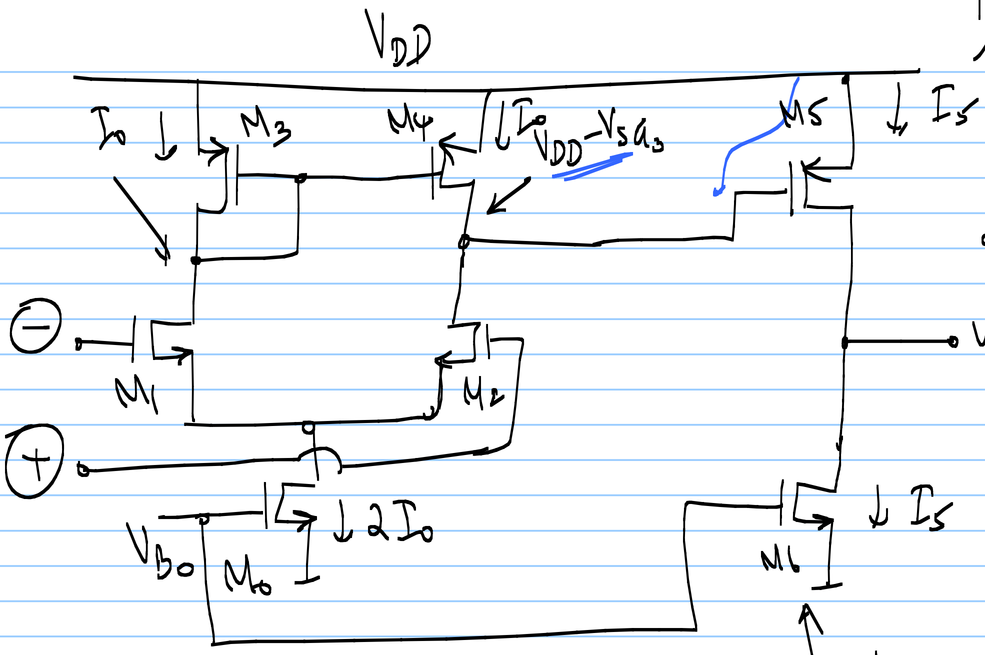
$i_c = sC(V_x - 0)$

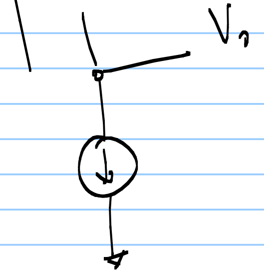
$i_{gm_2} = G_{m_2} V_x$

$i_c = i_{gm_2}$ @ z_1 : $z_1 = \frac{G_{m2}}{C}$

$\omega_u = \frac{G_{m_1}}{C} = \omega_d \cdot A_0$

Normally $z_1, p_2 > \omega_u \Rightarrow$ 1) $G_{m_2} >> G_{m_1}$





1) choose $\left(\frac{W}{L}\right)_6$ so that $I_{D6} = I_5$ desired

$(W/L)_6 : (W/L)_0 = I_5 : 2I_0$

2) $V_{DD} - V_{SG_3} = V_{DD} - V_{SG_5}$

$V_{SG_3} = V_{SG5}$

$V_{Dsat_6} = V_{Dsat_5}$

$$\sqrt{\frac{2I_0}{\beta(W/L)_3}} = \sqrt{\frac{2I_5}{\beta(W/L)_5}}$$

$$\boxed{\frac{I_0}{(W/L)_3} = \frac{I_5}{(W/L)_5}}$$

$Gm_1 = g_{m_1}$

$Gm_2 = g_{m_5}$

$g_{m5} \gg g_m$

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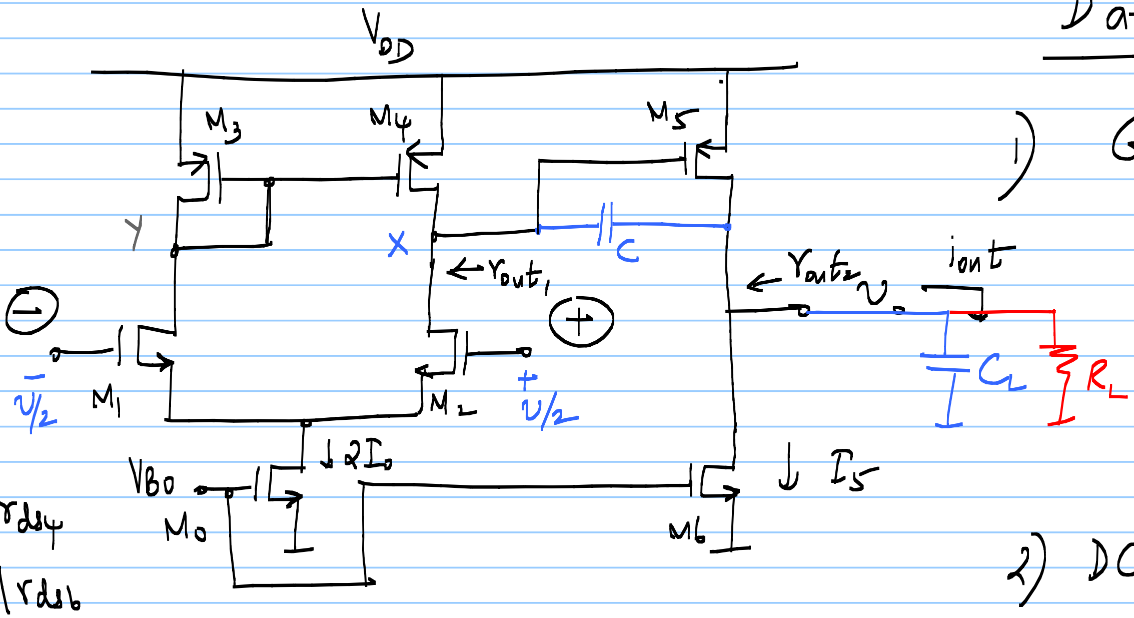
Lec 15



$r_{out_1} = r_{ds_2} || r_{ds_4}$

$r_{out_2} = r_{ds_5} || r_{ds_6}$

Datasheet of 2-stage opamp

1) $G_m = ?$

$v_x = -g_{m_1}(r_{out_1}) \cdot v$

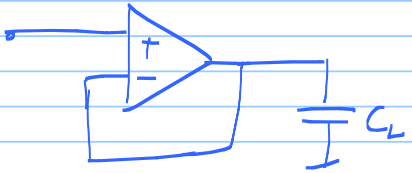
$i_{out} = + g_{m_1} r_{out_1} g_{m_5} \cdot v$

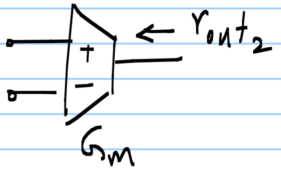
$G_m = g_{m_1} r_{out_1} g_{m_5}$

2) DC gain $A_0 = (g_{m_1} r_{out_1})(g_{m_5} r_{out_2})$

If R_L exists,

$A_0 = \underline{(g_{m_1} r_{out_1})}(g_{m_5}(r_{out_2} || R_L))$





3) $\omega_u = \frac{G_{m_1}}{C} = \frac{g_{m_1}}{C}$; $\omega_d = \frac{1}{R_1 \cdot (G_{m_2} R_2 \cdot C)} = \frac{1}{r_{out_1} \, (g_{m_5} r_{out_2} \cdot C)}$

4) ND poles & zeroes :

p_2 @ output node $\qquad$ p_3 @ node Y

z_1 @ $\frac{G_{m_2}}{C} = \frac{g_{m5}}{C}$ (RHP) $\qquad$ z_2 due to $+v/2$ through X path
$-v/2$ through Y-X path (LHP)

2 poles & 2 zeroes that are ND.

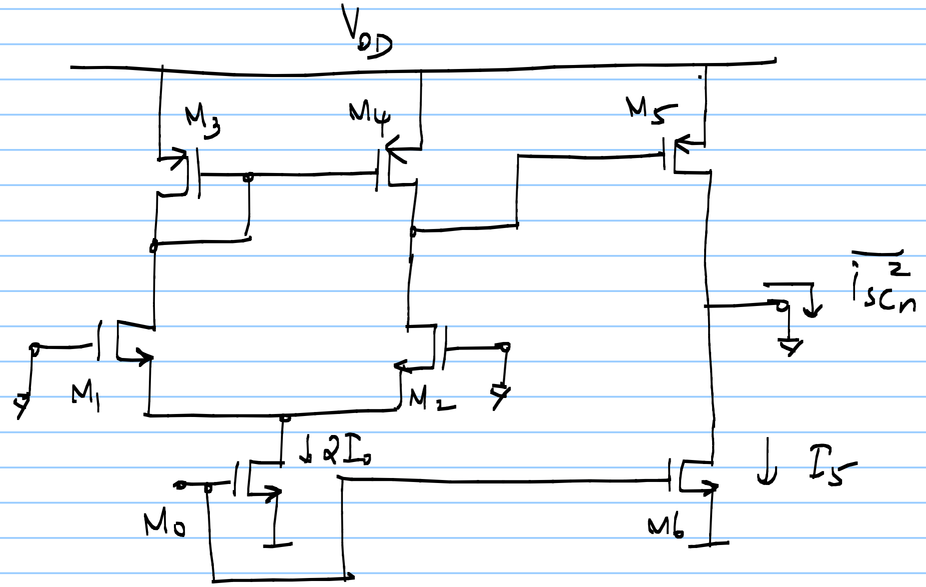
5) ICMR

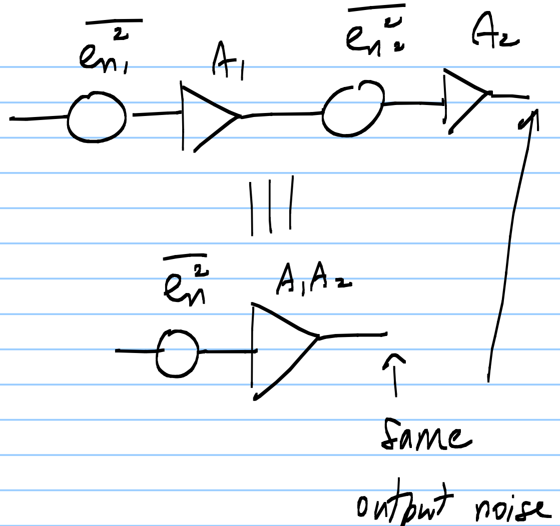
$$V_{CM,in} = \left\{ V_{GS_1}\Big|_{I_0} + V_{Dsat_0}\Big|_{2I_0} \,,\quad V_{DD} - V_{SG_3}\Big|_{I_0} + V_{T_1} \right\}$$

6) OCMR

$$V_{CM,out} = \left\{ V_{B_0} - V_{T_6} \,,\quad V_{DD} - V_{Dsat_5}\Big|_{I_5} \right\}$$

7) $\overline{e_n^2} =$





$$\overline{e_n^2}\cdot(A_1A_2)^2 = \overline{e_{n_1}^2}\cdot(A_1A_2)^2 + \overline{e_{n_2}^2}\cdot(A_2)^2$$

$$\frac{\overline{e_{n_1}^2}}{\Delta f} = \frac{16kT}{3}\left[\frac{1}{g_{m_1}} + \frac{g_{m_3}}{g_{m_1}^2}\right] \quad ; \quad A_1 = g_{m_1} r_{out_1}$$

$$\overline{e_n^2} = \overline{e_{n_1}^2} + \frac{1}{A_1^2}\cdot\overline{e_{n_2}^2}$$

$$\overline{e_{n_2}^2} = \frac{8kT}{3}\left[\frac{1}{g_{m5}} + \frac{g_{m6}}{g_{m5}^2}\right] \quad ; \quad A_2 = g_{m5}\, r_{out_2}$$

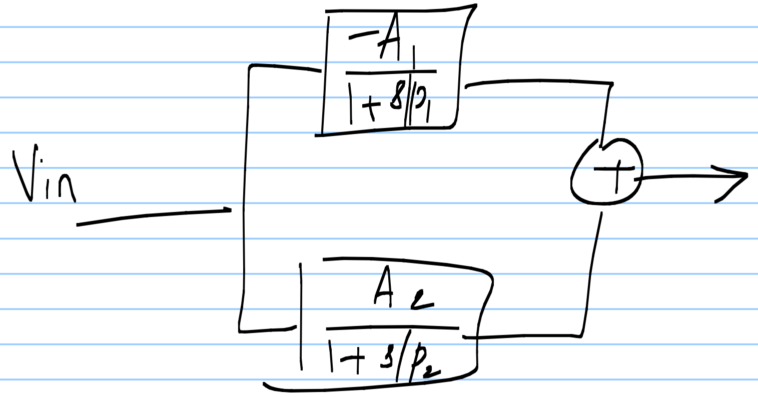
$$\overline{e_n^2} = \overline{e_{n_1}^2} + \frac{1}{A_1^2}\cdot \overline{e_{n_2}^2}$$

$$\overline{e_n^2} = \frac{16kT}{3}\left[\frac{1}{g_{m1}} + \frac{g_{m3}}{g_{m1}^2}\right] + \underbrace{\left(\frac{1}{g_{m1}\, r_{out_1}}\right)^2}_{} \cdot \underbrace{\frac{8kT}{3}\left[\frac{1}{g_{m5}} + \frac{g_{m6}}{g_{m5}^2}\right]}_{}$$

$g_{m5} \gg g_{m1}$ (to keep $Z_1 \gg \omega_u$)

↳ $\overline{e_{n_2}^2} \ll \overline{e_{n_1}^2}$

$\overline{e_n^2} \approx \overline{e_{n_1}^2}$



$$V_o = \frac{A_1 + A_2}{1 + s/p_1}$$

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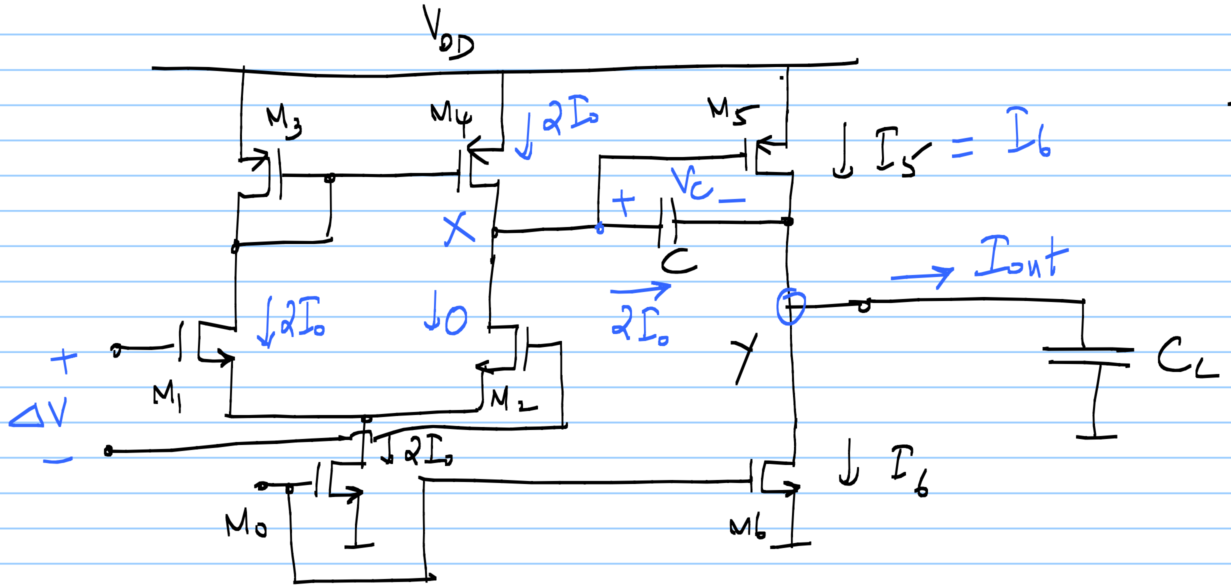
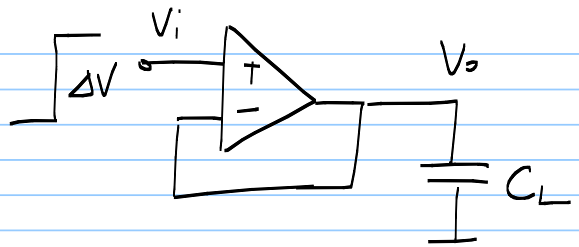
Lec 16

8) Offset voltage

$$\sigma_{os}^2 = \sigma_{V_{T_{1-2}}}^2 + \frac{g_{m_3}^2}{g_{m_1}^2} \sigma_{V_{T_{3-4}}}^2$$

9) Slew rate





$$\left.\frac{dV_o}{dt}\right|_{t=0^+} = ?$$

$$\frac{dV_c}{dt} = \frac{2I_o}{C}$$

$$\text{KCL @ } Y : \underset{\checkmark}{2I_o} + \underset{?}{I_5} = I_{out} + \underset{\checkmark}{I_6} \quad @ \; t = 0^+$$

$$V_{GS5} = V_X \quad ; \quad V_X(0^+) = V_X(0^-)$$

$$I_5(0^+) = I_5(0^-) = I_6 \Rightarrow I_{out} = 2I_o$$

$$\left.\frac{dV_o}{dt}\right|_{t=0^+} = \frac{2I_o}{C_L}$$

$$SR = \pm \frac{2I_o}{C_L}$$

$G_{m2} \gg G_{m1}$

$(z_1 \gg \omega_u)$

$g_{m5} \gg g_{m1}$

normally $I_{5,6} \gg 2I_o$

Design Example

Specs : $A_o \geq 80\,dB$; $f_u \geq 5\,MHz$; $V_{o_{pp}} \geq 4V$

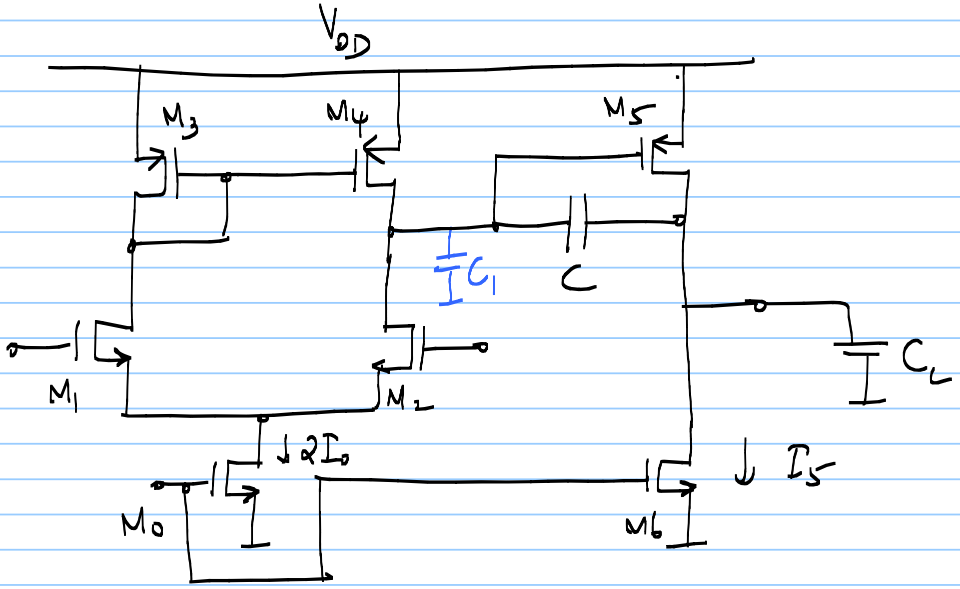
$SR \geq 10\,V/\mu s$; $PM \geq 60°$ (unity gain)

Design parameters : $V_{DD} = 5V$, $C_L = 10pF$, $\mu_n C_{ox} = 50\mu A/V^2$

$\mu_p C_{ox} = 25\mu A/V^2$, $V_{Tn} = V_{Tp} = 1V$,

$L_{min} = 2\mu m$, $C_{ox} = 1.5\,fF/\mu m^2$

$(\lambda L)_n = 0.04\,\mu m/V$; $(\lambda L)_p = 0.1\,\mu m/V$



$$SR = \frac{2I_o}{C_L} \Big/ \frac{2I_o}{C}$$

$$10V/\mu s = \frac{2I_o}{10pF}$$

$$2I_o = 100\mu A$$ ← overdesigned

$$I_o = 50\mu A$$

PM = 60°

$Z_1 >> \omega_u \Rightarrow g_{m5} >> g_{m1}$ e.g. $g_{m5} = 10\, g_{m1}$

Assume $C_1 << C \Rightarrow p_2 = \frac{g_{m5}}{C_L} = \sqrt{3}\,\omega_u = \sqrt{3} \times 2\pi \times 5e^6$

$g_{m5} = \sqrt{3} \times 2\pi \times 5\times10^6 \times 10\times10^{-12} = 543.8\,\mu S$

$g_{m1} = 54.4\,\mu S$

$g_{m1} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_0} \Rightarrow \left(\frac{W}{L}\right)_1 = 0.6$

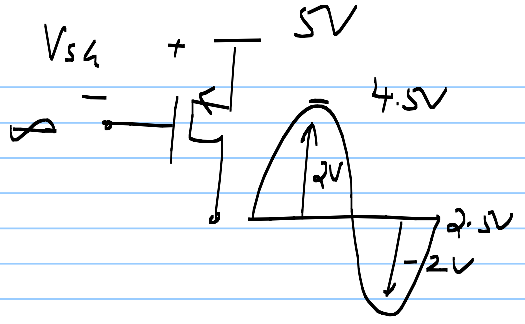
$\omega_u = \frac{g_{m1}}{C} \Rightarrow C = 1.73pF \longrightarrow$

$SR = \frac{2I_0}{C} = \frac{100\mu A}{1.73pF}$

$= 57.8\ V/\mu s$

$V_{o_{pp}} = 4V \Rightarrow V_{o_{max}} = 4.5V\ ;\ V_{o_{min}} = 0.5V$

$V_{Dsat_5} = 0.5V \qquad V_{Dsat_6} = 0.5$



$V_{SG_5} = 1.5V$

$$\left.\begin{aligned} g_{m_5} &= 543.8 \mu S \\ V_{Dsat_5} &= 0.5V \end{aligned}\right\} \quad \begin{aligned} I_5 &= 135 \mu A \\ \left(\frac{W}{L}\right)_5 &= 43.2 \end{aligned}$$

$I_6 = I_5$; $V_{Dsat_6} = 0.5V$

$$\Rightarrow \left(\frac{W}{L}\right)_6 = \frac{1}{2}\left(\frac{W}{L}\right)_5 = 21.6$$

$$\Rightarrow \left(\frac{W}{L}\right)_0 = \frac{2I_0}{I_5} \times \left(\frac{W}{L}\right)_6 = \frac{100\mu A}{135\mu A} \cdot 21.6 = 16$$

$$V_{SG_{3,4}}\Big|_{I_0} = V_{SG_5}\Big|_{I_5} \Rightarrow \frac{I_0}{(W/L)_3} = \frac{I_5}{(W/L)_5}$$

$$\Rightarrow \left(\frac{W}{L}\right)_{3,4} = \frac{50\mu A}{135\mu A} \cdot 43.2 = 16$$

$$A_0 = 80\,dB \Rightarrow A_1 \cdot A_2 = 10^4$$

$$g_{m_1}\underbrace{(r_{ds_2} || r_{ds_4})}_{r_{out_1}} \cdot g_{m_5}\underbrace{(r_{ds_5} || r_{ds_6})}_{r_{out_2}} = 10^4$$

If $L = L_{min}$, what are r_{ds} values?

$L_5 = L_{3,4}$

$L_{1,2} = L_{3,4} = L_5 = L_6 = 2\mu m$

$r_{ds} = \frac{1}{\lambda I}$

$\lambda L = $ const. ; $L = L_{min.}$

$r_{ds_{1,2}} = 1 M\Omega$

$r_{ds_{3,4}} = 0.4 M\Omega$

$r_{ds_5} = 0.148 M\Omega$

$r_{ds_6} = 0.37 M\Omega$

$\lambda_n = 0.02 /V$

$\lambda_p = 0.05 /V$

$r_{out_1} = 0.29 M\Omega$; $r_{out_2} = 0.11 M\Omega$

$A_1 = 54.4 \mu S \times 0.29 M\Omega = 15.8$

$A_2 = 543.8 \mu S \times 0.11 M\Omega = 59.8$

$A_o = A_1 A_2 = 944.8$ V/V

Set $r_{d_{3,4}} = 1M\Omega \Rightarrow L_{3,4} = 5\mu m = L_5$

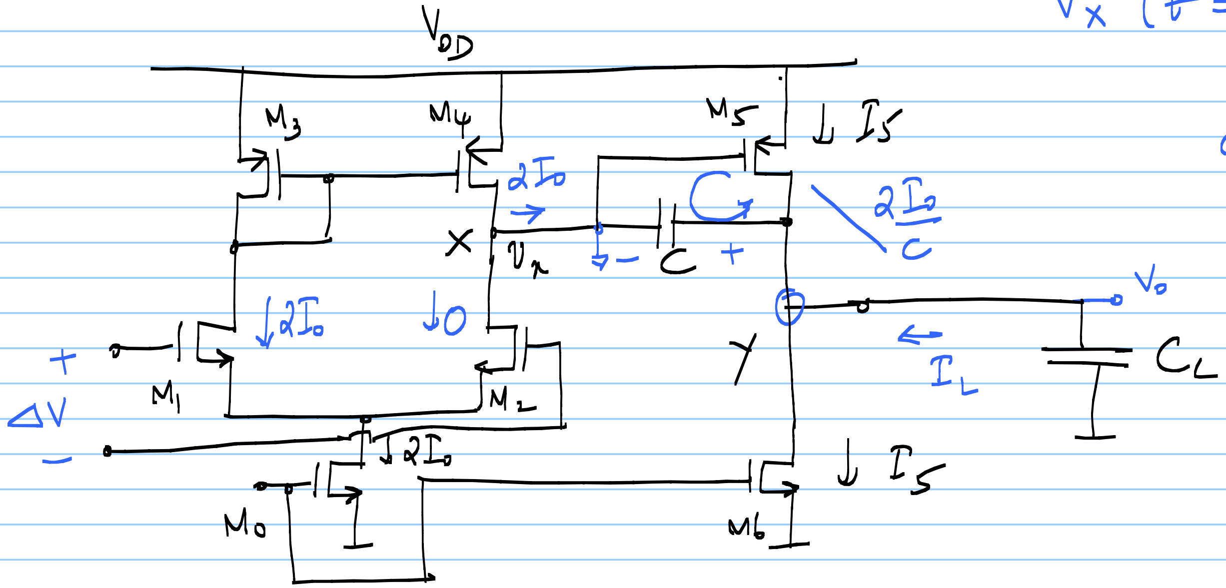
$r_{ds_5} = 0.37M\Omega$

$A_1 = g_{m_1} \cdot (0.5M\Omega) = 27.2$

$A_2 = g_{m_5} \cdot (0.185 M\Omega) = 100.6$

14/2/2020

Lec 17



$$V_x(t=0^+) = V_x(t=0^-)$$

$$\frac{dV_o}{dt}\Big|_{t=0^+} = \frac{2I_o}{C_L}$$

$V_x \approx 0$ after a short period of time due to fast neg. f.b. loop around M_5

$$-\frac{I_L}{C_L} = \frac{dV_o}{dt} \Rightarrow I_L = \frac{2I_o}{C}\cdot C_L \Rightarrow \frac{dV_o}{dt} = -\frac{2I_o}{C}$$

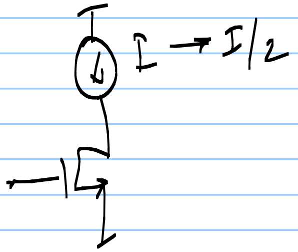
KCL @ Y $\Rightarrow$ $I_{D5} = I_5 - 2I_0 - I_L$

$$= I_5 - 2I_0\left[1 + \frac{C_L}{C}\right]$$

$SR = \frac{2I_0}{C} = 57.8\ V/\mu s$

We wanted $10\ V/\mu s$

$2I_0$ can be reduced from $100\mu A$ to $\frac{1}{5.78} \times 100\mu A = 17.3\mu A$



$r_{ds} = \frac{1}{\lambda I}$

gain = $g_m r_{ds}$

$\hookrightarrow = g_m' r_{ds}' = \frac{g_m}{\sqrt{2}} \cdot 2r_{ds} = \sqrt{2}\, g_m r_{ds}$

With all $L = L_{min}$, $A_1 = 15.8$, $A_2 = 59.8$

If $2I_0$ falls to $17.3\mu A$, $A_1' = \sqrt{5.78}\, A_1$
$= 38$ V/V

new $A_0 = A_1 A_2 = 2272.4$

$f_u' = \frac{g_{m1}'}{2\pi C} = \frac{f_u}{\sqrt{5.78}} = 2.1$ MHz $\rightarrow$ set $g_{m1}' = g_m$ by $\uparrow \left(\frac{W}{L}\right)_1$ by $\sqrt{5.78}$ $\boxed{g_{m1} \approx 59.8\mu S}$

$\Rightarrow$ set $\left(\frac{W}{L}\right)_1 = 1.44$

$\Rightarrow$ $A_1 = 5.78 \times 15.8 = 91.3$ V/V ; $A_2 = 59.8$ V/V

$A_o = A_1 A_2 = 5459.7$ V/V

$r_{ds_{1,2}} = 5.78 \times 1 M\Omega = 5.78 M\Omega$

$r_{ds_5} = 0.148 M\Omega$

$(\lambda L)_n = 0.04$

$\lambda_n = 0.02/V$

$r_{ds_{3,4}} = 5.78 \times 0.4 M\Omega$
$= 2.3 M\Omega$

$r_{ds_6} = 0.37 M\Omega$

$L_{3,4} = L_5 = 5 \mu m \Rightarrow$

$r_{ds_{3,4}} = 5.78 M\Omega$

$\{\lambda_p = 0.02\}$

$r_{ds_5} = 0.37 M\Omega$

$\left.\begin{matrix} \lambda_n = 0.02/V \\ \lambda_p = 0.05/V \end{matrix}\right\}$ for $L_{min} = 2 \mu m$

$r_{out_1}' = 2.89 M\Omega$

$r_{out_2}' = 0.185 M\Omega$

$A_1' = g_{m_1}' r_{out_1}' = 172.8$ V/V

$A_2' = g_{m_2} r_{out_2}' = 110.63$ V/V

$A_o' = A_1' A_2' = 19116.9$ V/V ✓

* Check $C_{gs_5} << g_{m_5} \cdot r_{out_2} \cdot C$

* Check Mirror pole $(M_{3,4})$ $>>$ ω_u

$$\hookrightarrow \simeq \frac{g_{m_{3,4}}}{2C_{gs_{3,4}}}$$

18/2/20

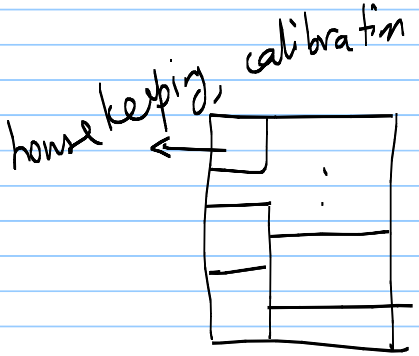
Lec 18

$b_{n-1} b_{n-2} \cdots b_0 = 10 \cdots 0$

$R_{nm} = 1k\Omega$

Process variations - how to tackle ?

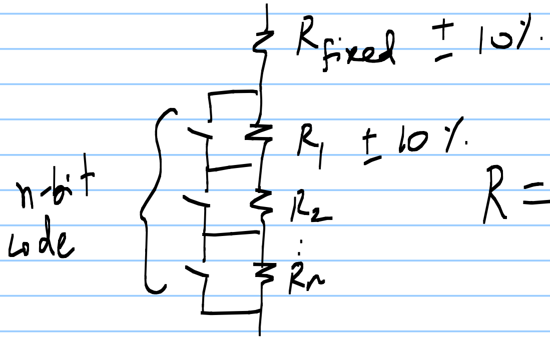
$R, \; C, \; RC, \; g_m, \; \ldots$



$R = 1k\Omega \pm 10\% \Rightarrow \underbrace{900\Omega \text{ to } 1.1k\Omega}$

$\Downarrow$

$\{ 990 - 1010 \, \Omega \}$



$R = R_{fixed} + R_{var.}$

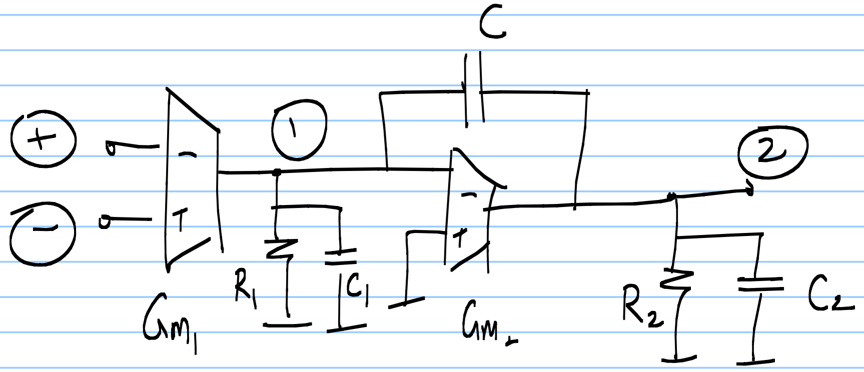
$\Downarrow$

$f(b_{n-1} b_{n-2} \cdots b_0)$

R- tuner, RC-tuner

C - tuner,

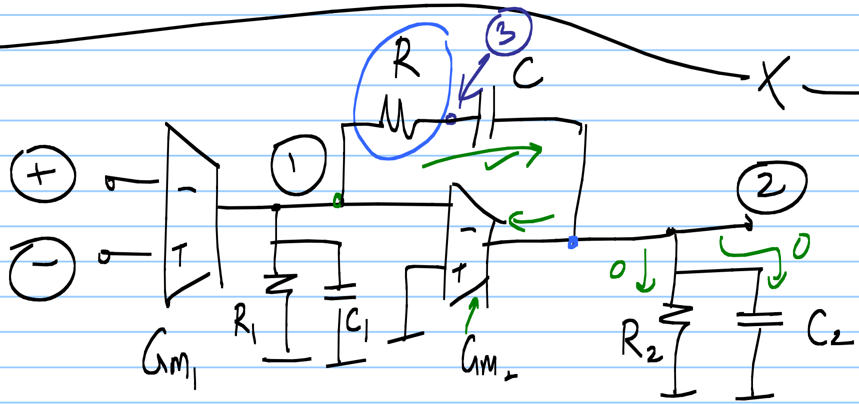
Pole zero cancellation compensation



$$p_1 \approx \frac{-1}{R_1 \cdot (Gm_2 R_2) C}$$

$$p_2 \approx \frac{-1}{\left[R_2 \| \left(\frac{C}{C+C_1}\right) \cdot \frac{1}{Gm_2}\right] \cdot \left[C_2 + \frac{CC_1}{C+C_1}\right]}$$

$$z_1 = \frac{+Gm_2}{C} \rightarrow$$

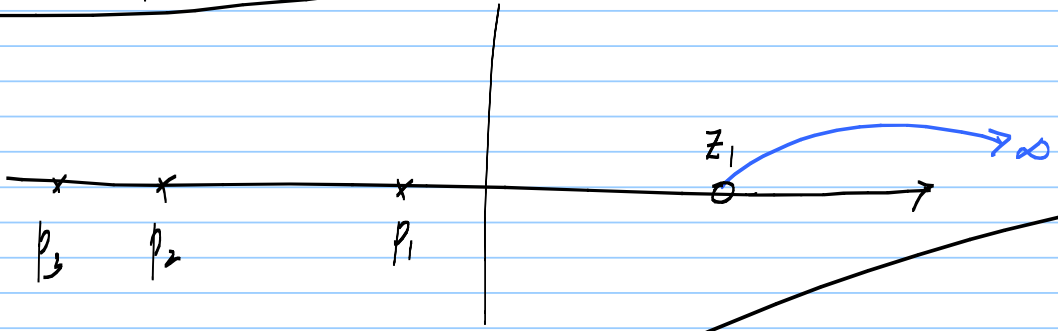


$$p_1 \approx \frac{-1}{R_1 (Gm_2 R_2) C}$$

$$p_2 \approx \frac{-Gm_2}{C_1 C_2 + CC_1 + CC_2}$$

$$Z_1 = \frac{+1}{C\left(\frac{1}{Gm_2} - R\right)} \quad ; \quad p_3 \approx \frac{-1}{R \cdot C_1}$$

1) <u>Push Z_1 to ∞</u>



set $R = \frac{1}{Gm_2} = \frac{1}{g_{m5}}$

$\Rightarrow Z_1 = \infty$

p_1 = stays the same

p_2 = ,,

$$p_3 = \frac{-1}{RC_1} = -\frac{Gm_2}{C_1}$$

2) What happens if $R > \frac{1}{G_{m_2}}$ ⇒ zero moves into LHP

⇒ option of cancelling p_2 or p_3

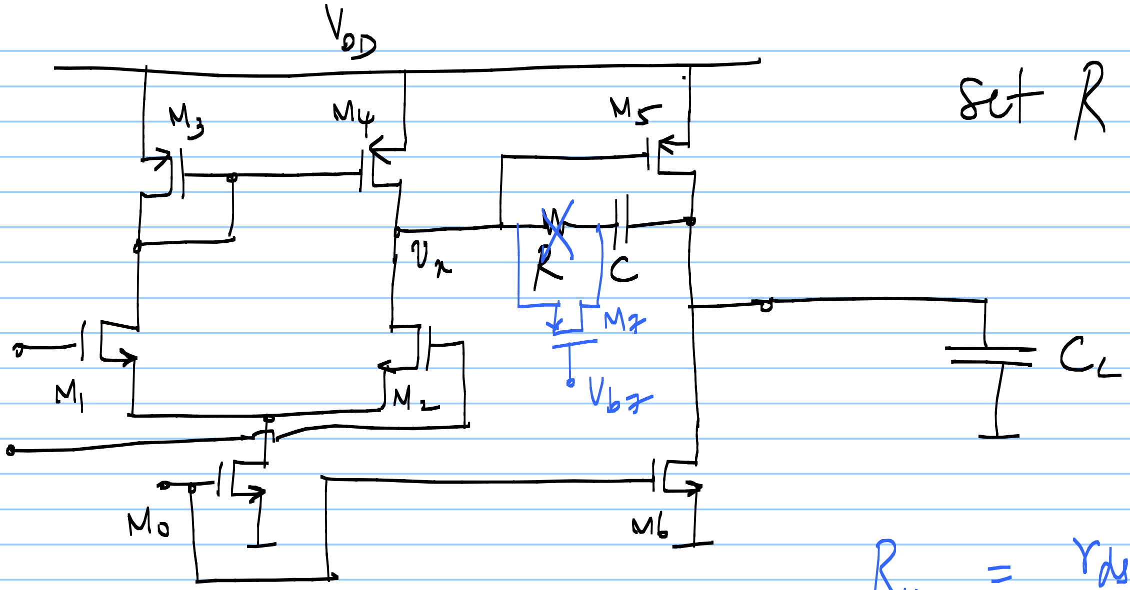
↑ first ND pole

If you cancel p_2, PM improves or BW improves

$$Z_1 = p_2 \Rightarrow \frac{1}{C\left(\frac{1}{G_{m_2}} - R\right)} = \frac{-G_{m2}}{\Sigma C_i C_j}$$ ⇒ gives value of R

$$R = f(G_{m_2})$$

1)



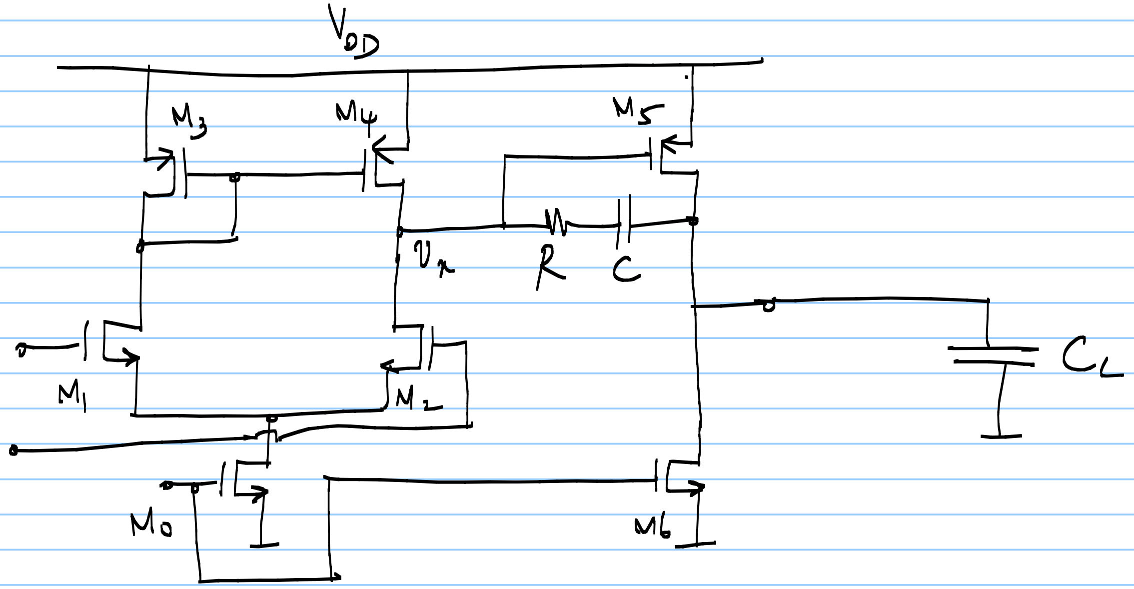
Set R to depend on G_{m_2} or g_{m_5}

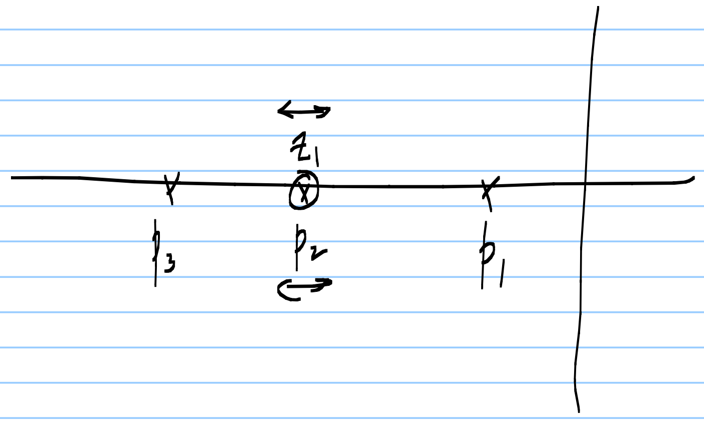
M_7 is in triode region

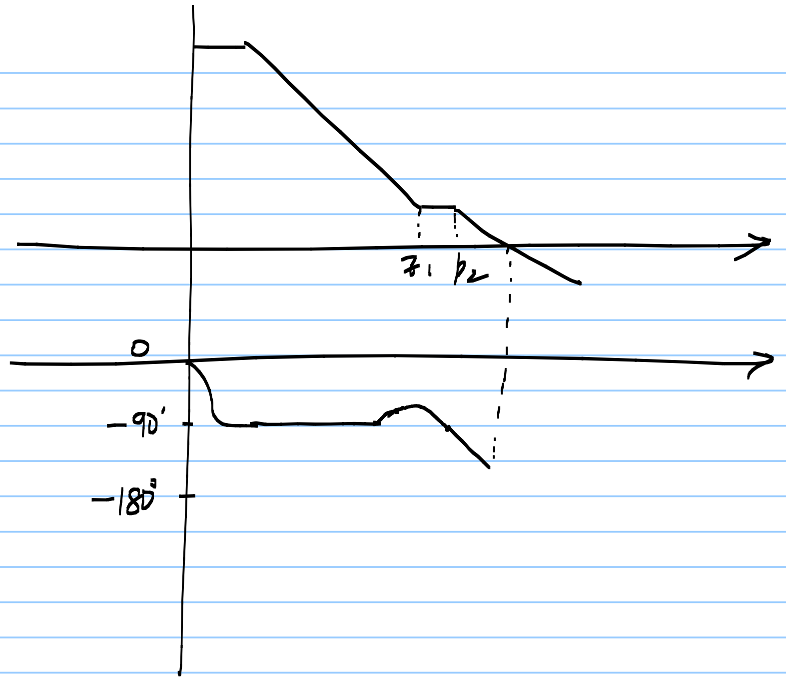
$$R_{M7} = r_{ds7} = \frac{1}{\mu_p C_{ox} \left(\frac{W}{L}\right)_7 \left(V_{SG7} - V_{T7} - V_{SD7}\right)}$$

$$g_{m5} = \mu_p C_{ox} \left(\frac{W}{L}\right)_5 \left(V_{SG5} - V_{T5}\right)$$

2)




z_1
p_3
p_2
p_1


z_1
p_2
0
-90°
-180°

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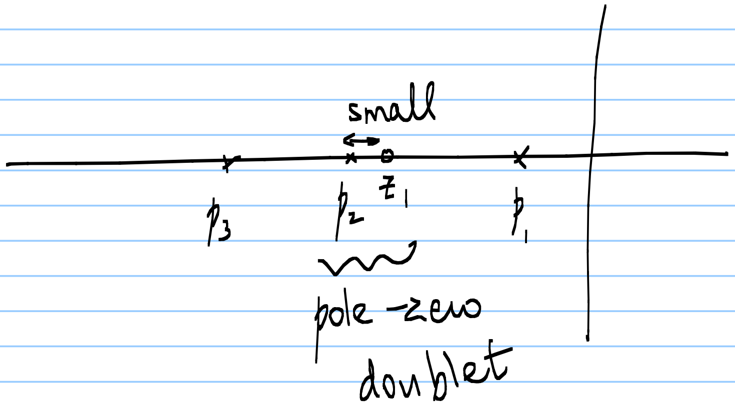
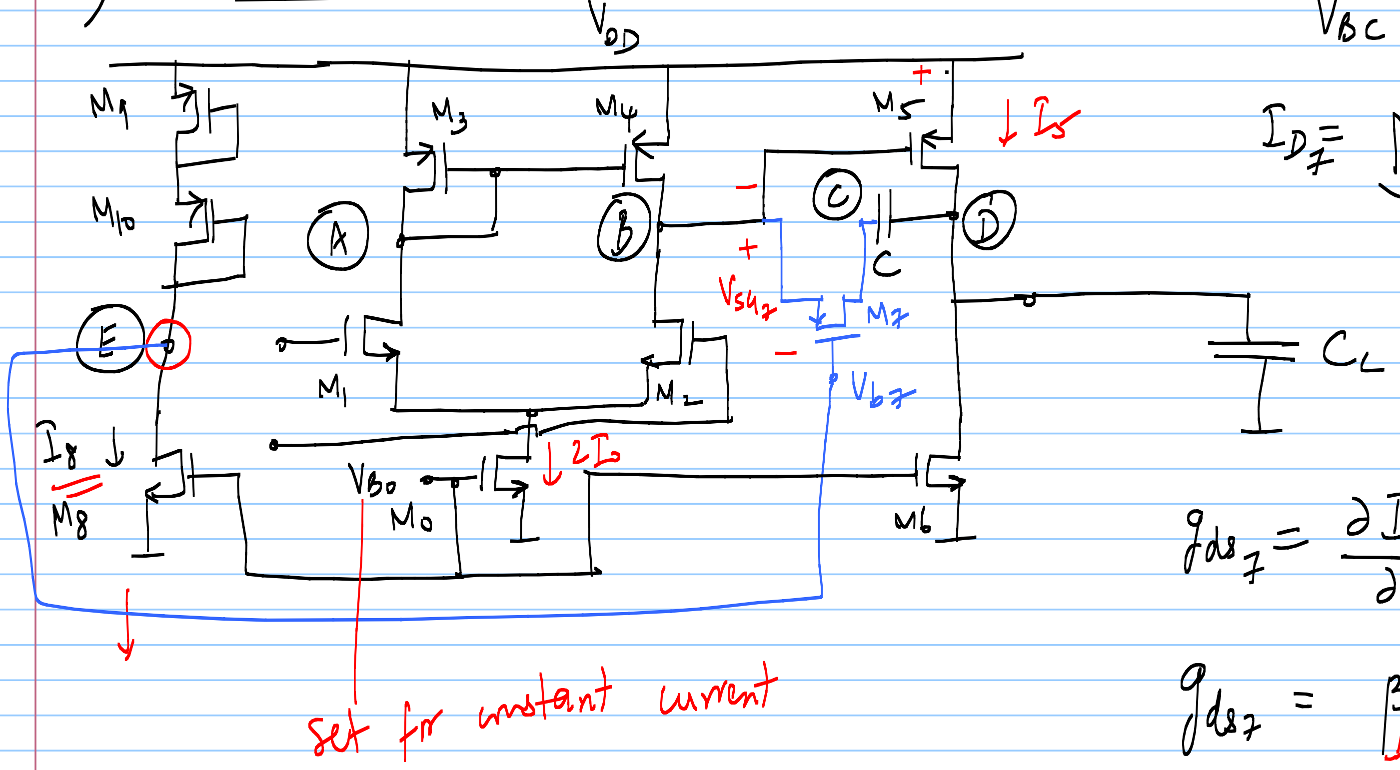


causes very slow-settling component in step response

1) Create R from MOSFET :



$$V_{BC} = 0 = V_{DS_7}$$

$$I_{D_7} = \underbrace{\mu_p C_{ox}}_{\beta_p} \left(\frac{W}{L}\right)_7 \left[(V_{SG_7} - V_{T_7})(V_{DS_7}) - \frac{V_{DS_7}^2}{2} \right]$$

$$g_{ds_7} = \frac{\partial I_{D_7}}{\partial V_{DS_7}} = \beta_p \left(\frac{W}{L}\right)_7 \left(V_{SG_7} - V_{T_7} - \cancelto{0}{V_{DS_7}}\right)$$

$$g_{ds_7} = \beta_p{\downarrow} \left(\frac{W}{L}\right)_7 \left(V_{SG_7}{\nearrow} - V_{T_7}\right)$$

Choose $g_{ds_7} = 1/R$ nominally

$$V_{b7} = V_{DD} - V_{SG5} - V_{SG7}$$

$$V_E = V_{DD} - V_{SG9} - V_{SG10}$$

set $V_{B7} = V_E$

e.g. $V_{SG9} = V_{SG5}$ i.e. $\dfrac{I_5}{(W/L)_5} = \dfrac{I_8}{(W/L)_9}$

$$\boxed{V_{SG7} = V_{SG10}}$$

choose M_{10} so that its $g_{m_{10}} = g_{ds_7}$

if $\left(\dfrac{W}{L}\right)_7 = \left(\dfrac{W}{L}\right)_{10}$

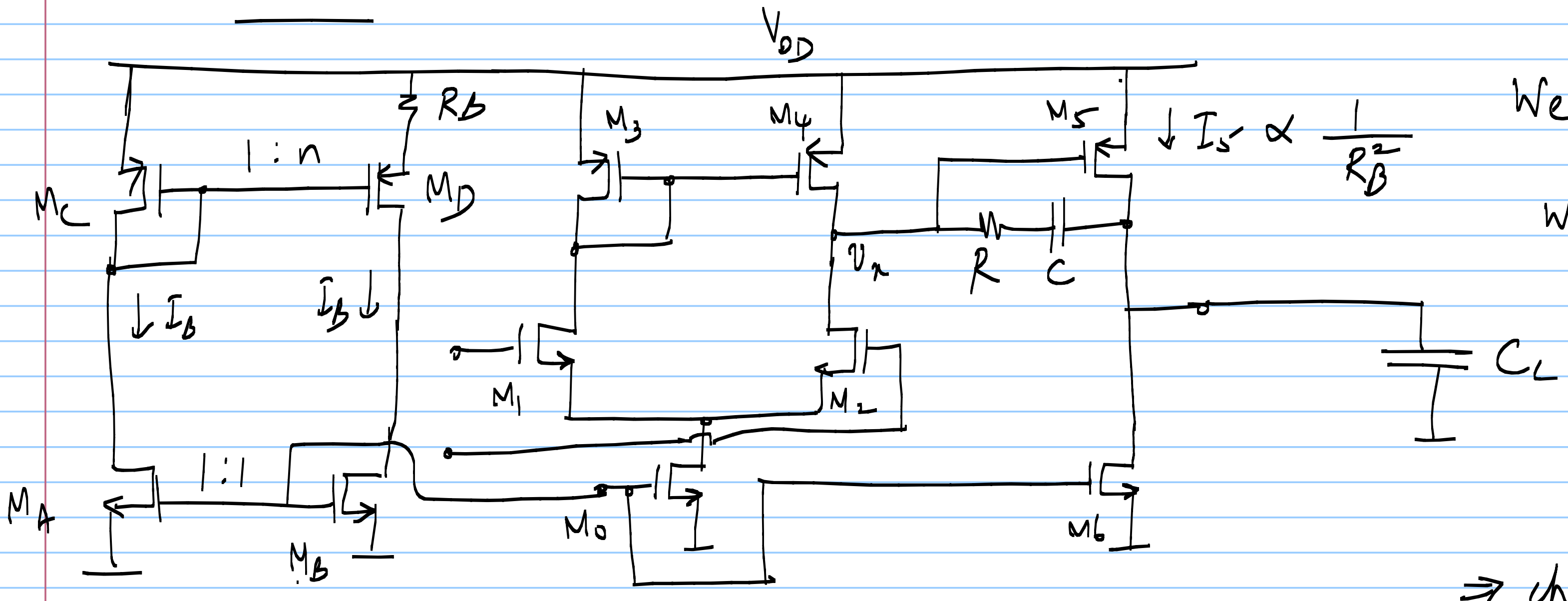
as Temp ↑ $\underline{\underline{I_D}} = \frac{1}{2} \mu_p{\downarrow} C_{ox} \left(\frac{W}{L}\right) (V_{SG}{\uparrow} - V_T)^2$

$\Rightarrow V_E = (V_{DD} - V_{SG9} - V_{SG10})$ has to decrease

V_{SG5} also increase due to ↓ in μ_p

V_{SG7} ↑ ⇒ g_{ds7} is constant ⇒ R is constant

Case 2 :



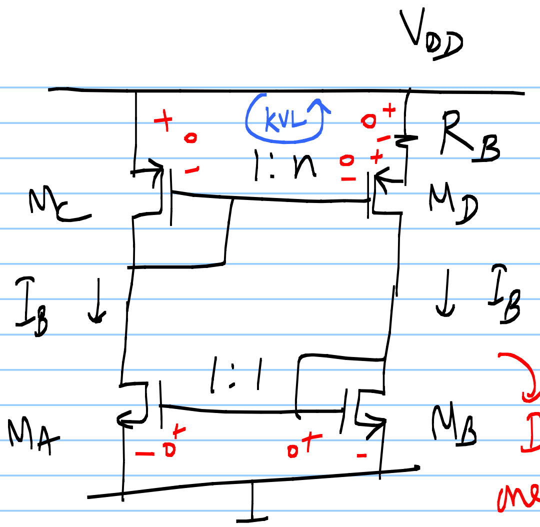
↓ $I_5 \propto \frac{1}{R_B^2}$

We want $R \propto \frac{1}{g_{m5}}$

We can also use a passive R, and force g_{m5} to track $\frac{1}{R}$

⇒ change I_5

Constant g_m bias circuit



$I_B = 0$ is one valid solution

KVL around M_C M_D loop:

$$V_{SG_C} = V_{SG_D} + I_B \cdot R_B$$

$$V_{T_C} + V_{ov_C} = V_{T_D} + V_{ov_D} + I_B \cdot R_B$$

$$V_{ov_C} = \sqrt{n} \cdot V_{ov_D}$$

$$V_{ov_C} = \frac{V_{ov_C}}{\sqrt{n}} + I_B R_B$$

$$g_m = \frac{2 I_{bias}}{V_{SG} - V_T} = \frac{2 I_{bias}}{V_{ov}} \Rightarrow \frac{2 I_B}{g_{m_C}} = \frac{1}{\sqrt{n}} \frac{2 I_B}{g_{m_C}} + I_B R_B$$

If $I_B \neq 0$; $\frac{2}{g_{m_C}} = \frac{1}{\sqrt{n}} \frac{2}{g_{m_C}} + R_B$ { Need a startup circuit }

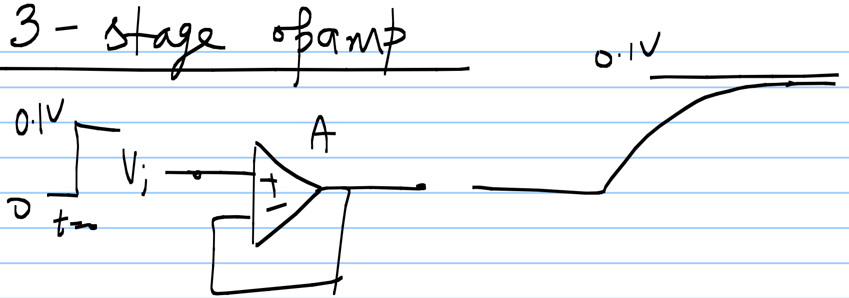
$$g_{m_C} = \frac{2\left(1 - 1/\sqrt{n}\right)}{R_B}$$ ← set g_{m5} based on this

$$\sqrt{2\beta_p \left(\frac{W}{L}\right)_C I_B} = \frac{2\left(1 - 1/\sqrt{n}\right)}{R_B}$$

$$I_B \propto \frac{1}{\beta_p R_B^2}$$

choose R_B & R to be same type of resistor!

3-stage opamp



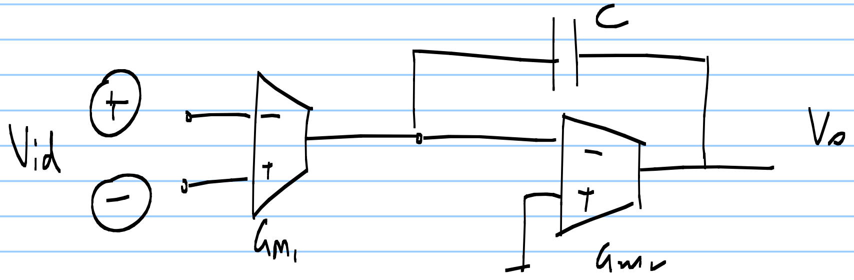
$\frac{A}{1+A} \cdot V_i$

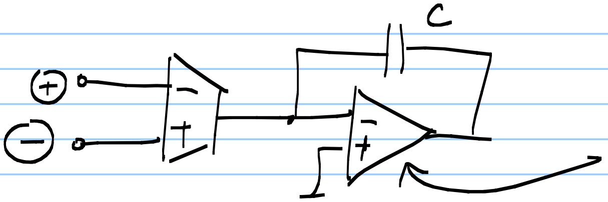
Ideal Opamp → $R_{in} = \infty$, $R_o = 0$, $A = \infty$ VCVS

Ideal OTA → $R_{in} = \infty$, $R_o = \infty$, $G_m = \infty$ VCCS

* reduce V_e even further

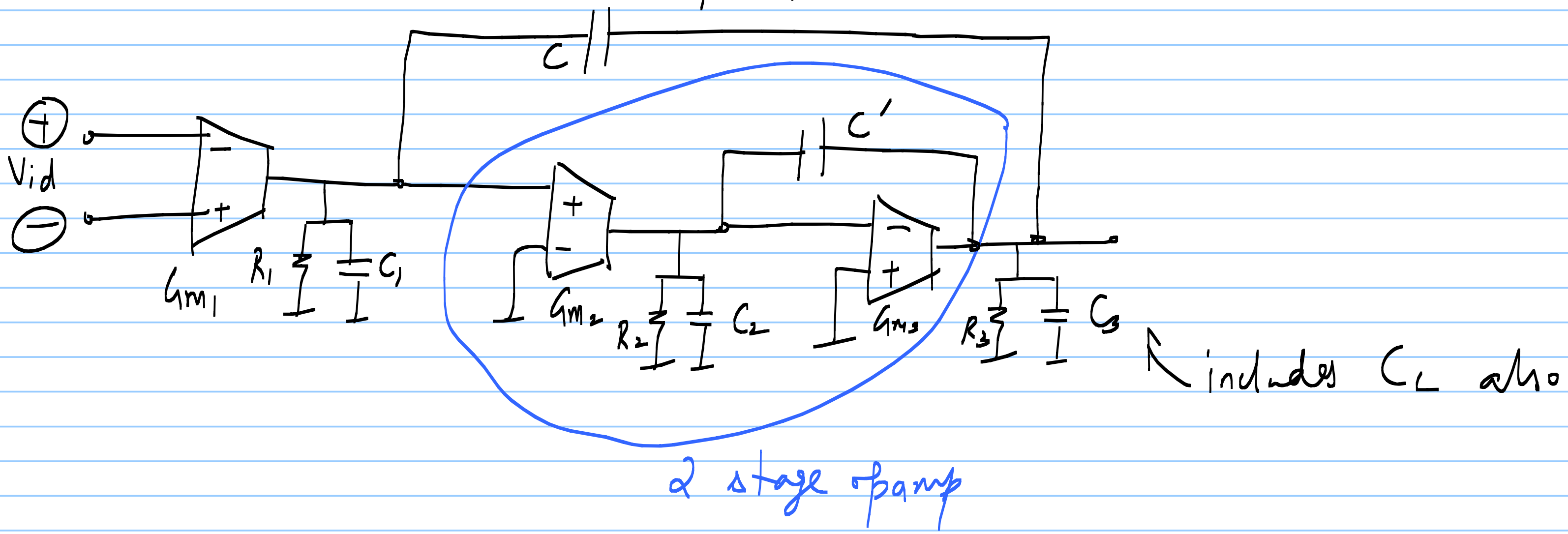
2stage opamp





Replace G_{m_2} by 2-stage opamp

Nested Miller 3-stage opamp



includes C_L also

DC gain $A_0 = (G_{m_1} R_1)(G_{m_2} R_2)(G_{m_3} R_3)$

$\omega_u = \frac{G_{m_1}}{C}$; $\omega_d = p_1 = \frac{\omega_u}{A_0}$

2 ND poles : p_2, p_3 ; at least 2 zeroes

Conditions for a 'good' design:

1) $$\underbrace{\frac{G_{m2}}{C'}}_{\text{VGF of inner opamp}} \gg \omega_u$$

$\Rightarrow$ Speed of 3-stage opamp is typically lower than that of 2-stage opamp.

2) $$p_3 \gg \underbrace{\frac{G_{m2}}{C'}}_{\omega_u'}$$

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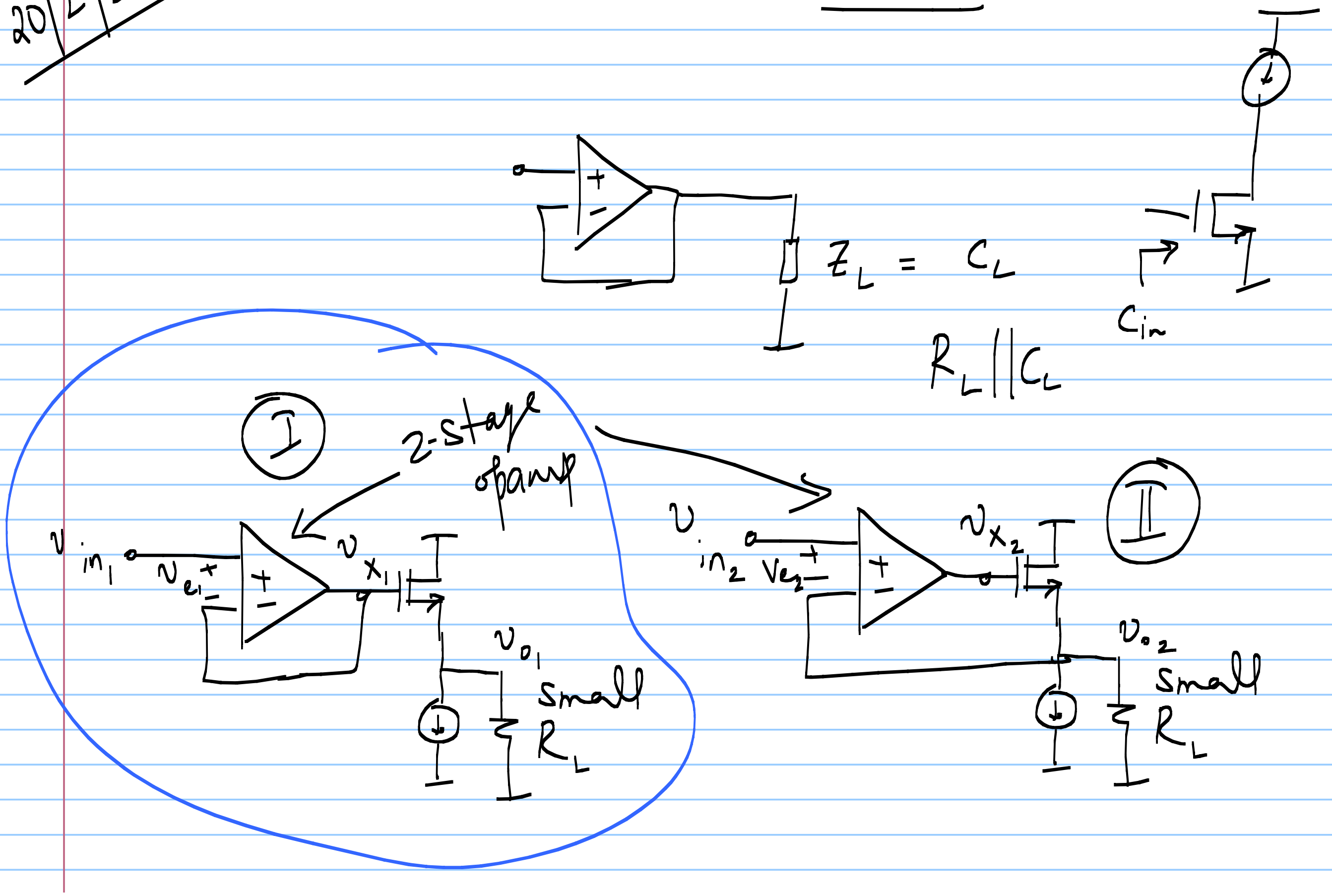
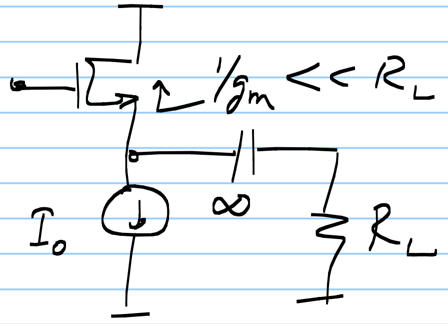


drive small R_L

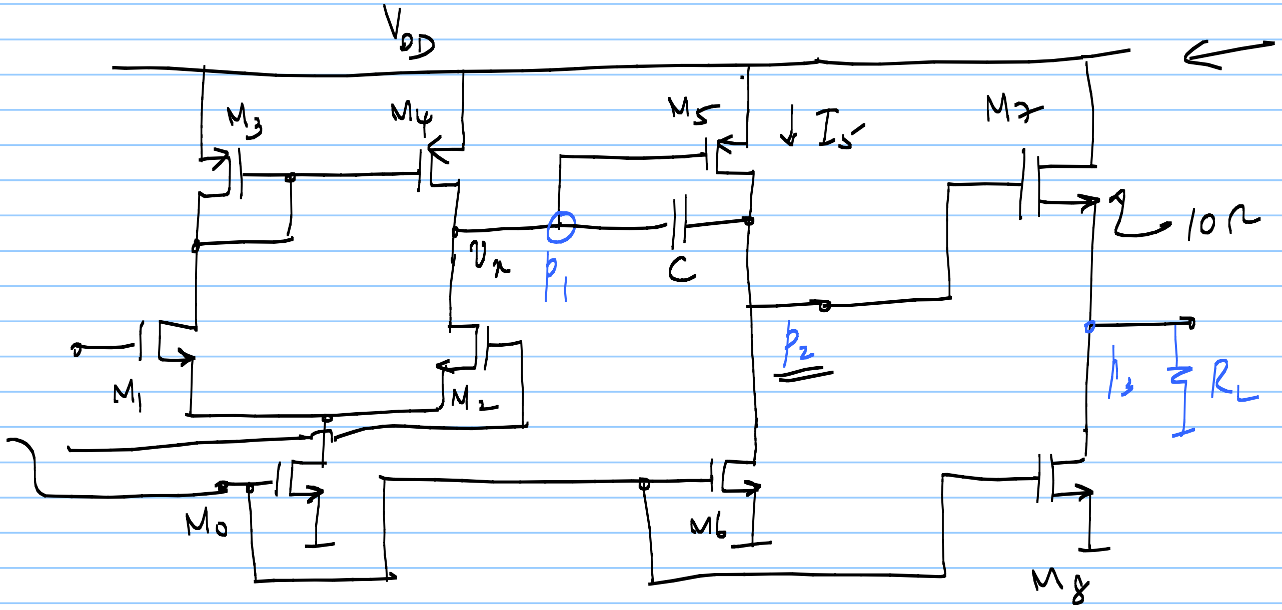
$\Downarrow$

CDA



gain < 1

"3"-stage opamp



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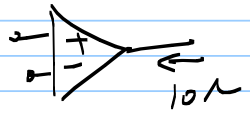
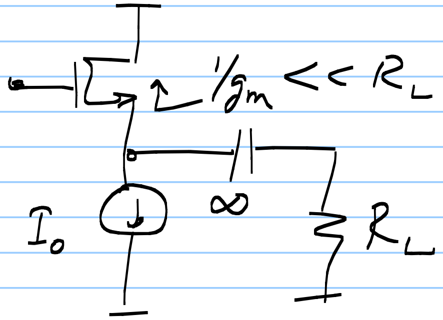




gain < 1

$g_m R_L = 10$

$$\text{gain} = \frac{g_m R_L}{1+g_m R_L} = \frac{10}{11} \approx 0.91$$

Case I : $v_{x_1} = v_{in_1} - v_{e_1}$

$v_{o_1} = 0.91 v_{x_1}$

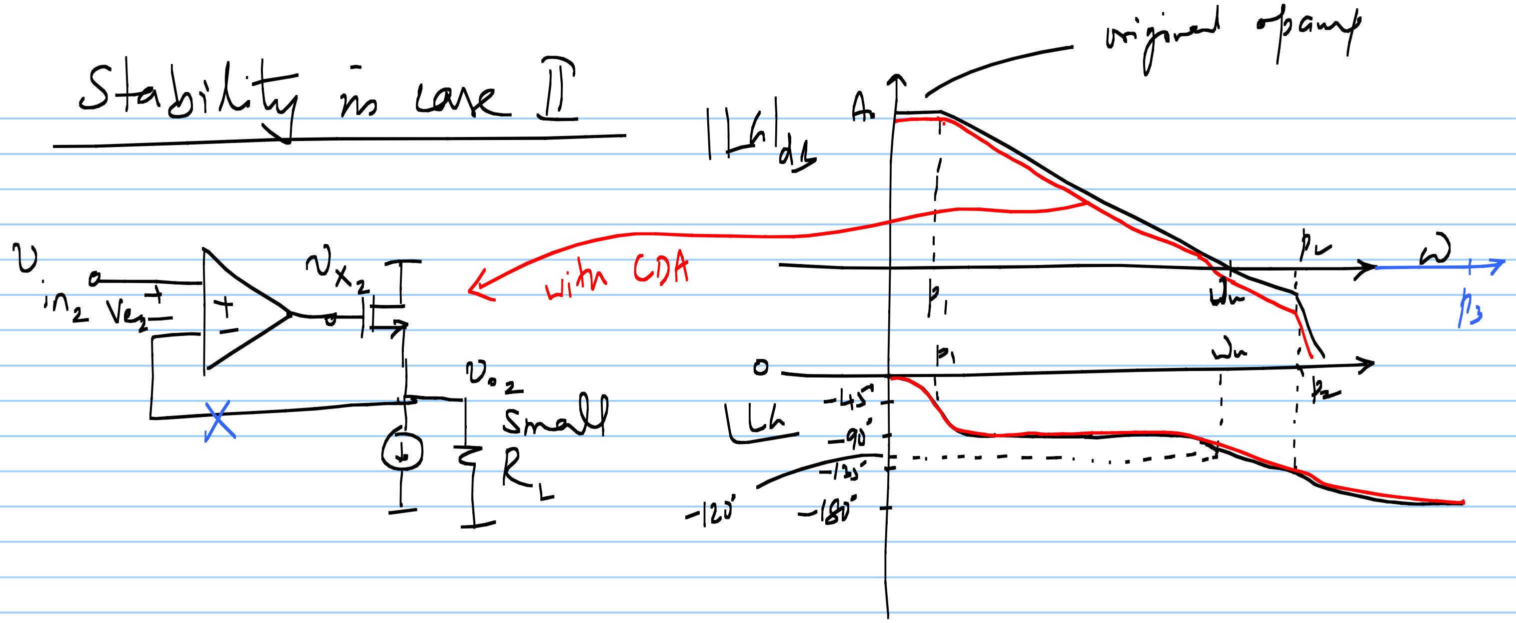
case II : $v_{o_2} = v_{in_2} - v_{e_2}$

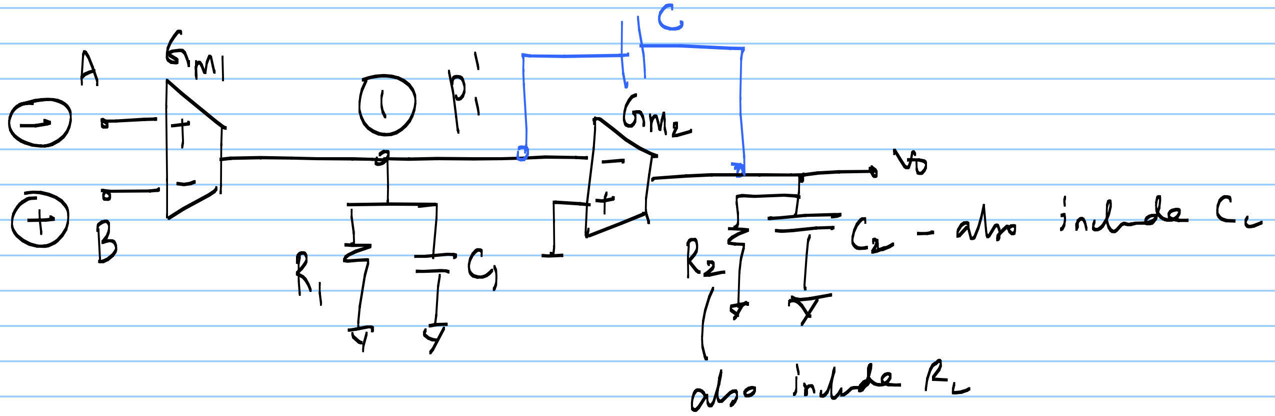
$v_{x_2} = 1.1 v_{o_2}$ ✓

v_{e_1} vs v_{e_2} ← will be about the same

Stability in case II

e.g. PM = 60°





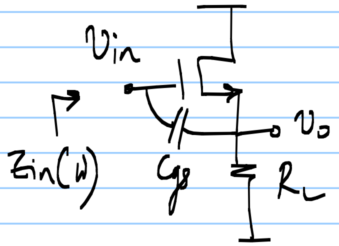
In our example, $G_{m_2} = g_{m_5} \approx 544\mu S \approx 0.5mS$

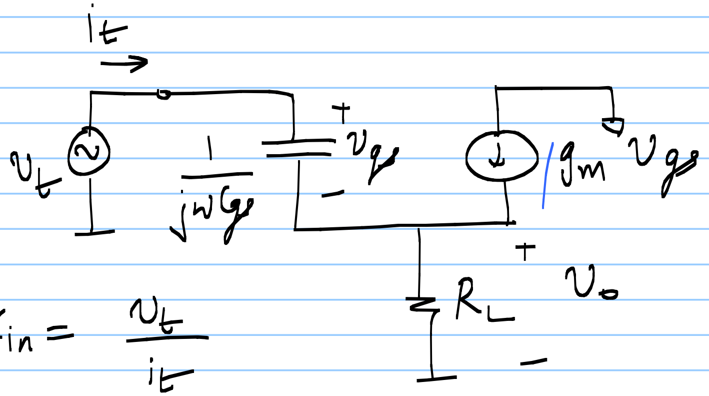
If $R_L = 2k\Omega$ is the load

$\Rightarrow A_2 = G_{m_2}(R_2 || R_L) = 0.5mS \times (130k\Omega // 2k\Omega)$

≈ 1

$A_1 \approx 170$





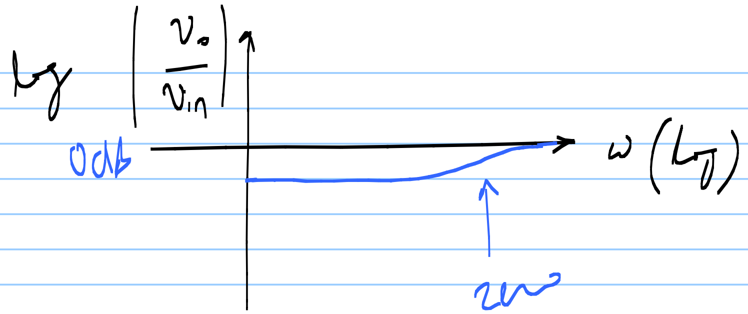
$$i_t = j\omega C_{gs} \cdot v_{gs} \qquad Z_{in} = \frac{v_t}{i_t}$$

$$v_t = v_{gs} + v_o$$

$$= v_{gs} + \left[j\omega C_{gs} \cdot v_{gs} + g_m v_{gs} \right] \cdot R_L$$

$$v_t = v_{gs} \left[1 + g_m R_L + j\omega C_{gs} R_L \right]$$

$$= \frac{1 + g_m R_L + j\omega C_{gs} R_L}{j\omega C_{gs}} \cdot i_t \Rightarrow Z_{in}(j\omega) = \frac{1 + g_m R_L + j\omega C_{gs} R_L}{j\omega C_{gs}}$$


log $\left|\frac{v_o}{v_{in}}\right|$
0dB
ω (Log)
2ω₀

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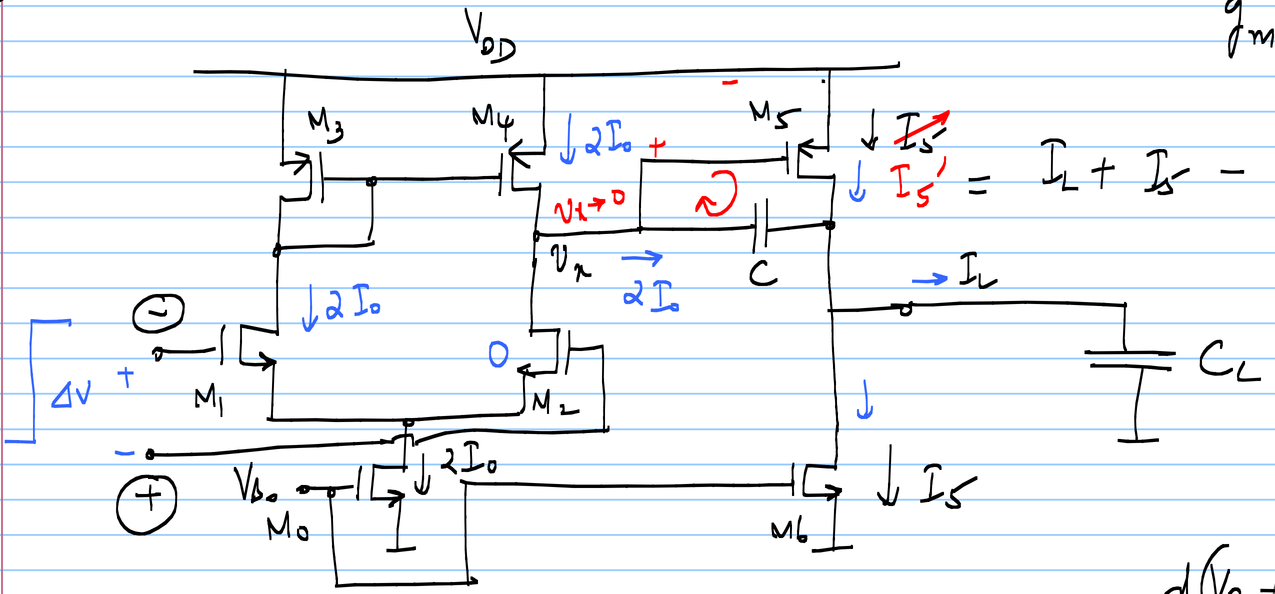
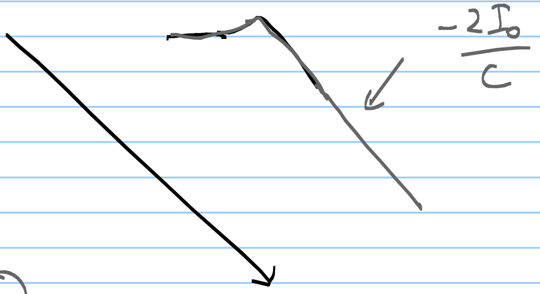
$g_{m5} >> g_{m1} \Rightarrow I_5 >> I_o$



$$I_5' = I_L + I_5 - 2I_o = I_5 - 2I_o - 2I_o \cdot \frac{C_L}{C}$$

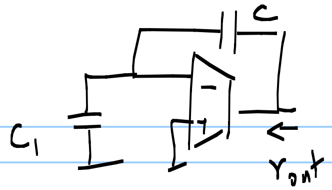


$$-C \frac{d(V_o - V_x)}{dt} = 2I_o$$

V_x = virtual ground

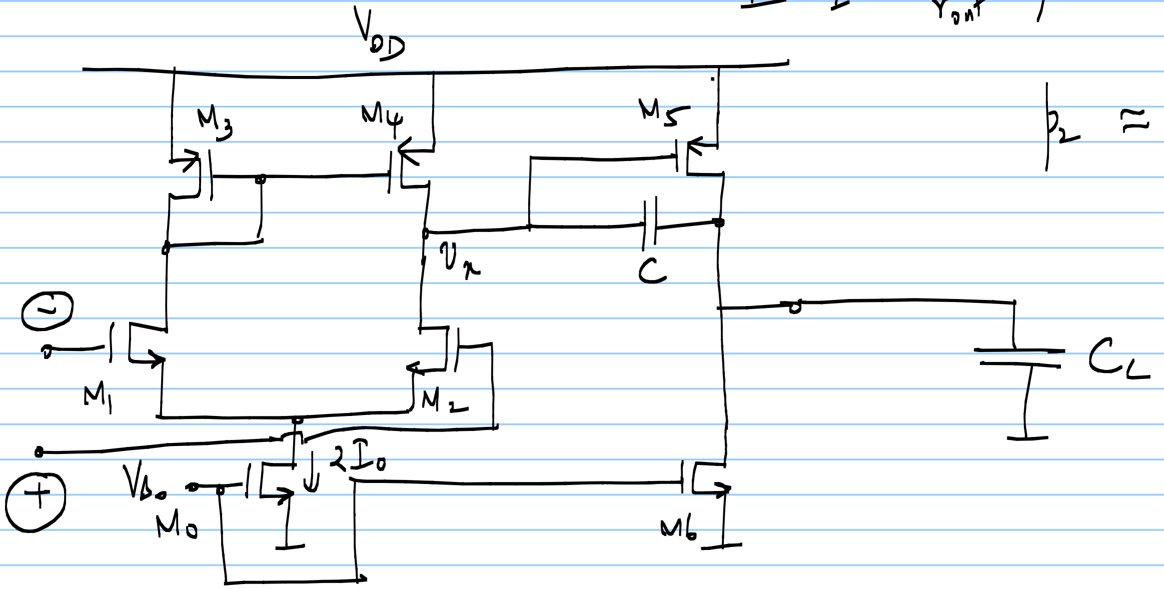
$$\frac{dV_o}{dt} = \frac{-2I_o}{C}$$

$$C_L \, dV_o/dt = I_L = -\frac{2I_o}{C} \cdot C_L$$



$$p_1 \approx \frac{-1}{r_{out_1} \cdot (g_{m5} r_{out_2})\, C}$$

$$p_2 \approx \frac{-1}{\left(\frac{C+C_1}{C} \cdot \frac{1}{g_{m5}}\right) C_L}$$



$$Z_1 = \frac{+\,g_{m5}}{C}$$

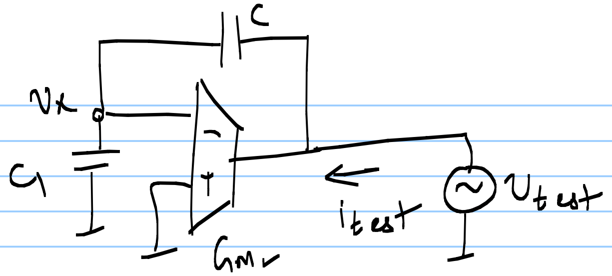
$$\omega_u = \frac{g_{m1}}{C}$$

$$r_{out} = \frac{(C+C_1)}{C} \cdot \frac{1}{G_{M2}} \quad ✓$$

$$\sim$$

$$\frac{C}{C+C_1} \cdot \frac{1}{G_{m2}} \quad ✗$$

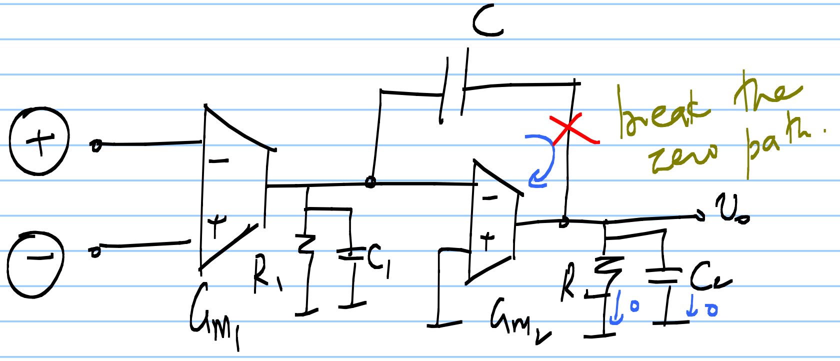
Set $g_{m5} \gg g_{m1}$



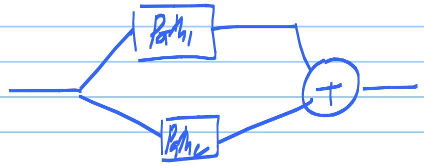
$$v_x = \frac{C}{C+C_1} \cdot v_{test}$$

$$i_{test} = G_{m2} \cdot v_x = \frac{G_{m2} \cdot C}{C+C_1} \cdot v_{test}$$

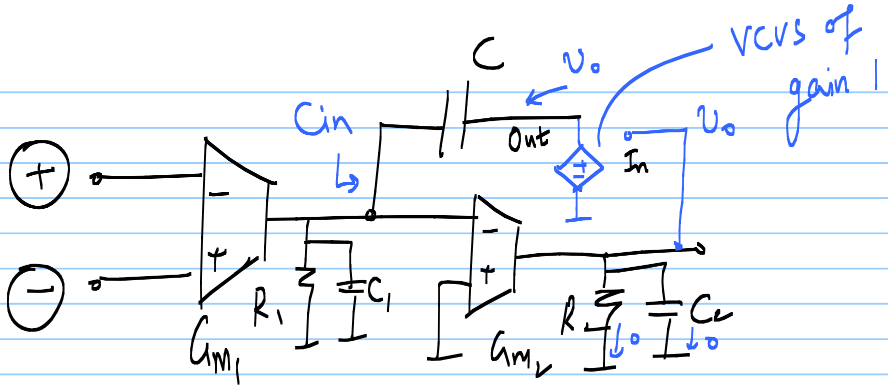
$$r_{out} = \left(\frac{C+C_1}{C}\right) \cdot \frac{1}{G_{m2}}$$



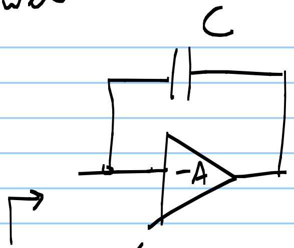
$$Z_1 = + \frac{G_{m2}}{C}$$



@ Z_1 : $v_o = 0$

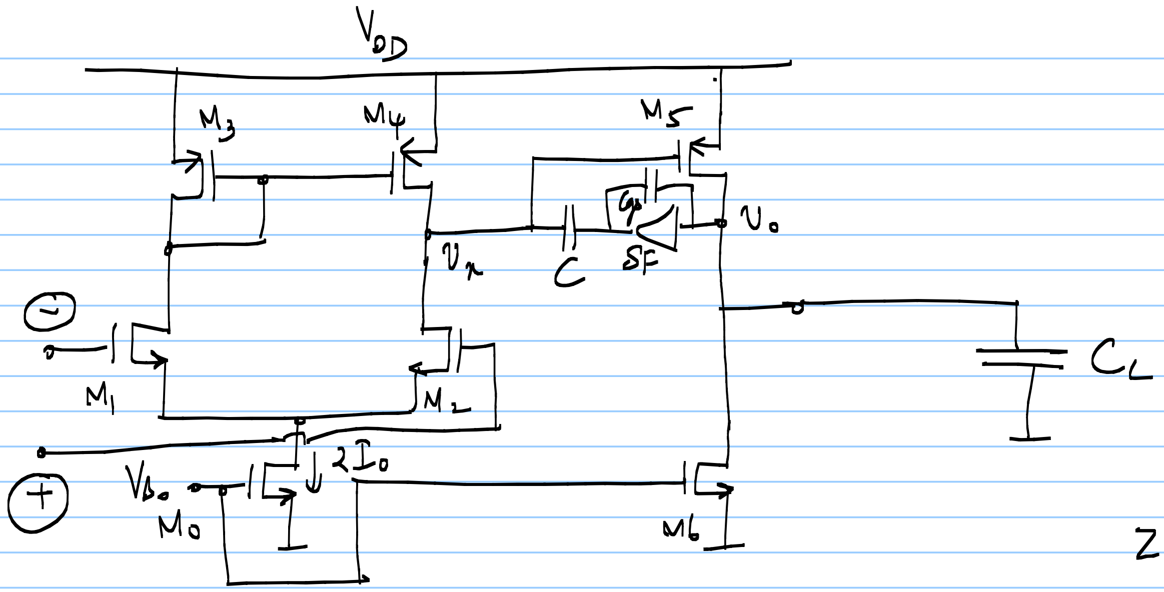


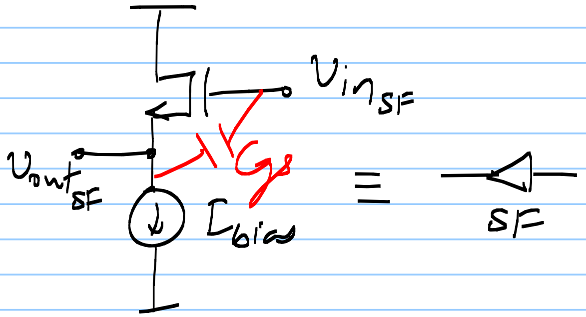
= source follower



$C_{in} = (1+A)\,C$

$C_{in} = (1 + G_{m_2} R_2)\,C$





$$\text{zero} = \frac{+\,g_{m5}}{\left(\frac{C \cdot C_{gs}}{C + C_{gs}}\right)} \gg \omega_u$$

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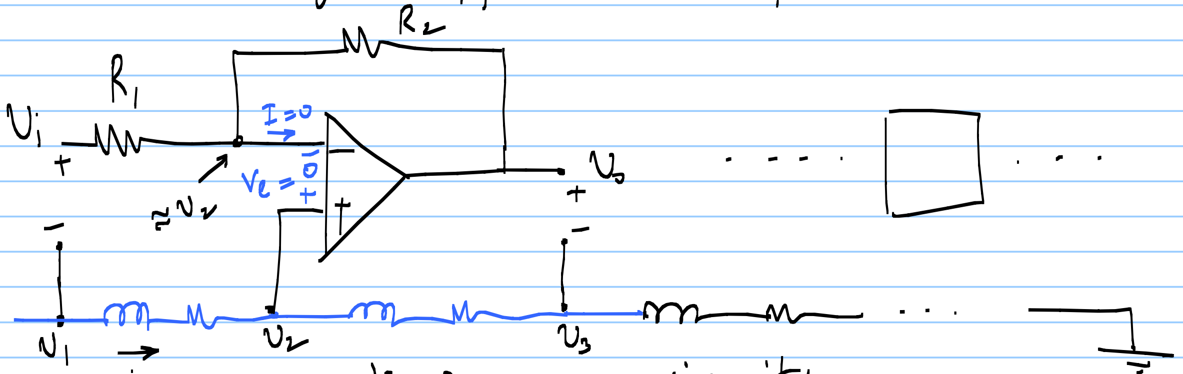
Lec22

Fully Differential Opamps

1)



We want $\frac{v_o}{v_i} = -\frac{R_2}{R_1}$

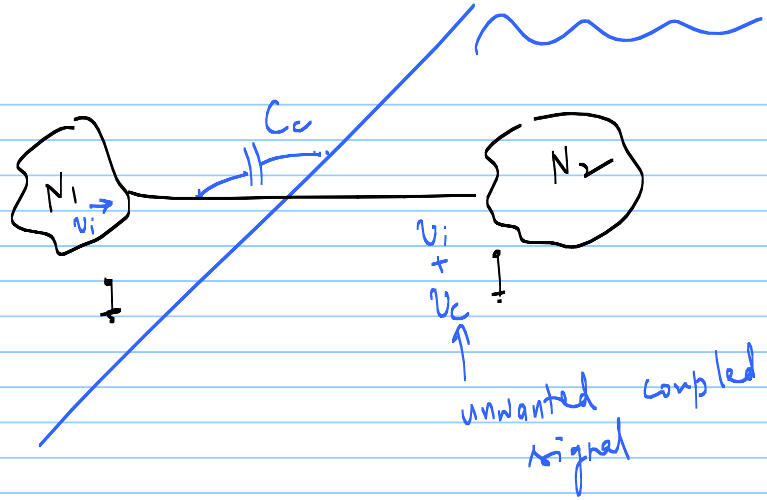
$+ (\quad)$

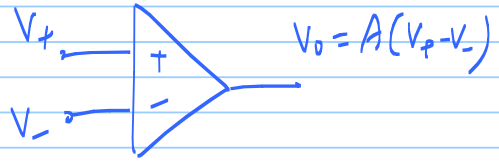
i_x = currents of many circuits

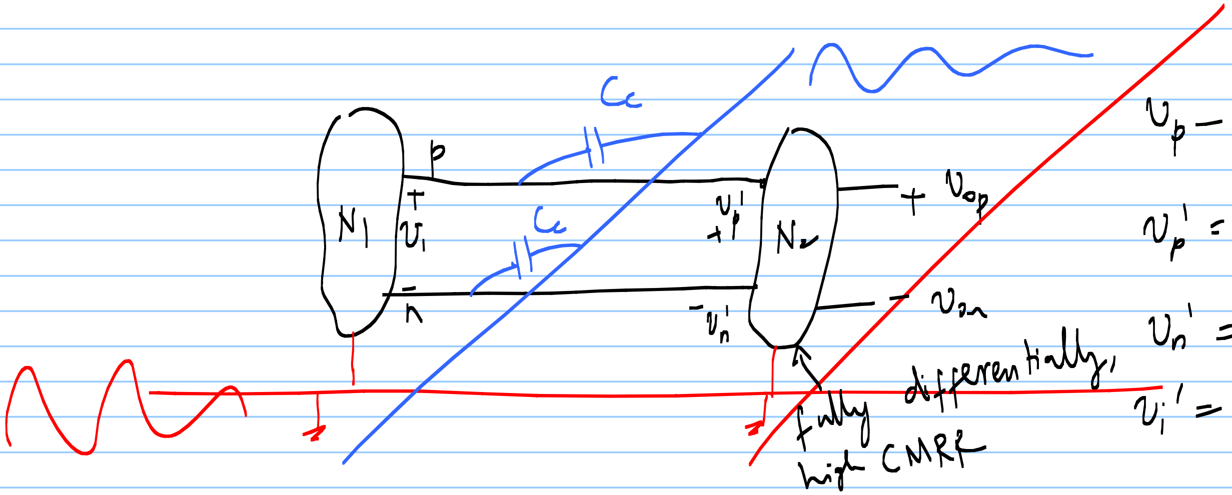
v_1, v_2, v_3 = functions of i_x

v_o = function of i_x

2)





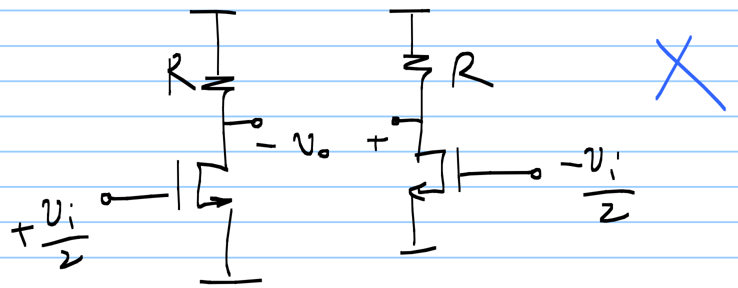


$v_p - v_n = v_i$

$v_p' = v_p + v_c$ ← common-mode signal to p & n lines

$v_n' = v_n + v_c$

$v_i' = v_p - v_n$

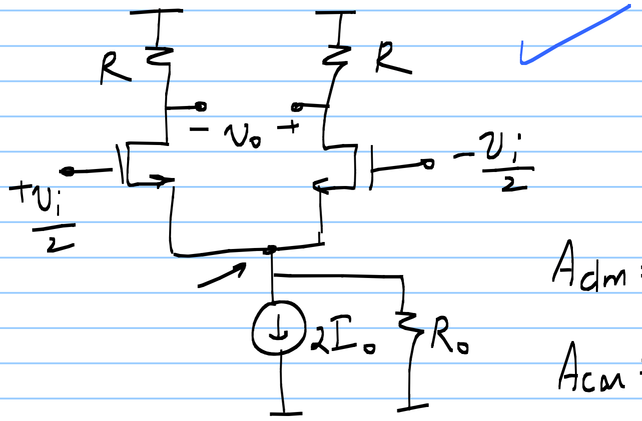


$$A_{dm} = \frac{v_o}{v_i} = g_m R$$

$$A_{cm} = A_{dm}$$

$$CMRR = 1 = 0dB$$

"pseudo-differential" amplifier

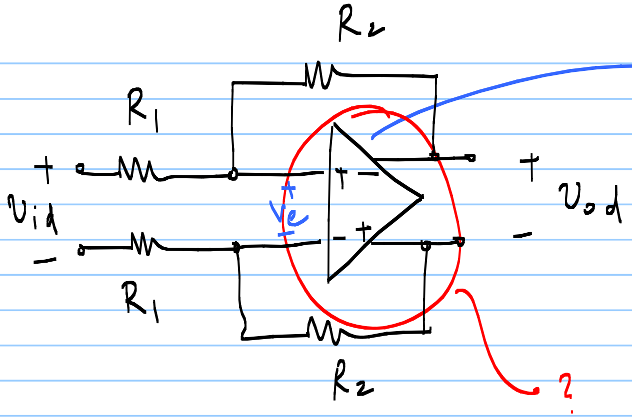


$$A_{dm} = \frac{v_o}{v_i} = g_m R$$

$$A_{cm} = \frac{g_m R}{1 + 2 g_m R_o}$$

high CMRR

"fully differential" amplifier



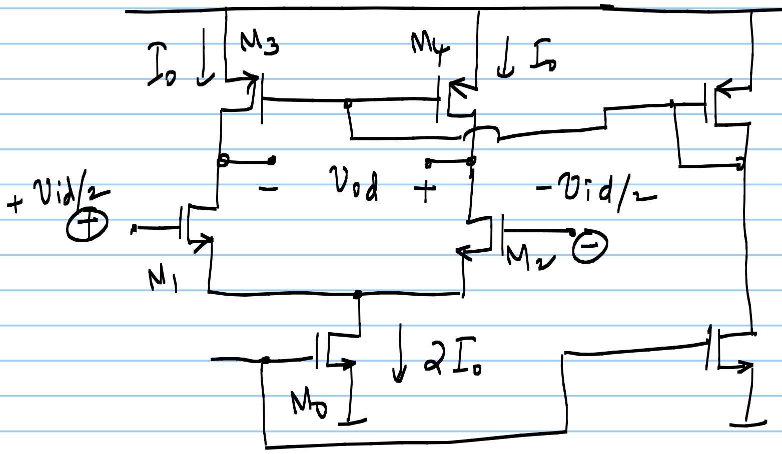
$$v_{od} = \frac{-R_2}{R_1} \cdot v_{id}$$

fully differential opamp (CMOS)

$R_{in} = \infty$

$R_o =$ large (OTA)

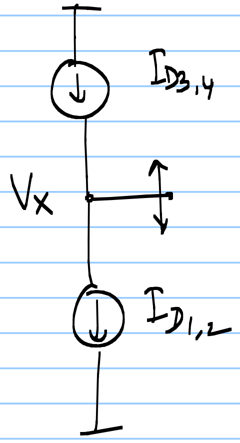
$A =$ large $\Rightarrow V_e \rightarrow 0$



$M_5 = M_{3,4}$

$M_6 = \frac{1}{2} M_o$

Slight mismatches in $M_o - M_6$ or $M_5 - M_{34}$ will cause $I_{D_{34}}$ & $I_{D_{12}}$ to be different



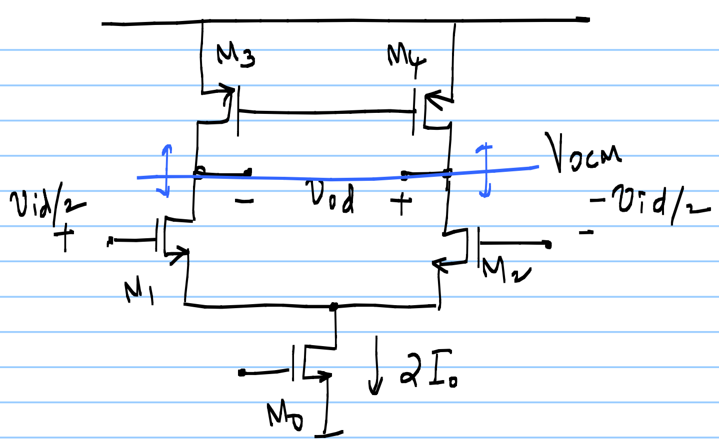
If $I_{D3,4} > I_{D1,2} \Rightarrow V_x \uparrow$

$< \Rightarrow V_x \downarrow$

$M_{1,2}$ or $M_{3,4}$ could move into triode region

* Output common-mode is not set properly.

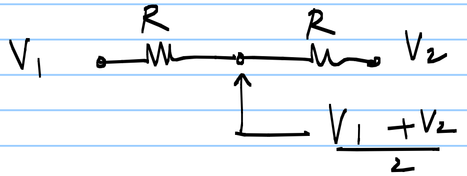
$$V_{OCM} = \frac{V_{O+} + V_{O-}}{2}$$

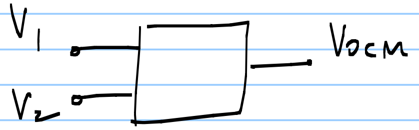


Use Negative f.b. to set V_{OCM}

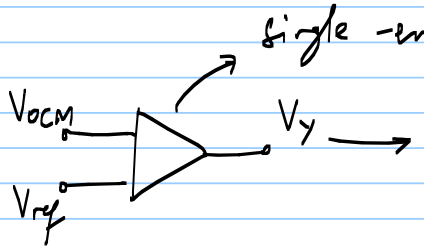
- Sense V_{OCM}
- Compare V_{OCM} to desired value V_{ref}
- Employ feedback to drive $V_{OCM} \to V_{ref}$

1) Sense V_{ocm}:

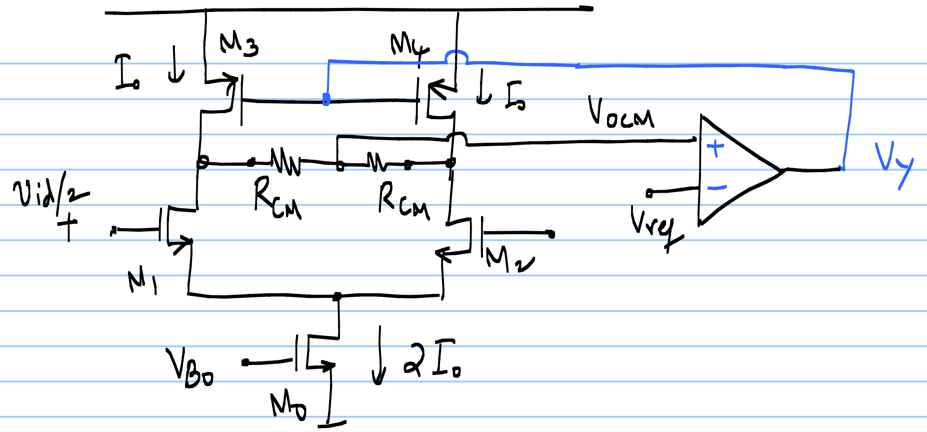




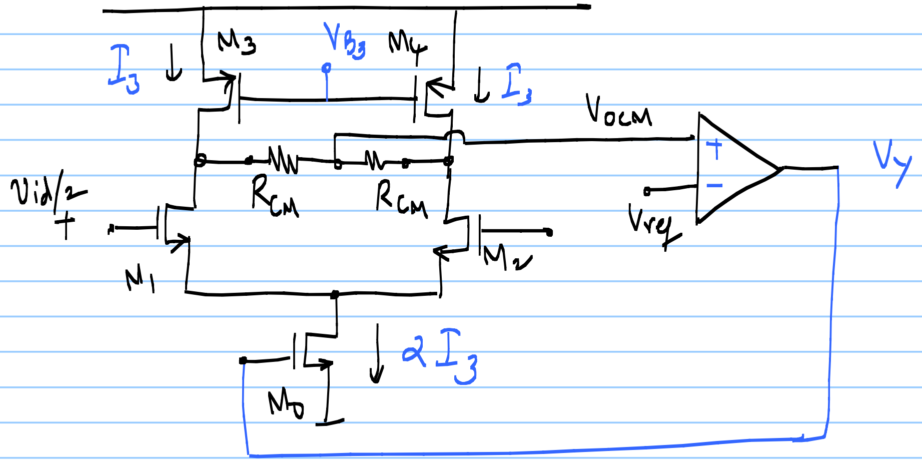
2) Compare V_{OCM} to V_{ref}:



connect V_y so that $V_{OCM} \to V_{ref}$



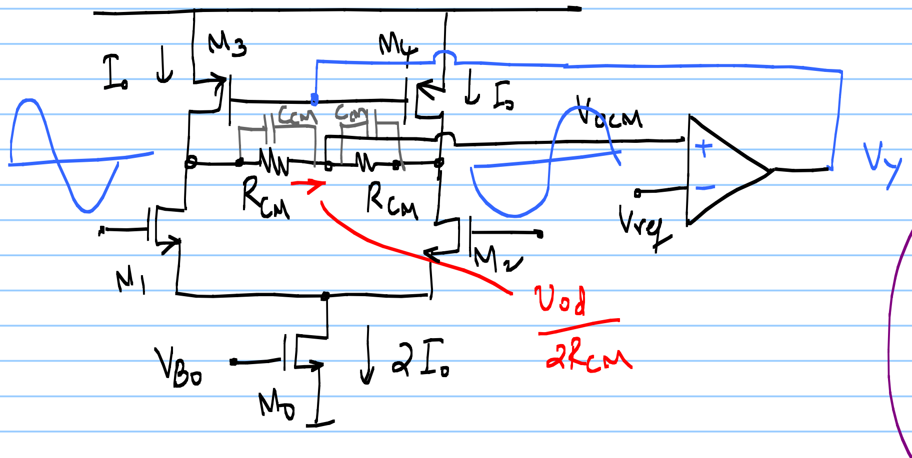
→ Set M_0 current & f.b. to $V_{G3,4}$

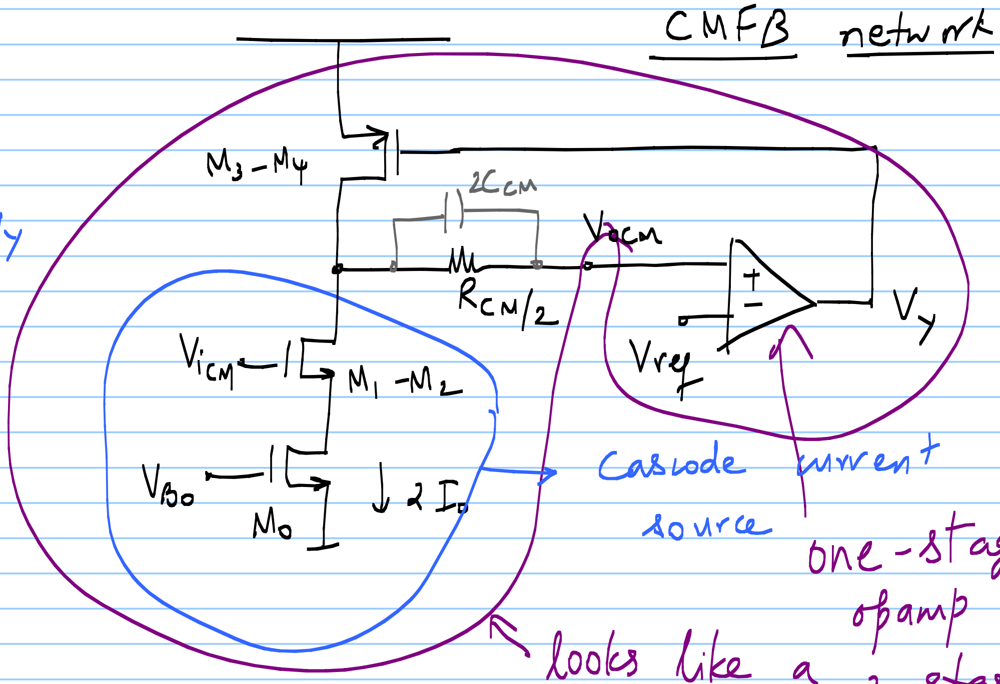


27/2/20

Lec 23

CMFB network





cascode current source

one-stage opamp

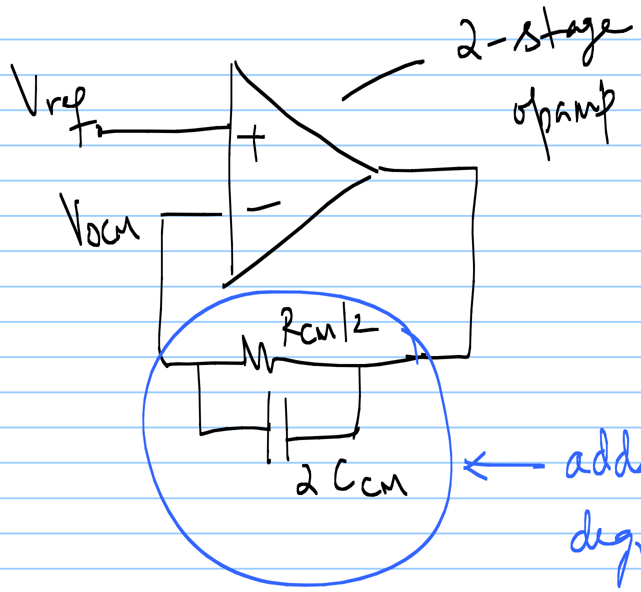
looks like a 2-stage opamp

diffl. gain $\frac{v_{od}}{v_{id}} = g_{m_1}(r_{ds_2} || r_{ds_4} || R_{CM})$

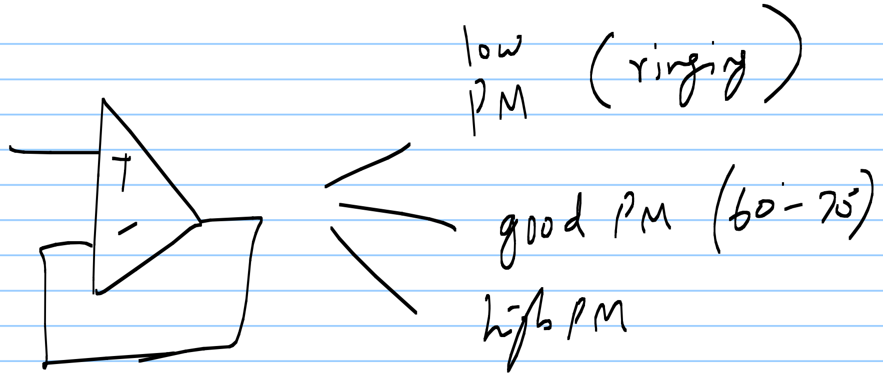
R_{CM} – chosen to be much larger than r_{ds_2}, r_{ds_4}

$R_{CM} \sim M\Omega$ is possible

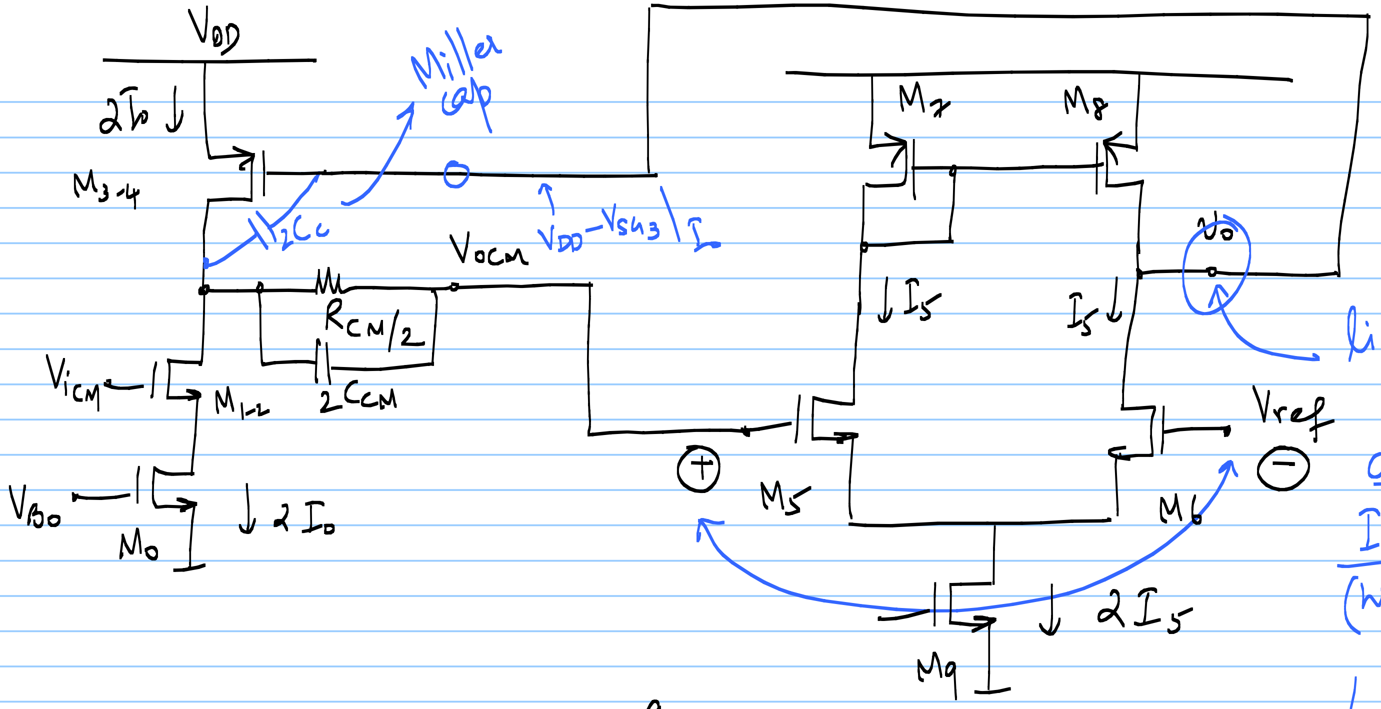
→ comes with parasitic cap. C_{CM}



adds phase shift, degrades PM further



* You have to compensate the CMFB loop



op amp

likes to have $V_{DD}-V_{SG7}|_{I_5}$

choose

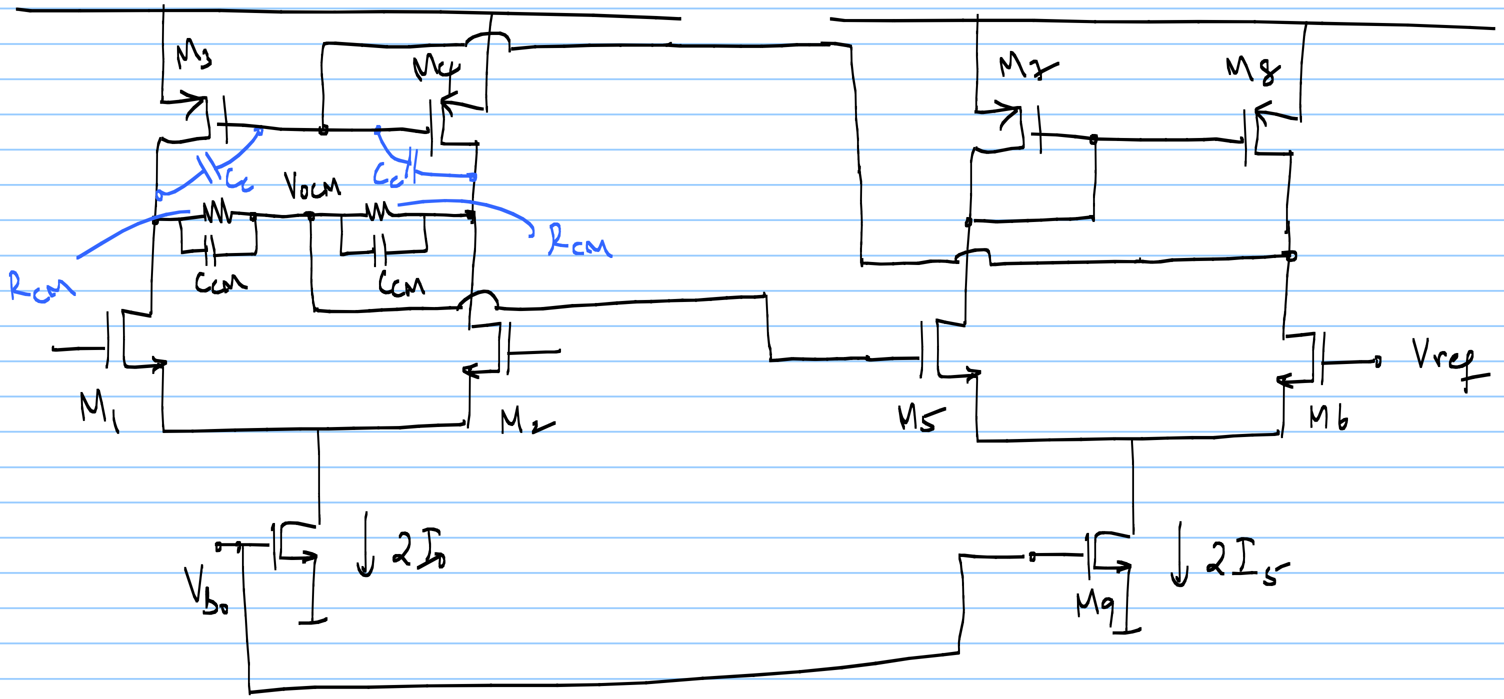
$$\frac{I_5}{(W/L)_7} = \frac{I_0}{(W/L)_3}$$

$L_7 = L_3$

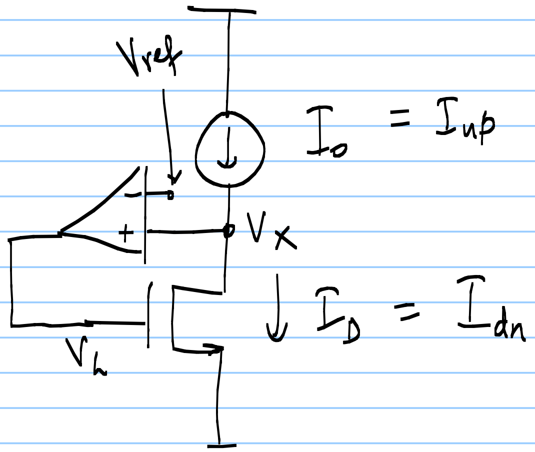
Normally $I_5 << I_0$

CMFB : $\omega_{u_{CM}} = \frac{g_{m5}}{2C_c}$

DC gain = $g_{m5}(r_{ds6}||r_{ds8}) \cdot g_{m_{3-4}}(r_{ds_{3-4}}||(g_{m_{1-2}} r_{ds_{1-2}} \cdot r_{ds0}))$

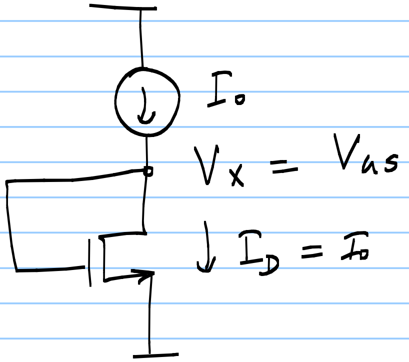

M3
M4
Cc
Cc
VOCM
RCM
RCM
CCM
CCM
M1
M2
M5
M6
M7
M8
Vref
Vb0
2I0
2I5
M9

1)

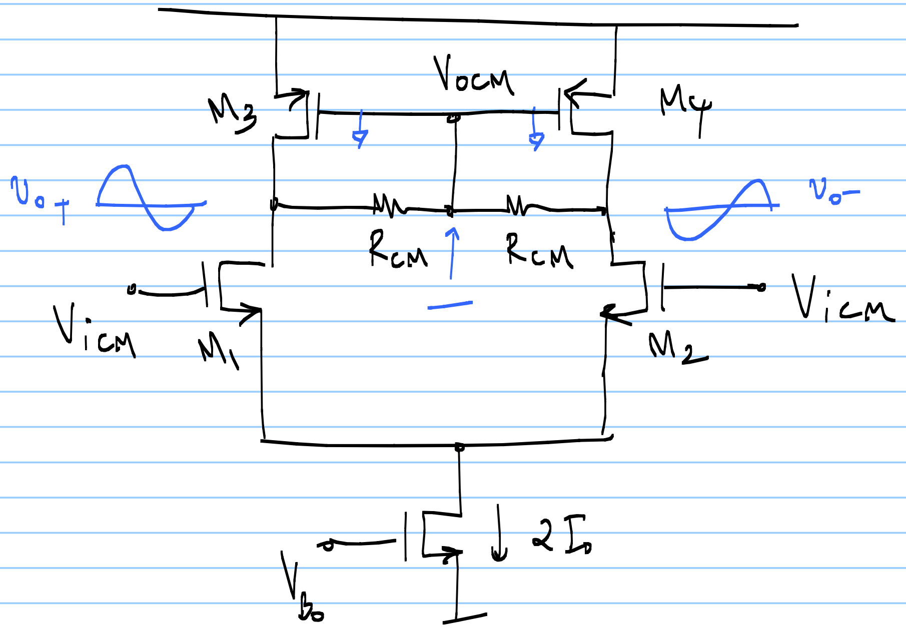


$V_x = V_{ref}$

2)



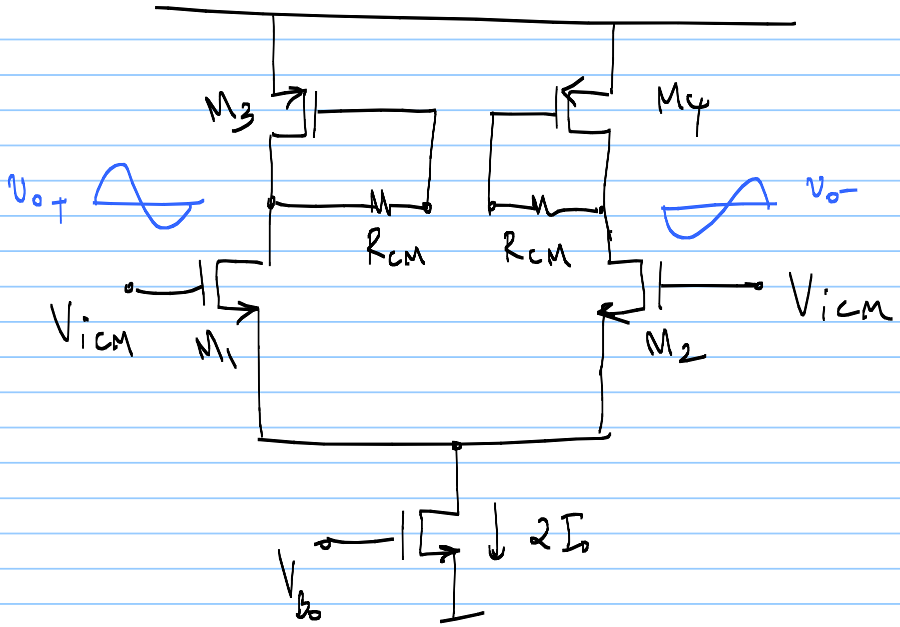
$V_x = V_{gs}$



$$V_{OCM} = V_{DD} - V_{SG_3}\Big|_{I_0}$$

$$\frac{v_{od}}{v_{id}} = g_{m_1}\left(r_{ds_2} \| r_{ds_4} \| R_{CM}\right)$$

R_{CM} cM det.
is linear



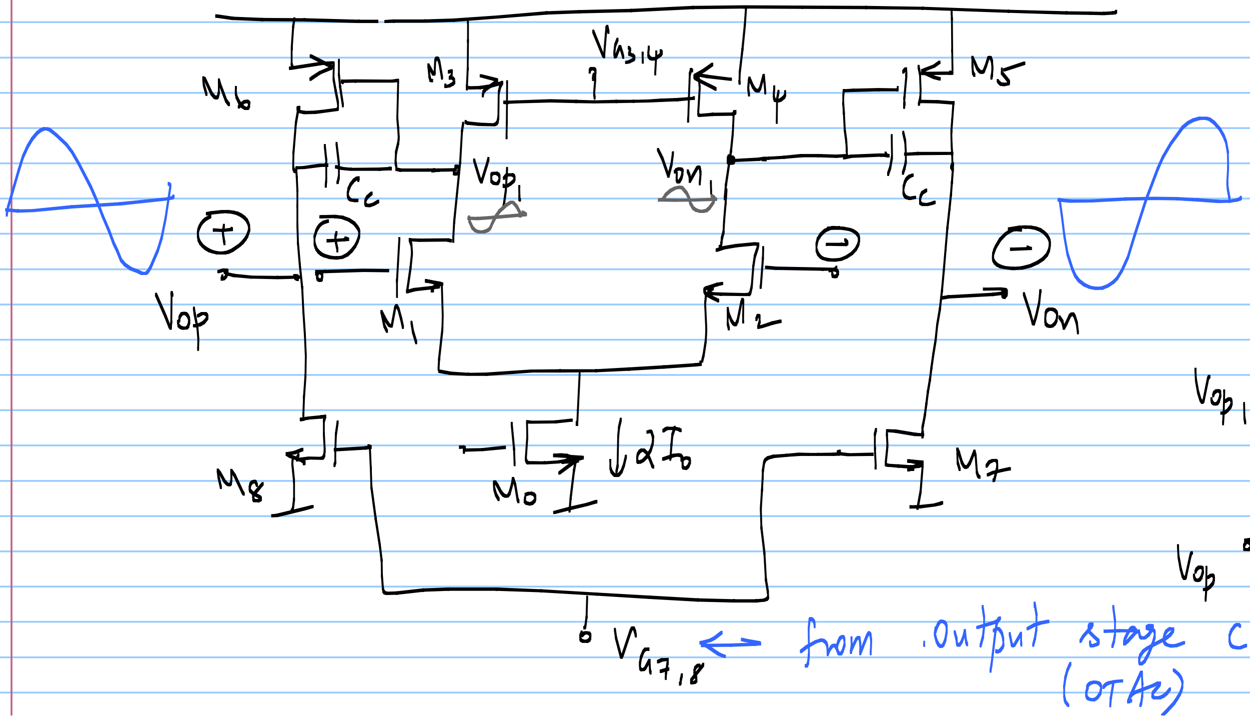
$$V_{OCM} = V_{DD} - V_{SG_3}\Big|_{I_0}$$

$$\frac{v_{od}}{v_{id}} \simeq \frac{g_{m_1}}{g_{m_3}}$$

3/3/20

Lec 24

Two-stage fully differential opamps

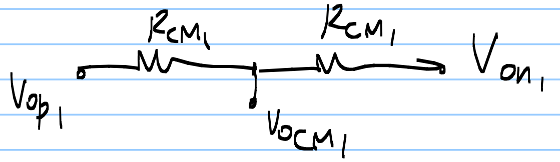


← from output stage CMFB (OTA2)

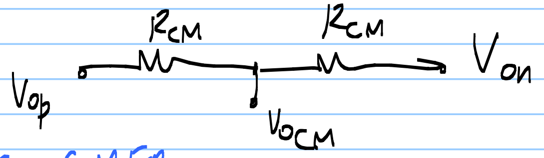
Need to set

$$V_{OCM_1} = \frac{V_{op_1} + V_{on_1}}{2}$$

$$V_{OCM} = \frac{V_{op} + V_{on}}{2}$$



? ✗



? ✓

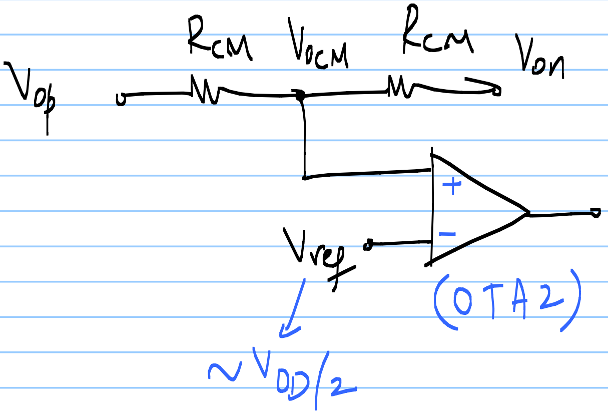
1) V_{op}, V_{on} — 100s of mV to V of swing ← Use Resistive CM detector

V_{op1}, V_{on1} — mV to 10s of mV of swing ← can use active CM detector

2) Two-stage opamp — resistive loads ← you get gain from 1st stage

large gain, small error voltages

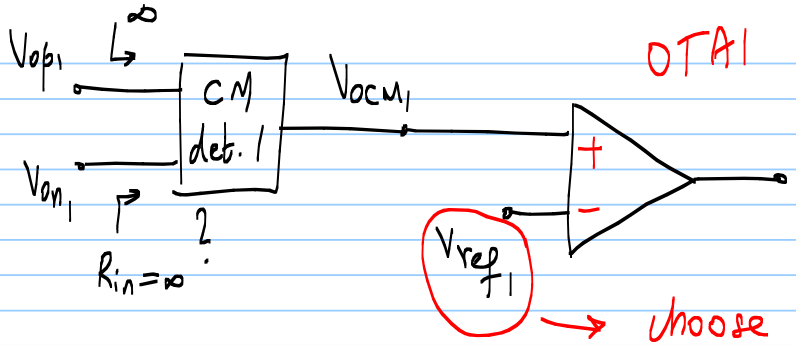
use active CM detector with $R_{in} = \infty$



goes to $V_{a7,8}$

→ * choose PMOS input pair + NMOS CM load

* NMOS CM has same current density as $M_7 - M_8$

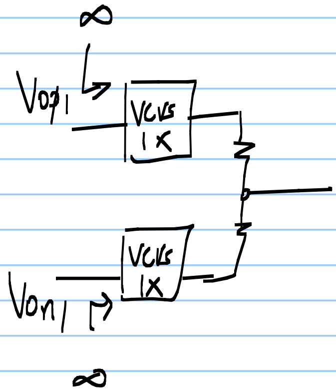


goes to $V_{a3,4}$

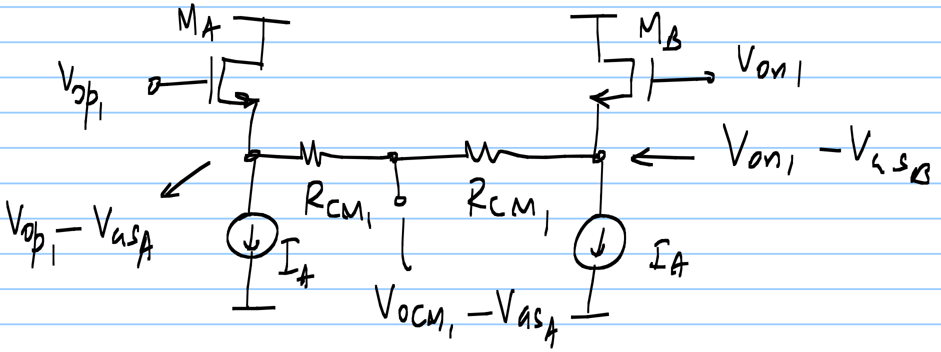
OTA1 → NMOS input pair + PMOS CM load and ensure PMOS CM has same current density as $M_3 - M_4$

→ choose $V_{ref_1} = V_{DD} - V_{SG_5}\Big|_{I_5}$

so that M_5 & M_6 have a current I_5 each



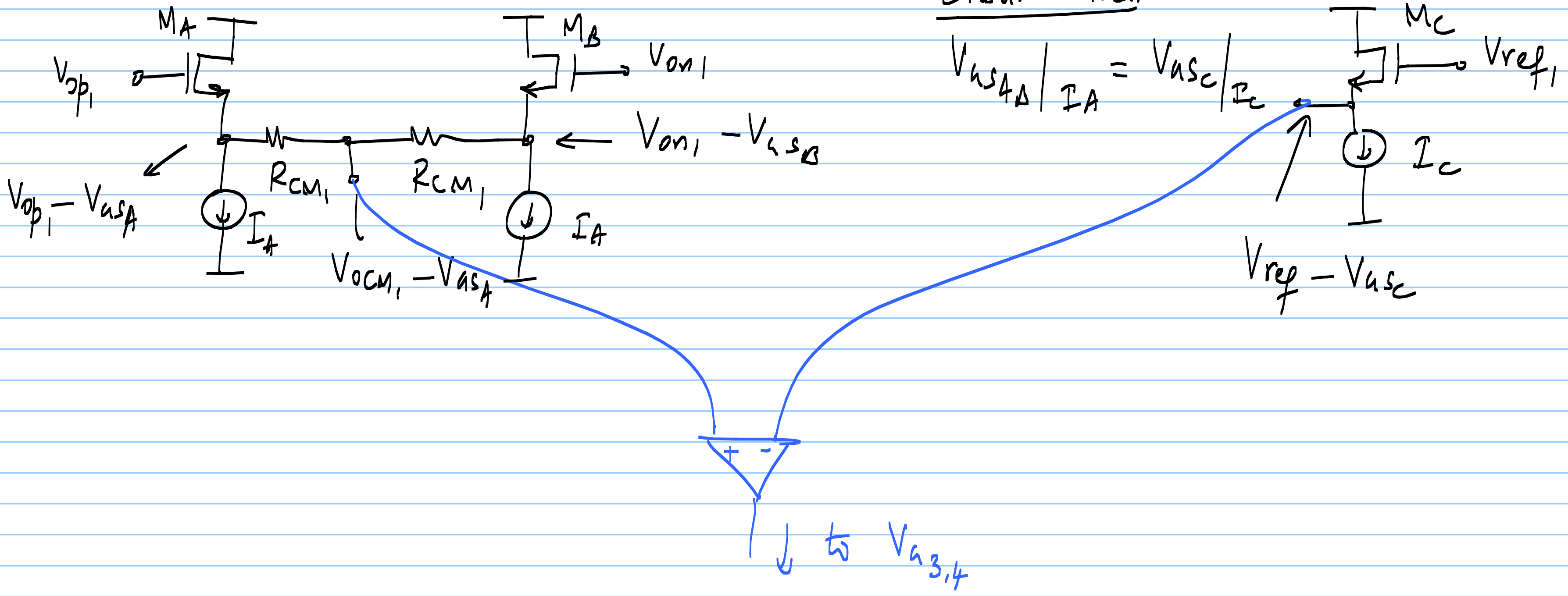
$$\frac{V_{op_1} + V_{on_1}}{2} = V_{ocm_1}$$



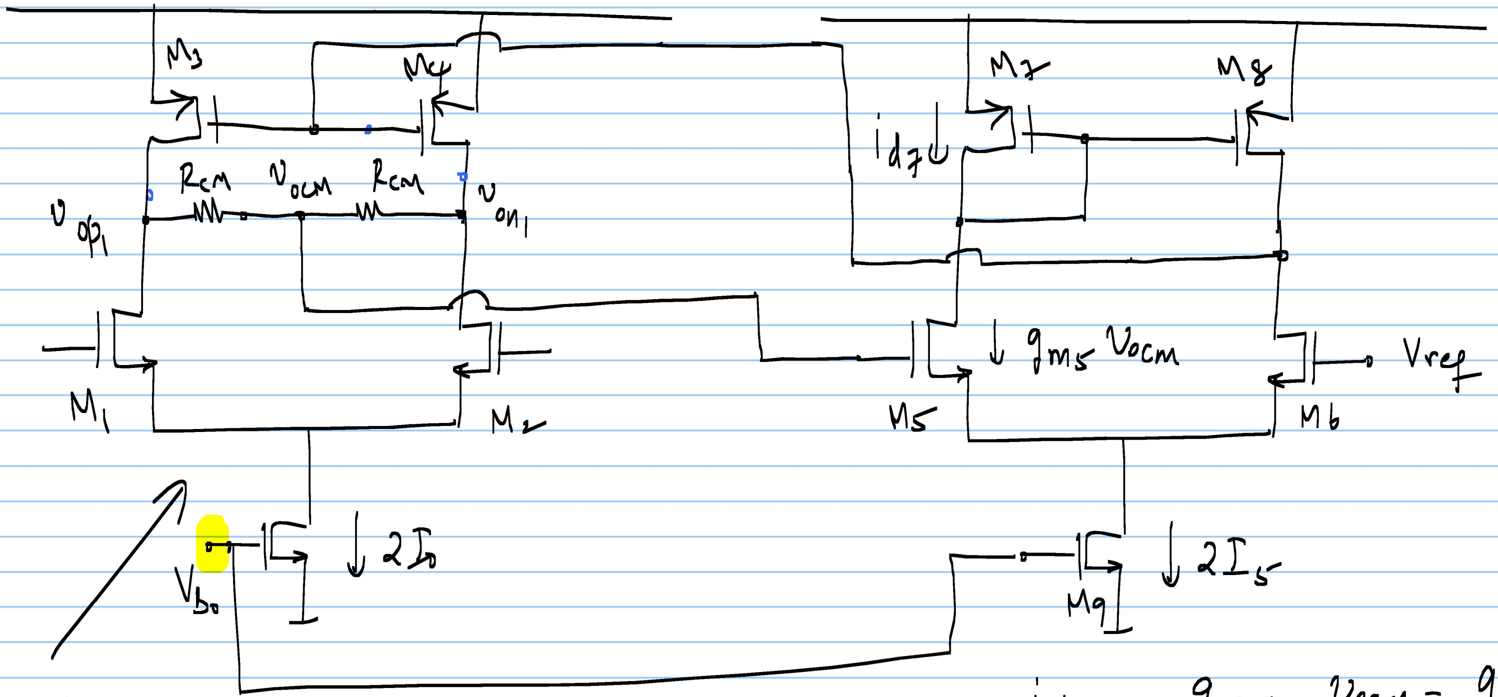
Method 1

Ensure that

$V_{GS_{A,B}}|_{I_A} = V_{GS_C}|_{I_C}$



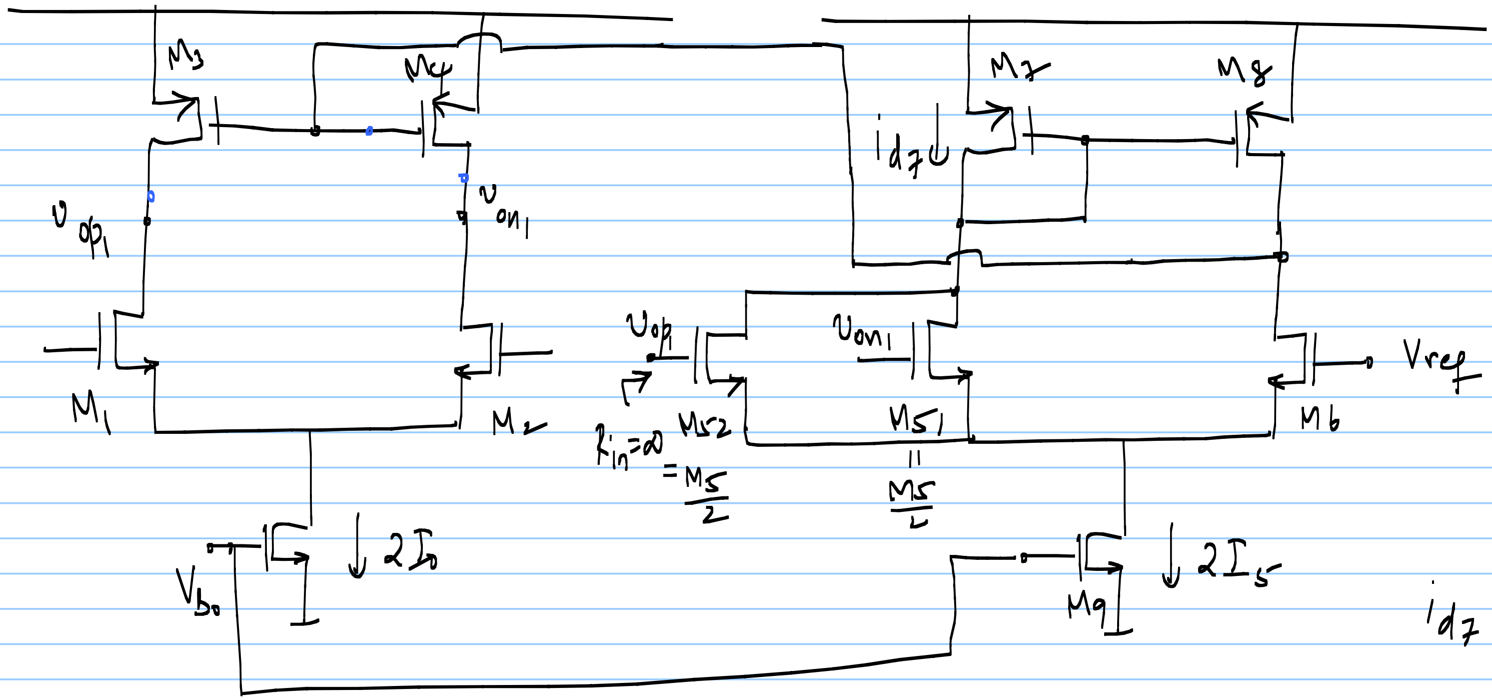
Method 2 :



1st stage of 2-stage opamp

$$i_{d5} = g_{m5} \cdot v_{ocm} = g_{m5}\left(\frac{v_{op_1}+v_{on_1}}{2}\right)$$

$$= i_{d7}$$



$$i_{d52} = g_{m52} \cdot v_{op1}$$

$$i_{d51} = g_{m51} \cdot v_{on1}$$

$$g_{m51} = g_{m52} = \frac{g_{m5}}{2}$$

$$i_{d7} = i_{d51} + i_{d52}$$

$$= \frac{g_{m5}}{2}\left(v_{op1} + v_{on1}\right)$$

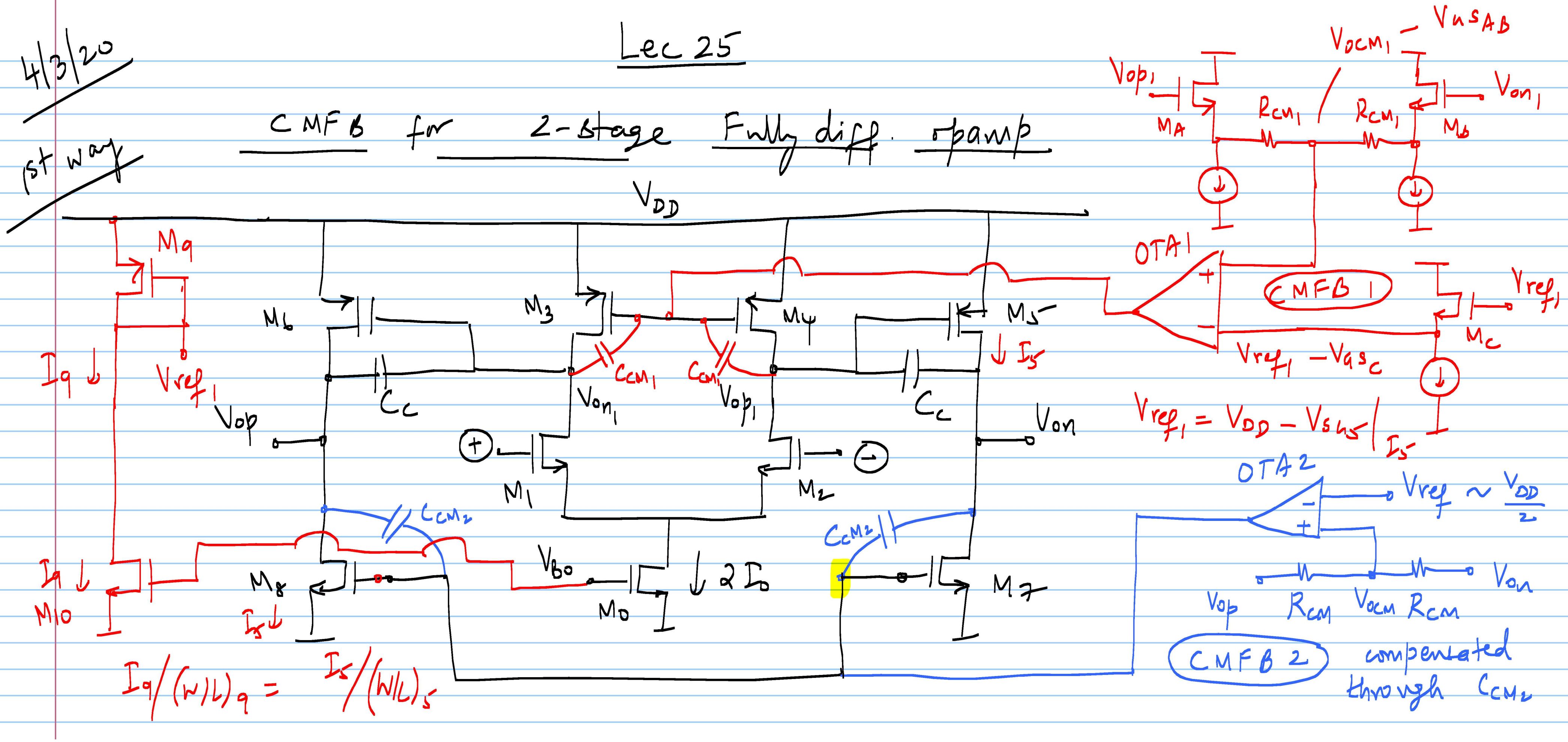
Same as before

4/3/20

Lec 25

CMFB for 2-stage Fully diff. opamp

1st way

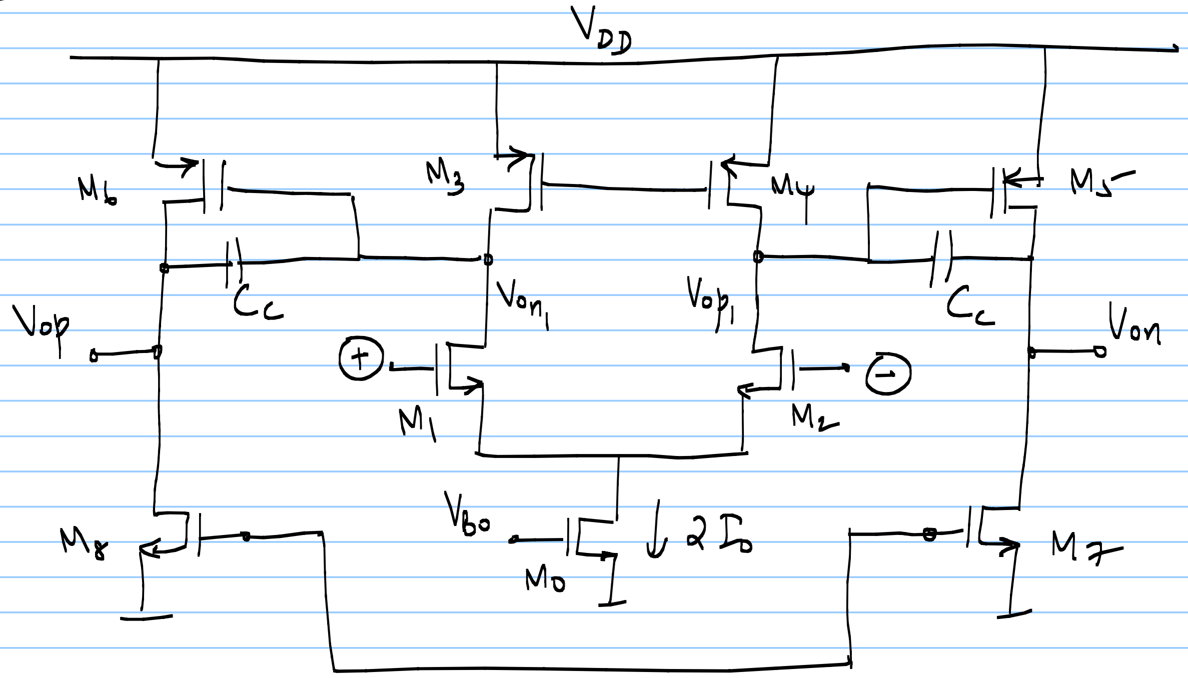


$V_{ref_1} = V_{DD} - V_{SG5}\Big|_{I_5}$

$I_9/(W/L)_9 = I_5/(W/L)_5$

compensated through C_{CM_2}

2nd way



* M_5 & M_6

→ pseudo diff. amplifier

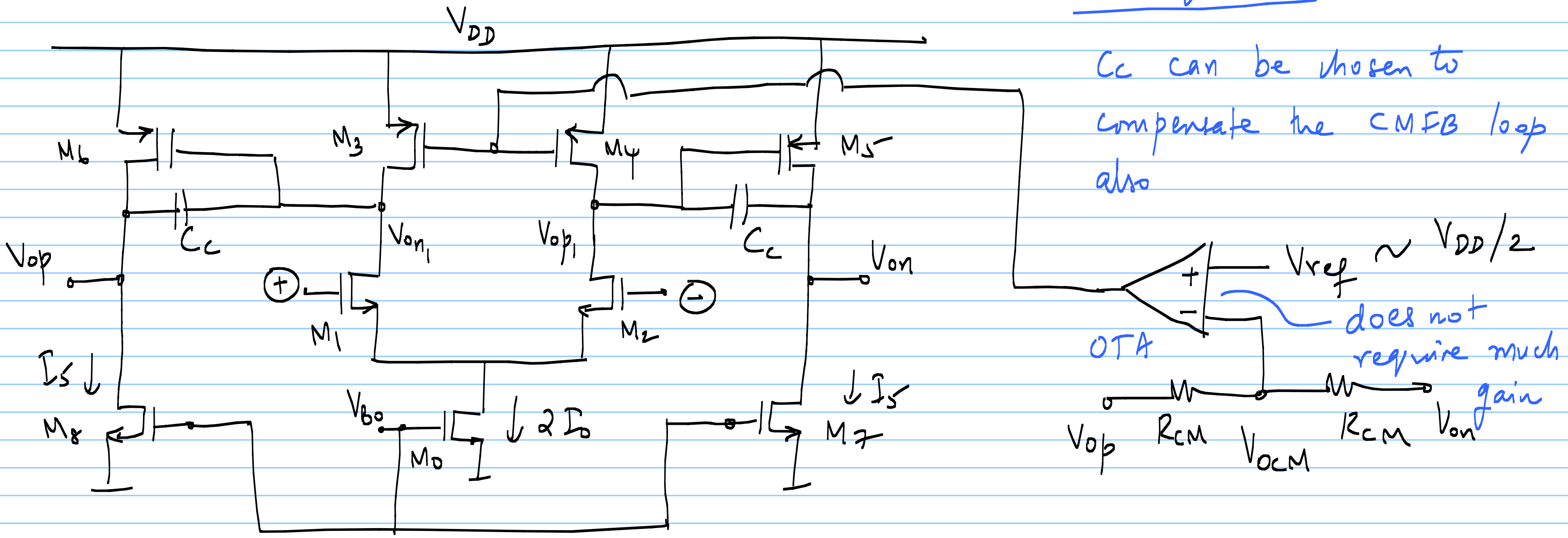
→ has CM gain

* Changing V_{OCM_1}

→ will also change V_{OCM}

3 stage loop

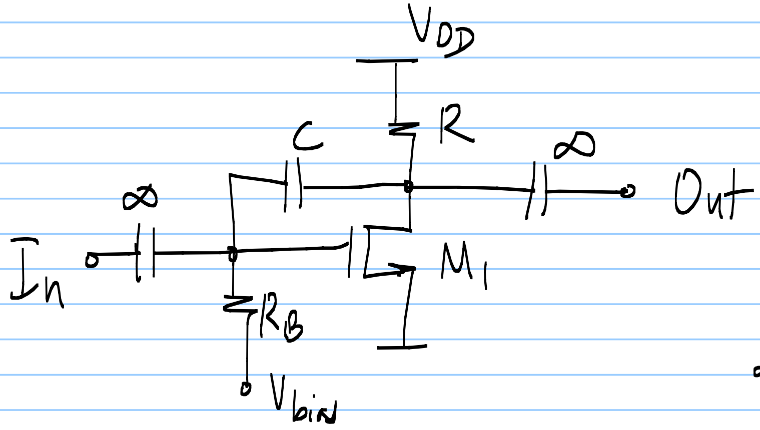
Cc can be chosen to compensate the CMFB loop also



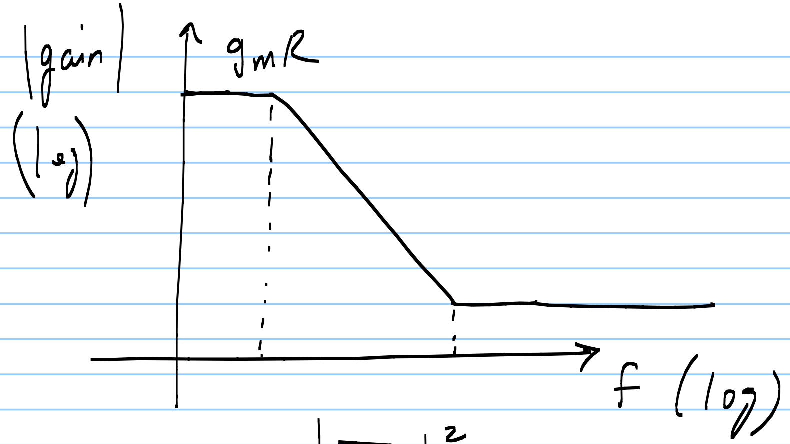
does not require much gain

* CMFB will set V_{g34} so that $\frac{V_{op1}+V_{on1}}{2} = V_{DD} - V_{SG_5}\Big|_{I_5}$

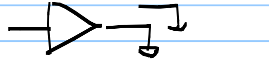
Quiz 1 discussion



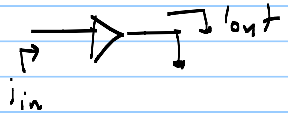
1)



2) $\overline{e_n^2} = \dfrac{|\overline{i_{sc}}|^2}{|I_{out}/V_{in}(\omega)|^2}$

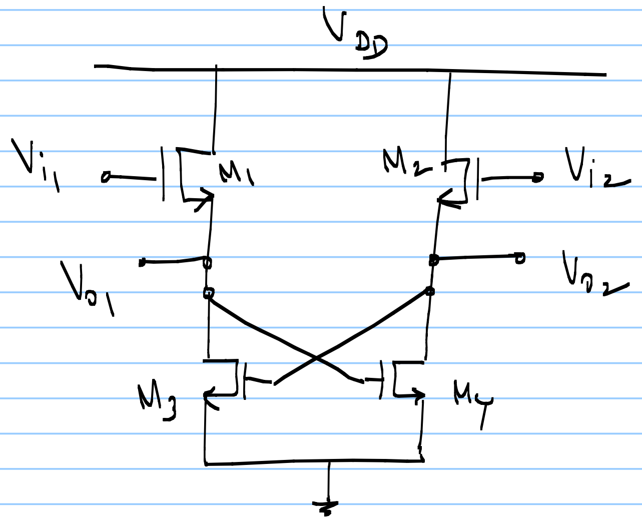


3) $\overline{i_n^2} = \dfrac{|\overline{i_{sc}}|^2}{|I_{out}/I_{in}(\omega)|^2}$



4) R_B adds no component to $\overline{e_n}$

Quiz 2 discussion



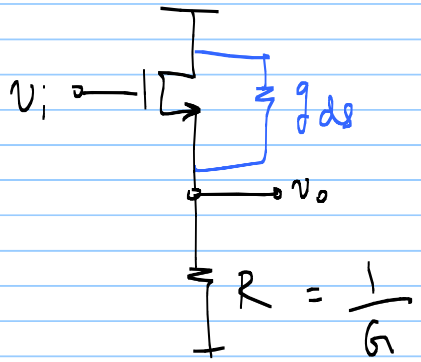
1) V_{icM} so that bias current = 1mA

$$V_{icM} = V_{GS_1} + V_{GS_4} = V_{GS_2} + V_{GS_3}$$

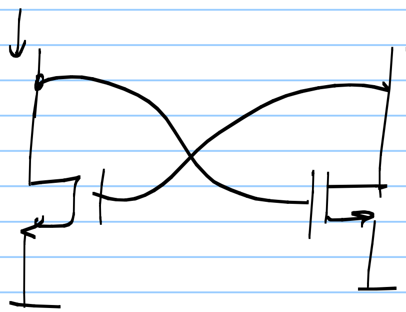
$$\left(\frac{W}{L}\right) = 80/2$$

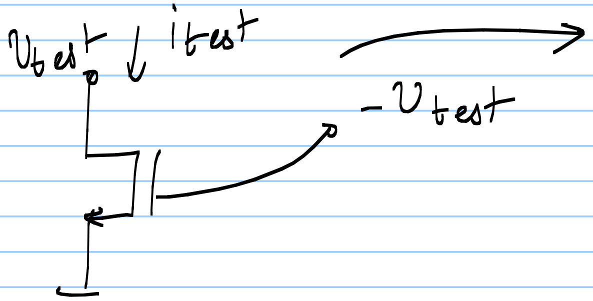
$$V_{icM} = 2V_{GS}\Big|_{1\,mA} = 4V$$

2) $$\frac{v_{out_1} - v_{out_2}}{v_{in_1} - v_{in_2}} = g_m$$

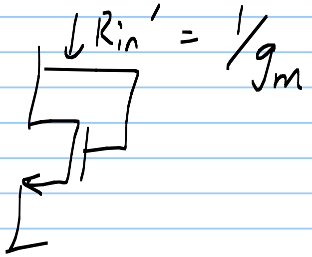


$$\frac{v_o}{v_i} = \frac{g_m R}{1 + g_m R} = \frac{g_m}{g_m + G + g_{ds}}$$





$$R_{in} = \frac{-1}{g_m}$$



$$\frac{v_{od}}{v_{id}} = \frac{g_{m_1}}{g_{m_1} + g_{ds_1} - g_{m_3} + g_{ds_3}} = \frac{g_{m_1}}{2\, g_{ds_1}}$$

3) $\overline{e_n^2}$: calculate $\frac{i_{sc}^2}{|G_m|^2}$ or $\frac{\overline{v_{on}^2}}{|A|^2}$

4) $\delta V_{T_{1-2}} \leq 100mV$ & $\Delta V_{T_{3-4}} \leq 100mV$

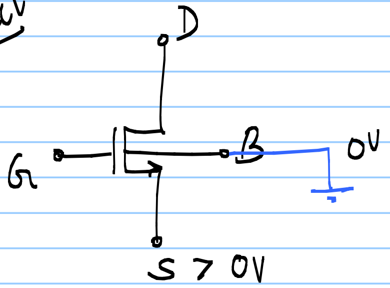
worst case :

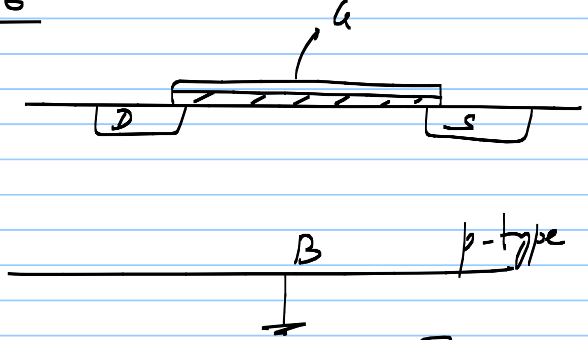
$$\left.\begin{aligned} V_{T_1} &= V_{T_2} + 0.1V \\ V_{T_4} &= V_{T_3} + 0.1V \end{aligned}\right\} \quad \begin{aligned} V_{i_1} &= V_{GS_1} + V_{GS_4} \\ V_{i_2} &= V_{GS_3} + V_{GS_2} \end{aligned}$$

5/3/20

Lec 26

Body Effect

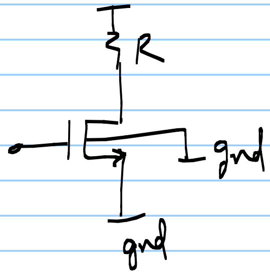




$$V_T = V_{T_0} + \gamma\left[\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F}\right]$$

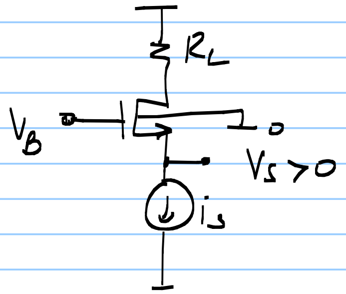
$V_{SB} > 0 \Rightarrow V_T > V_{T_0}$

1)



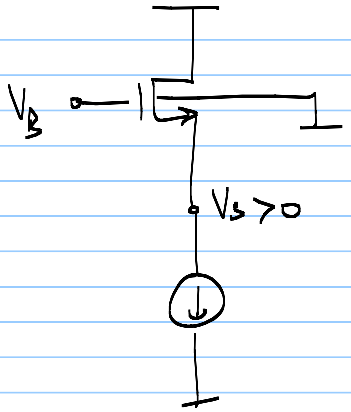
$V_{SB} = 0, \quad V_T = V_{T_0}$

2)



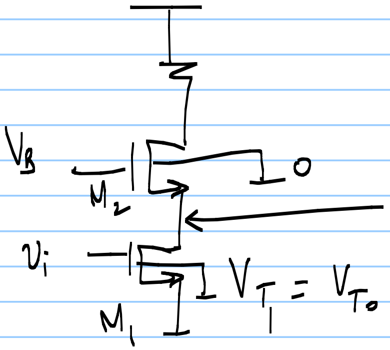
$V_T > V_{T_0}$

3)



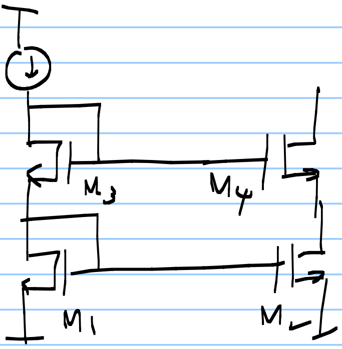
$V_T > V_{T0}$

4)



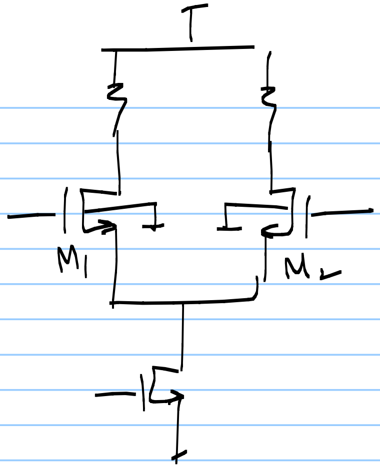
$V_{S2} > 0 \Rightarrow V_{T_2} > V_{T0}$

5)

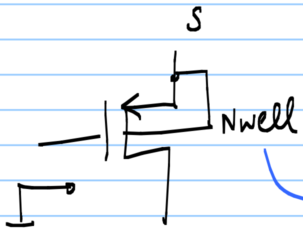


$V_{T_{3,4}} > V_{T_{1,2}}$

6)

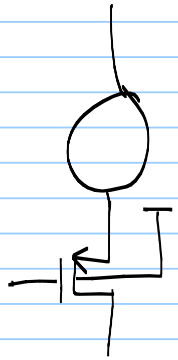


$V_{T_{1,2}} > V_{T_0}$

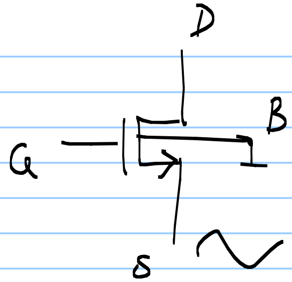


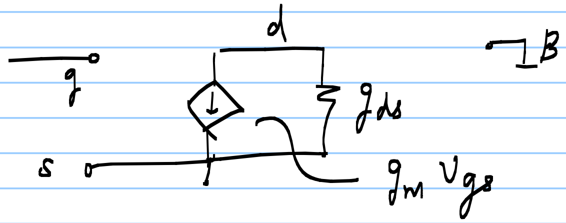
$V_{SB} = 0 \Rightarrow V_{T_p} = V_{T_{p0}}$

has relatively larger parasitic caps



$V_{T_p} > V_{T_{p0}}$





$$g_m = \frac{\partial I_D}{\partial V_{GS}}$$

$$g_{ds} = \frac{\partial I_D}{\partial V_{DS}}$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_T)^2 (1 + \lambda V_{DS})$$

$$\frac{\partial I_D}{\partial V_{SB}} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) \times 2 \left(V_{GS} - V_T\right) \times -1 \cdot \frac{\partial V_T}{\partial V_{SB}} = -g_m \cdot \frac{\partial V_T}{\partial V_{SB}}$$

$$\frac{\partial I_D}{\partial V_{SB}} = \underset{-\partial I_D / \partial V_{GS}}{\underset{\parallel}{\frac{\partial I_D}{\partial V_T}}} \cdot \frac{\partial V_T}{\partial V_{SB}}$$

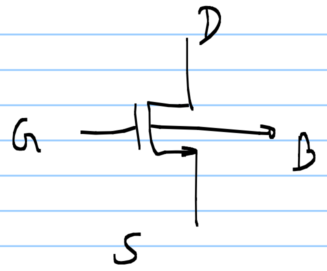
$$V_T = V_{T_0} + \gamma \left[\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F} \right]$$

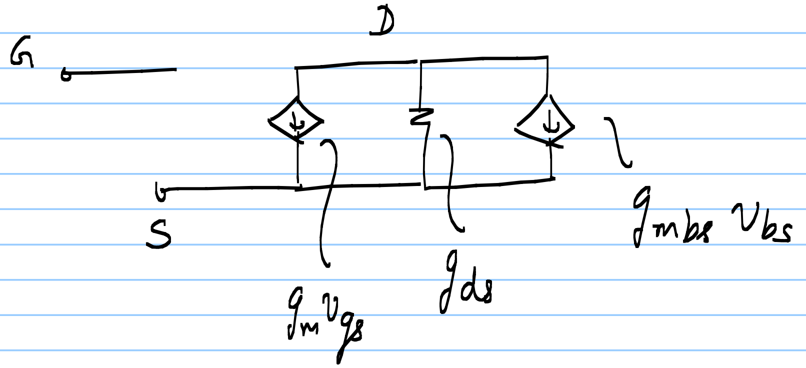
$$\frac{\partial V_T}{\partial V_{SB}} = \frac{\gamma}{2} \cdot \frac{1}{\sqrt{2\phi_F + V_{SB}}}$$

normally $\alpha << 1$

$$\frac{\partial I_D}{\partial V_{SB}} = - \frac{g_m \gamma}{2} \cdot \frac{1}{\sqrt{2\phi_F + \underbrace{V_{SB}}_{\text{DC value}}}} = - \alpha g_m$$

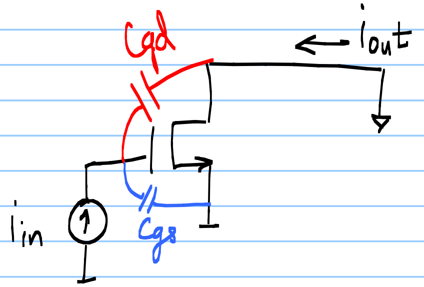
$$g_{mbs} = \frac{\partial I_D}{\partial V_{BS}} = \alpha g_m$$





$$g_m \gg g_{mbs} \gg g_{ds}$$

Transit frequency



$$i_{out} = g_m \cdot v_{gs} = g_m \cdot i_{in} \cdot \frac{1}{j\omega C_{gs}}$$

$$\left|\frac{i_{out}}{i_{in}}\right| = \frac{g_m}{\omega C_{gs}} \rightarrow = 1 \quad @ \quad \omega = \omega_T$$

$$f = f_T$$

$$\omega_T = \frac{g_m}{C_{gs}} \; ; \quad f_T = \frac{g_m}{2\pi C_{gs}}$$

$$\left\{ \text{Note: with } C_{gd}, \quad \omega_T = \frac{g_m}{C_{gs} + C_{gd}} \right\}$$

$$\omega_T = \frac{\mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_T)}{\frac{2}{3} W L C_{ox}} = \frac{3}{2} \mu_n \frac{(V_{GS} - V_T)}{L^2}$$

High speed

1) NMOS ckts ← $\mu_n > \mu_p$

2) Small L { but, leads to lower $g_m r_{ds}$ }

3) larger $(V_{GS} - V_T)$ { but, leads to lower swings }

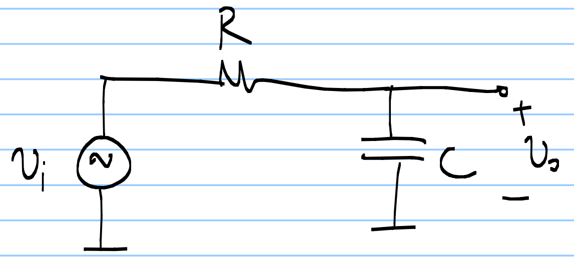
Short-channel effect – Velocity saturation $\quad vel. = \mu_n \mathcal{E}$

$$I_D = \frac{W C_{ox}}{2} (V_{GS} - V_T) \overset{\text{velocity}}{V_{sat}} \quad \text{in saturation region}$$

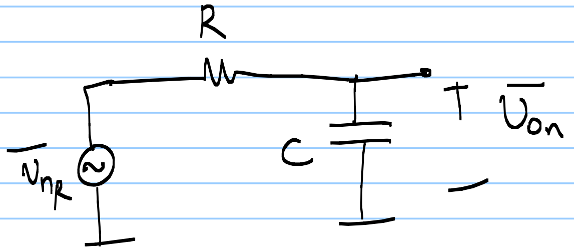
$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \frac{W C_{ox}}{2} \cdot V_{sat}.$$

$$\omega_T = \frac{g_m}{C_{gs}} = \frac{\frac{W C_{ox}}{2} V_{sat}}{\frac{2}{3} W L C_{ox}} = \frac{3}{4} \frac{V_{sat}}{L}$$

Noise in an RC LPF



$$\frac{v_o}{v_i} = \frac{1}{1+sCR}$$



$$\overline{v_{on}^2} = \overline{v_{nR}^2} \cdot \left| \frac{1}{1+j2\pi fCR} \right|^2$$

$$= \frac{4kTR}{1+4\pi^2 f^2 C^2 R^2}$$

Total integrated noise $= \displaystyle\int_0^\infty \frac{4kTR}{1+4\pi^2 f^2 C^2 R^2}\cdot df$

Set $x = 2\pi f CR$

$$= \int_0^\infty \frac{4kTR}{1+x^2}\cdot\frac{dx}{2\pi CR}$$

$$\tan^{-1}x \Big|_0^\infty = \pi/2$$

Total integrated noise $= \dfrac{4kTR}{2\pi CR}\cdot\dfrac{\pi}{2} = \dfrac{kT}{C}$ ← Build LPF with small R, large C keeping RC constant

$$g_m = \frac{2I_D}{V_{ov}} \Rightarrow \frac{g_m}{I_D} = \frac{2}{V_{ov}}$$

assumes pure square law operation

From 17/3/20 :

Tue 3:30 - 4:45pm

Thu 3:30 - 4:45pm

Fri 3:30 - 4:45pm

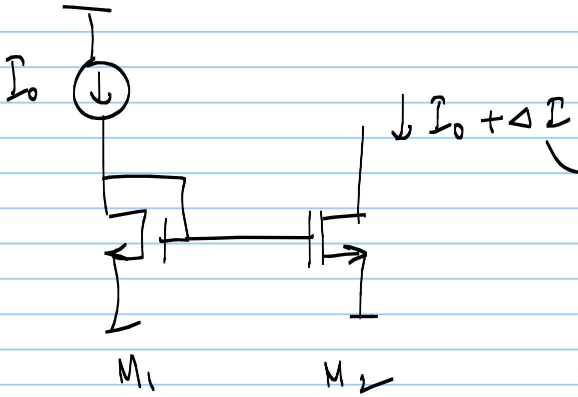
Next week :

Wed (11/3) 3:30 - 4:45pm

Thu (12/3) 3:30 - 4:45pm

11/3/2020

Lec 27



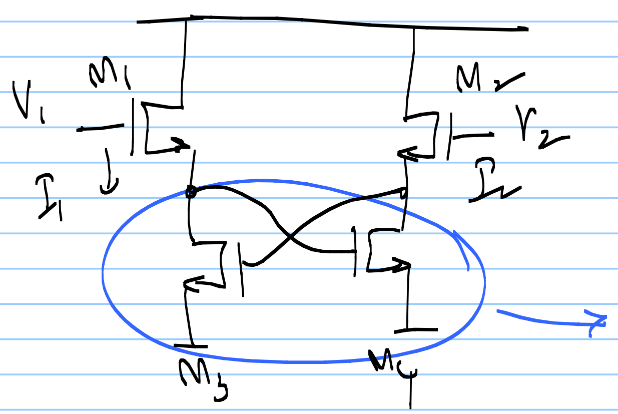
$$\underbrace{\Delta I = -g_{m_2} \cdot \Delta V_T}$$

ΔV_T = small signal

$$V_{T_2} = V_{T_1} + \Delta V_T$$

$$I_D = \frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_T)^2 \leftarrow$$

$$\times\ (1 + \lambda V_{DS})$$



$I_1 = I_2 = 0$

$$V_1 = \overset{V_{T_1} + V_{Dsat_1}}{V_{GS_1}} + \overset{V_{T_2} + V_{Dsat_2}}{V_{GS_4}}$$

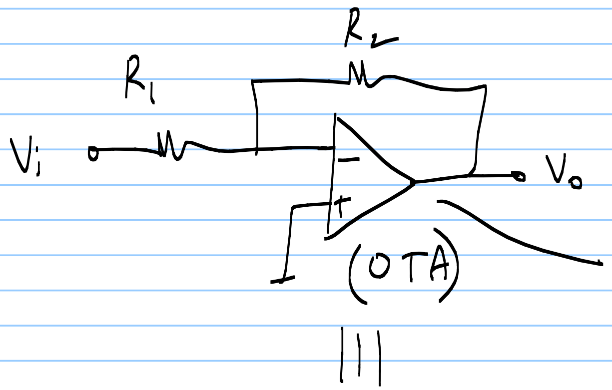
⋮

$$V_2 = V_{GS_2} + V_{GS_3}$$

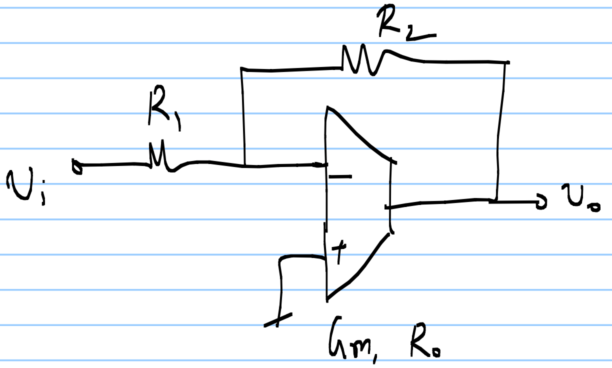
Use CLM in DC eqn or small-signal analysis

Switched capacitor Circuits

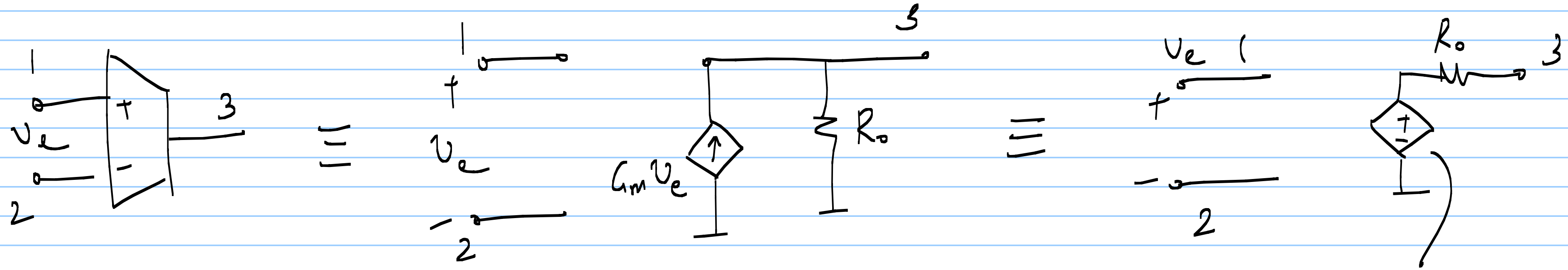
$$\frac{v_o}{v_i} = \frac{-R_2}{R_1}$$



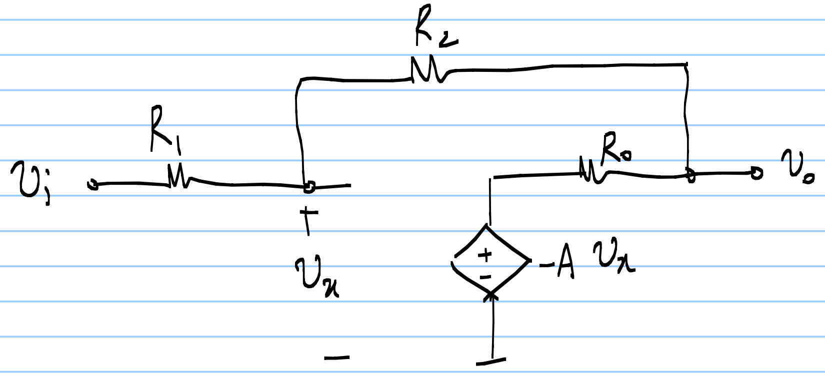
we wanted high gain, ∞ R_{in} & 0 R_{out}



we got high gain, ∞ R_{in}, but large R_{out}



$A \cdot v_e = G_m R_o v_e$



$$\frac{v_o}{v_i} = \frac{-R_2}{R_1} .$$

$$v_x = v_i + \frac{v_o - v_i}{R_1 + R_2} \cdot R_1$$

$$-Av_x - R_o \cdot \left[\frac{v_o - v_i}{R_1 + R_2} \right] = v_o$$

$$-A \left[v_i + \frac{v_o - v_i}{R_1 + R_2} \cdot R_1 \right] - R_o \cdot \left[\frac{v_o - v_i}{R_1 + R_2} \right] = v_o$$

$$\frac{v_o}{v_i} = \frac{-R_2}{R_1} \cdot \frac{1 - \frac{R_o}{AR_2}}{1 + \frac{R_1 + R_2 + R_o}{AR_1}} = \frac{-R_2}{R_1} \cdot \frac{A - \frac{R_o}{R_2}}{1 + A + \frac{R_2}{R_1} + \frac{R_o}{R_1}}$$

Opamp with gain A and $R_o = 0$: $$\frac{v_o}{v_i} = -\frac{R_2}{R_1} \cdot \frac{A}{1+A+\frac{R_2}{R_1}}$$

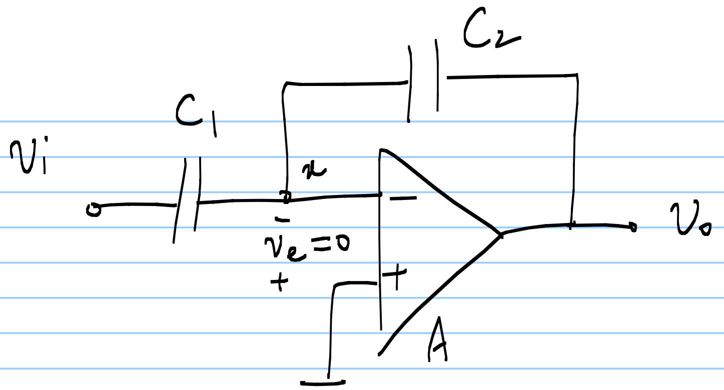
1) R_1 & R_2 load OTA : gain error

choose large R_1 & R_2

2) R_{in} of closed loop amplifier = R_1 : loads previous stage

3) R_1 & R_2 add thermal noise

4) large area for R_1 & R_2, large parasitics



ideal opamp

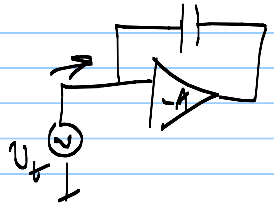
$$\frac{v_o}{v_i} = -\frac{C_1}{C_2}$$

OTA : no steady state current drawn from OTA ✓

$V_{eMin} + 0$
(DC) (ac)

Issues :

1) Gain is a function of freq. ?

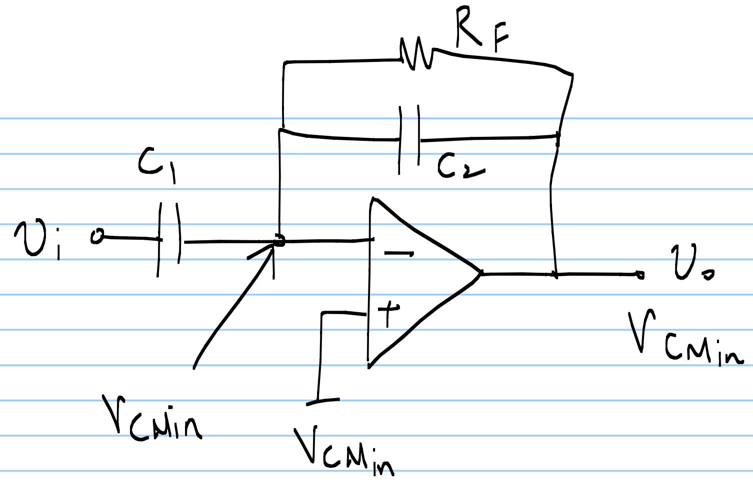


$R_{in} = \infty$ ✓

noise $= 0$ ✓

2) DC gain $= 0$; Use only for ac signals

3) No DC feedback ! V_x is undefined

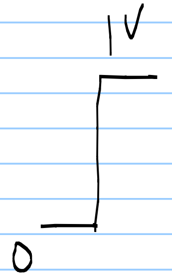


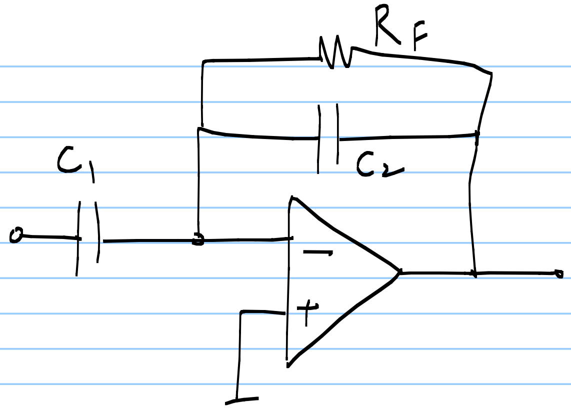
$$\frac{v_o}{v_i}(s) \approx -\frac{R_F \cdot \frac{1}{sC_2}}{R_F + 1/sC_2} \div \frac{1}{sC_1} = -\frac{sC_1R_F}{1 + sC_2R_F}$$

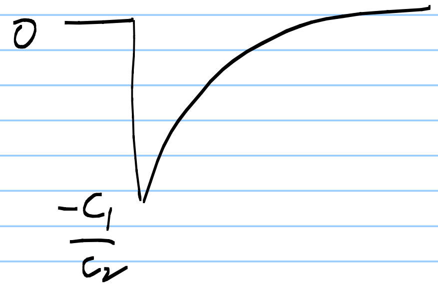
$$= -\frac{sC_1}{\frac{1}{R_F} + sC_2}$$

1) No longer a wideband amplifier

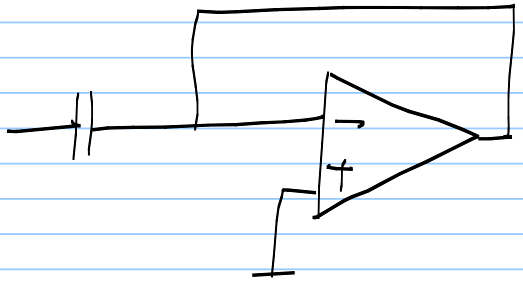
2) Choose large R_F : $\frac{v_o}{v_i} \approx -\frac{C_1}{C_2}$



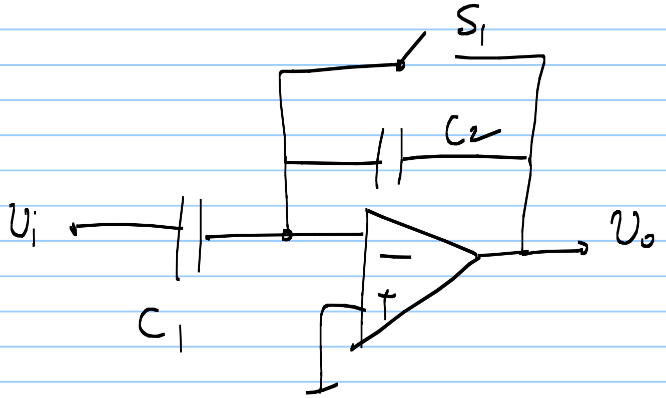




Avoid large R_F!

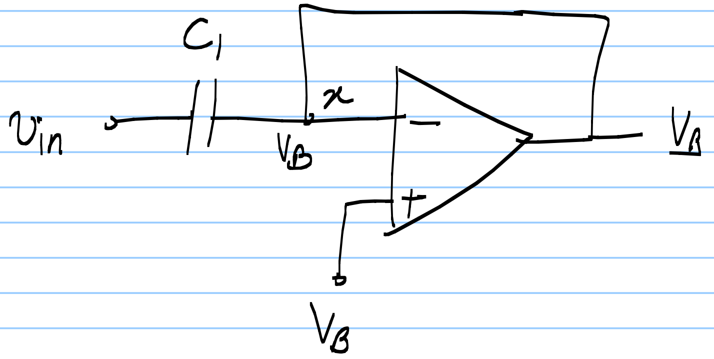


- sets DC f.b.
- but no ac gain



→

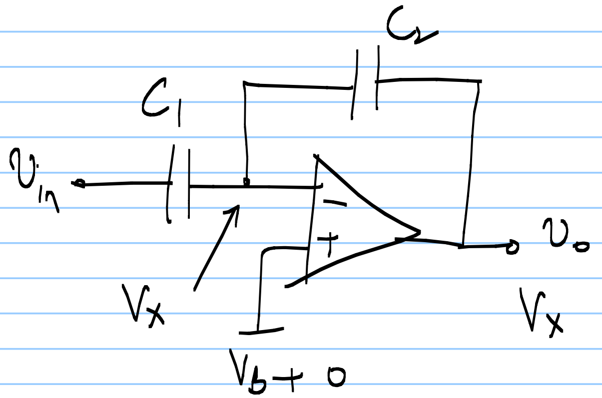
<u>phase1</u> : close S_1



$$v_{C_1} = v_{in}$$

↓

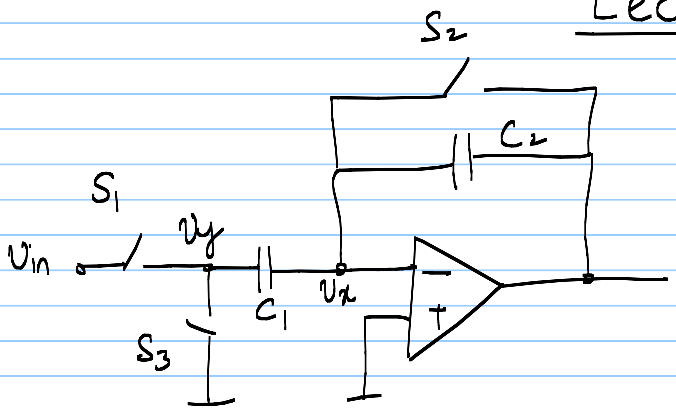
⇐ <u>phase 2:</u> open S_1



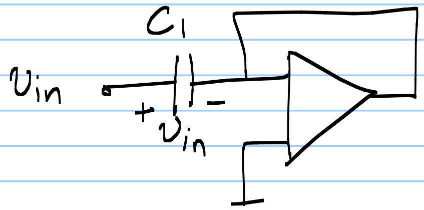
V_x stays the same (charge is held)

12/3/20

Lec 28



phase 1 : close S_1 & S_2, open S_3



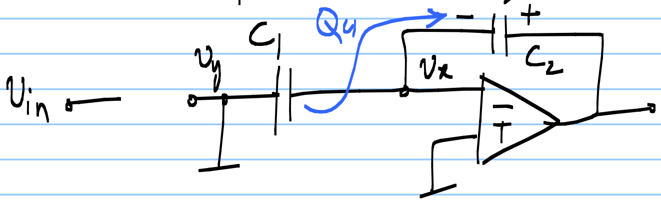
$v_y = v_{in}$

$v_x = 0$

$v_{c_1} = v_{yx} = v_{in}$

"sampling mode"

phase 2 : open S_1 & S_2, close S_3 @ $t = t_0$



$v_y = 0$, $v_x = 0$, $v_{xy} = 0$

at $t = t_0 - \delta t$, $Q_{c_1}(t_0 - \delta t) = c_1 v_{in}(t_0)$

at $t = t_0 + \delta t$, $Q c_1 (t_0 + \delta t) = 0$

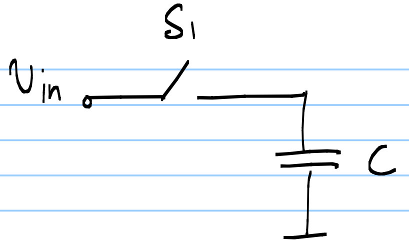
$$Q_{C_2}(t+\delta t) = Q_{C_1}(t-\delta t) = C_1 \, v_{in}(t_0)$$

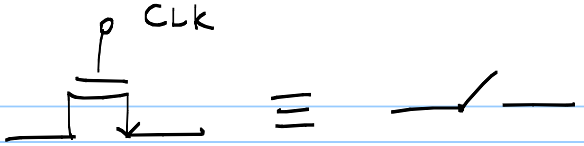
$$v_{C_2} = \frac{C_1}{C_2} v_{in}(t_0)$$

$$v_{out} = \frac{C_1}{C_2} v_{in}(t_0)$$: amplifier with gain $\frac{C_1}{C_2}$

Build discrete time amplifiers using switches, OTAs & capacitors

- → sampling phase ✓
- → amplification phase

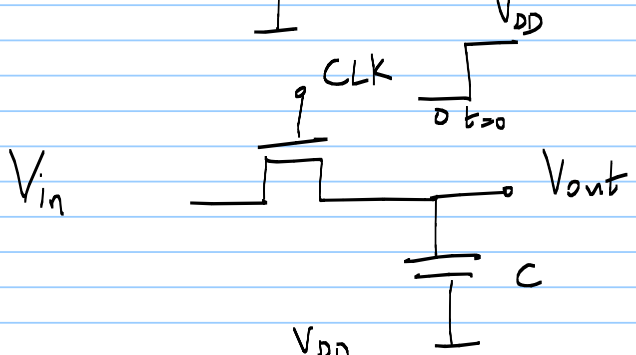




CLK = V_{DD} ⇒ switch is ON

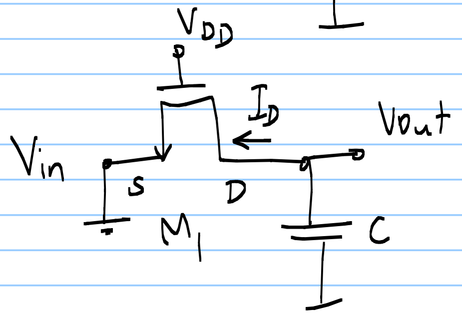
CLK = 0 ⇒ swith is off

Case 1)



$V_c(t=0^-) = V_{DD}$

$V_{in} = 0$



@ $t = 0^+$, $V_{GS} = V_{DD}$, $V_{DS} = V_{DD}$ } MOSFET is in saturation

$\beta = \mu_n C_{ox}\left(\frac{W}{L}\right)$

$$I_D = \frac{\beta}{2}(V_{DD} - V_T)^2 = -C\frac{dV_{out}}{dt}$$

$$V_{out}(t) = V_{DD} - \frac{\beta}{2C}(V_{DD}-V_T)^2 \cdot t$$

@ $t = t_1$, M_1 goes into triode : $V_{out} = V_{DD} - V_T$

$$V_{DD} - V_T = V_{DD} - \frac{\beta}{2C}(V_{DD}-V_T)^2 \cdot t_1$$

<u>$t > t_1$</u> :

$$I_D = \beta\left((V_{DD}-V_T)\cdot V_{out} - \frac{V_{out}^2}{2}\right) = -C\frac{dV_{out}}{dt}$$

$V_{out}(t) = ?$

$$\frac{dV_{out}}{(V_{DD}-V_1)V_{out} - \frac{V_{out}^2}{2}} = \frac{-\beta}{C}dt$$

$$\frac{dV_{out}}{2(V_{DD}-V_T)V_{out} - V_{out}^2} = \frac{-\beta}{2C}\,dt$$

$$\frac{dV_{out}}{\left[2(V_{DD}-V_T) - V_{out}\right]V_{out}} = \frac{-\beta}{2C}\,dt$$

$$\left[\frac{1}{V_{out}} + \frac{1}{2(V_{DD}-V_T) - V_{out}}\right] \cdot \frac{dV_{out}}{2(V_{DD}-V_T)} = \frac{-\beta}{2C}\,dt$$

$$\ln(V_{out}) - \ln\left[2(V_{DD}-V_T) - V_{out}\right] = -\frac{\beta(V_{DD}-V_T)}{C}\cdot t$$

$$\ln\left[\frac{V_{out}}{2(V_{DD}-V_T) - V_{out}}\right] = -\frac{\beta}{C}(V_{DD}-V_T)\cdot t$$

$$\frac{V_{out}}{2(V_{DD}-V_T)-V_{out}} = \exp\left(\frac{-\beta}{C}(V_{DD}-V_T)\cdot t\right)$$

$$V_{out} = 2(V_{DD}-V_T)\cdot\exp\left(\frac{-\beta}{C}(V_{DD}-V_T)\cdot t\right) - V_{out}\cdot\exp\left(\frac{-\beta}{C}(V_{DD}-V_T)\cdot t\right)$$

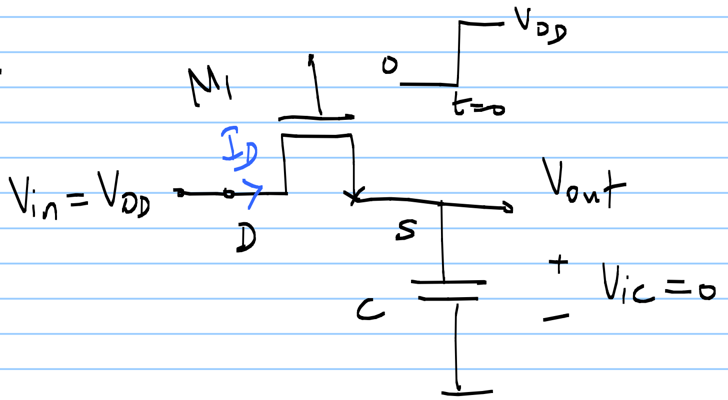
$\underline{t > t_1}$

$$V_{out}(t) = \frac{2(V_{DD}-V_T)\cdot\exp\left(-\beta/C\,(V_{DD}-V_T)\cdot t\right)}{1+\exp\left(\frac{-\beta}{C}(V_{DD}-V_T)\cdot t\right)}$$

replace with $t-t_1$

$$@\ t=\infty : \quad V_{out}(\infty) = 0$$

Case 2:



$$V_{GS} = V_{DD} - V_{out}$$

at $t = 0^+$: M_1 is in saturation

$$I_D = \frac{\beta}{2}\left(V_{DD} - V_{out} - V_T\right)^2 = \frac{C\,dV_{out}}{dt}$$

$$V_{out}(t) = ?$$

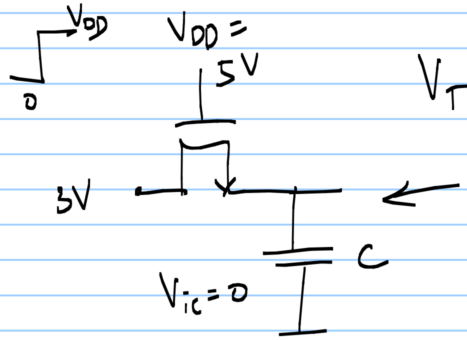
$$\frac{dV_{out}}{\left(V_{DD} - V_{out} - V_T\right)^2} = \frac{\beta}{2C}\,dt$$

$$\left.\frac{1}{\left(V_{DD} - V_{out} - V_T\right)}\right|_0^{V_{out}} = \left.\frac{\beta}{2C}\,t\,\right|_0^{t}$$

$$\frac{1}{V_{DD}-V_{out}-V_T} - \frac{1}{V_{DD}-V_T} = \frac{\beta \cdot t}{2C}$$

$$V_{out} = V_{DD} - V_T - \frac{1}{\frac{\beta t}{2C} + \frac{1}{V_{DD}-V_T}}$$

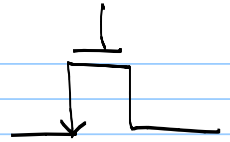
@ $t=\infty$: $V_{out} = V_{DD} - V_T$



$V_T = 1V$

$V_{out}(\infty) = 3V$

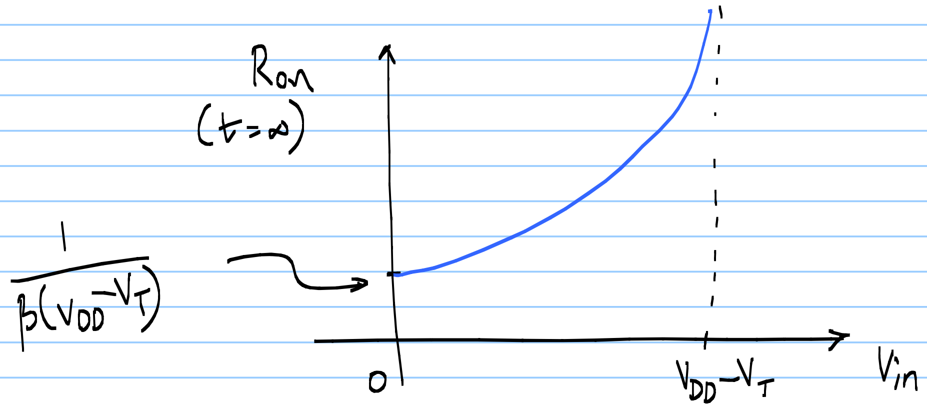
@ $t=0^+$, M_1 is in triode

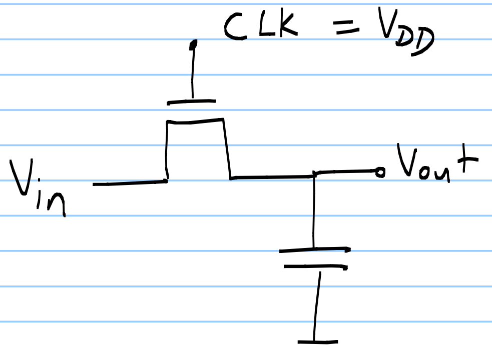


Ron in Triode

$$I_D = \beta\left((V_{GS}-V_T)V_{DS} - \frac{V_{DS}^2}{2}\right)$$

$$R_{on} = r_{ds} = \frac{1}{\partial I_D/\partial V_{DS}} = \frac{1}{\beta(V_{GS}-V_T-V_{DS})}$$





Case 1 : $V_{in} = 0$

Case 2 : $V_{in} = V_{DD}$

Case 3 : $V_{in} = 0 < V_{in} < V_{DD}$

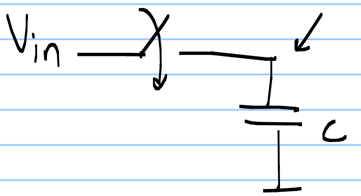
Case 4 : $V_{in} = V_{DD} - V_T$

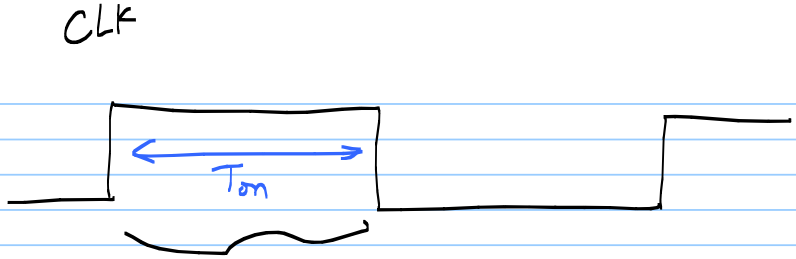
$V_{out}(\infty) = V_{DD}-V_T$
$V_{DS}(\infty) = 0$
$V_{GS}(\infty) = V_T$

Why R_{on} $(t=\infty)$ ←

$V_{out}(\infty) = V_{in}$

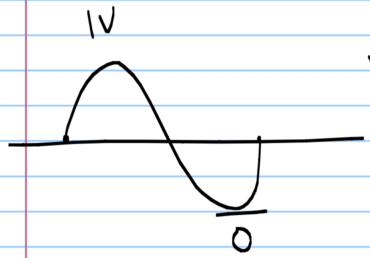
Sampler

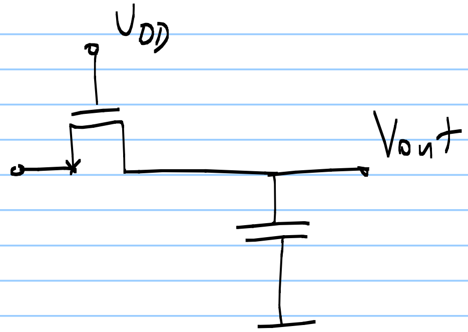




sampling phase

within T_{on}, we want $V_{out} = V_{in} - \underbrace{\delta V}_{\text{precision specification}}$





Note Title 24-03-2020

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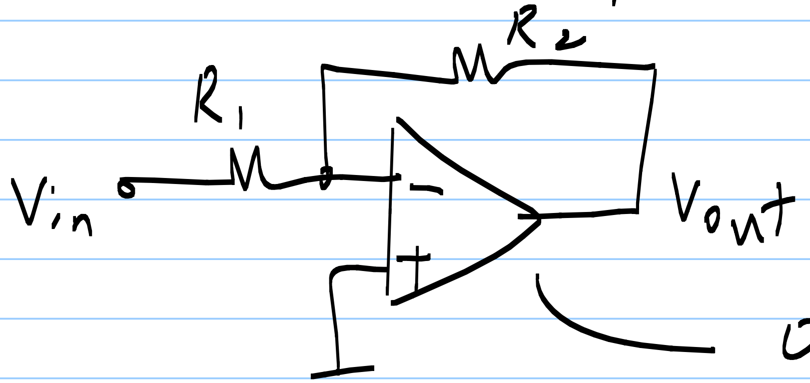
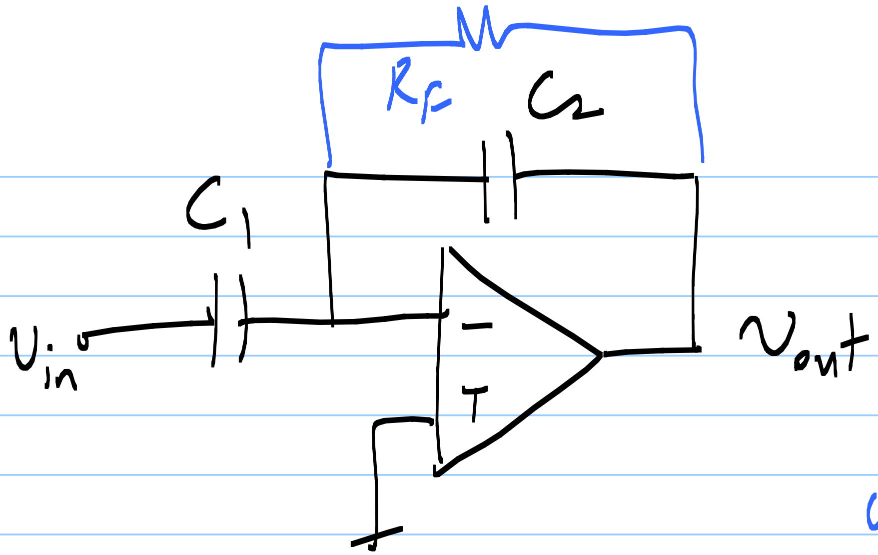
Switched Capacitor Circuits



OTA, R_{out} high

$R_{in} \neq \infty$ (loading on previous stage)

R_1, R_2 – thermal noise

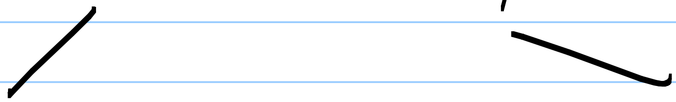


choose large $R_F >> \frac{1}{s C_2}$

$$\frac{V_o}{V_i} = -\frac{C_1}{C_2}$$

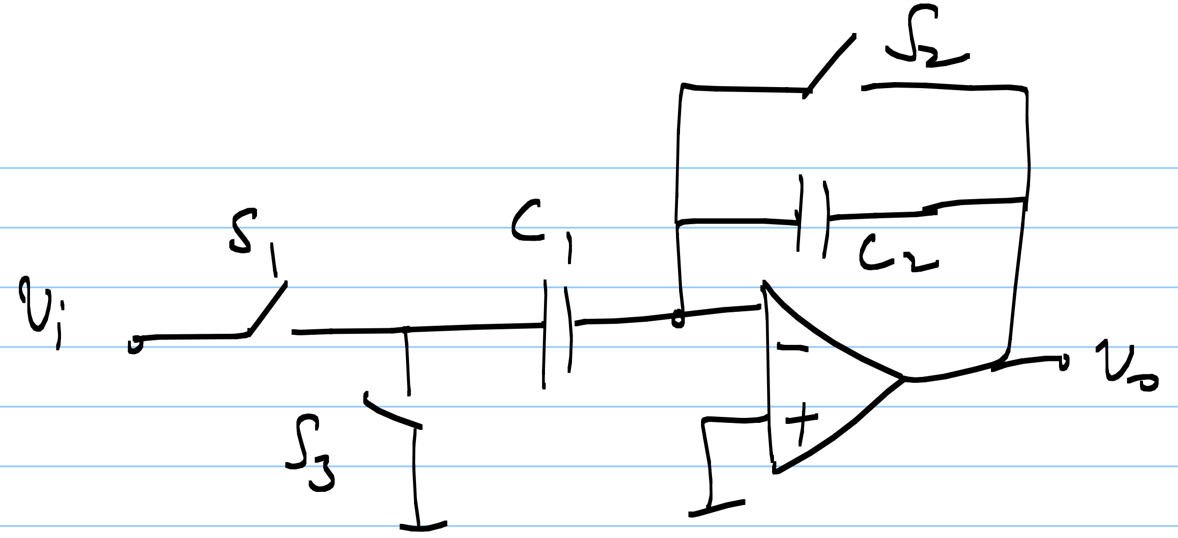
* No DC feedback

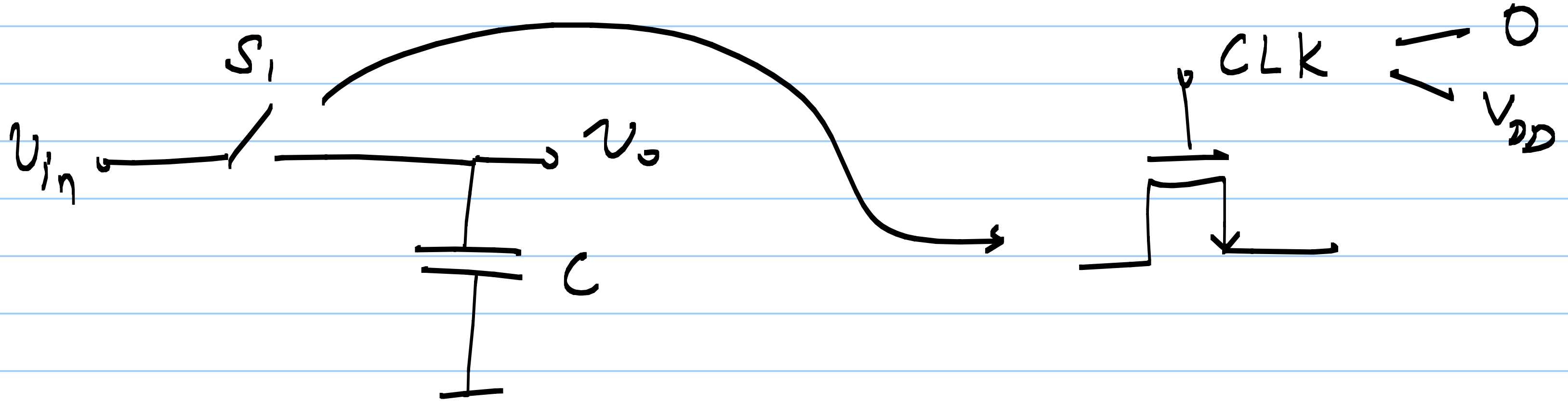
Discrete time amplification

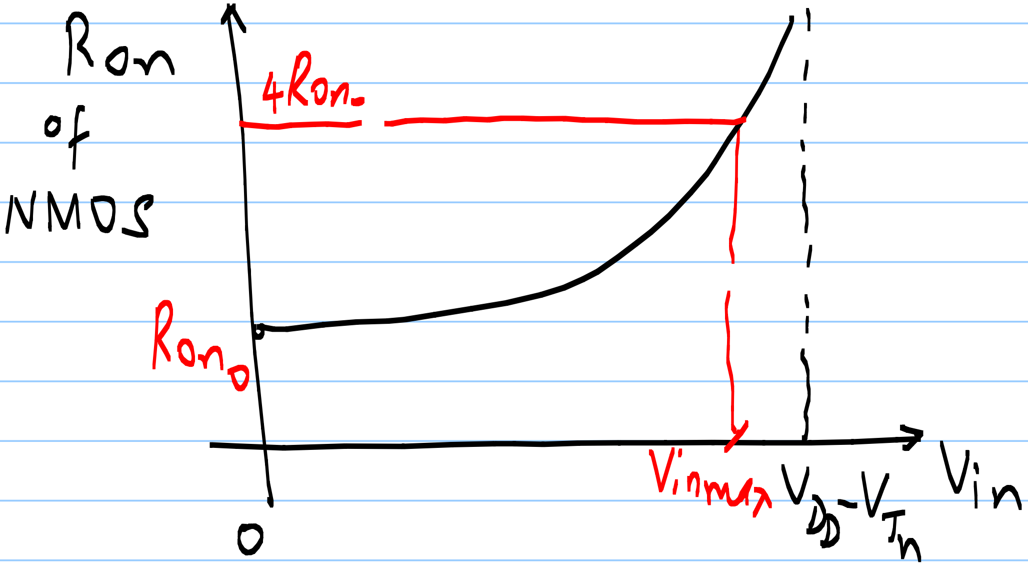


sampling phase

amplification phase

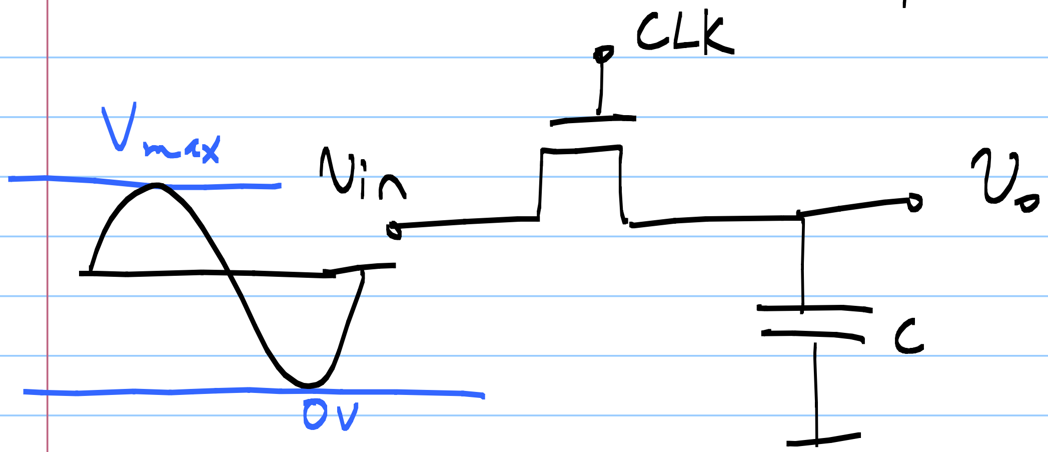

S₂
S₁
C₁
C₂
v_i
S₃
−
+
v_o


S₁
v_in
v_o
C
CLK
0
V_DD



$$\beta_n = \mu_n C_{ox}\left(\frac{W}{L}\right)_n$$

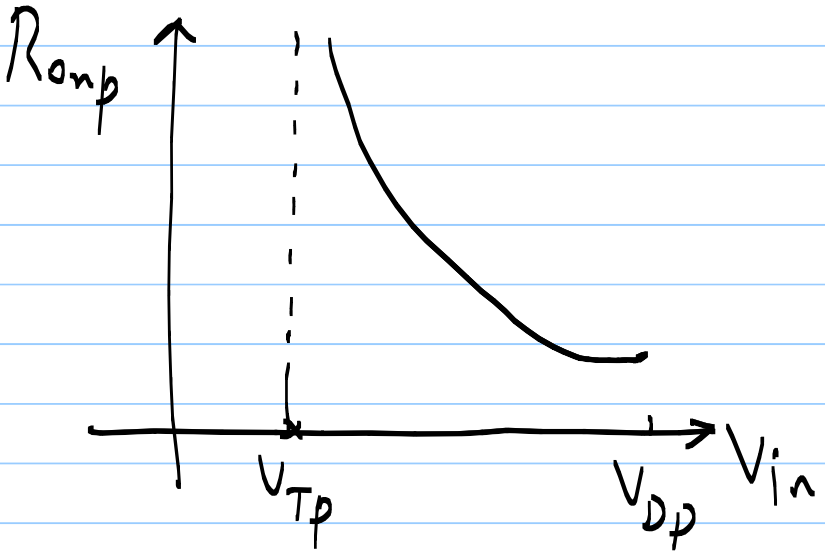
$$R_{on} = r_{ds} = \frac{1}{\beta_n\left(V_{GS}-V_T\right)}$$

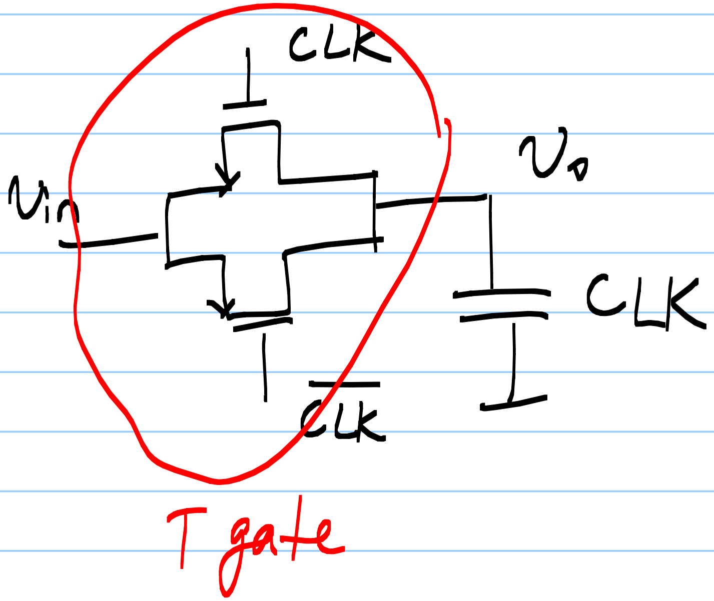


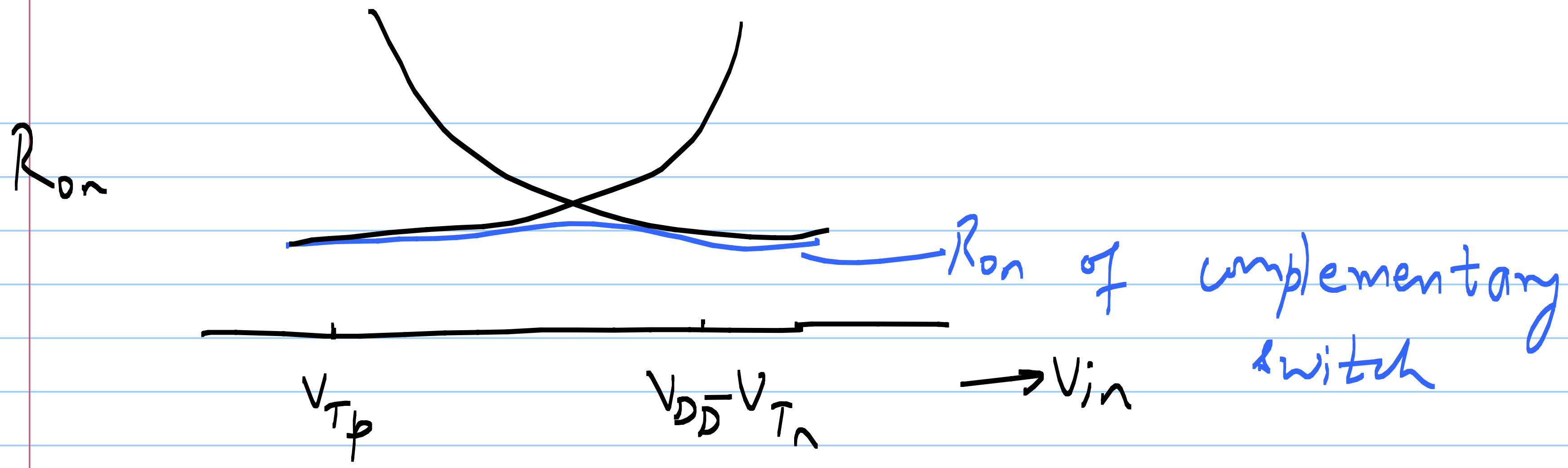
$$R_{max} = 4x\ R_{on}\left(V_{in}=0\right)$$

$$\frac{1}{\beta_n(V_{DD}-V_{in_{max}}-V_{Tn})}=\frac{4}{\beta_n(V_{DD}-V_{Tn})}$$

$$\Rightarrow \quad V_{in_{max}}=\frac{3}{4}(\underbrace{V_{DD}}_{5V}-\underbrace{V_{Tn}}_{1V}) \approx 3V$$







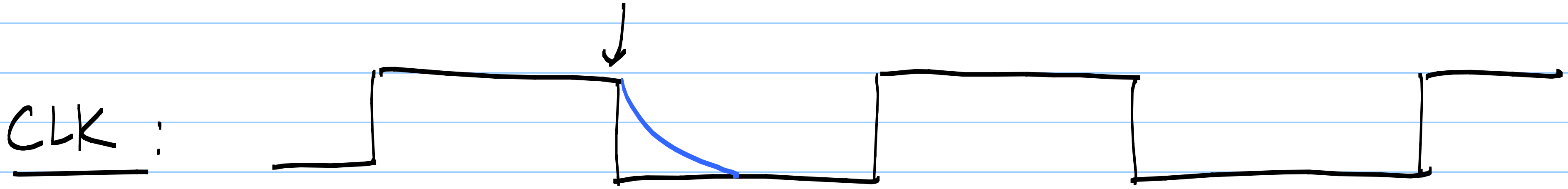
$$R_{on} = R_{on_n} \| R_{on_p}$$

$$= \frac{1}{\beta_n \left(V_{DD} - V_{in} - V_{T_n}\right)} \Big\| \frac{1}{\beta_p \left(V_{in} - V_{T_p}\right)}$$

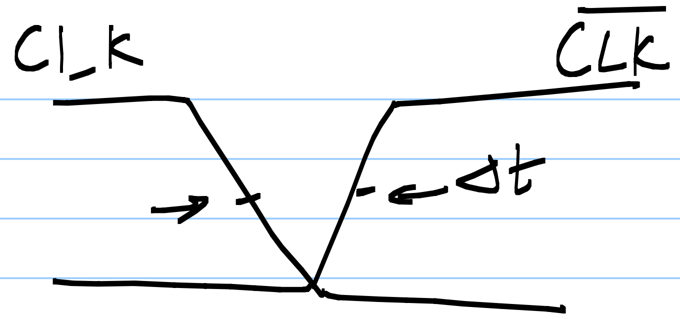
$$= \frac{1}{\beta_n\left(V_{DD} - V_{T_n}\right) - \beta_p\left(V_{T_p}\right) - \underline{\left(\beta_n - \beta_p\right) \cdot V_{in}}}$$

If $\beta_n = \beta_p$ i.e. $\mu_n C_{ox}\left(\frac{W}{L}\right)_n = \mu_p C_{ox}\left(\frac{W}{L}\right)_p$

$\Rightarrow R_{on}$ is independent of V_{in}

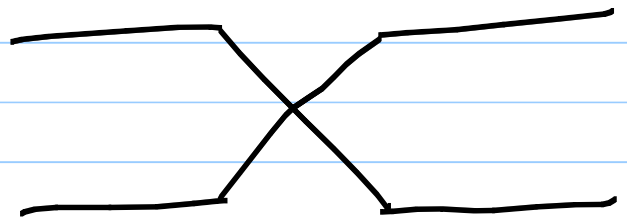


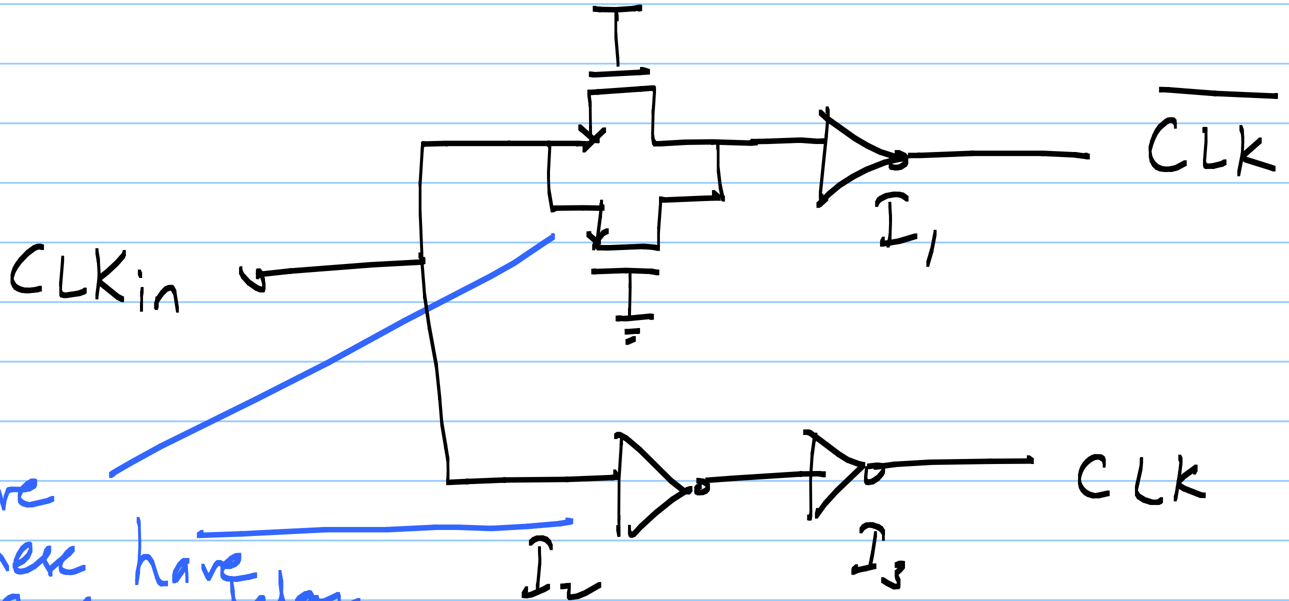
Higher speed → $\underbrace{\text{low } R_{on}}_{\text{large } (W/L)}$ and/or C



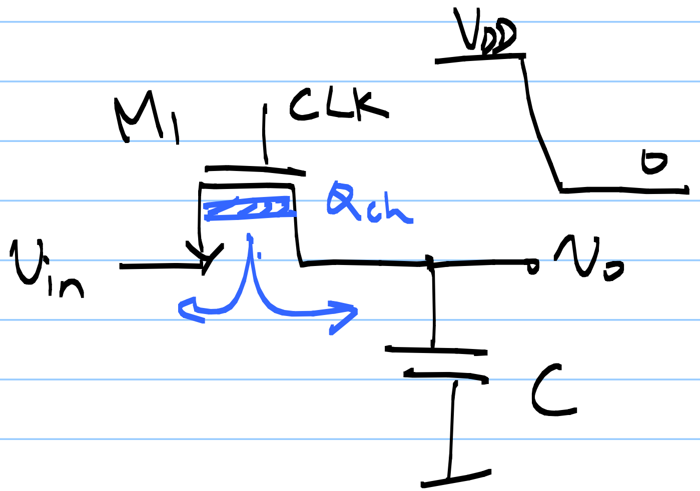
NMOS turns off first

Good clock signals





ensure that these have same delay



$$Q_{ch} = WL\,C_{ox}\,(V_{DD} - V_{in} - V_{Tn})$$

estimate $\frac{Q_{ch}}{2}$ each goes into V_{in} & V_{out} nodes.

$$\Delta V_{out} = \frac{WL\,C_{ox}\,(V_{DD} - V_{in} - V_{Tn})}{2C}$$

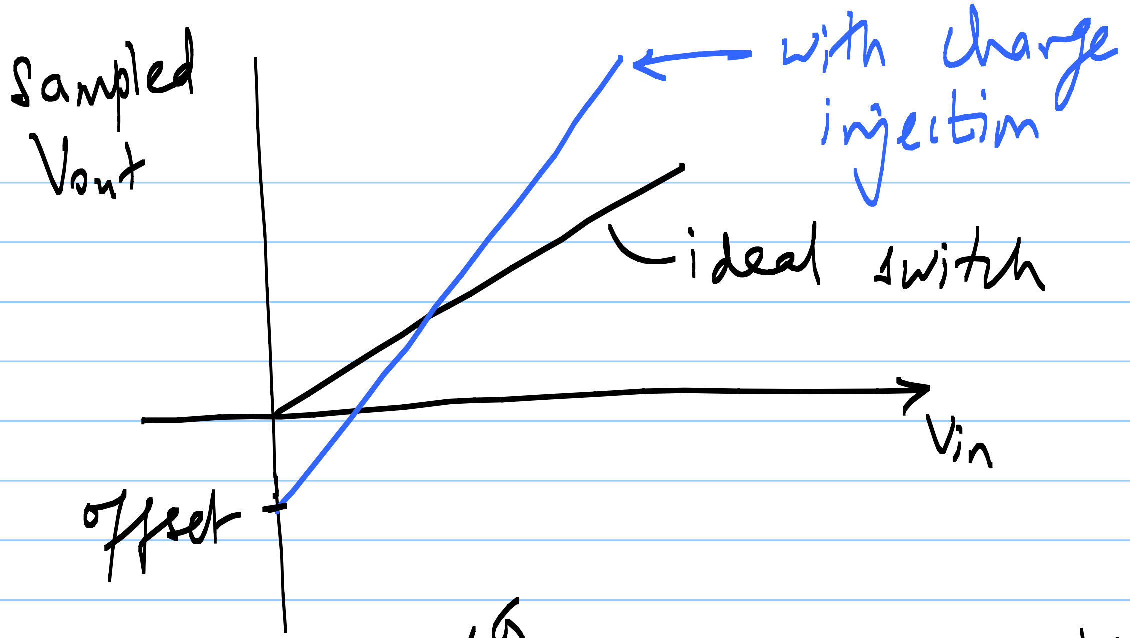
Worst case: all of Q_{ch} goes into C

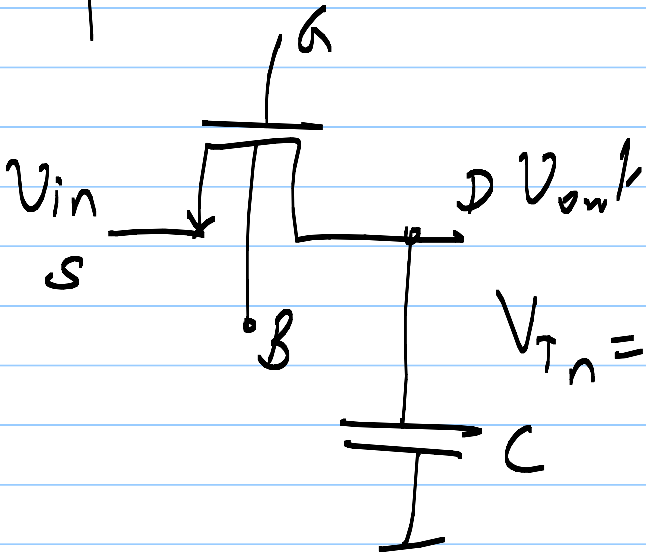
$$\Delta V_{out} = \frac{WLC_{ox}\left(V_{DD}-V_{in}-V_{Tn}\right)}{C}$$

$$V_{out} = V_{in} - \Delta V_{out}$$

$$= V_{in} - \frac{WLC_{ox}\left(V_{DD}-V_{in}-V_{Tn}\right)}{C}$$

$$= V_{in}\underbrace{\left(1+\frac{WLC_{ox}}{C}\right)}_{\text{gain error}} - \underbrace{\frac{WLC_{ox}}{C}\left(V_{DD}-V_{Tn}\right)}_{\text{offset}}$$





$\Longrightarrow$ V_{Tn} is not constant

$$V_{Tn} = V_{Tno} + \gamma\left(\sqrt{2\phi_B + \underbrace{V_{SB}}_{V_{in}}} - \sqrt{2\phi_B}\right)$$

$$V_{out} = V_{in} - \frac{WLC_{ox}}{C}\left[V_{DD} - V_{in} - V_{To} - \gamma\sqrt{2\phi_B + V_{in}} + \gamma\sqrt{2\phi_B}\right]$$

$$= V_{in}\underbrace{\left(1+\frac{WLC_{ox}}{C}\right)}_{\text{gain error}} + \gamma\,\frac{WLC_{ox}}{C}\cdot\underbrace{\sqrt{2\phi_B + V_{in}}}_{\text{non-linearity}}$$

$$\underbrace{-\,\frac{WLC_{ox}}{C}\left(V_{DD}-V_{Tn_0}+\gamma\sqrt{2\phi_B}\right)}_{\text{ffset}}$$

$$\text{FoM} \quad F = \frac{1}{\tau\cdot\Delta V}$$

$$\tau = R_{on}\cdot C = \frac{C}{\beta_n\left(V_{DD}-V_{in}-V_{Tn}\right)}$$

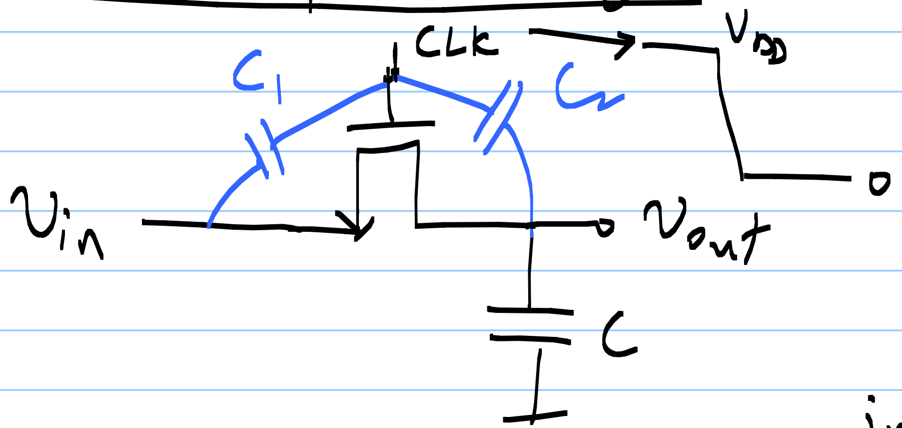
$$\Delta V = \frac{WLC_{ox}}{C}\left(V_{DD}-V_{in}-V_{Tn}\right)$$

$$\tau\cdot\Delta V = \frac{\cancel{C}}{\mu_n C_{ox}\left(\frac{W}{L}\right)\cancel{\left(V_{DD}-V_{in}-V_{Tn}\right)}}\cdot\frac{WLC_{ox}}{\cancel{C}}\cancel{\left(V_{DD}-V_{in}-V_{Tn}\right)}$$

$$F = \frac{1}{\tau \cdot \Delta V}$$

$$= \frac{\mu_n}{L^2}$$

Clock feedthrough



$$\Delta V_{out} = \frac{V_{DD} \cdot C_2}{C + C_2}$$

independent of V_{in}

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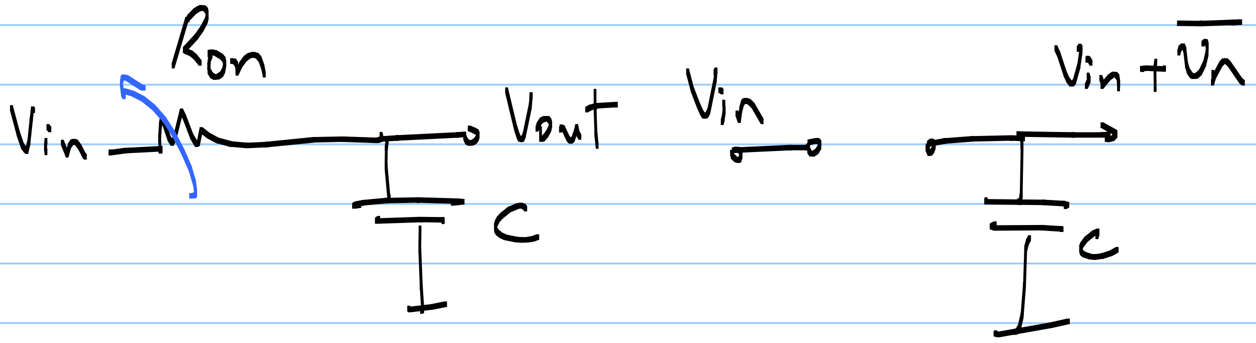
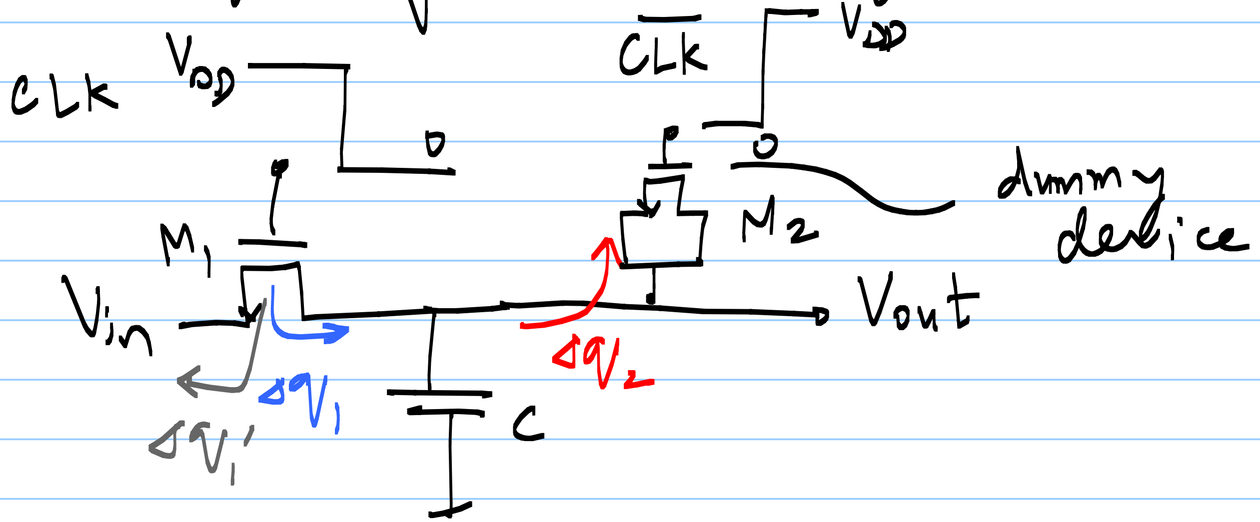


integrated $\overline{v_{on}^2} \approx \frac{kT}{C}$

high precision ⇒ low noise ⇒ C should be as large as possible

⇒ loading in preceding circuit

Charge injection – mitigation

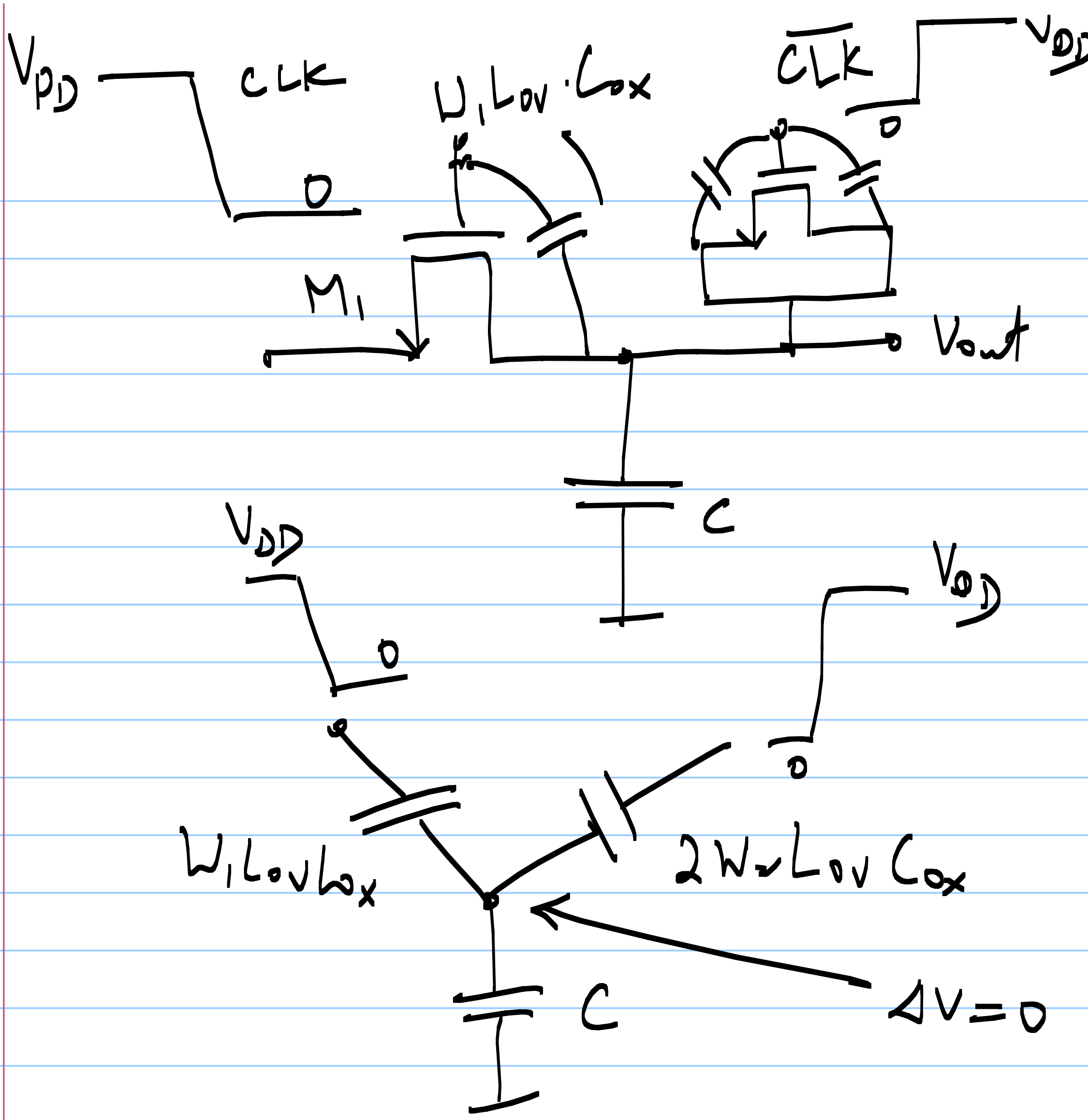


$$\Delta Q_1 = \frac{W_1 L_1 C_{ox}}{2} \left(V_{DD} - V_{in} - V_{Tn_1}\right)$$

$$\Delta Q_2 = W_2 L_2 C_{ox} \left(V_{DD} - V_{in} - V_{Tn_2}\right)$$

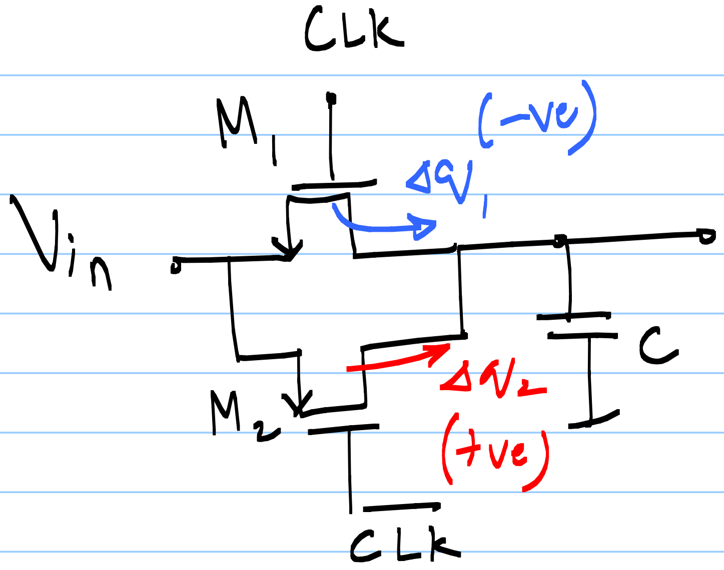
$$L_1 = L_2, \quad W_2 = 0.5 W_1 \Rightarrow \Delta Q_2 = \Delta Q_1$$

* Will work well only if $\Delta Q_1 = \Delta Q_1'$


V_DD
CLK
0
W_1 L_ov · C_ox
$\overline{CLK}$
V_DD
0
M_1
V_out
C
V_DD
0
V_DD
0
W_1 L_ov C_ox
2W_2 L_ov C_ox
C
ΔV=0

$2W_2 = W_1$

⇒



$$\Delta Q_1 = W_1 L_1 C_{ox} \left(V_{DD} - V_{in} - V_{Tn}\right)$$

$$\Delta Q_2 = W_2 L_2 C_{ox} \left(V_{in} - V_{Tp}\right)$$

for charge cancellation, $\Delta Q_1 = \Delta Q_2$

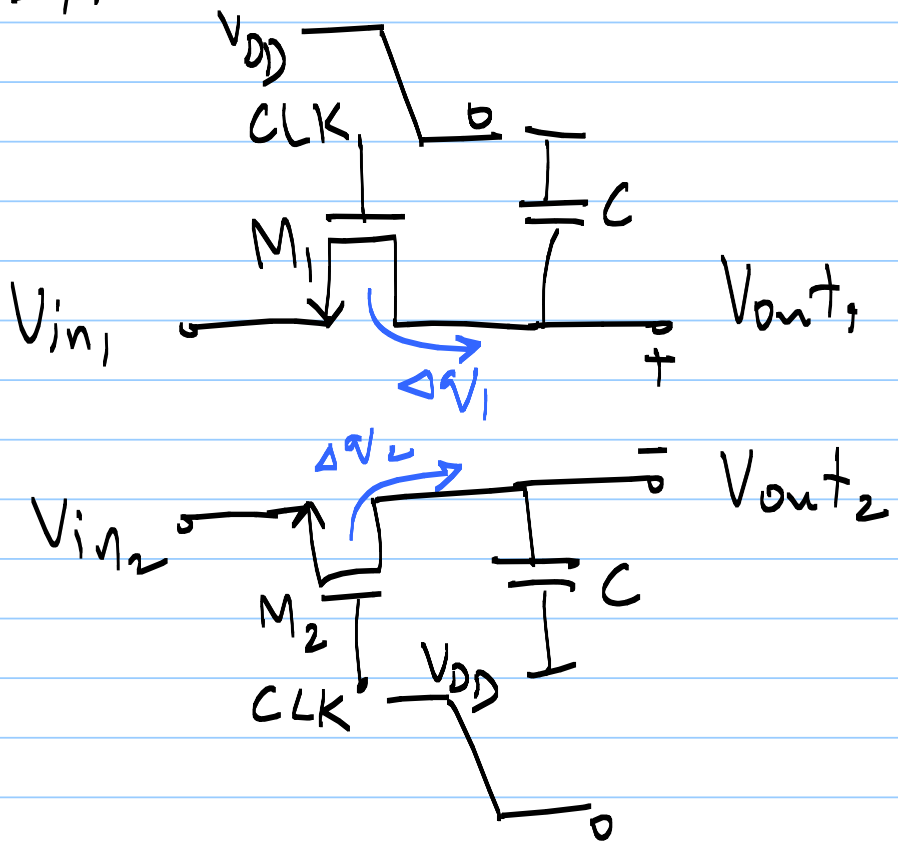
$$W_1 L_1 C_{ox} \left(V_{DD} - V_{in} - V_{Tn}\right) = W_2 L_2 C_{ox} \left(V_{in} - V_{Tp}\right)$$

⇒ valid for only one value of V_{in}

* CLK feedthrough – does not cancel completely

$L_{ovp} \neq L_{ovn}$

Differential Circuit



$$\Delta Q_1 = WL\,C_{ox}\left(V_{DD} - V_{in_1} - V_{T_n}\right)$$

$$\Delta Q_2 = WL\,C_{ox}\left(V_{DD} - V_{in_2} - V_{T_n}\right)$$

We want $\Delta Q_1 = \Delta Q_2$ for diff'l. charge cancellation

$\Delta Q_1 = \Delta Q_2$ only if $V_{in_1} = V_{in_2}$ {which is not true for diff'l. opern}

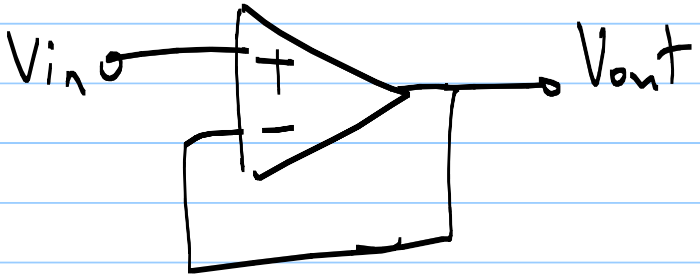
$$\Delta q_1 - \Delta q_2 = WLC_{ox}\left[(V_{in_2}-V_{in_1}) + (V_{Tn_2}-V_{Tn_1})\right]$$

$$= WLC_{ox}\left[\underbrace{(V_{in_2}-V_{in_1})} + \gamma\left(\sqrt{2\phi_f+V_{in_2}} - \sqrt{2\phi_f+V_{in_1}}\right)\right]$$

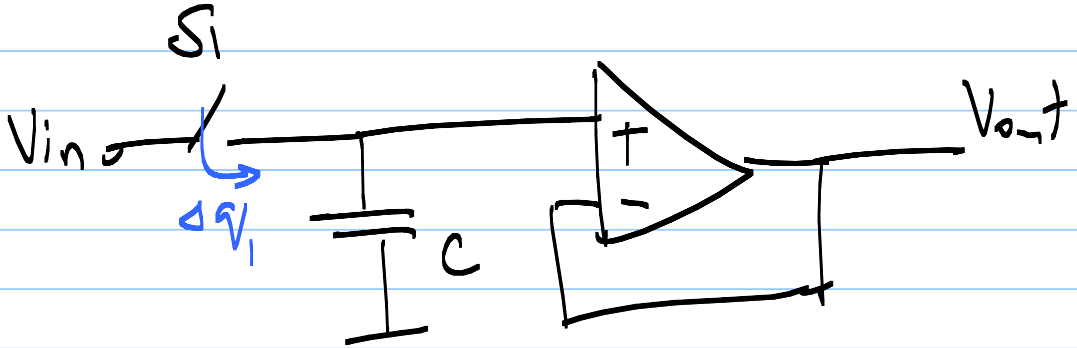
effects get cancelled

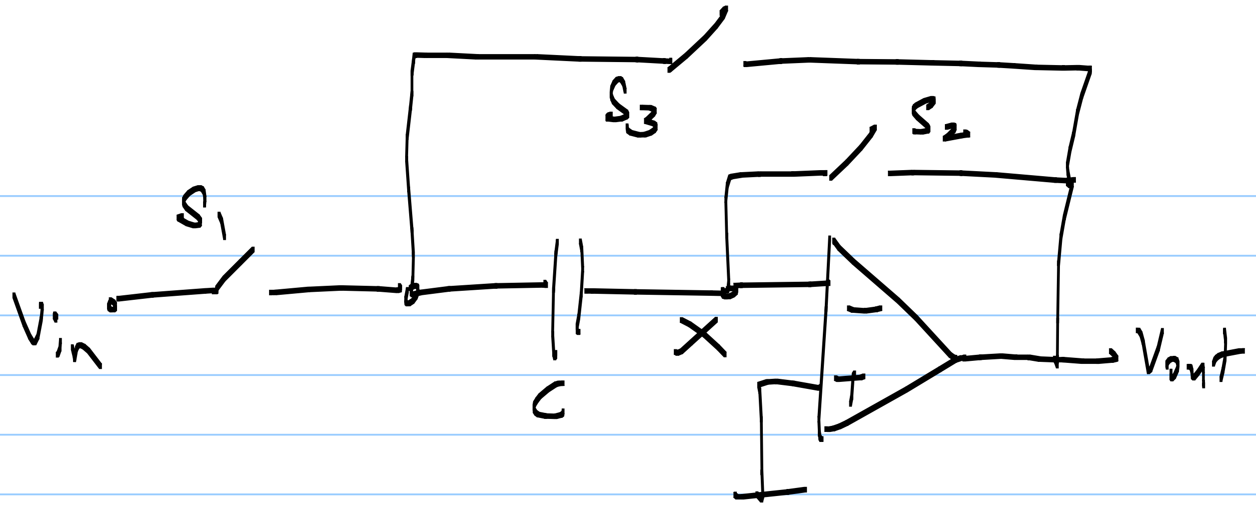
SC amplifiers

1) Unity gain buffer

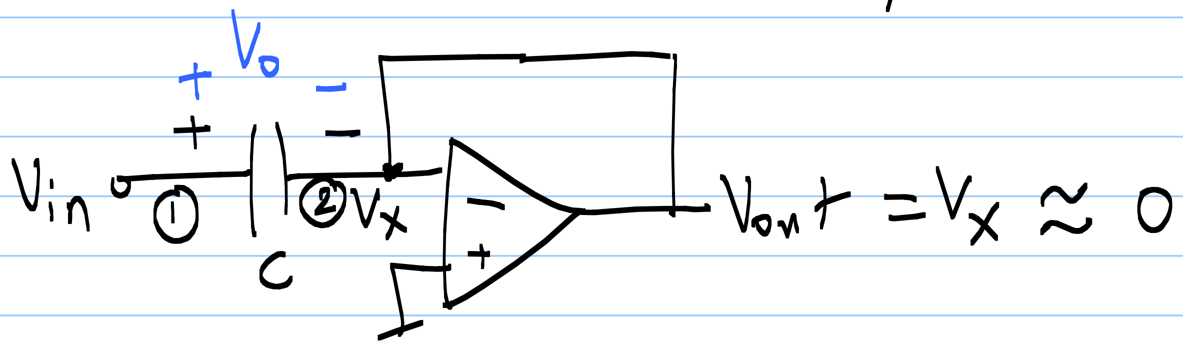


CT version





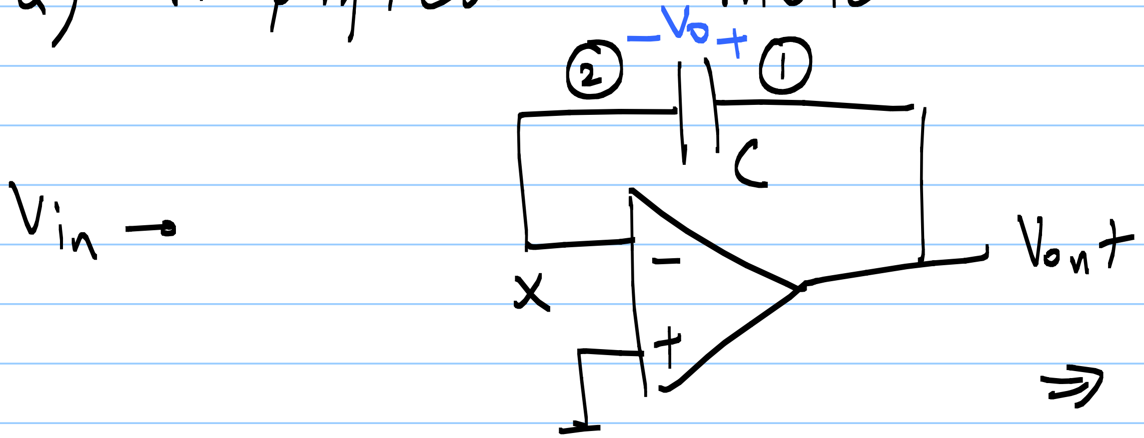
1) sampling mode: S_1, S_2 on;
S_3 off



$V_{12} = V_c$ will track V_{in}

@ $t = t_0$, open S_1 & S_2 } say
close S_3 } $V_{in} = V_0$

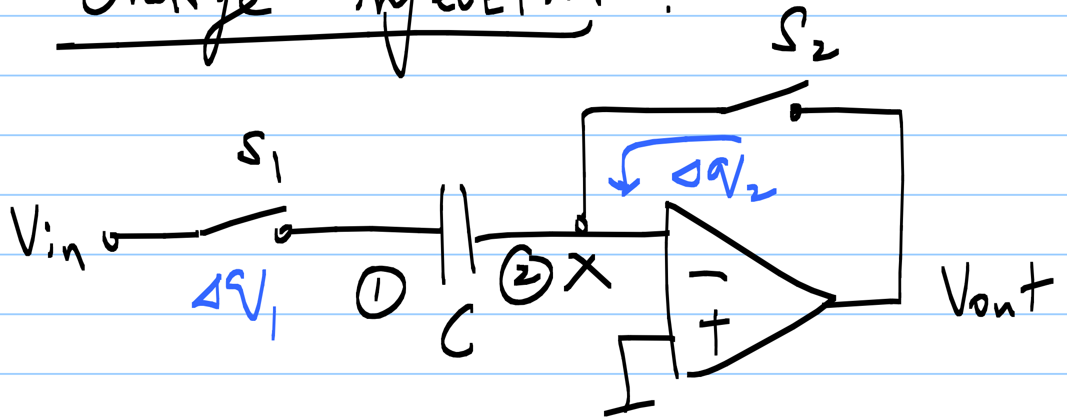
2) Amplification mode



$V_X \approx 0$ due to ac −ve f.b. through C

$\Rightarrow V_{out} = V_o$

Charge injection :

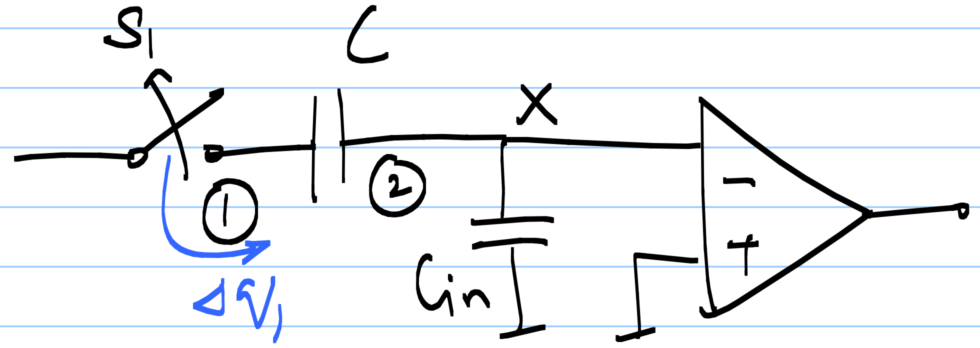


turn off S_2 slightly before S_1

$\Delta V_c = \dfrac{\Delta q_2}{C}$ → independent of V_{in} because X is a virtual gnd

$\Delta q_2 = WLC_{ox}(V_{DD} - V_X - V_{T_n})$ ⇒ leads to offset

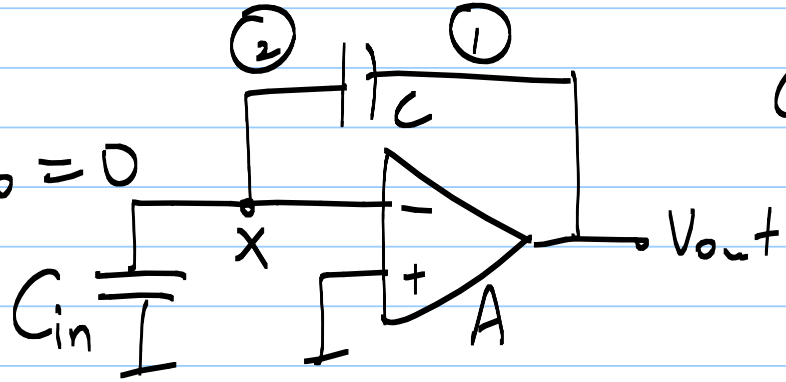
⇒ Can be tackled using differential operation



C & Cin both have charges ΔQ_1

$\Downarrow$ S_3 closed

assume $V_{in}(t_0) = V_0 = 0$



$$C_{in}V_x - (V_{out} - V_x)C = 0 \quad — ①$$

$$V_x = -\frac{V_{out}}{A} \quad — ②$$

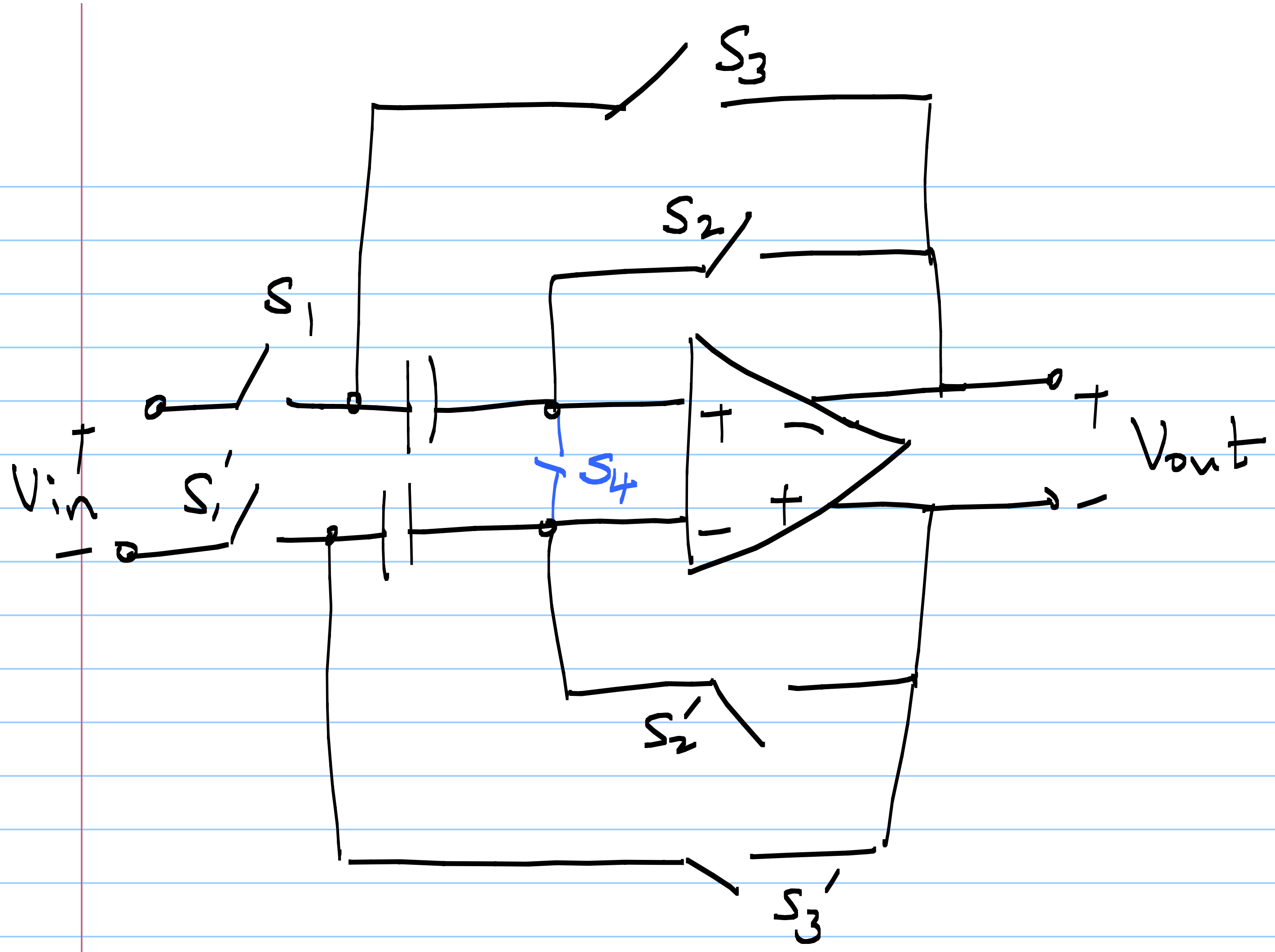
①: $+(C_{in}+C)V_x - V_{out}C = 0$

$$-(C_{in}+C)\cdot\frac{V_{out}}{A} - V_{out}C = 0$$

$\Rightarrow V_{out} = 0$ (independent of δq_{V_1})

<u>S_3</u> turns on

$\Rightarrow$ channel charge needs to come from somewhere


S_3
S_2
S_1
$+$
V_{in}
$-$
S_1'
S_4
$+$
$-$
$-$
$+$
$+$
V_{out}
$-$
S_2'
S_3'

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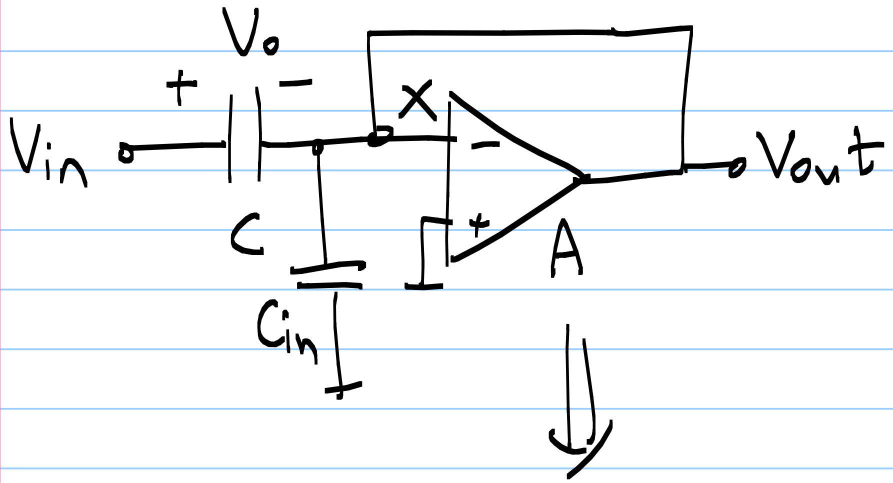
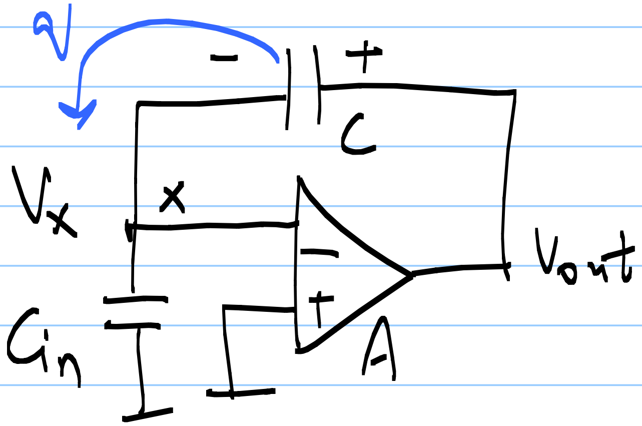


Sampling mode

Ampl. mode



$V_x \neq 0$ (A is finite)

C_{in} has been charged to $C_{in} \cdot V_x$

* charge is conserved @ X ⇒ charge on C = $CV_0 + C_{in}V_x$

⇒ Voltage across C

$$= \frac{C V_0 + C_{in} V_x}{C}$$

$$V_x = V_{out} - \left[\frac{C V_0 + C_{in} V_x}{C}\right] \qquad \text{———} \; \textcircled{1}$$

$$V_x = -\frac{V_{out}}{A} \qquad \text{———} \; \textcircled{2}$$

$$-\frac{V_{out}}{A} = V_{out} - V_0 - \frac{C_{in}}{C} \cdot \left(-\frac{V_{out}}{A}\right)$$

$$V_{out}\left[1 + \frac{1}{A}\left(1 + \frac{C_{in}}{C}\right)\right] = V_0$$

$$V_{out} = \frac{V_o}{1 + \frac{1}{A}\left(\frac{C_{in}}{c} + 1\right)}$$

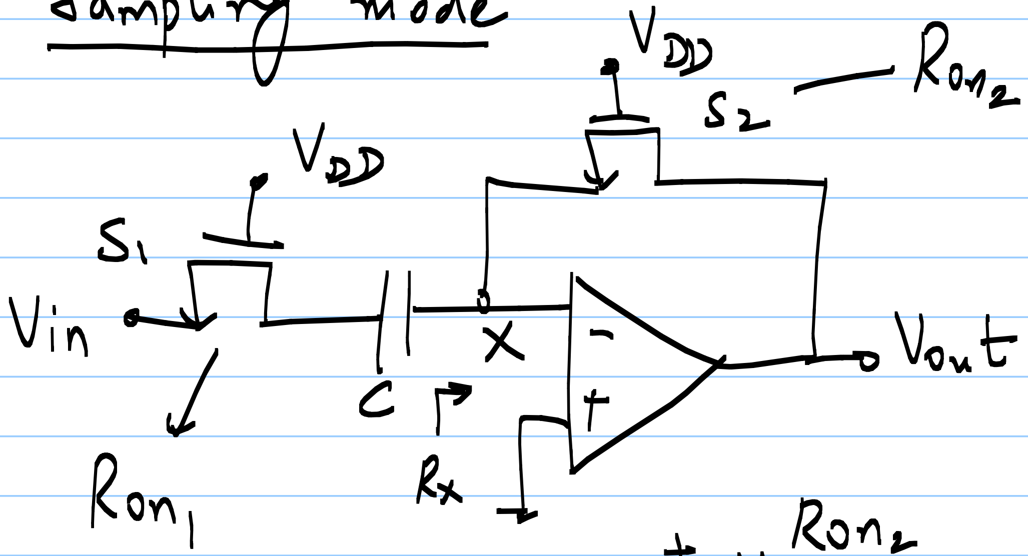
$$\approx V_o\left[1 - \overbrace{\frac{1}{A}\left(\frac{C_{in}}{c} + 1\right)}^{\text{err}}\right]$$

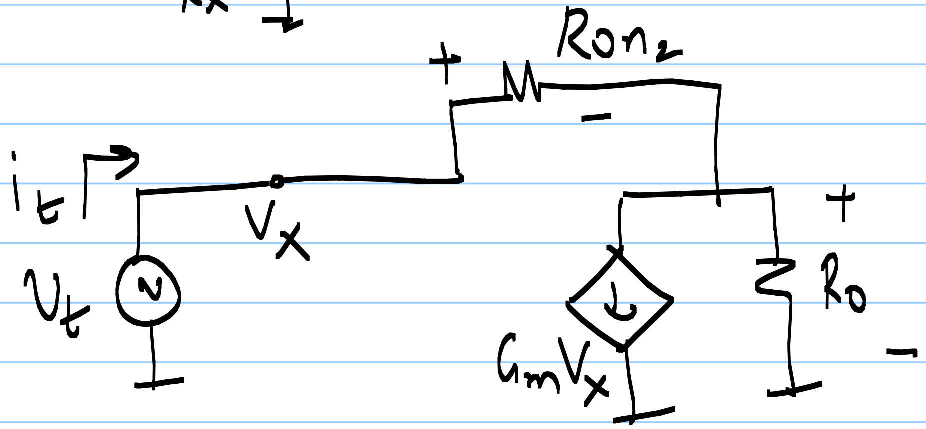
If $\frac{C_{in}}{c} \ll 1 \Rightarrow V_{out} \approx V_o\left(1 - \frac{1}{A}\right)$

* minimise C_{in} ⇒ tradeoff precision with offset, power, noise etc.

Speed :

1) Sampling mode





$$R_x = \frac{v_t}{i_t}$$

$$\left(i_t - G_m \cdot v_t\right) R_0 + i_t \cdot Ron_2 = v_t$$

$$R_X = \frac{R_o + R_{on_2}}{1 + G_m R_o}$$

$R_o >> R_{on_2}$

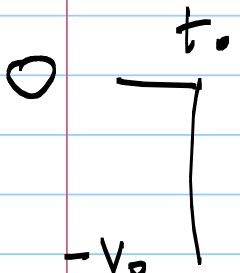
$G_m R_o >> 1$

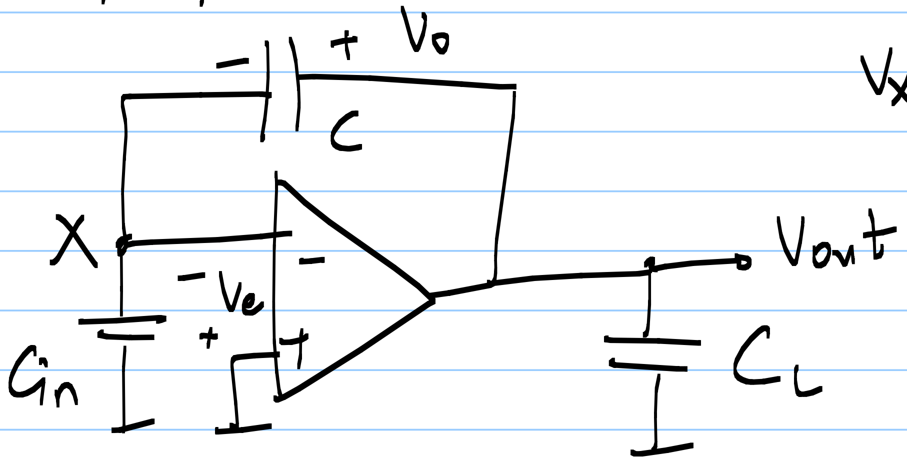
$$R_X \approx \frac{1}{G_m}$$

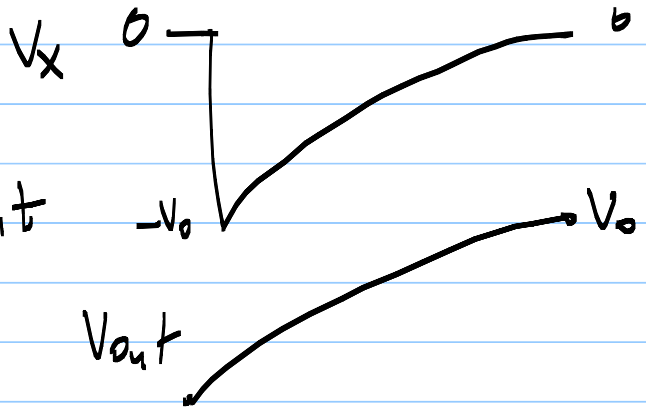
$$\tau_{sampl.} = \left(R_{on_1} + \frac{1}{G_m}\right) \cdot C$$

small enough
for V_c to settle
to desired precision

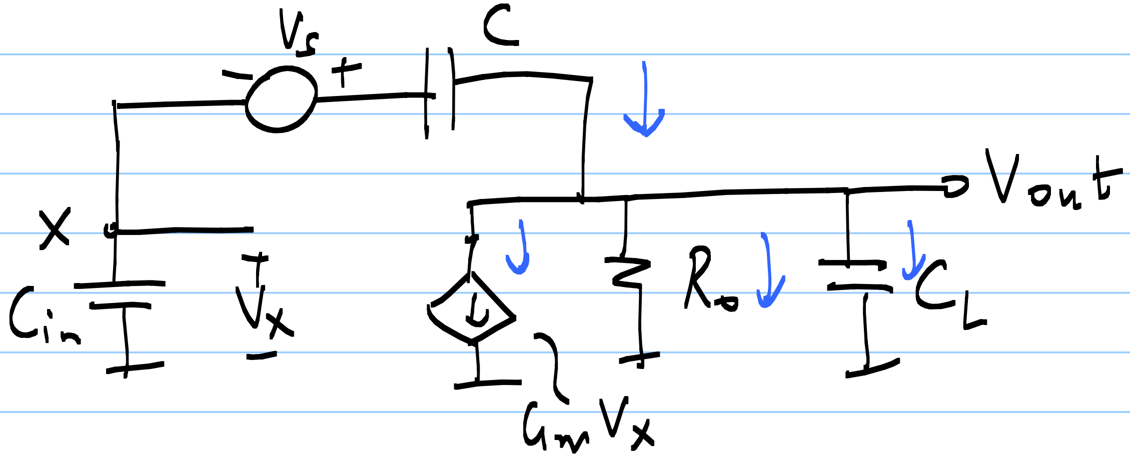
2) Amplification mode ($t=t_0$)







- $V_{out} \simeq 0$ at beginning of ampl. phase $(t=t_0^+)$
- $V_e(t=t_0^+) = V_0$



We want

$$\frac{V_{out}}{V_s}(s)$$

$$V_{out}\left(\frac{1}{R_o}+sC_L\right)+G_mV_x$$

$$=\left(V_s+V_x-V_{out}\right)sC \quad \text{—} ③$$

$$i_{C_{in}}=V_x\cdot sC_{in}$$ → also flows through C

$$V_{out}=\frac{V_x\cdot sC_{in}}{sC}+V_x+V_s \quad \text{—} ④$$

$$V_{out}-V_s=V_x\left(1+\frac{C_{in}}{C}\right)$$

$$V_x=\left(\frac{C}{C+C_{in}}\right)\left(V_{out}-V_s\right)$$ ← plug into ③

$$V_{out}\left(\frac{1}{R_o} + sC_L\right) + \frac{G_m C}{C + C_{in}}\left(V_{out} - V_s\right)$$

$$= \left[V_s - V_{out} + \left(\frac{C}{C + C_{in}}\right)\left(V_{out} - V_s\right)\right] sC$$

$$\frac{V_{out}}{V_s}(s) = R_o \frac{\left(G_m + sC_{in}\right) C}{R_o\left(C_L C_{in} + C C_{in} + C C_L\right)s \boxed{+ G_m R_o C + C + C_{in}}}$$

<u>Assume</u>

$$G_m R_o C \gg C + C_{in}$$

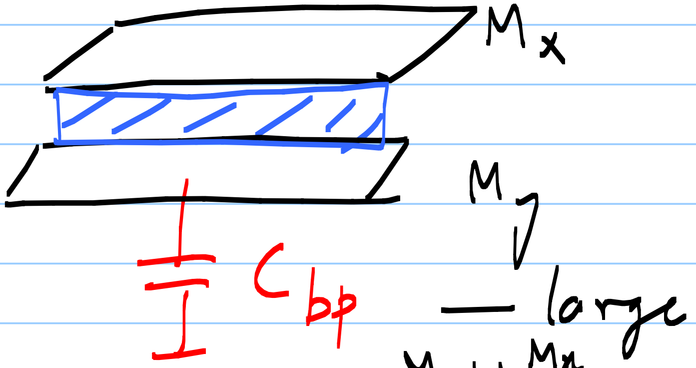
$$\frac{V_{out}}{V_s}(s) = \frac{\left(G_m + s C_{in}\right) C}{G_m C + s\left(C_L C_{in} + C C_{in} + C C_L\right)}$$

$$\tau_{ampl.} = \frac{C_L C_{in} + C C_{in} + C C_L}{G_m C}$$

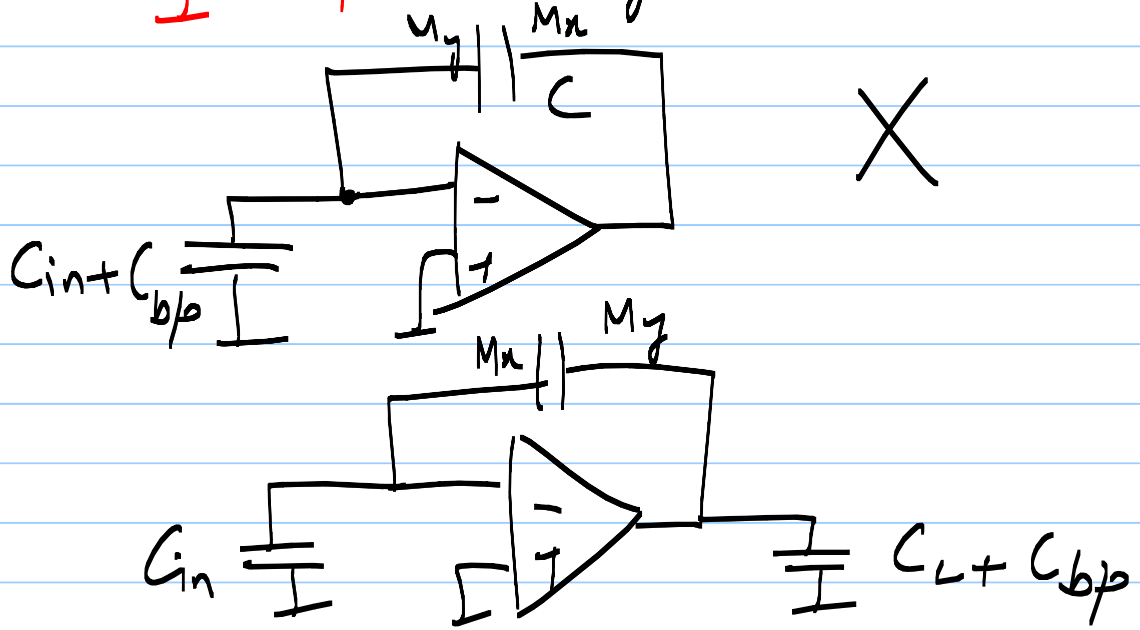
$$\tau_{ampl.} = \frac{1}{G_m}\left[C_{in} + \left(1+\frac{C_{in}}{C}\right)C_L\right]$$

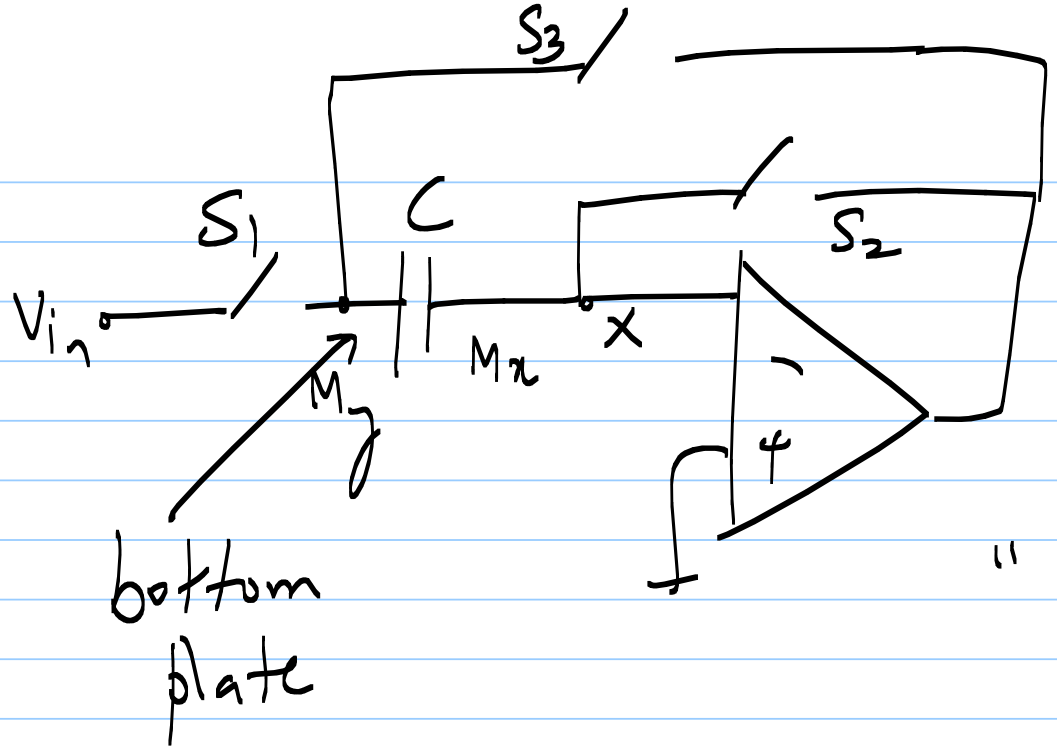
larger C_{in} → lower speed

$C \rightarrow$



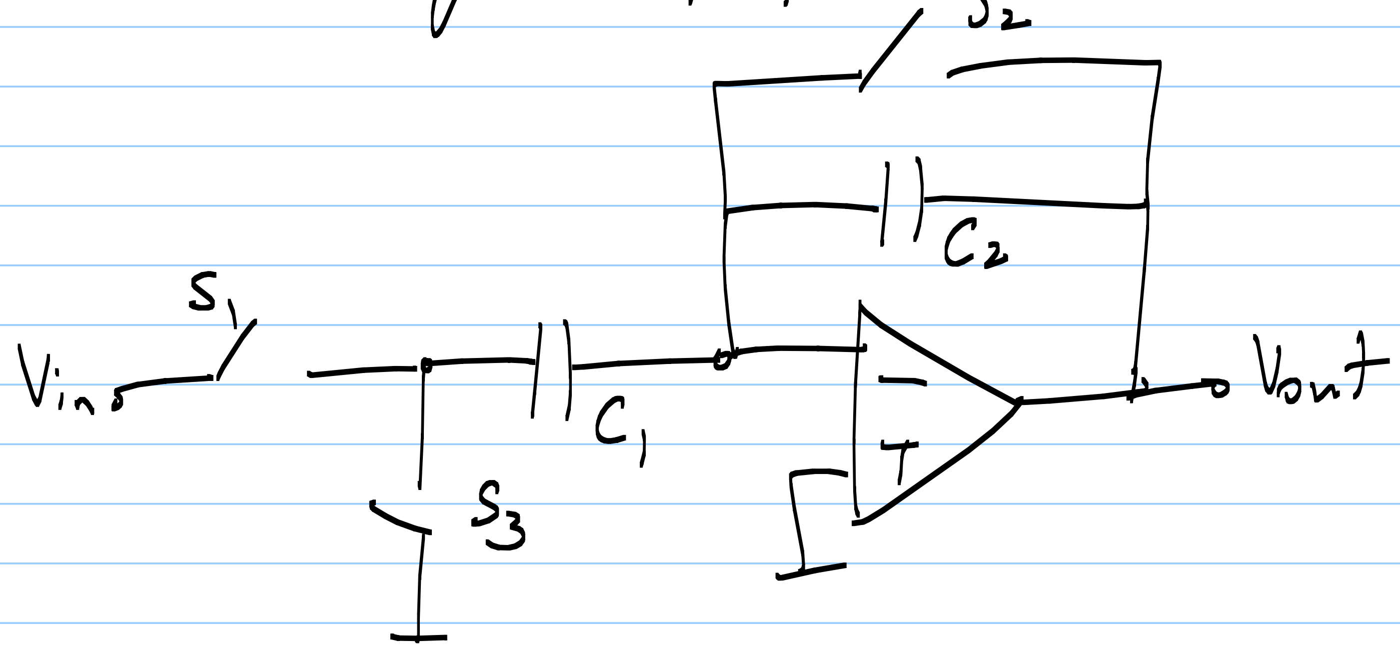
MIM capacitor



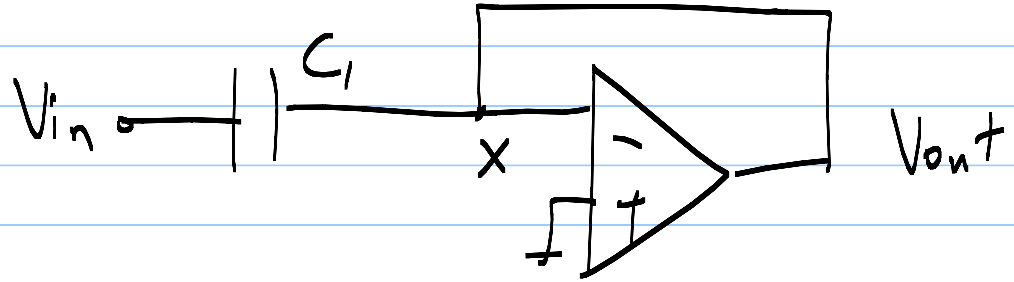


" bottom plate sampling "

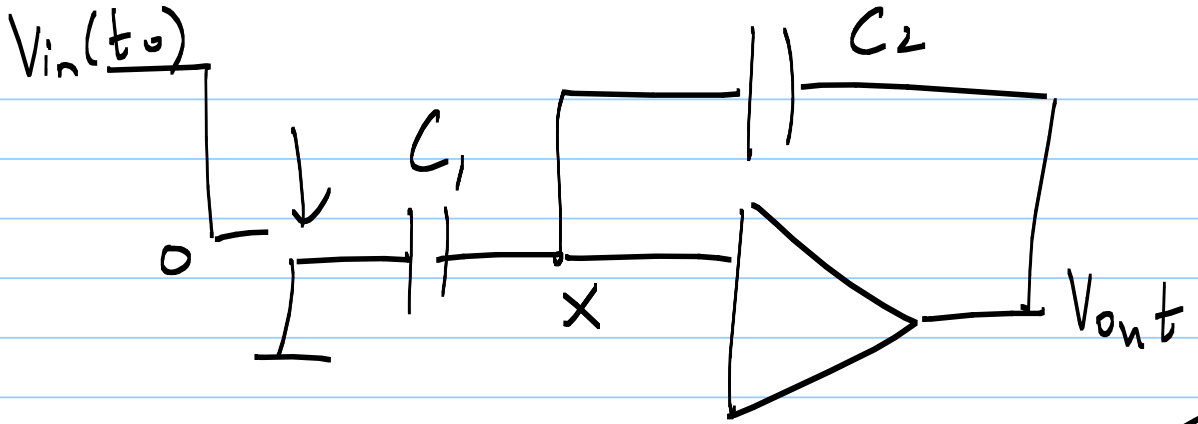
Non-inverting amplifier

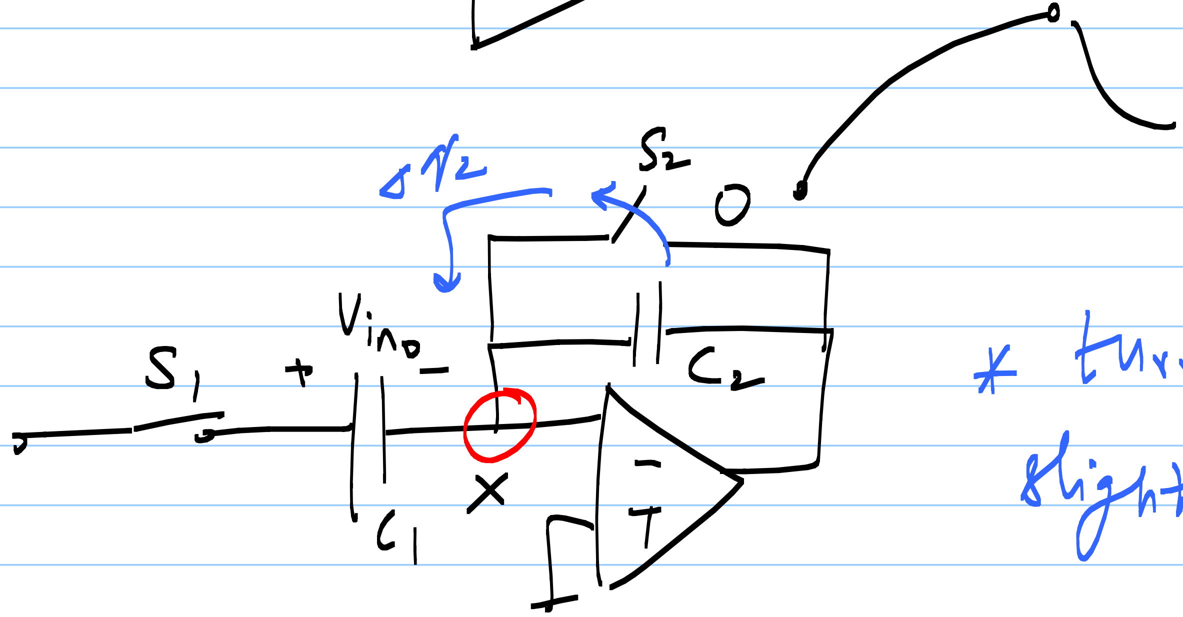


Sampling → S_1, S_2 On ; S_3 off



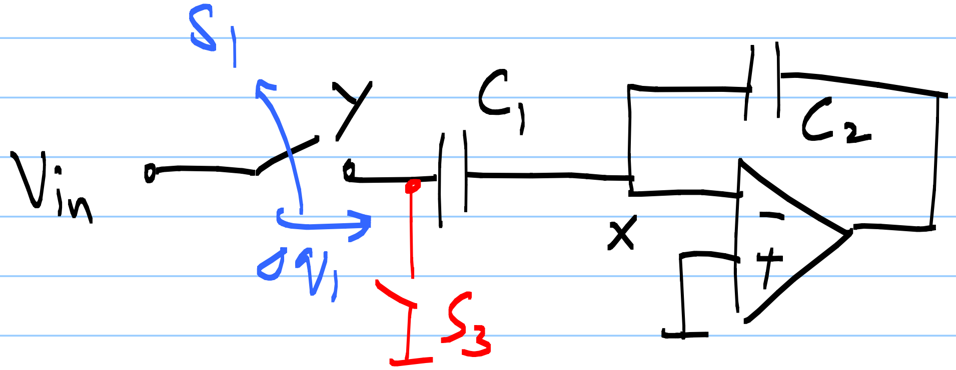
Ampl. mode : S_1, S_2 off ; S_3 ON





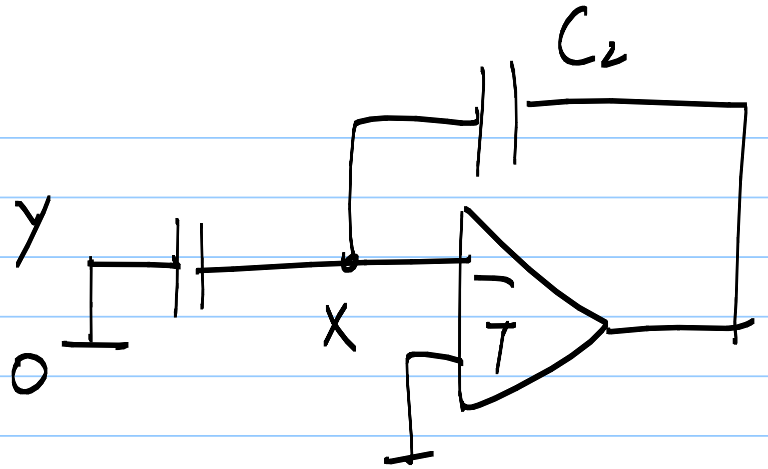
$\frac{C_1}{C_2}(V_{in}(t_0))$

* turn off S_2 slightly before S_1

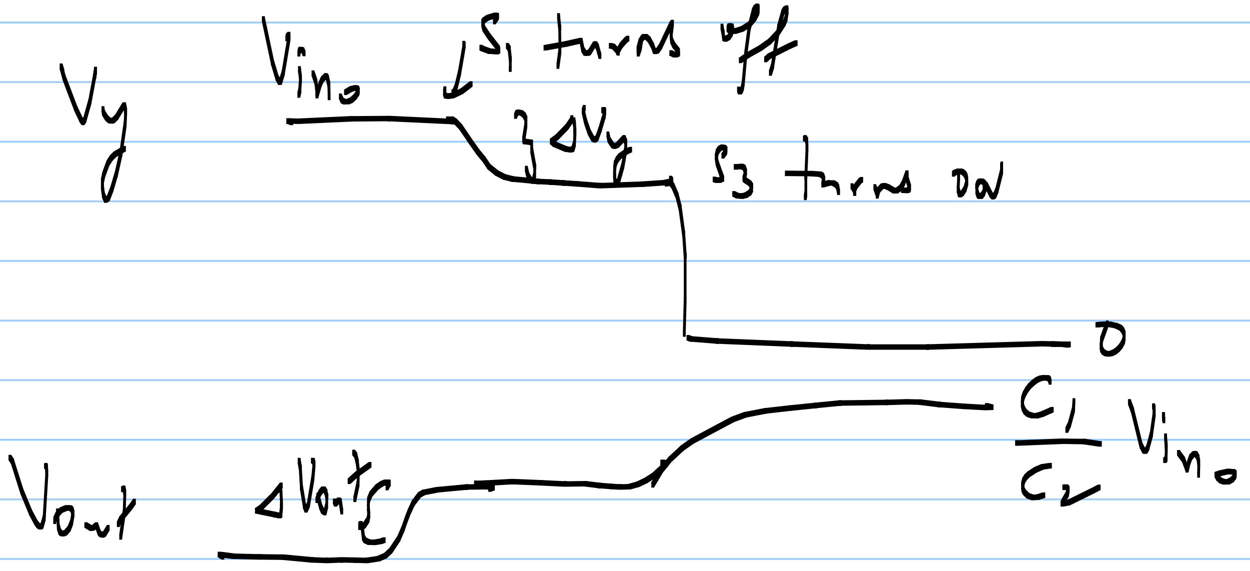


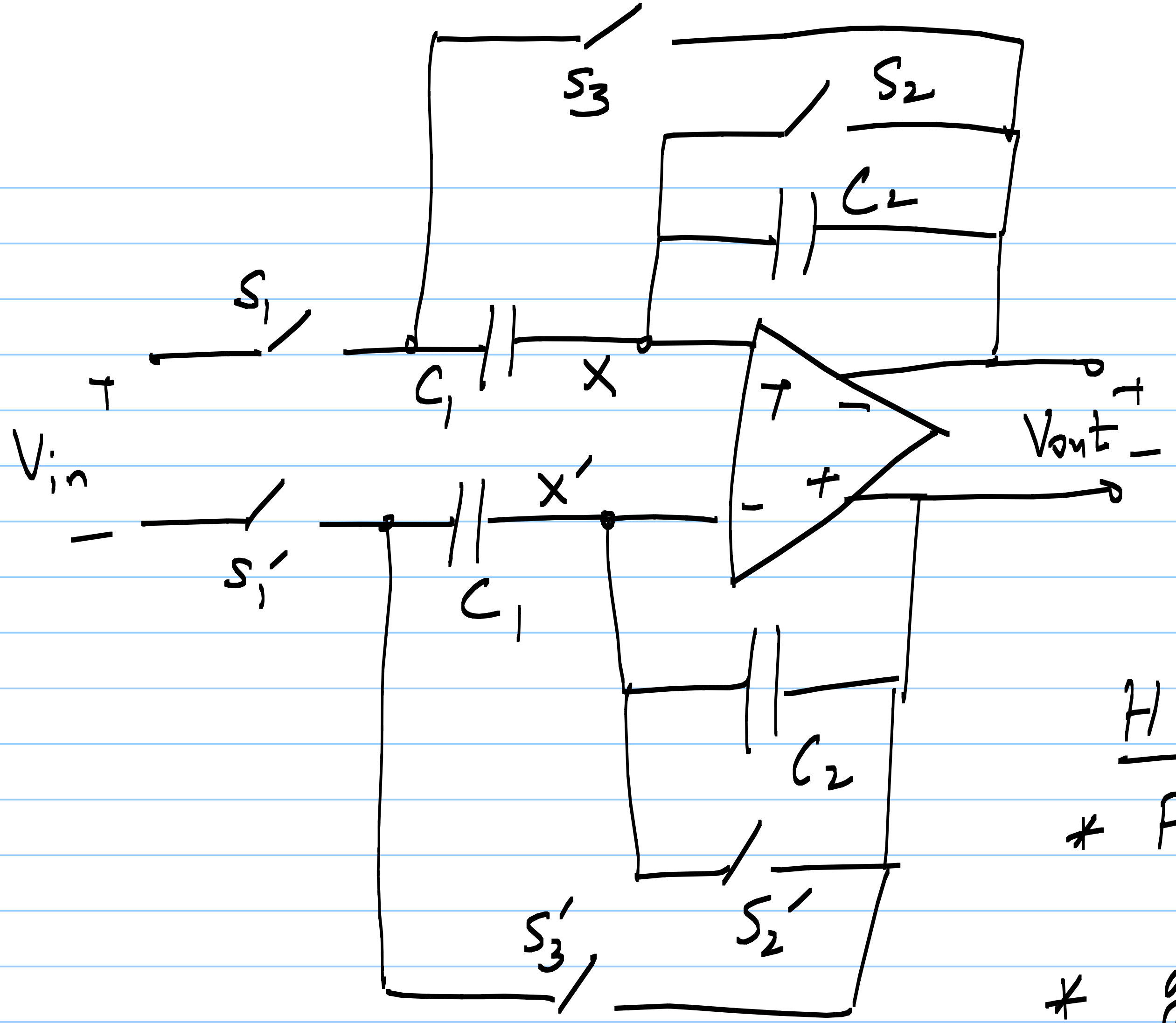
$$\Delta V_y = \frac{\Delta Q_1}{C_1}$$

$$\Delta V_{out} = \Delta V_y \cdot (\quad)$$



$V_{out} = \frac{C_1}{C_2} V_{in_0}$





HW

* Finite A &
  non-zero C_{in}

* gain error

* $\tau_{sampl.}$ & $\tau_{ampl.}$

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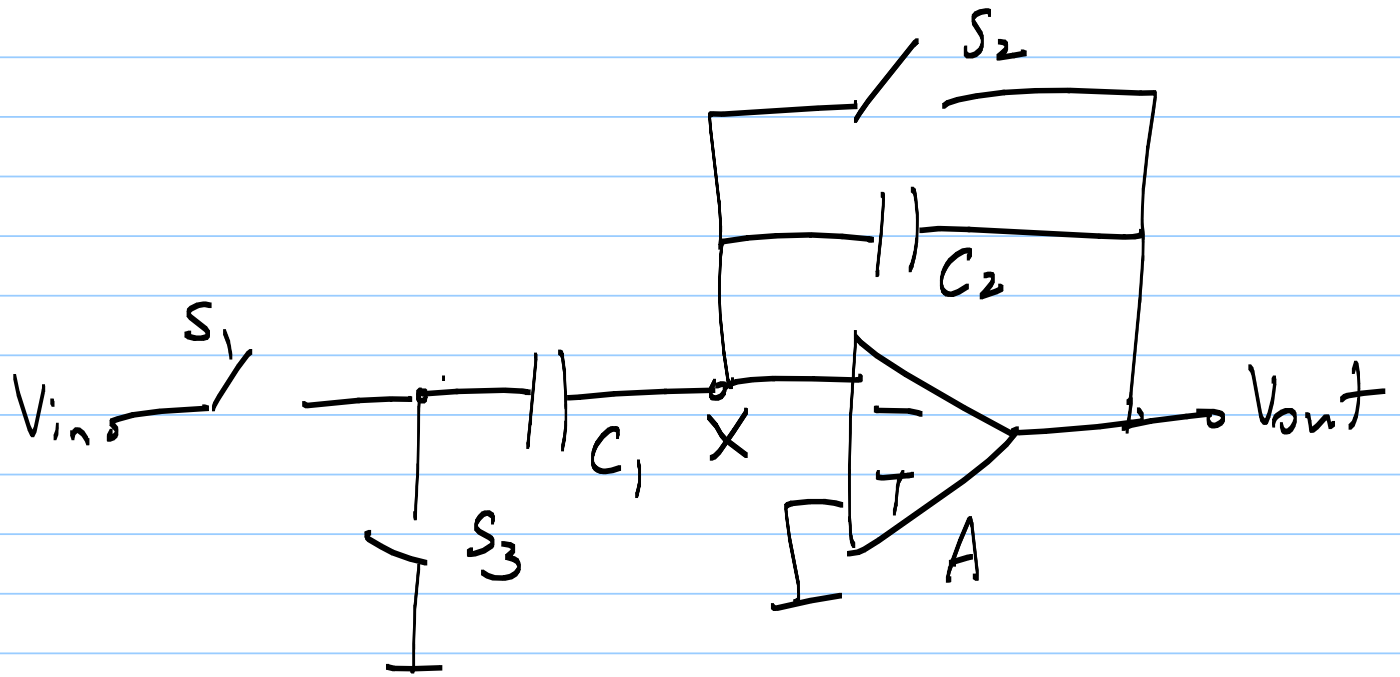
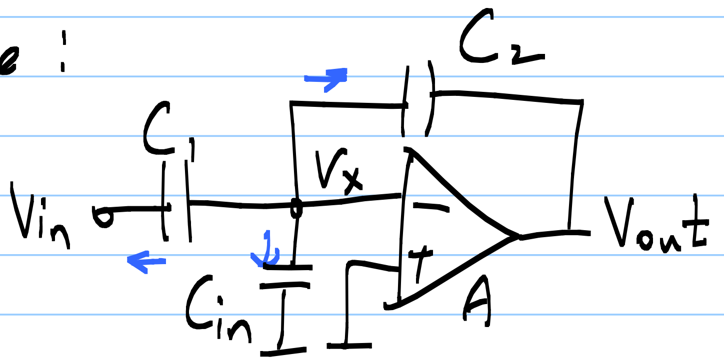
1) Precision



In ampl. mode:



$$V_x = \frac{-V_{out}}{A}$$

KCL @ x:

$$(V_x - V_{in})\, sC_1 + V_x \cdot sC_{in}$$

$$+ (V_x - V_{out})\, sC_2 = 0$$

$$\left(-\frac{V_{out}}{A} - V_{in}\right) sC_1 - \frac{sC_{in}V_{out}}{A} - V_{out}\left(1+\frac{1}{A}\right) sC_2 = 0$$

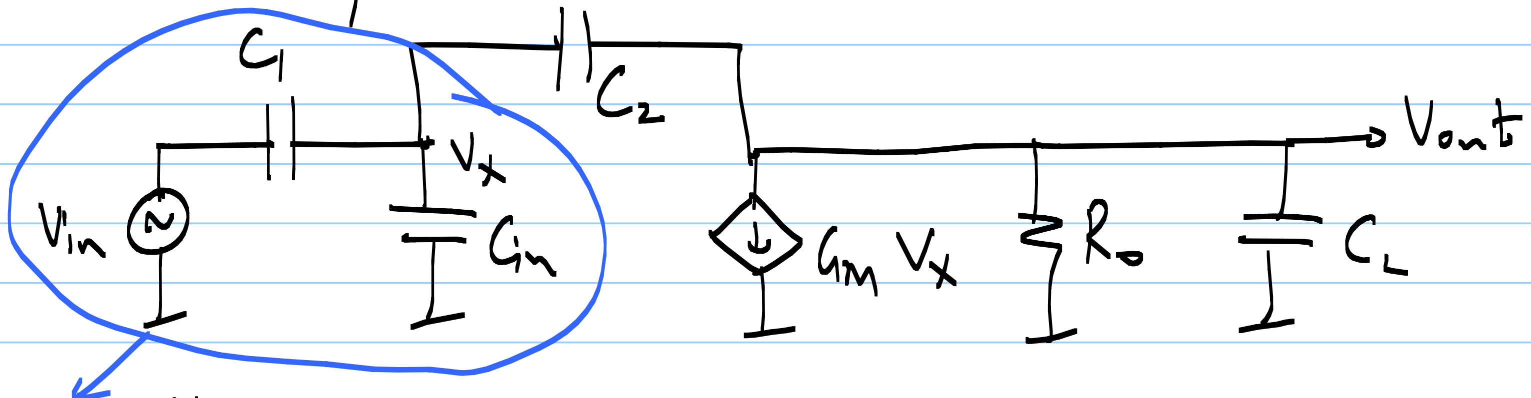
$$-V_{in}\cdot C_1 = V_{out}\left(C_2 + \frac{C_2}{A} + \frac{C_{in}}{A} + \frac{C_1}{A}\right)$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{C_1}{C_2 + \frac{1}{A}(C_1 + C_2 + C_{in})}$$

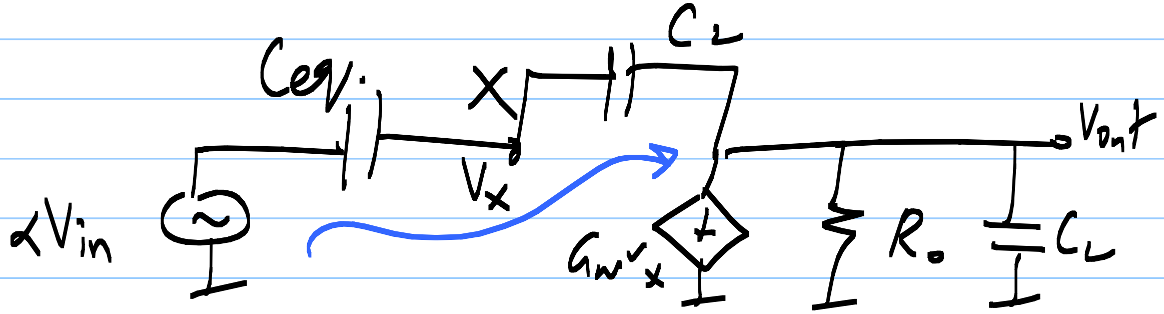
$$= \frac{C_1}{C_2} \cdot \frac{1}{1 + \frac{1}{A}\cdot\frac{1}{C_2}(C_1 + C_2 + C_{in})}$$

$$\left|\frac{V_{out}}{V_{in}}\right| \approx \frac{C_1}{C_2}\left[1 - \underbrace{\frac{1}{A}\,\frac{C_1+C_2+C_{in}}{C_2}}_{\text{gain error (increases with } C_1/C_2)}\right]$$

2) Speed in ampl. mode



Replace with thevenin eq. ckt @ X



$$\alpha = \frac{C_1}{C_1 + C_{in}} \; ; \qquad C_{eq.} = C_1 + C_{in}$$

$$V_x = \left(\alpha V_{in} - V_{out}\right) \cdot \frac{C_{eq.}}{C_2 + C_{eq.}} + V_{out}$$

<u>KCL @ output node</u>

$$G_m V_x + V_{out}\left[\frac{1}{R_o} + sC_L\right] = sC_2 \cdot \underbrace{\left(V_x - V_{out}\right)}_{\substack{\parallel \\ \frac{C_{eq}}{C_2 + C_{eq.}} \cdot \left(\alpha V_{in} - V_{out}\right)}}$$

$$\left((\alpha V_{in} - V_{out}) \frac{C_{eq.}}{C_2 + C_{eq.}} + V_{out} \right) G_m$$

$$V_{out} \left(\overset{+}{\frac{1}{R_o}} + sC_L \right) = (\alpha V_{in} - V_{out}) \frac{C_{eq.}}{C_2 + C_{eq.}} \cdot sC_2$$

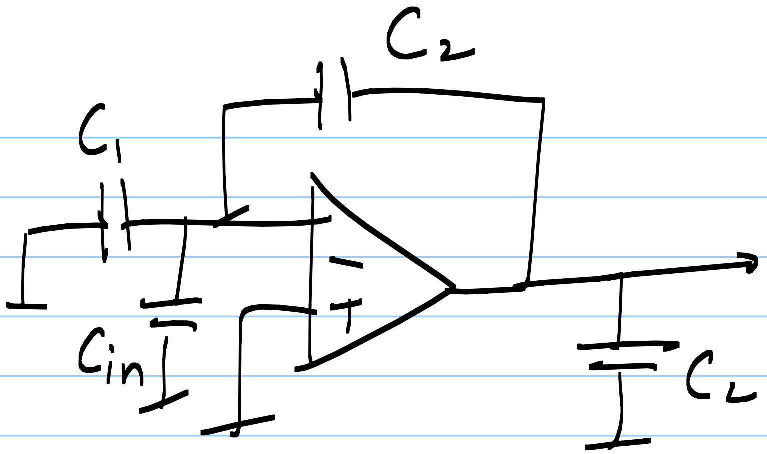
$$\frac{V_{out}}{V_{in}}(s) = \frac{- C_{eq.} \cdot \frac{C_1}{C_1 + C_{in}} (G_m - sC_2) \cdot R_o}{sR_o \left[C_2 C_{eq} + C_2 C_L + C_L C_{eq} \right] + C_2 G_m R_o + C_{eq.} + C_2}$$

If $G_m R_o >> 1 \Rightarrow G_m R_o C_2 >> C_{eq.} + C_2$

$$\frac{V_{out}}{V_{in}}(s) \approx \frac{-C_{eq.}\frac{C_1}{C_1+C_{in}}(G_m-sC_2)}{s(C_2C_{eq}+C_LC_{eq.}+C_LC_2)+G_mC_2}$$

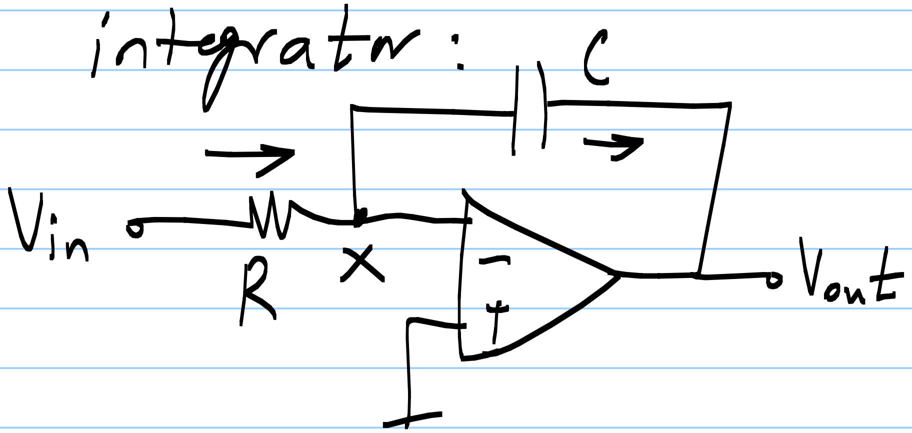
$$\tau_{ampl.} = \frac{C_2C_{eq.}+C_LC_{eq.}+C_LC_2}{G_mC_2}$$

$$\tau_{ampl.} = \underbrace{\frac{C_1+C_2+C_{in}}{C_2}\cdot\frac{\overbrace{C_L+\frac{C_2(C_{in}+C_1)}{C_2+C_{in}+C_1}}^{\text{eq. cap.}}}{G_m}}_{\text{equivalent resistance}}$$



Switched-capacitor integrator

CT integrator:



$$\frac{V_{in}}{R} = -V_{out} \cdot sC$$

$(\text{ideally } V_x = 0)$

$$\frac{V_{out}}{V_{in}}(s) = \frac{-1}{sCR}$$

$$V_{out}(t) = \frac{-1}{RC}\int_{-\infty}^{t} V_{in}\,dt$$

Issues

1) $R_{in} = R$ {loads previous ckt}

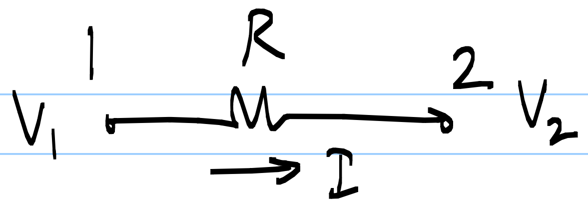
2) $$\frac{V_{out}}{V_{in}}(s) = \frac{-1}{s}\cdot\frac{1}{RC}$$

±20%

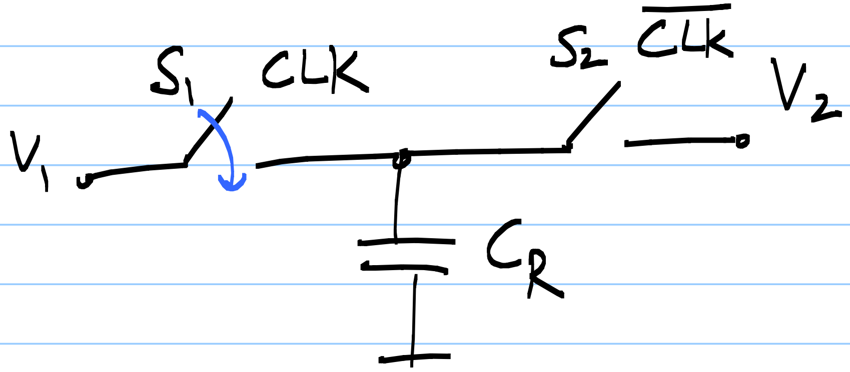
±15%

R & C Vary independently

3) R adds thermal noise

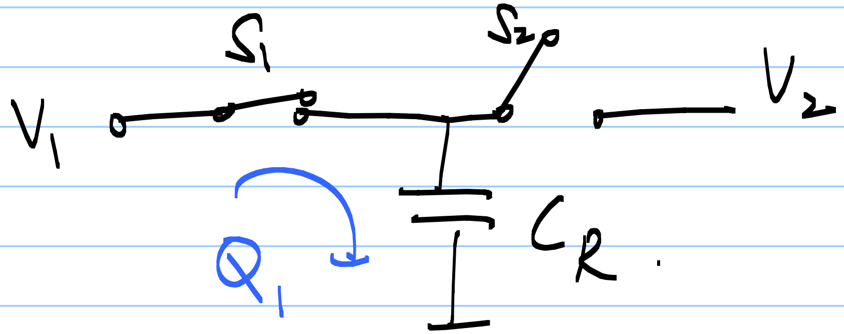


$$I = \frac{V_1 - V_2}{R}$$



<u>phase 1</u> : CLK ON, S_1 is ON

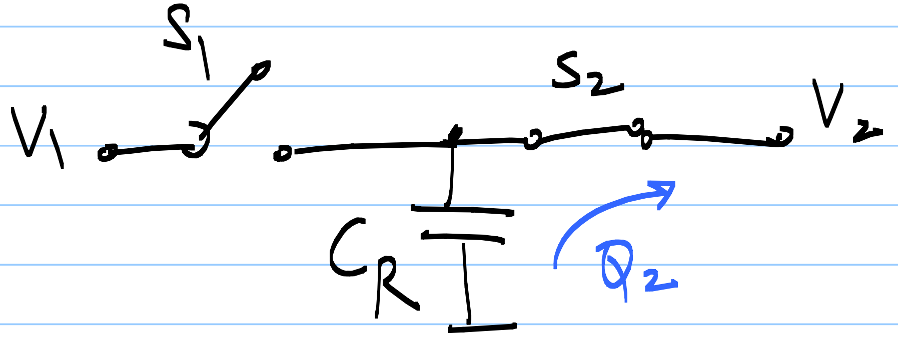
S_2 is off



Q_1 from V_1 = $C_R \cdot V_1$

phase 2 : $\overline{CLK}$ ON , S_1 OFF

S_2 ON



new charge on C_R = $C_R \cdot V_2$

net charge moved from V_1 to V_2 :

$$= C_R(V_1 - V_2)$$

Done over a time period $= T = \dfrac{1}{f_{CLK}}$

$$I_{avg.} = C_R(V_1 - V_2)/T$$

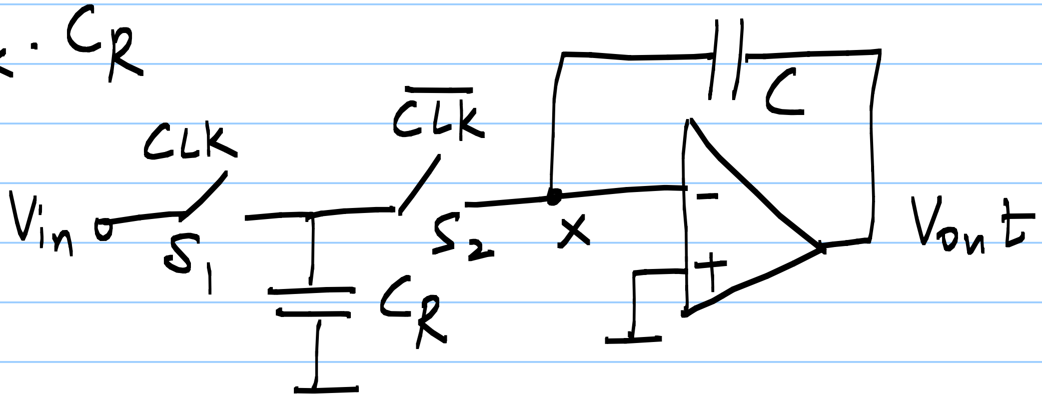
$$I_{avg.} = f_{CLK} \cdot C_R (V_1 - V_2)$$

compare with resistor

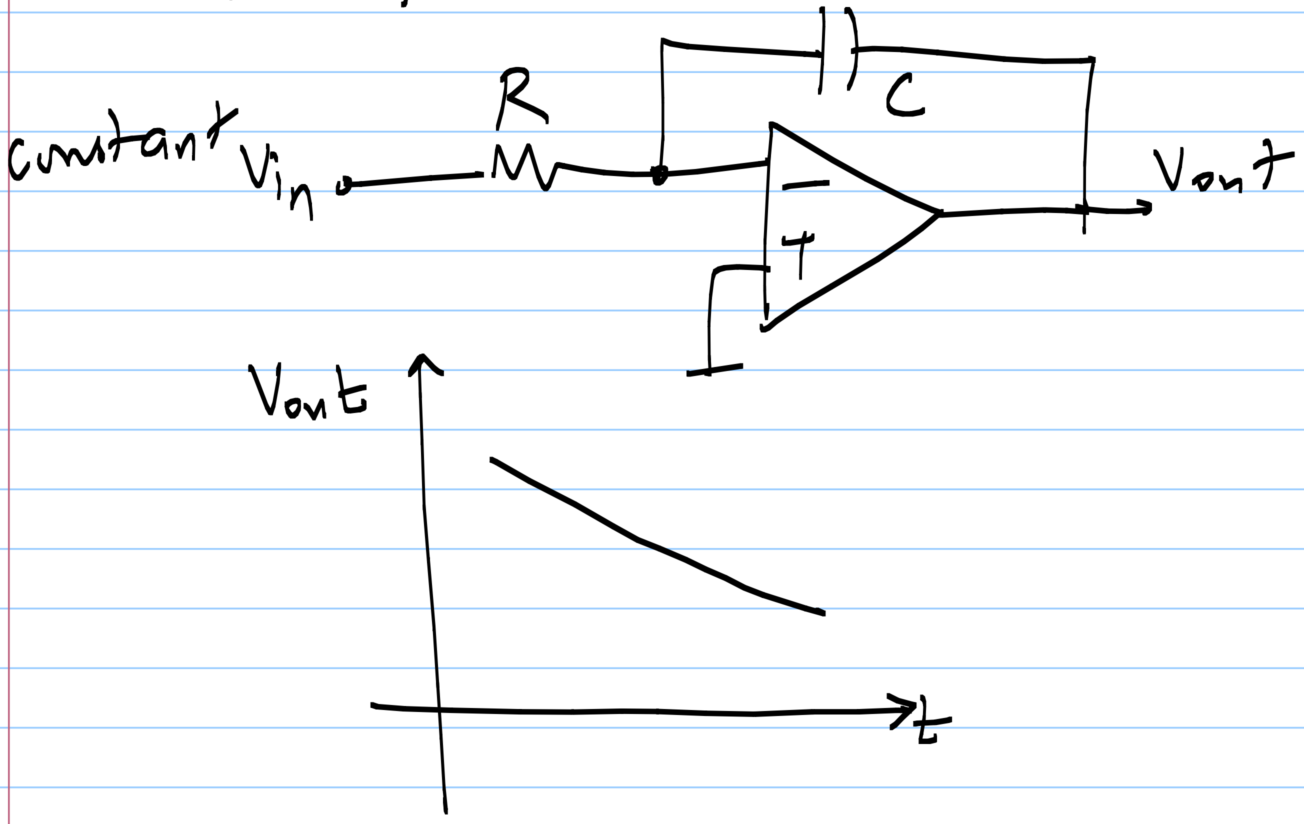
$$I_R = \frac{V_1 - V_2}{R}$$

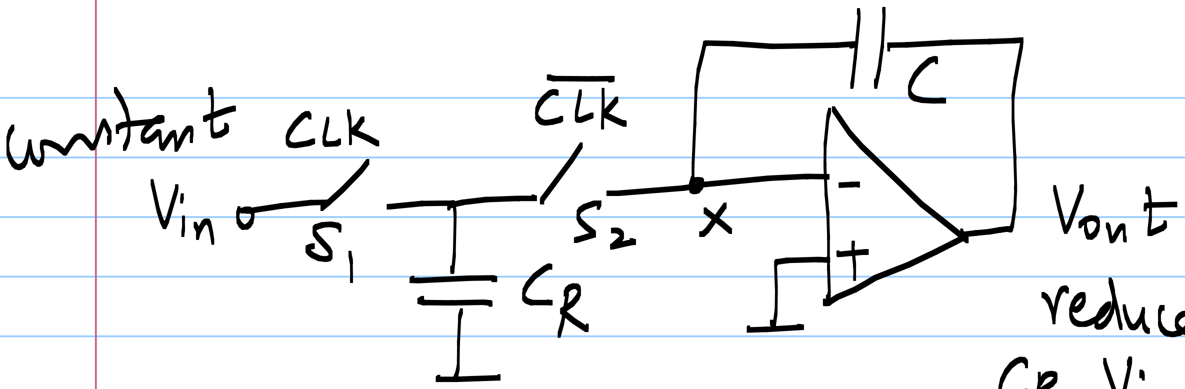
Switched cap. $R_{eq.} = \dfrac{1}{f_{CLK} \cdot C_R}$

Discrete time integrator :

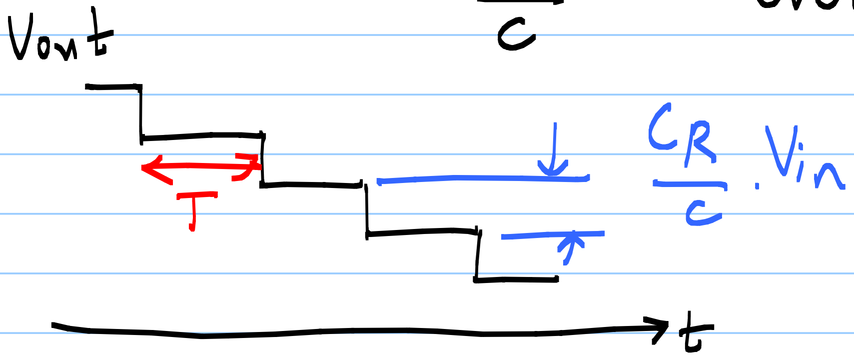


e.g. if V_{in} is constant:





reduces by $\frac{C_R}{C} V_{in}$ every clock cycle



* gain of integrator $\propto \frac{C_R}{C}$

* presents cap. input impedance

ratio caps tracks very well across PVT

$$V_{out}(kT_{CLK}) = V_{out}((k-1)T_{CLK}) - V_{in}((k-1)T_{CLK}) \cdot \frac{C_R}{C}$$

$\Rightarrow$ Z-domain analysis

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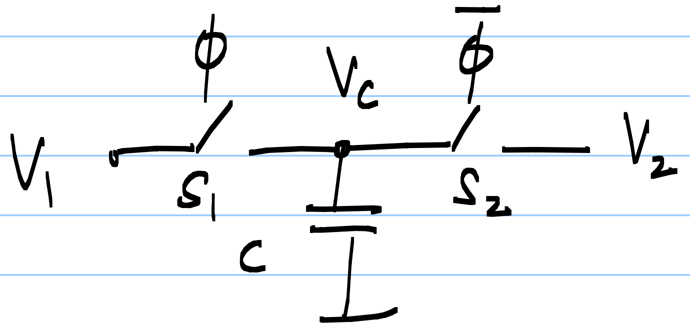
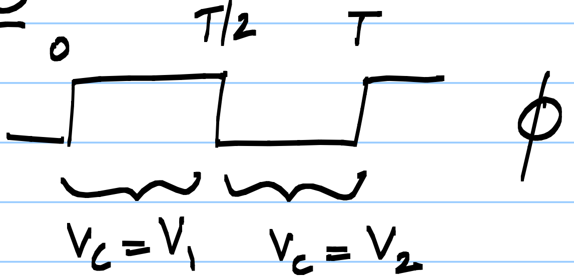




$$T = \frac{1}{f_{clk}}$$

ϕ: t = 0 to T/2

$Q_c = C V_1$

$\bar{\phi}$: t = T/2 to T

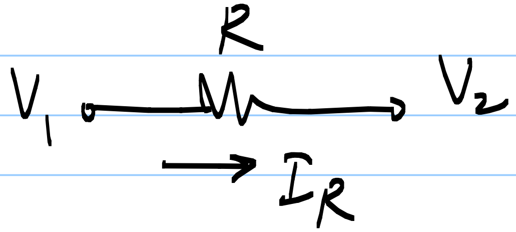
$Q_c = C V_2$

$\Delta Q_c = C(V_1 - V_2)$ = charge transferred from V_1 to V_2

$\Delta t = T - 0 = T$

$$I_{avg.} = \frac{\Delta Q_c}{\Delta t} = \frac{C(V_1 - V_2)}{T}$$

$$\left\{ \text{CT current } I = \frac{dQ}{dt} \right\}$$

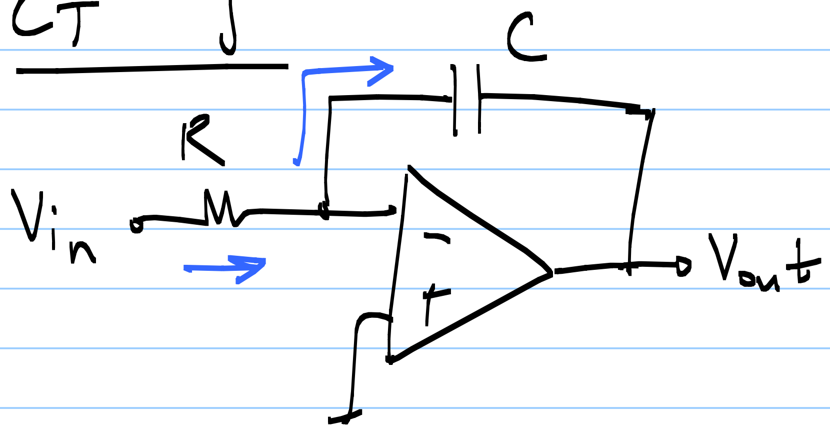


$$I_R = \frac{V_1 - V_2}{R} = I_{avg}'$$

$$I_{avg}' = I_{avg} \Rightarrow \frac{C(V_1 - V_2)}{T} = \frac{V_1 - V_2}{R}$$

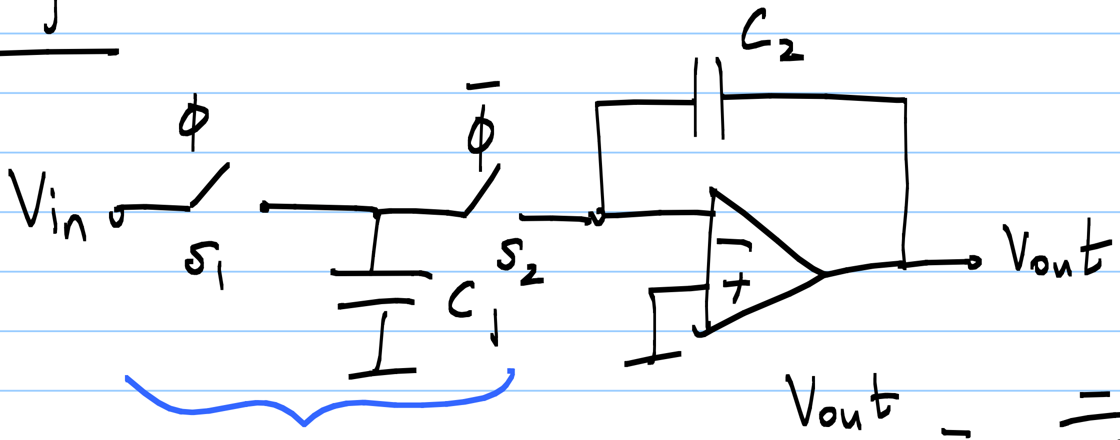
$$R = \frac{T}{C} = \frac{1}{C f_{clk}}$$

CT ∫



$$\frac{V_{in}}{R} = -V_{out} \cdot sC \Rightarrow \frac{V_{out}}{V_{in}} = \frac{-1}{sCR}$$

DT ∫



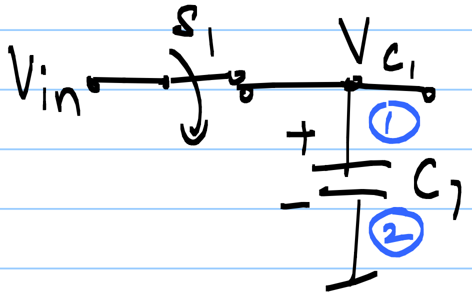
Req.

$$\frac{V_{out}}{V_{in}} = \frac{-1}{sC_2R_{eq}.} = \frac{-C_1 f_{clk}}{sC_2}$$

ratio of caps

very tightly controlled across PVT

1) $\phi : 0 \leq t \leq T/2$



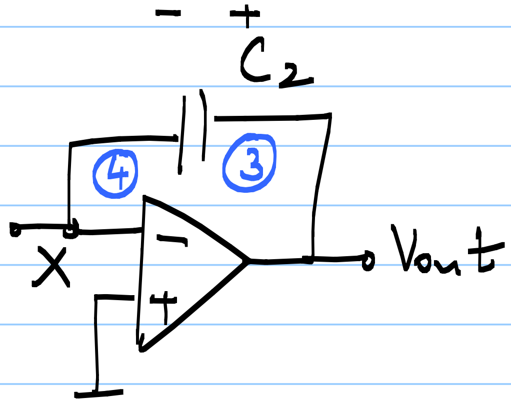
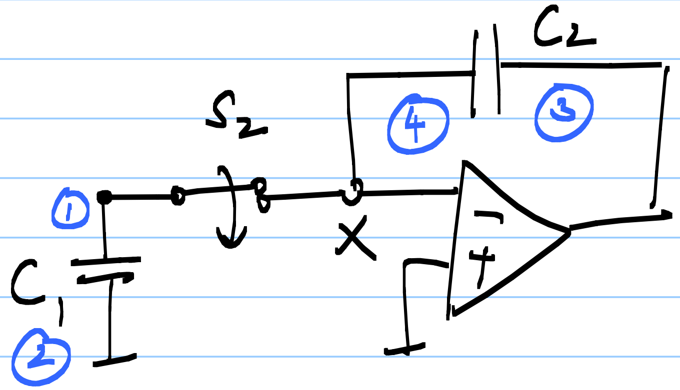


plate ① of C_1 : $q_1 = C_1 \cdot V_{in}(T/2)$

plate ③ of C_2 : $q_2 = C_2 \cdot V_{out}(T/2) = C_2 V_{out}(0)$

{No charge transferred from Vin so far} ✓

2) $\bar{\phi}$: $T/2 \leq t \leq T$ $\qquad V_x \simeq 0$



Total charge on plates ① & ④ is conserved

$$C_1 V_{in}(T/2) - C_2 V_{out}(0) = \overset{V_x \simeq 0}{0} - C_2 V_{out}(T)$$

plate ④ is -ve plate of C_2

$$\underset{\overset{||}{V_{out}(3T/2)}}{V_{out}(T)} = \underset{\overset{||}{V_{out}(T/2)}}{V_{out}(0)} - \frac{C_1}{C_2} \underset{\overset{||}{V_{in}(0)}}{V_{in}(T/2)}$$

(or) $$V_{out}(3T/2) = V_{out}(T/2) - \frac{C_1}{C_2} V_{in}(T/2)$$

In general, we can write

$$V_{out}(nT) = V_{out}(\overline{n-1}\,T) - \frac{C_1}{C_2} V_{in}(\overline{n-1}\,T)$$

In Z-domain :

$$V_{out}(z) = Z^{-1} \cdot V_{out}(z) - \frac{C_1}{C_2} \cdot Z^{-1} \cdot V_{in}(z)$$

where $Z = e^{jW_d T}$; $T = \frac{1}{f_{clk}}$

W_d = DT freq.

$$\frac{V_{out}}{V_{in}}(z) = \frac{-C_1}{C_2} \cdot \frac{Z^{-1}}{1-Z^{-1}} = \frac{-C_1}{C_2} \cdot \frac{1}{Z-1}$$

$$= -\frac{C_1}{C_2} \cdot \frac{1}{e^{jW_d T} - 1}$$

$$e^{j\omega_d T} = 1 + j\omega_d T + \frac{(j\omega_d T)^2}{2!} + \cdots$$

$$\frac{V_{out}}{V_{in}}(z) = -\frac{C_1}{C_2} \cdot \frac{1}{j\omega_d T + \frac{(j\omega_d T)^2}{2!} + \cdots}$$

If $\omega_d T$ is very small,

$$\frac{V_{out}}{V_{in}}(z) \approx -\frac{C_1}{C_2} \cdot \frac{1}{j\omega_d T}$$

$$= -\underbrace{\frac{C_1}{T}}_{= 1/R_{eq}} \cdot \frac{1}{C_2} \cdot \frac{1}{j\omega_d}$$

$$\frac{V_{out}}{V_{in}}(z) \approx \frac{-1}{j\omega_d \cdot R_{eq} \cdot C_2}$$

In general,

$$H(s) \longrightarrow H(z)$$

Non linear transformation : complex s-plane mapped to complex z-plane

forward : $s = \frac{2}{T} \frac{z-1}{z+1}$

reverse : $z = \frac{2+sT}{2-sT}$

$$H(s) = \frac{-1}{sRC} \qquad \left\{ H(j\omega) = \frac{-1}{j\omega RC} \right\}$$

We want

$$H_d(j\omega_d) = H(s)$$

$$= H\left[\frac{2}{T} \cdot \frac{z-1}{z+1}\right]$$

$$= H\left[\frac{2}{T} \frac{e^{j\omega_d T} - 1}{e^{j\omega_d T} + 1}\right]$$

$$= H\left[j \cdot \frac{2}{T} \frac{\left(e^{j\omega_d T/2} - e^{-j\omega_d T/2}\right)/2j}{\left(e^{j\omega_d T/2} + e^{-j\omega_d T/2}\right)/2}\right]$$

$$= H\left[j \cdot \frac{2}{T} \cdot \tan\left(\frac{\omega_d T}{2}\right)\right]$$

We want $H\left(j \cdot \frac{2}{T} \cdot \tan\left(\frac{\omega_d T}{2}\right)\right) = H(j\omega)$

i.e. DT $H_d(j\omega_d)$ = CT $H(j\omega)$

If $\frac{\omega_d T}{2} << 1 \Rightarrow \tan\left(\frac{\omega_d T}{2}\right) \approx \frac{\omega_d T}{2}$

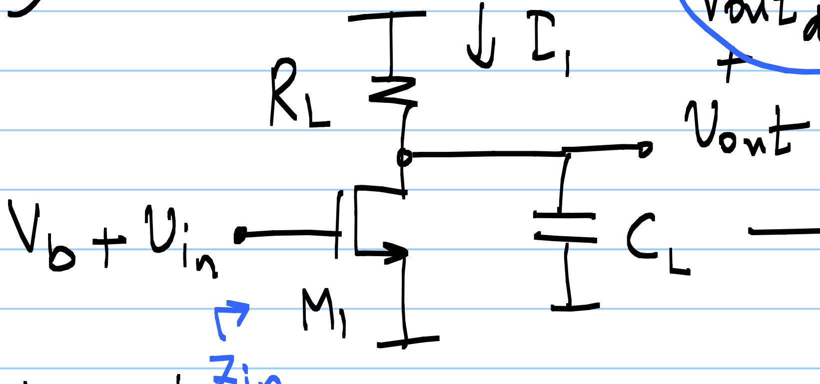
$$H\left(j \cdot \frac{2}{T} \tan\left(\frac{\omega_d T}{2}\right)\right) \approx H\left(j \cdot \frac{2}{T} \cdot \frac{\omega_d T}{2}\right)$$

$$= H(j\omega_d) = H(j\omega)$$

$$\boxed{\omega_d T << T}$$

High Speed Amplifiers

1) CSA

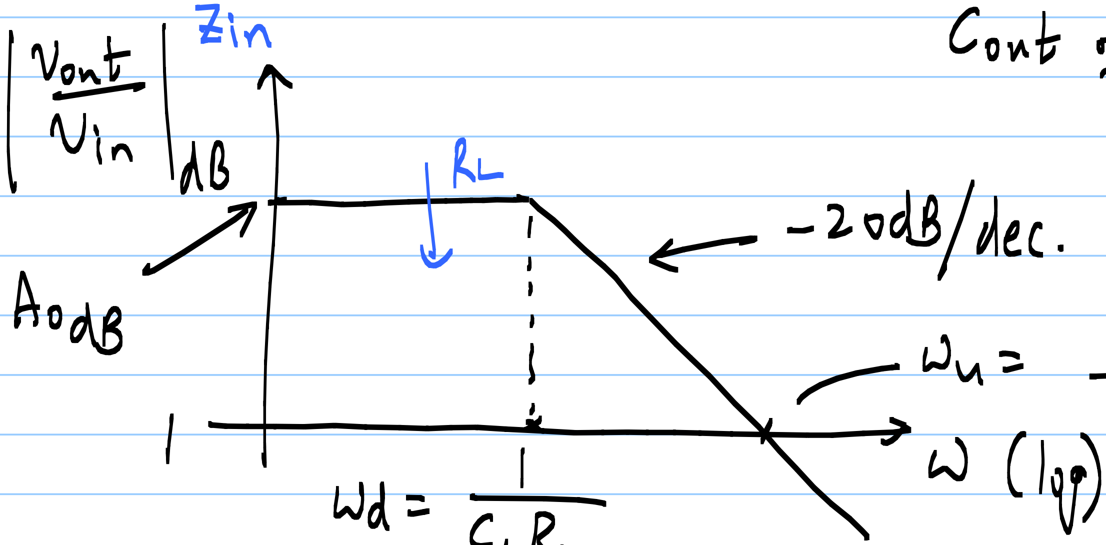


$\boxed{V_{out_{dc}}} = V_{DD} - I_1 R_L$

C_L includes → C_{in} of next stage

\+

C_{out} of this stage



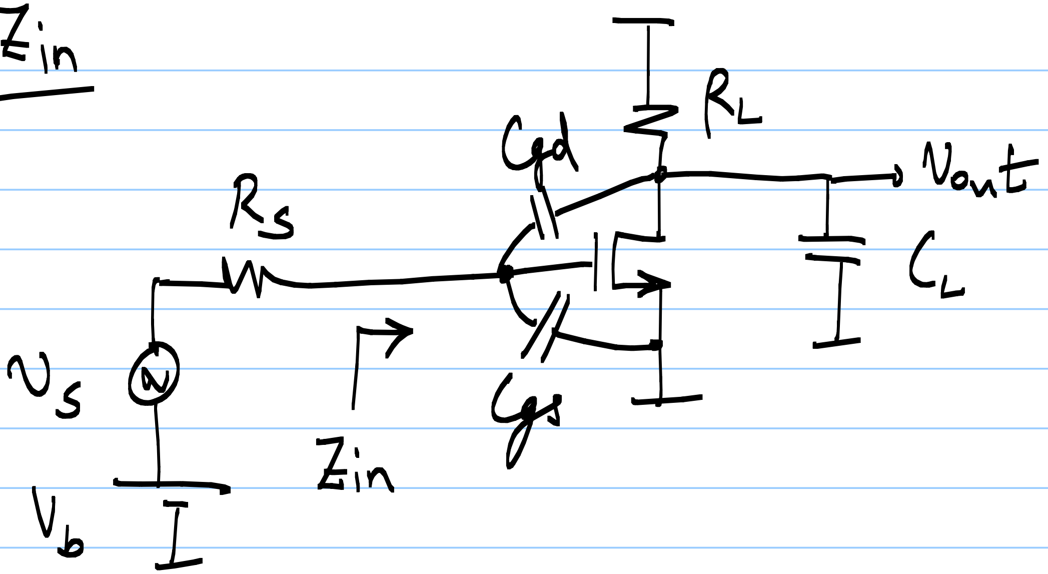
$|A_0| = g_{m_1} R_L$

$\omega_u = \frac{g_{m_1}}{C_L}$

$\omega_d = \frac{1}{C_L R_L}$

$$\frac{v_{out}}{v_{in}} = -g_{m_1}\left(R_L \| \frac{1}{j\omega C_L}\right) = \frac{-g_{m_1} R_L}{1 + j\omega C_L R_L}$$

Z_{in}



$$\text{gain} = -A$$
$$= -g_m R_L$$

$L \sim L_{min}$ for high speed

C_{gd} is significant compared to C_{gs}

e.g. in deep submicron processes,
@ L_{min}, $C_{gd} \approx 50\%$ of C_{gs}

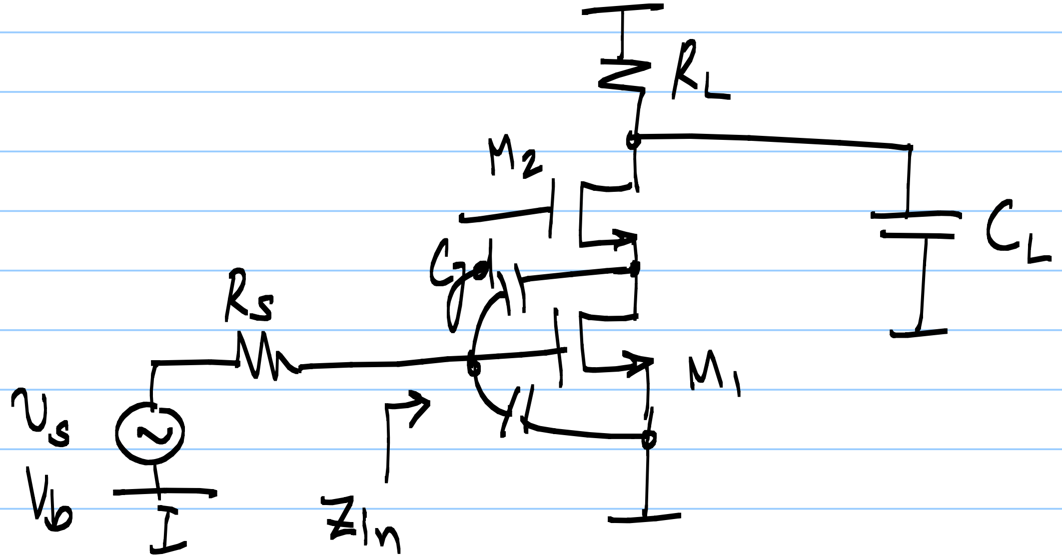
$$Z_{in} \approx \frac{1}{s(C_{gs} + C_{gd}(1+A))} = \frac{1}{sC_{gs}\left[1+\frac{C_{gd}}{C_{gs}}(1+g_mR_L)\right)}$$

e.g. $g_m R_L = 3$

$$Z_{in} \approx \frac{1}{sC_{gs}(1+0.5(4))} = \frac{1}{3sC_{gs}}$$

input cap has almost tripled

2) Add a cascode device (Cascoded CSA)



$M_2 = M_1$

gain from gate of M_1 to drain of M_1

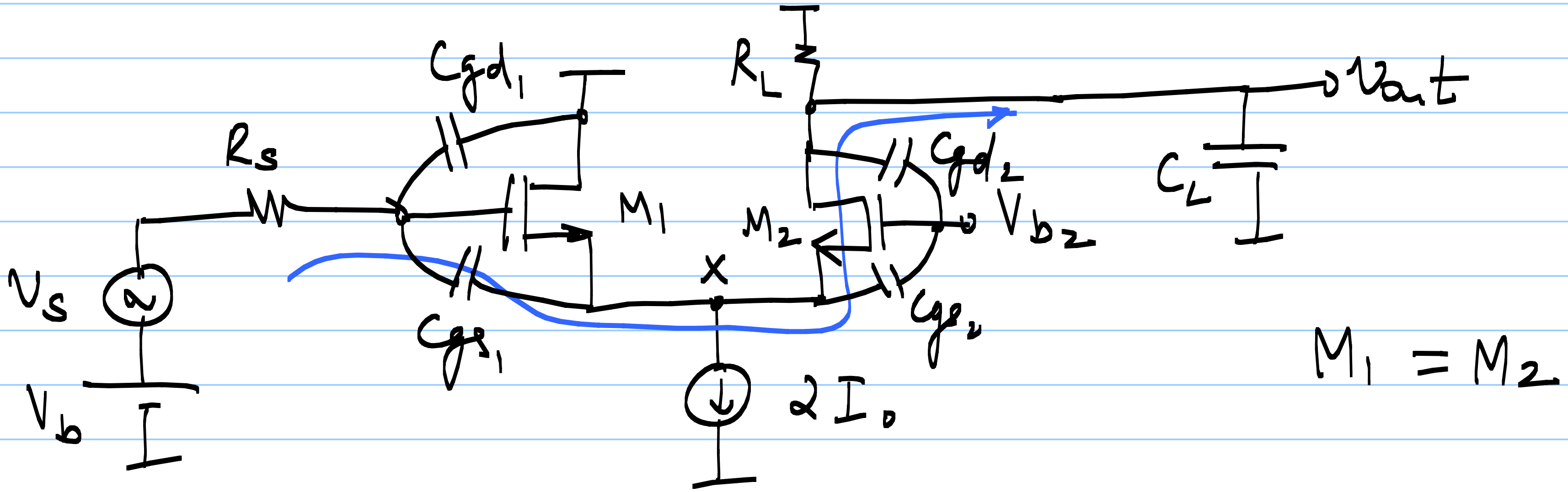
$$= -\frac{g_{m1}}{g_{m2}} \approx -1$$

$$Z_{in} \approx \frac{1}{sC_{gs}\left[1+0.5(2)\right]} \approx \frac{1}{2sC_{gs}}$$ ← C_{in} has reduced by about 30%.

Issue: Reduced swing at Vout

3) Source coupled amplifier

- * Miller effect → sending signal through Drain of M_1
- * Avoid drain node
- * Send signal through source node



* C_{gd_1} – is not Miller multiplied

* effect of C_{gs_1} is halved

→ C_{gs_1} in series with C_{gs_2}

* $$Z_{in} \approx \frac{1}{s\left(C_{gd} + \frac{C_{gs_1}}{2}\right)} = \frac{1}{sC_{gs}\left(0.5 + \frac{C_{gd}}{C_{gs}}\right)}$$

$$Z_{in} \approx \frac{1}{sC_{gs}}$$

* Issues : 1) parasitic cap @ node X is high

2) Power consumed is now 2X

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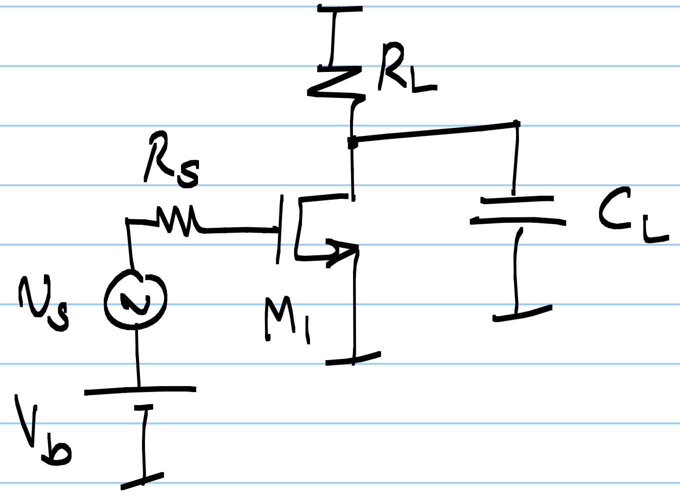
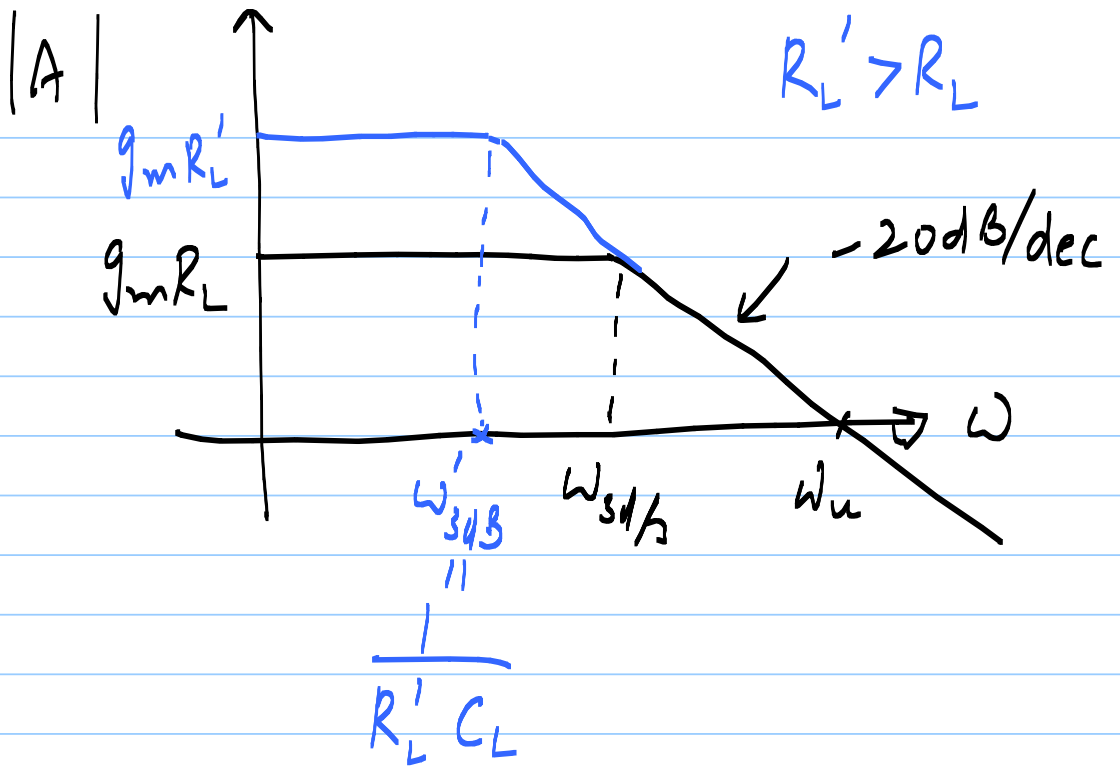


DC gain

$$A_0 = -g_m R_L$$

$$\omega_{3dB} = \frac{1}{R_L C_L}$$

$$\omega_u = \frac{g_m}{C_L}$$



We want to increase $\omega_u = \frac{g_m}{C_L}$

eg. load this amplifier with another identical amplifier

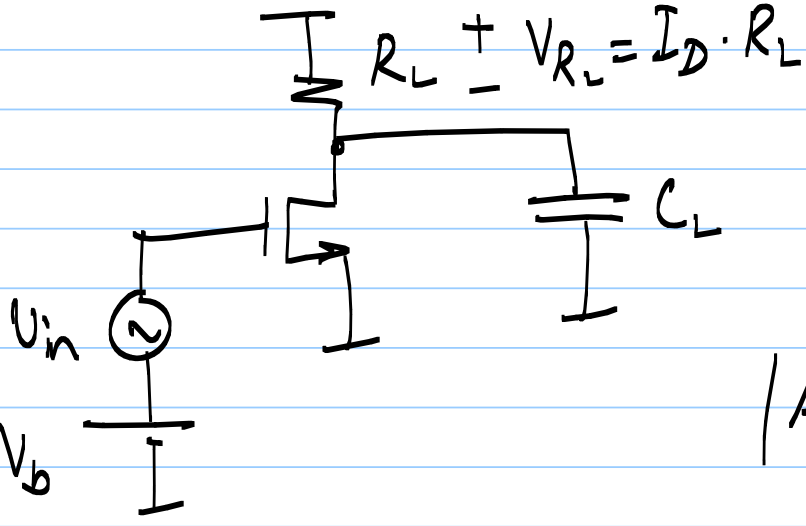
$C_L \propto W$

$$g_m = \mu_n C_{ox} \left(\frac{W}{L}\right)(V_{GS}-V_T)$$

$$\omega_u = \frac{g_m}{C_L} \propto \frac{(V_{GS}-V_T)}{L}$$

* to maximize ω_u → $L = L_{min}$
→ bias M_1 @ max $(V_{GS}-V_T)$

* In velocity saturation: ↑ $V_{GS}-V_T$ does not ↑ g_m



$$g_m = \frac{2 I_D}{V_{GS} - V_T}$$

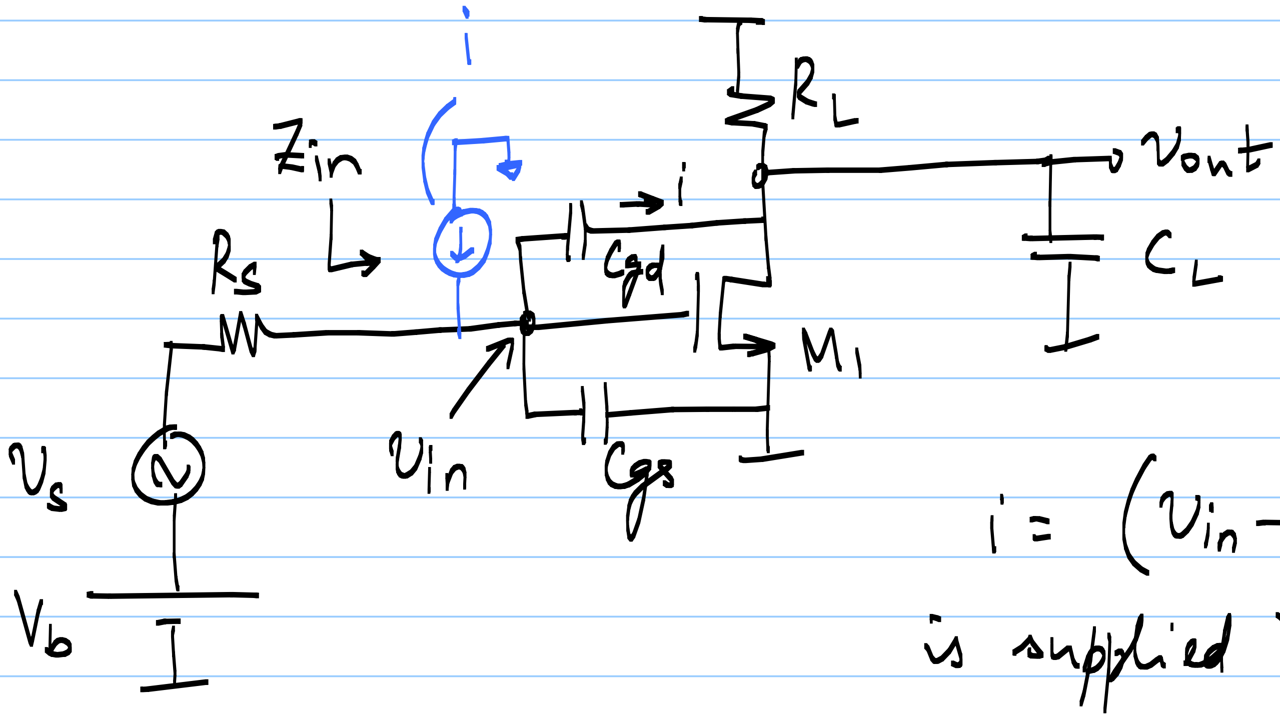
$$|A| = g_m R_L = \frac{2 I_D}{V_{GS} - V_T} \cdot R_L$$

$$= \frac{2 V_{R_L}}{V_{GS} - V_T}$$

$$\max A \approx \frac{2 V_{DD}}{V_{GS} - V_T}$$

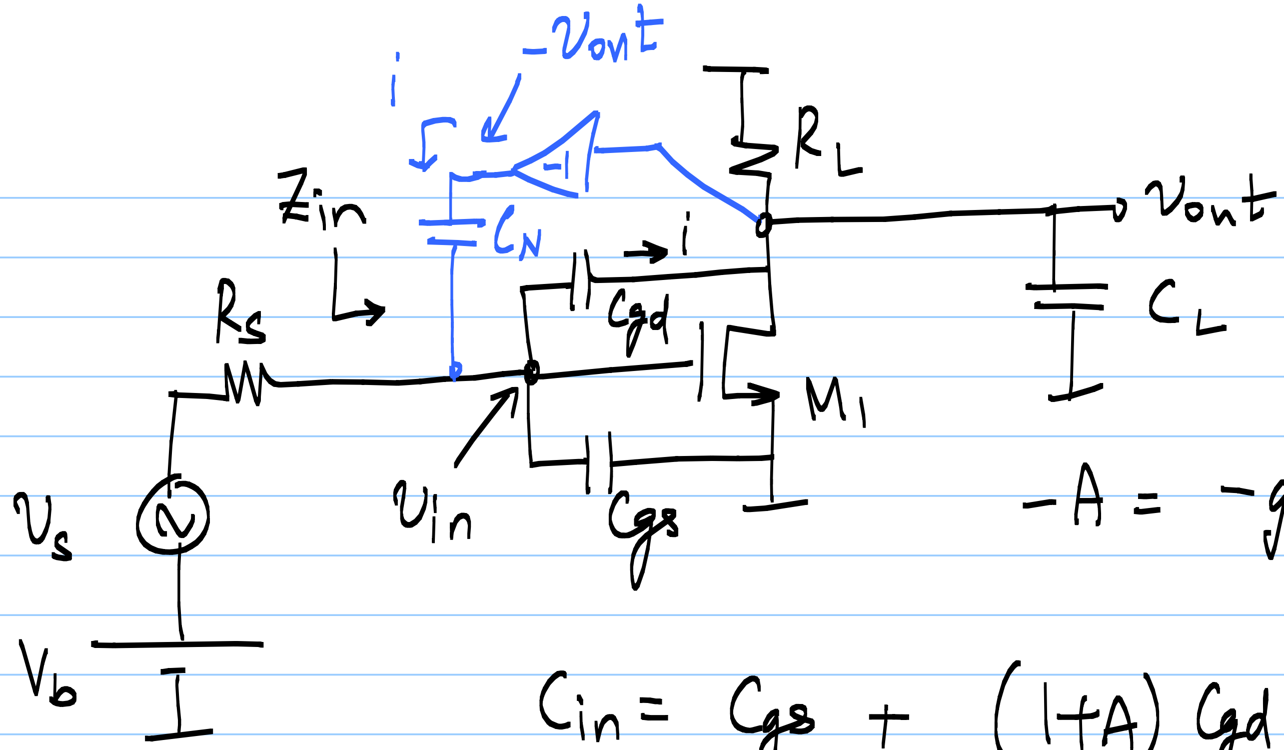
high ω_u → large $V_{GS} - V_T$ → small A_{max}

4) Neutralisation



$$i = (v_{in} - v_{out})\, sC_{gd}$$

is supplied by v_s
for the original amplifier



$i = (v_{in} - v_{out}) sC_{gd}$

$-A = -g_m R_L$

$$C_{in} = C_{gs} + (1+A) C_{gd} + (1-A) C_N$$

* choose $C_N = C_{gd}$

$$C_{in} \simeq C_{gs} + (1+A) C_{gd} + (1-A) C_{gd}$$

$= C_{gs} + 2 C_{gd}$ { Same as that for cascode } ✓

- $C_N < C_{gd}$
  - ↳ not fully neutralised
  - ↳ $C_{in} > C_{gs} + 2C_{gd}$

- $C_N > C_{gd}$

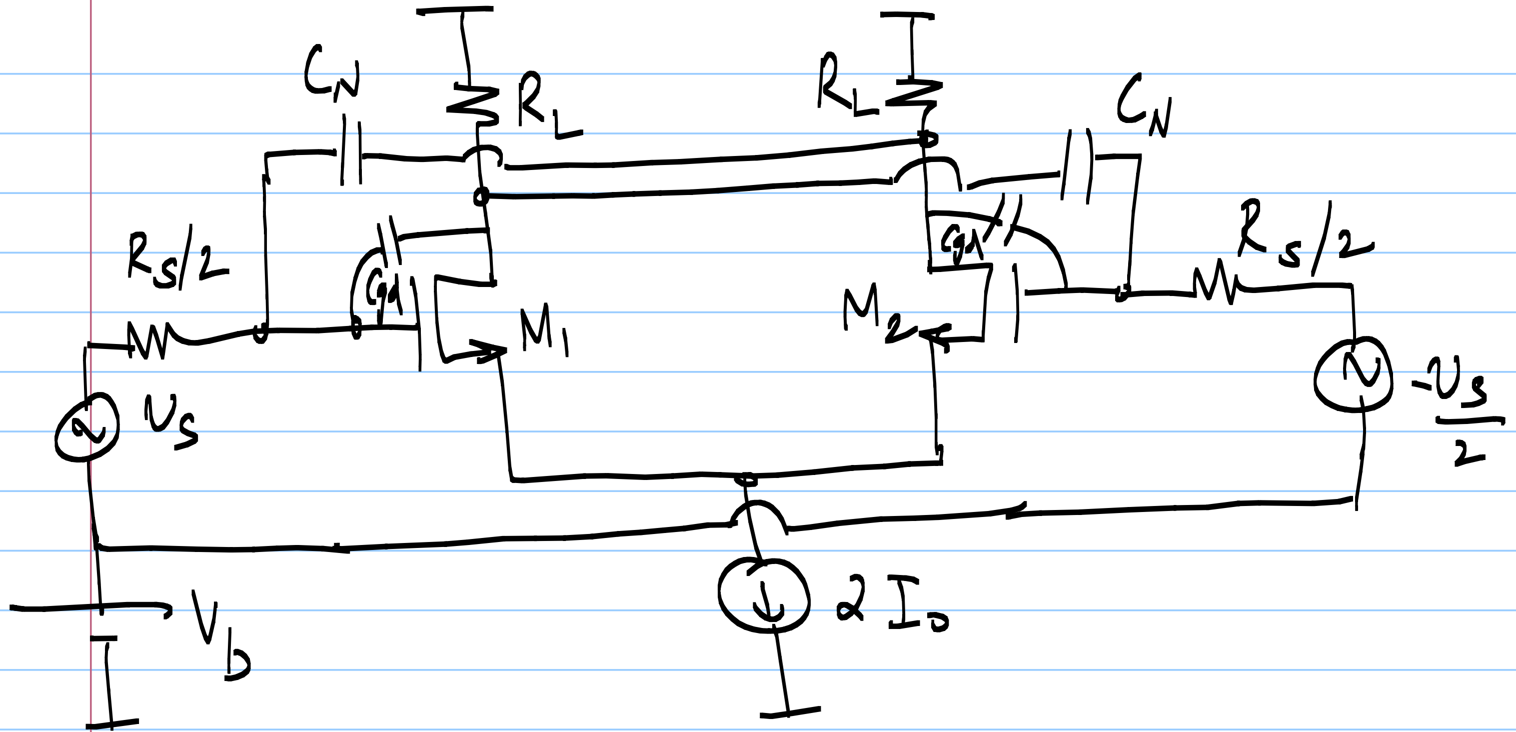
$$C_{in} = C_{gs} + (1+A)\,C_{gd} + (1-A)\,C_N$$

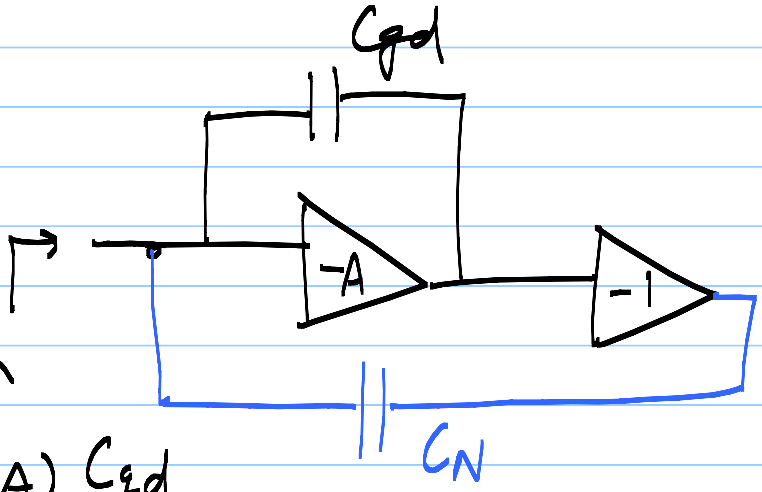
$$= C_{gs} + (1+g_mR_L)\,C_{gd} + \boxed{(1-g_mR_L)\,C_N}$$

adds −ve cap

→ can reduce effect of C_{gs}

- If C_N becomes too large → C_{in} is −ve
  - ↳ Can degrade stability
  - ↳ Can present inductive input impedance

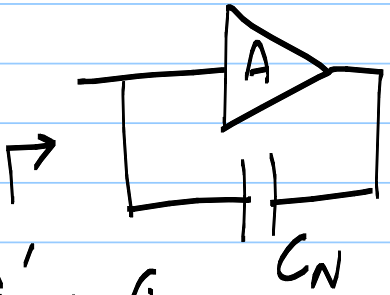




C_{in}

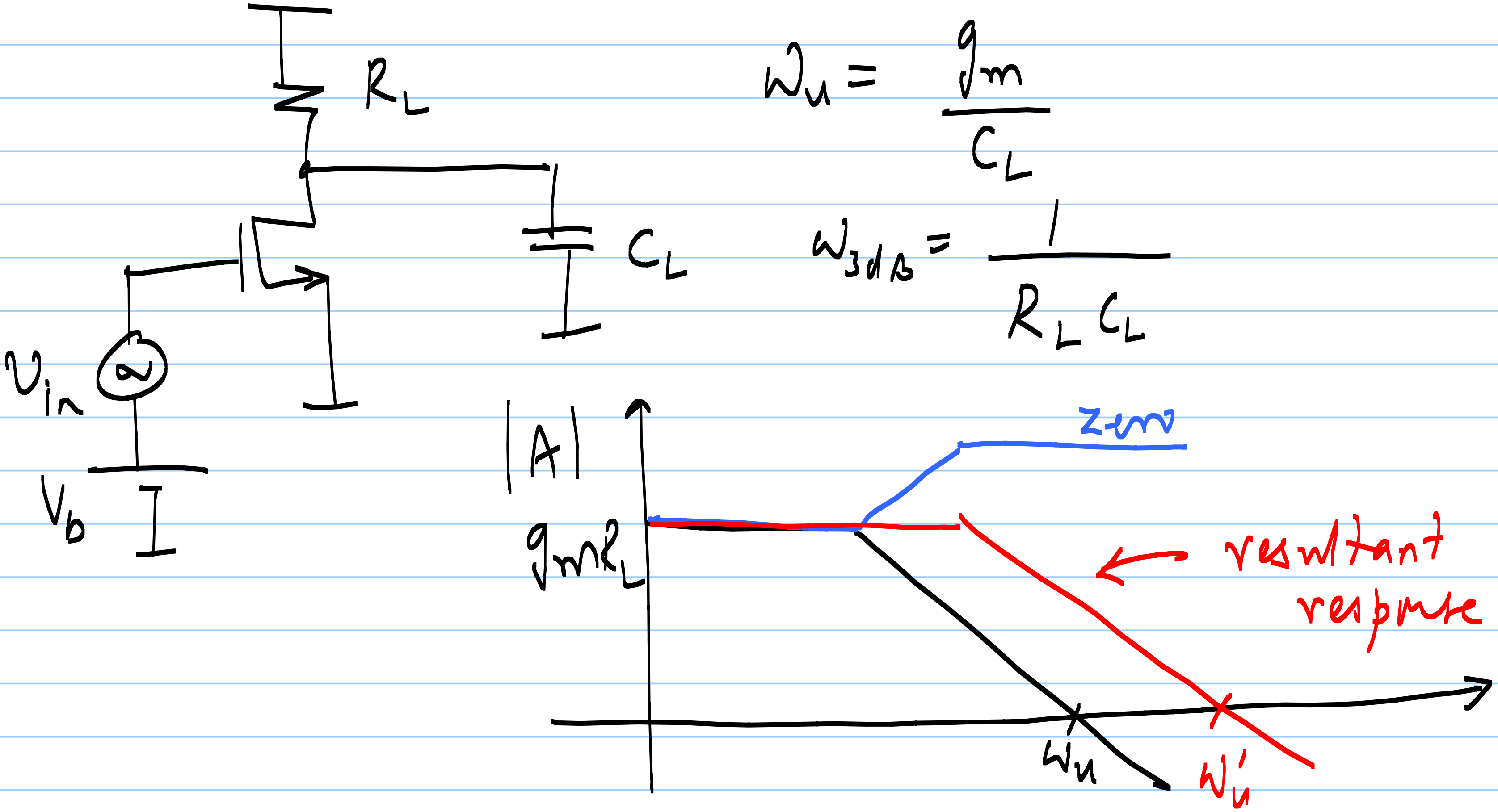
$||$

$(1+A)\,C_{gd}$



$C_{in}' = (1-A)\,C_N$

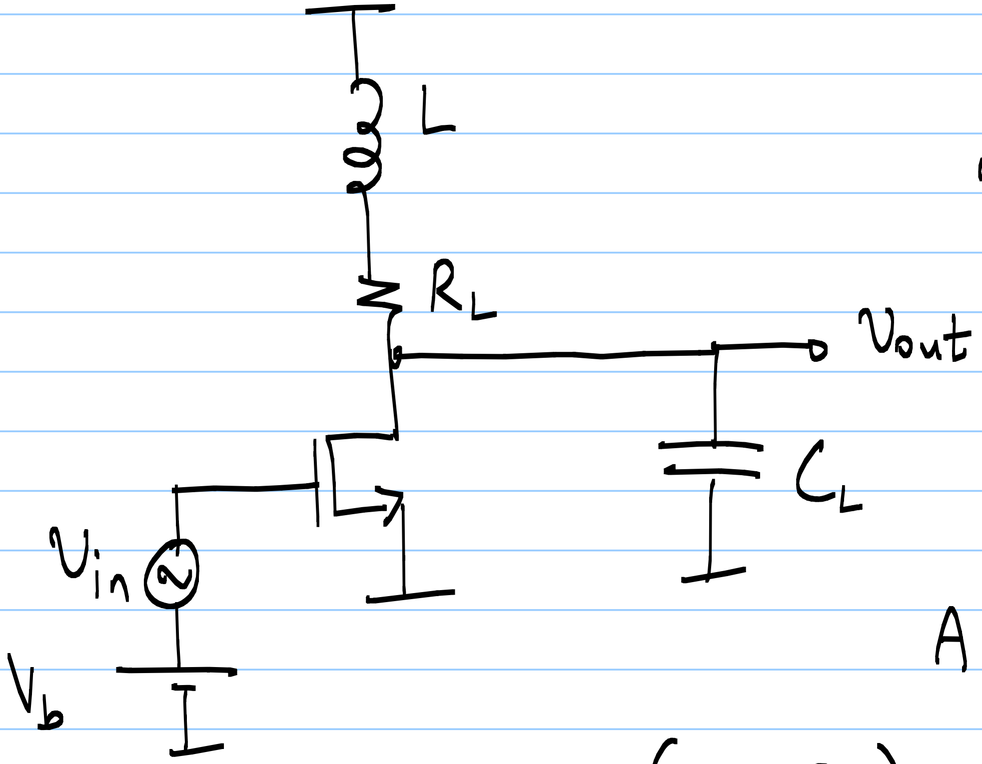
Bandwidth Extension

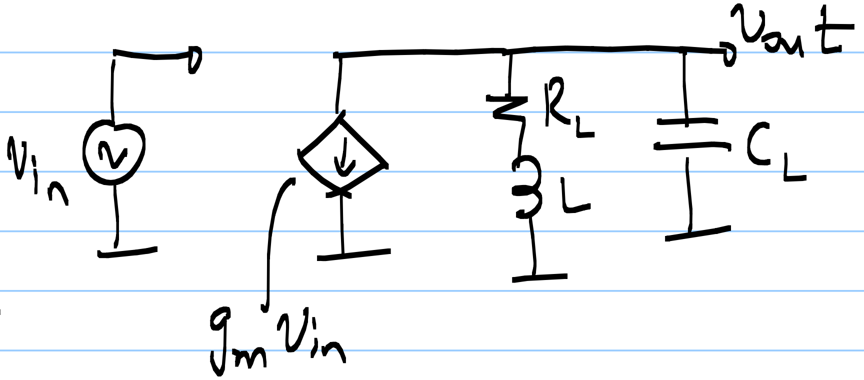


$$\omega_u = \frac{g_m}{C_L}$$

$$\omega_{3dB} = \frac{1}{R_L C_L}$$

Use an inductor to add a zero





$$A = -g_m \left[(sL + R_L) \,||\, (1/sC_L) \right]$$

$$A = -g_m \frac{(sL + R_L) \cdot 1/sC_L}{sL + R_L + \frac{1}{sC_L}}$$

$$= -g_m \frac{sL + R_L}{s^2 L C_L + s R_L C_L + 1}$$

$$A = -g_m R_L \frac{s\left(\frac{L}{R_L}\right) + 1}{s^2 L C_L + s R_L C_L + 1}$$

2 time constants:

$$\tau_0 = R_L C_L = \frac{1}{\omega_0} \quad ; \quad \tau_1 = \frac{L}{R_L}$$

set $m = \frac{\tau_0}{\tau_1} = \frac{R_L C_L}{L/R_L} = \frac{R_L^2 C_L}{L}$

$$A = -g_m R_L \frac{s\,\tau_1 + 1}{s^2 \cdot \tau_1^2 \cdot m}$$

$$\tau_1^2 = \frac{L^2}{R_L^2} \quad ; \quad \tau_1^2 \cdot m = \frac{L^2}{R_L^2} \cdot \frac{R_L^2 C_L}{L} = L C_L$$

<u>replace</u> $\tau_1 = \tau_0/m = \frac{1}{m\omega_0}$ and set $s = j\omega$

$$|A| = g_m R_L \cdot \left| \frac{j \frac{\omega}{m\omega_0} + 1}{-\left(\frac{\omega}{m\omega_0}\right)^2 \cdot m + j\left(\frac{\omega}{m\omega_0}\right) \cdot m + 1} \right|$$

Let new $\omega_{3dB} = \omega_1$ $\{$ old $\omega_{3dB} = \omega_0 \}$

$$\left| \frac{j \frac{\omega_1}{m\omega_0} + 1}{-\left(\frac{\omega_1}{m\omega_0}\right)^2 \cdot m + j \frac{\omega_1}{\omega_0} + 1} \right| = \frac{1}{\sqrt{2}}$$

* we want to find $\frac{\omega_1}{\omega_0}$ and maximize it

set $\alpha = \frac{\omega_1}{\omega_0}$ = "BW extension ratio"

$$\left| \frac{j\frac{\alpha}{m}+1}{j\alpha+1-\left(\frac{\alpha}{m}\right)^2\cdot m} \right| = \frac{1}{\sqrt{2}}$$

$$\frac{1+\frac{\alpha^2}{m^2}}{\alpha^2+\left(1-\frac{\alpha^2}{m}\right)^2} = \frac{1}{2}$$

$$2+\frac{2\alpha^2}{m^2} = \alpha^2+1+\frac{\alpha^4}{m^2}-\frac{2\alpha^2}{m}$$

$$\frac{\alpha^4}{m^2}+\alpha^2\left(1-\frac{2}{m}-\frac{2}{m^2}\right)-1=0$$

$$\alpha^4 + \alpha^2 \left(m^2 - 2m - 2\right) - m^2 = 0$$

$$\alpha^2 = \frac{\left(2+2m-m^2\right) + \sqrt{\left(2+2m-m^2\right)^2 + 4m^2}}{2}$$ larger root

$$= \left(-\frac{m^2}{2} + m + 1\right) + \sqrt{\left(-\frac{m^2}{2} + m + 1\right)^2 + m^2}$$

$$\alpha = \frac{W_1}{W_0} = \sqrt{\left(-\frac{m^2}{2} + m + 1\right) + \sqrt{\left(-\frac{m^2}{2} + m + 1\right)^2 + m^2}}$$

- m sets amount of BW extension

- $L = \dfrac{R_L^2 C_L}{m}$ is the desired inductor value

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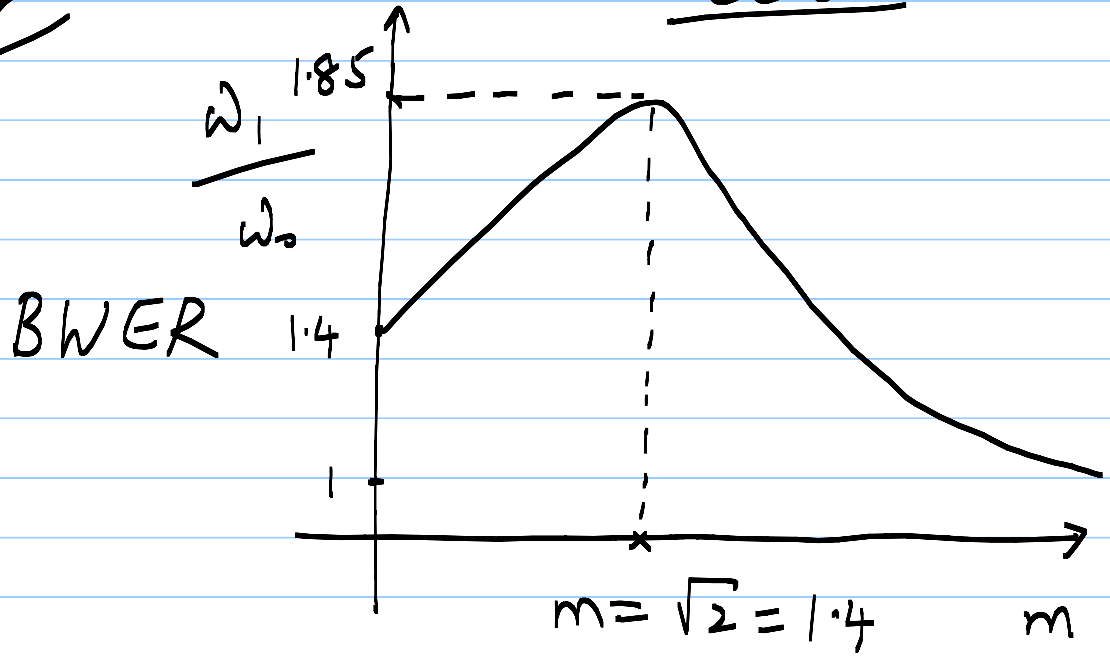
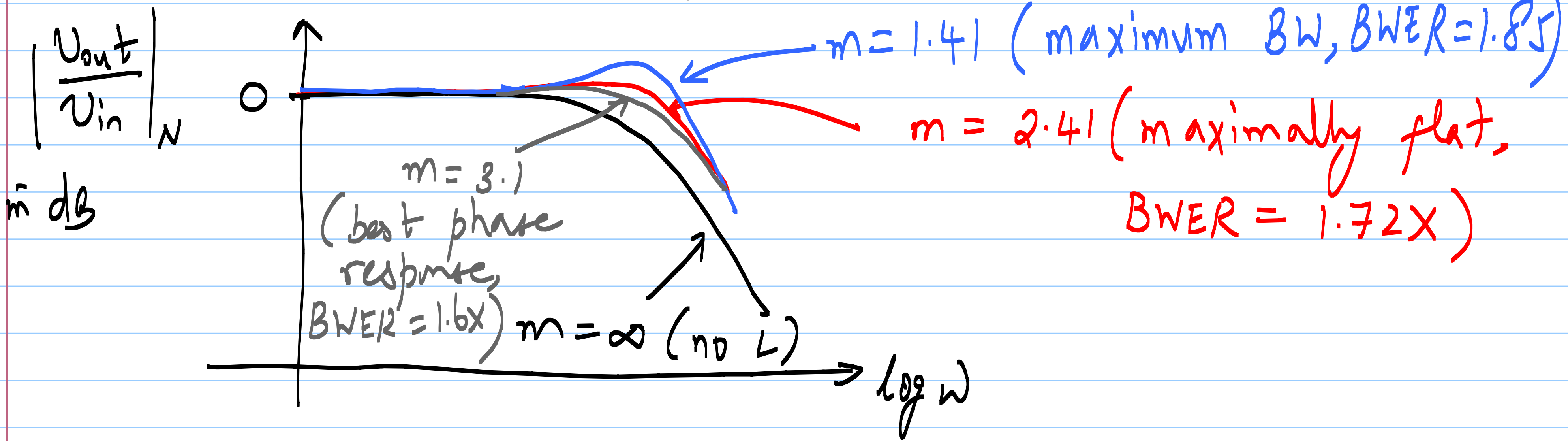


highest BWER = 1.85x

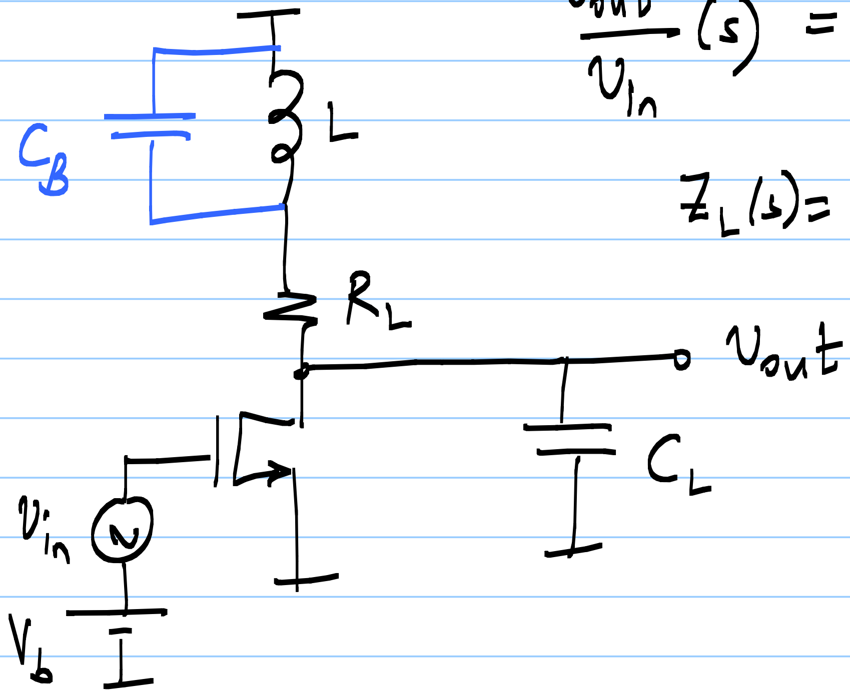
@ m = 1.41



$$T_D(\omega) = -\frac{d\phi(\omega)}{d\omega}$$

If delay is constant over as wide a BW as possible → maximally flat time delay (best phase response)

2) Bridged Shunt Peaking



$$\frac{v_{out}}{v_{in}}(s) = -g_m Z_L(s)$$

$$Z_L(s) = \left[\left(sL \| \frac{1}{sC_B}\right) + R_L\right] \| \left(\frac{1}{sC_L}\right)$$

$$Z_L(s) = \left[R_L + \left(\frac{sL \cdot \frac{1}{sC_B}}{sL + \frac{1}{sC_B}}\right)\right] \| \left(\frac{1}{sC_L}\right)$$

$$Z_L(s) = \left[R_L + \left(\frac{sL}{1+s^2LC_B}\right)\right] \parallel \frac{1}{sC_L}$$

$$= \left[\frac{R_L + sL + s^2LC_BR_L}{1+s^2LC_B}\right] \parallel \left(\frac{1}{sC_L}\right)$$

$$= \frac{\frac{(R_L + sL + s^2LC_BR_L)}{1+s^2LC_B} \times \frac{1}{sC_L}}{\frac{R_L + sL + s^2LC_BR_L}{1+s^2LC_B} + \frac{1}{sC_L}}$$

$$Z_L(s) = \frac{R_L + sL + s^2 L C_B R_L}{1 + s\,C_L R_L + s^2(LC_L + LC_B) + s^3 L C_B C_L R_L}$$

← 2 zeroes

Parameterize:

$$m = \frac{R_L^2 C_L}{L} \; ; \qquad \omega_0 = \frac{1}{R_L C_L} \; ; \qquad k_B = \frac{C_B}{C_L}$$

$$Z_L(s) = R_L \frac{1 + \frac{sL}{R_L} + s^2 L C_B}{1 + s C_L R_L + s^2 L(C_L + C_B) + s^3 L C_B C_L R_L}$$

$$\frac{L}{R_L} = \frac{1}{R_L/L} = \frac{1}{\frac{R_L^2 C_L}{L} \cdot \frac{1}{R_L C_L}} = \frac{1}{m\omega_0}$$

$$L C_B = \frac{1}{1/(L C_B)} = \frac{1}{\frac{R_L^2 C_L}{L} \cdot \frac{1}{R_L^2 C_L C_B}}$$

$$= \frac{1}{m \cdot \frac{1}{R_L^2 C_L^2} \cdot \frac{C_L}{C_B}} = \frac{1}{m \cdot \omega_0^2 \cdot \frac{1}{k_B}}$$

$$= \frac{k_B}{m \omega_0^2}$$

$$L\left(C_L + C_B\right) = \frac{1}{{}^{1}/\left(L\left(C_L + C_B\right)\right)} = \frac{1}{\frac{R_L^2 C_L}{L} \cdot \frac{1}{R_L^2 C_L} \cdot \frac{1}{\left(C_L + C_B\right)}}$$

$$= \frac{1}{m \cdot \frac{1}{R_L^2 C_L^2} \cdot \frac{1}{\left(1 + C_B/C_L\right)}} = \frac{k_B + 1}{m \omega_0^2}$$

$$L C_B C_L \cdot R_L = L \cdot \frac{C_B}{C_L} \cdot R_L C_L^2 = L \cdot k_B \cdot \frac{1}{R_L} \cdot \left(C_L^2 R_L^2\right)$$

$$= \frac{k_B}{\omega_0^2} \cdot \frac{L}{R_L} = \frac{k_B}{\omega_0^2} \cdot \frac{1}{m\omega_0} = \frac{k_B}{m\omega_0^3}$$

$$\frac{v_{out}}{v_{in}}(s) = -g_m R_L \cdot \underbrace{Z_N(s)}_{\text{normalized impedance}}$$

$$Z_N(s) = \frac{1 + \left(\frac{1}{m}\right)\frac{s}{\omega_0} + \left(\frac{k_B}{m}\right)\frac{s^2}{\omega_0^2}}{1 + \frac{s}{\omega_0} + \left(\frac{k_B+1}{m}\right)\frac{s^2}{\omega_0^2} + \left(\frac{k_B}{m}\right)\frac{s^3}{\omega_0^3}}$$

$m = \dfrac{R_L^2 \cdot C_L}{L}$

$k_B = 0$, $m = \infty$ ⇒ BWER $= 1X$

$k_B = 0$, $m = 1.41$ ⇒ $1.83X$, 1.5 dB peaking

$k_B = 0.3$, $m = 2.4$ ⇒ $1.83x$, maximally flat

* as $k_B \uparrow$ ($\because C_B \uparrow$), change m to keep BWER at $1.83X$ ⇒ amount of peaking reduces

<u>e.g.</u> final BW $= 1.5\,GHz = f_1$

phase modulation → best group delay case

⇒ $m = 3.1$

say $C_L = 1.5\,pF$; min $R_L = 100\Omega$
(limited by power consumption)

original BW $f_o = \frac{1}{2\pi R_L C_L} = 1.06$ GHz

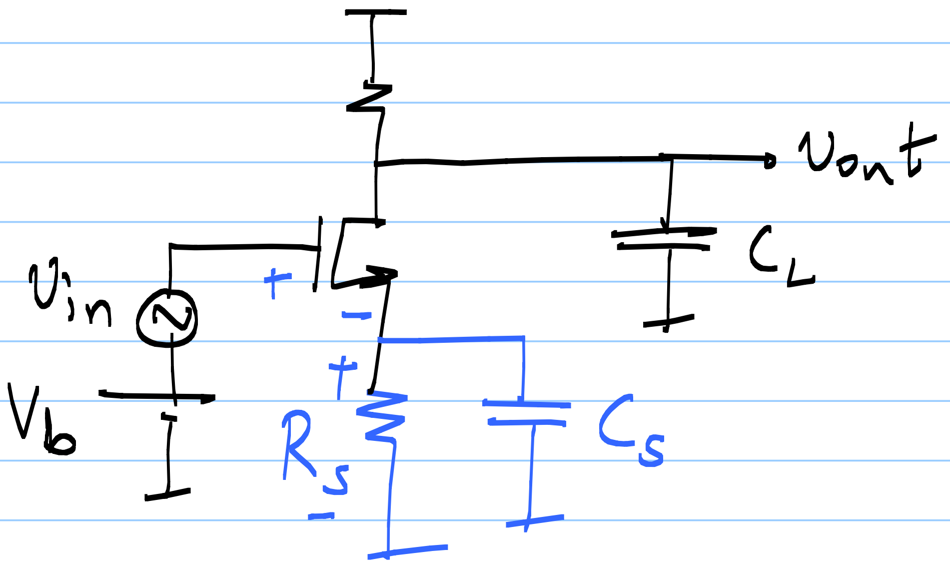
$m = 3.1 \Rightarrow L = \frac{R_L^2 C_L}{3.1} = 4.8$ nH

$f_1 = 1.6 f_o = 1.7$ GHz

$Q = \frac{w_o L}{R_L} = 0.5$ @ 1.7GHz

much
lower
than Q
of inductor
itself

3) Create zero without L



Zero → low gain @ low freq., high gain @ higher freq.

gain @ low freq. $= \dfrac{g_m R_L}{1+g_m R_s}$

$$Z_L = R_L \,\|\, \frac{1}{sC_L} \quad ; \quad Z_s = R_s \,\|\, \frac{1}{sC_s}$$

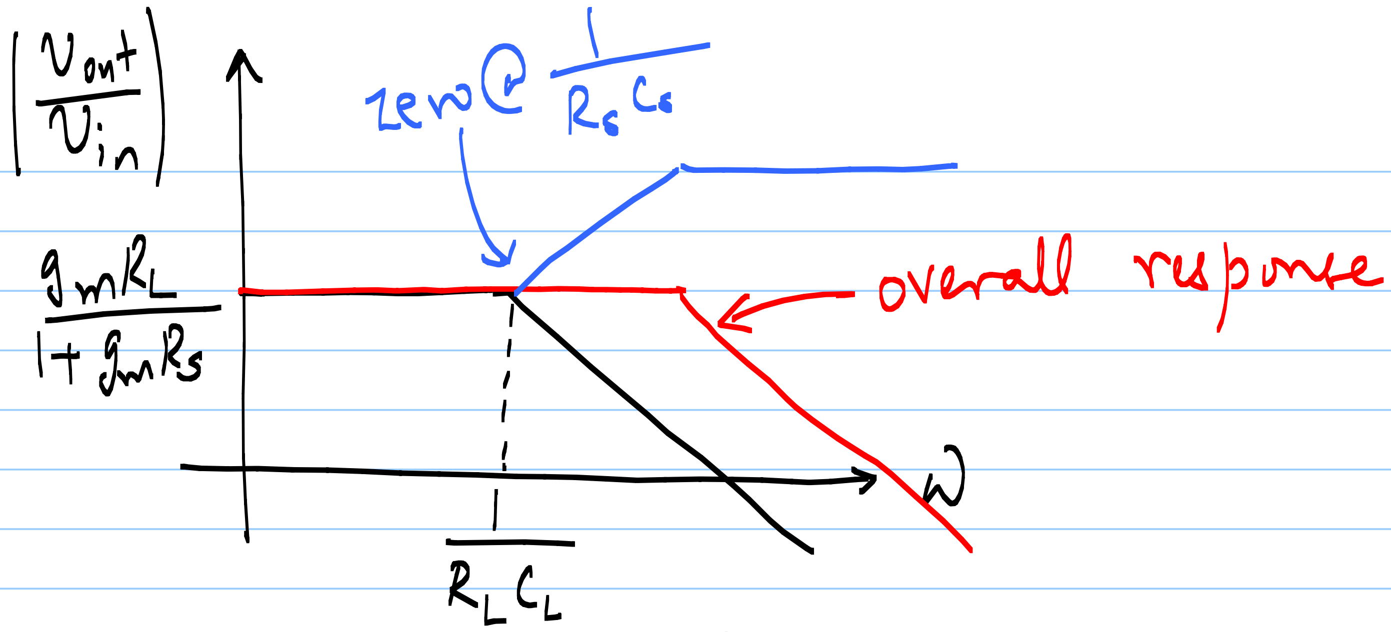
$$\frac{V_{out}}{V_{in}}(s) = -\frac{g_m Z_L}{1+g_m Z_s + sC_{gs} Z_s}$$

If $\omega << \omega_T \Rightarrow g_m >> sC_{gs}$

$$\frac{v_{out}}{v_{in}}(s) \approx -g_m R_L \frac{1 + sR_s C_s}{(1+sR_L C_L)(1+sR_s C_s + g_m R_s)}$$

Zero @ $\frac{1}{R_s C_s}$

2nd pole @ $\frac{1+g_m R_s}{R_s C_s}$



* Cancel pole @ $\frac{1}{R_L C_L}$ by zero @ $\frac{1}{R_s C_s}$

$$\Rightarrow \boxed{R_s C_s = R_L C_L}$$

$$\frac{V_{out}}{V_{in}} = -\frac{g_m R_L}{1 + sR_s C_s + g_m R_s}$$

new $BW = \frac{1 + g_m R_S}{R_S C_S} = \frac{1+g_m R_S}{R_L C_L}$

dc $A_o = \frac{g_m R_L}{1+g_m R_S}$

But : If we had designed original CSA for ~~same~~ same DC gain

choose $R_L' = \frac{R_L}{1+g_m R_S}$

$BW = \omega_2 = \frac{1}{R_L' C_L} = \frac{1+g_m R_S}{R_L C_L}$ = same as zero-peaked amplifier

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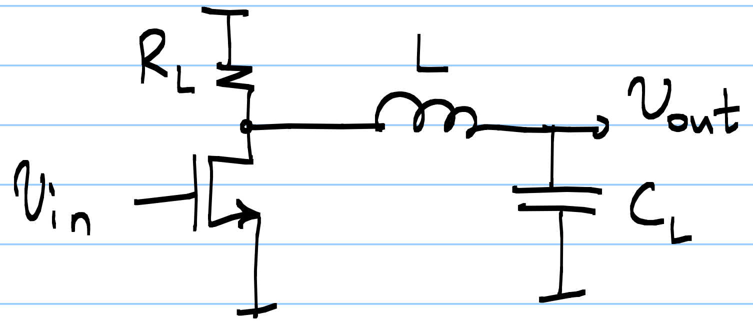
4) **Series peaking** (neglect drain parasitics)



$$\frac{V_{out}}{V_{in}}(s) = -g_m R_L \cdot Z_N(s)$$

$$\omega_0 = \frac{1}{R_L C_L} \; ; \quad m = \frac{R_L^2 C_L}{L}$$

$$Z_N(s) = \frac{1}{1 + \frac{s}{\omega_0} + \frac{s^2}{m\omega_0^2}}$$

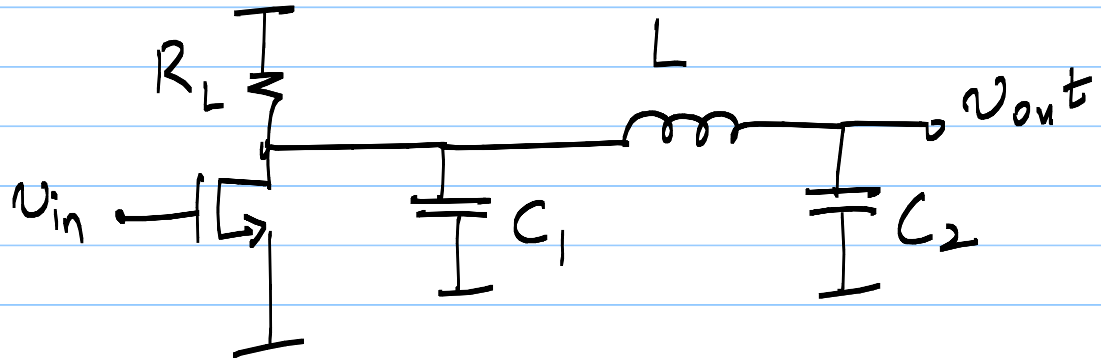
* inferior to shunt peaking due to lack of zero

* maximum BWER = 1.41X @ m=2

5) Series peaking with cap. load splitting

* C_L = drain parasitics + input cap of next stage
  C_1 C_2

$$C_L = C_1 + C_2$$



$$\omega_0 = \frac{1}{R_L C_L}$$

$$m = \frac{R_L^2 C_L}{L}$$

$$k_c = \frac{C_1}{C_L}$$

* no zeroes
* 3rd order system

$$Z_N(s) = \frac{1}{1 + \frac{s}{\omega_0} + \left(\frac{1-k_c}{m}\right)\frac{s^2}{\omega_0^2} + \left[\frac{k_c(1-k_c)}{m}\right]\frac{s^3}{\omega_0^3}}$$

$k_c = 0$, $m = 2$ $\Rightarrow$ BWER = 1.41X

$k_c = 0.3$, $m = 2.4$ $\Rightarrow$ BWER = 2.52x , no peaking

$k_c = 0.4$, $m = 1.9$ $\Rightarrow$ BWER = 2.75x, 1dB peaking

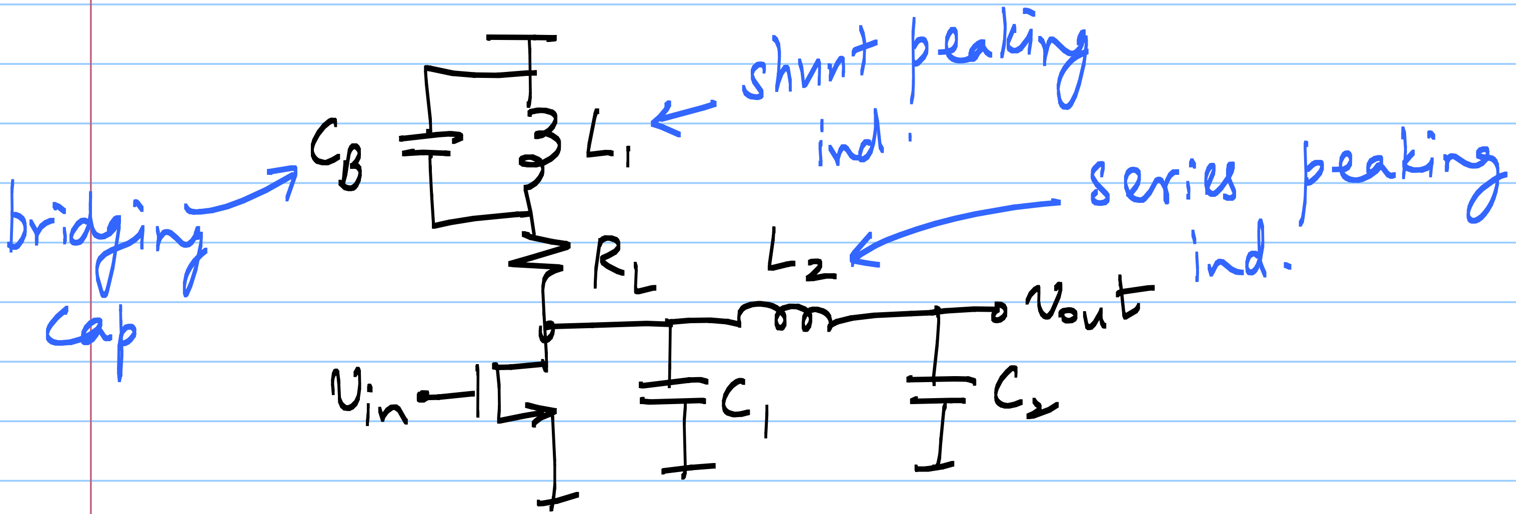
$k_c = 0.4$, $m = 2.5$ $\Rightarrow$ BWER = 3.17x, 3 dB peaking

Can you combine shunt & series peaking ?

Yes

6) Bridged shunt-series peaking

* adds $\underbrace{\text{2 zeroes}}_{\text{shunt } L + C_B}$ to $Z_N(s)$ in (5) above



$$\omega_0 = \frac{1}{R_L C_L}$$

$$C_L = C_1 + C_2$$

$$k_C = \frac{C_1}{C_L}$$

$$m_1 = \frac{R_L^2 C_L}{L_1}$$

$$m_2 = \frac{R_L^2 C_L}{L_2}$$

$$k_B = \frac{C_B}{C_L}$$

$$Z_N(s) = \frac{\text{2 zeroes}}{\text{5 poles}}$$

$$Z_N(s) = \frac{1 + \left(\frac{1}{m_1}\right)\frac{s}{\omega_o} + \left(\frac{k_B}{m_1}\right)\frac{s^2}{\omega_o^2}}{1 + \frac{s}{\omega_o} + \left[\frac{1+k_B}{m_1} + \frac{1-k_c}{m_2}\right]\frac{s^2}{\omega_o^2} + \left[\frac{k_B}{m_1} + \frac{k_c(1-k_c)}{m_2}\right]\frac{s^3}{\omega_o^3} + \left[\frac{(k_c+k_B)(1-k_c)}{m_1 m_2}\right]\frac{s^4}{\omega_o^4} + \left[\frac{k_B k_c(1-k_c)}{m_1 m_2}\right]\frac{s^5}{\omega_o^5}}$$

* $k_c = 0.4$, $m_1 = 8$, $m_2 = 2.4$, $k_B = 0.3$:

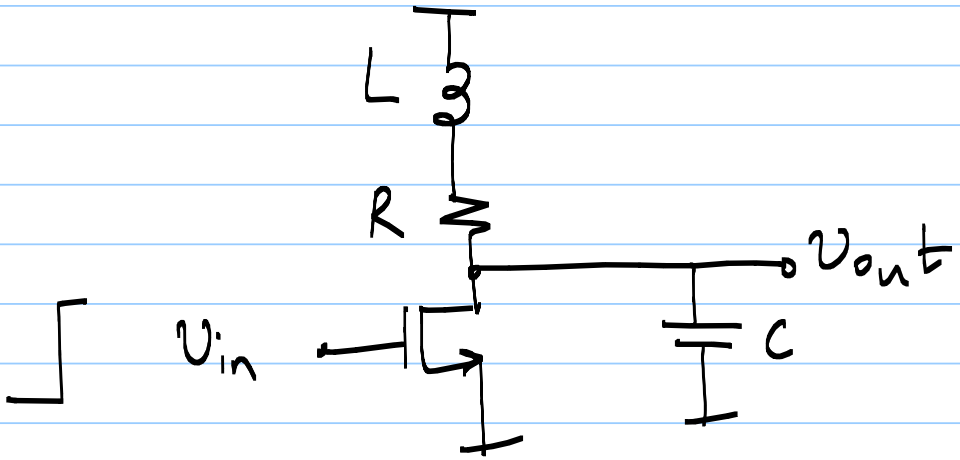
BWER = 3.92x , 0 dB peaking

* $k_c = 0.4$, $m_1 = 6$, $m_2 = 2.4$, $k_B = 0.2$:

BWER = 4x, 2 dB peaking

7) shunt + double series peaking

revisit basic shunt peaking



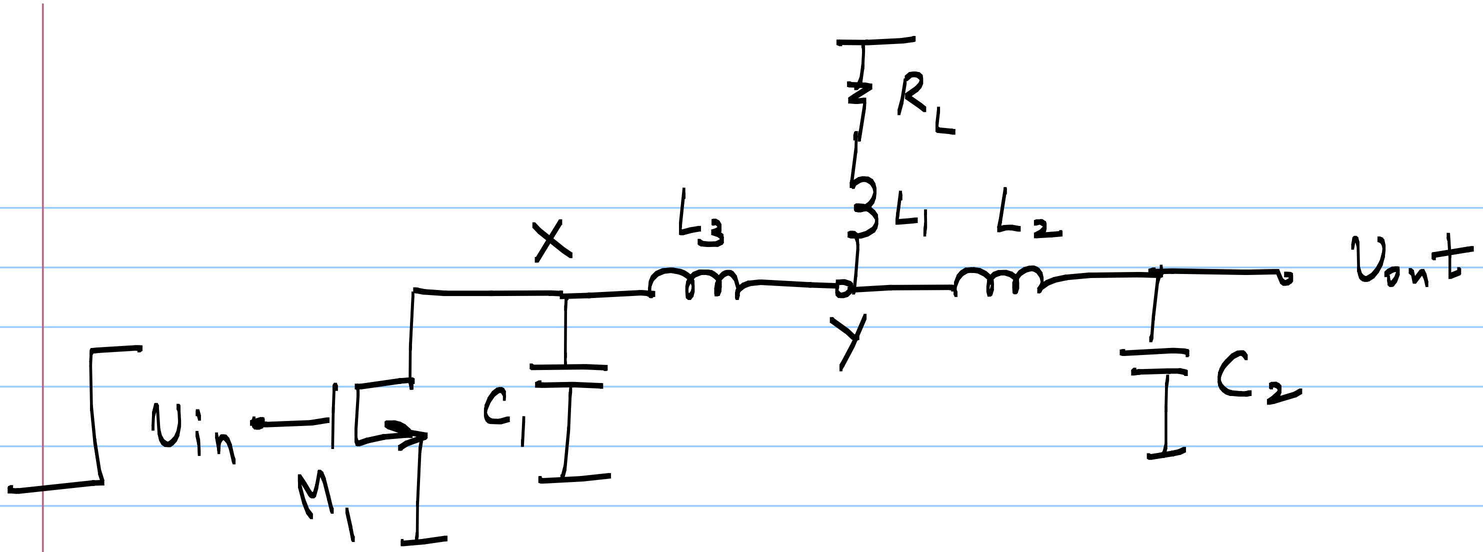
* BW can be inferred from step response

faster step response ⟷ higher BW

* L delays current flow through R

→ more current available to charge C

→ lower rise time, higher BW



- * L_1 delays current through R_L
- * L_3 delays current to rest of the network
- * M_1 drive only self cap. C_1 for some time
  → rise time @X (drain of M_1) improves
- * After some time, V_Y rises
- * Finally, V_{out} starts to rise (as current starts to flow through L_2)

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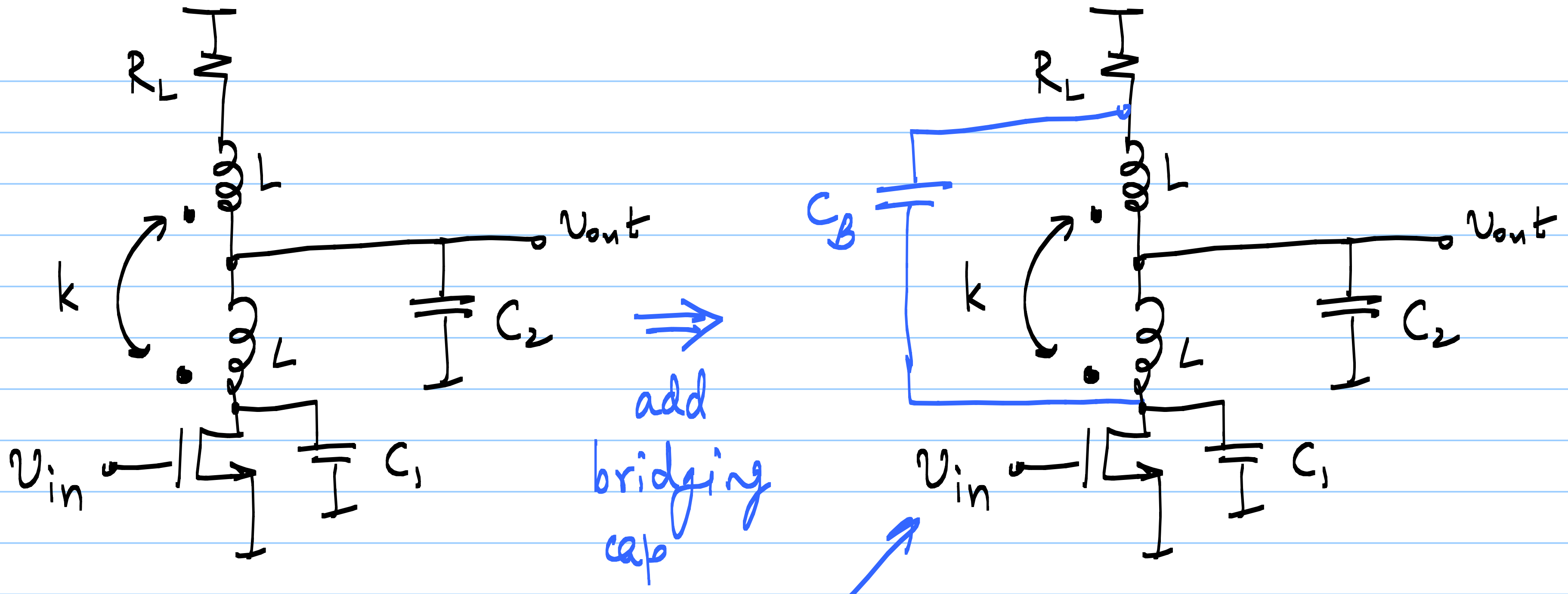
- * shunt + double series peaking network trades off increased delay for improved BW

8) T-coil BW enhancement

- * Realize $L_1-L_2-L_3$ as a transformer
  - → smaller die area
  - → Eq. ckt is the same
- * Use a symmetric Xfmr
- * works well if $C_1 << C_2$



9) Bridged T-coil BW enhancement

- * C_B creates parallel resonance across Xfmr
- * Circulating currents allow faster charging of node voltages (↑ BW)

$$Z_N(s) = \frac{1}{1 + 2\zeta\left(\frac{s}{\omega_n}\right) + \frac{s^2}{\omega_n^2}}$$

$$L = \frac{R_L^2 C_L}{2(1+k)}$$ = primary & secondary self-ind.

$$C_B = \frac{C_L}{4} \frac{1-k}{1+k}$$

k - decides magnitude response

* $k = \frac{1}{3} \Rightarrow$ Butterworth response (maximally flat)

$$\left(\zeta = \frac{1}{\sqrt{2}}\right)$$

$BWER = 2\sqrt{2} = 2.83\times$

* $k = \frac{1}{2}$ ⇒ maximally flat group delay

BWER = 2.7X

* $C_2 >> C_1$

* If $C_1 >> C_2$, interchang T-coil connections

e.g. min. $R_L = 100\Omega$, $C_L = 1.5pF$

$$f_0 = \frac{1}{2\pi R_L C_L} = 1.06 GHz$$

phase modulated data ⇒ $k = \frac{1}{2}$

$$L = \frac{R_L^{\,2} C_L}{2(1+k)} = 5\,nH$$

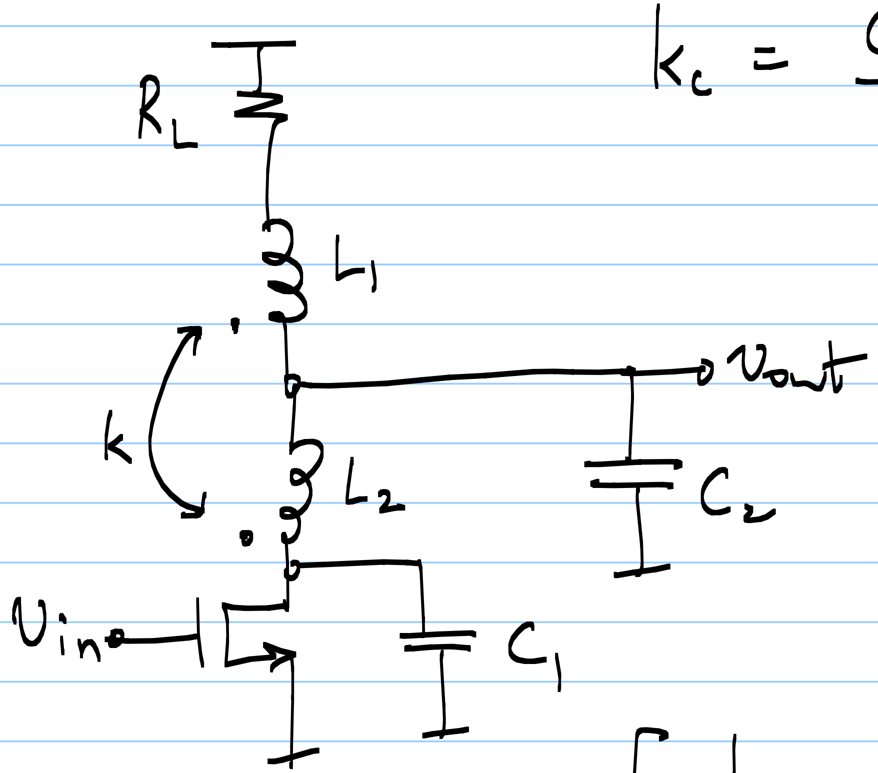
$$C_B = \frac{C_L}{4}\,\frac{1-k}{1+k} = 125\,fF$$

Can be absorbed as parasitic cap of Xfmr

$$\text{new BW} = 2.7 \times 1.06\,GHz$$
$$= 2.86\,GHz$$

⇒ no increase in power

10) Asymmetric T-coil



$$k_c = \frac{C_1}{C_L} \; ; \quad m_1 = \frac{R_L^2 C_L}{L_1} \; ;$$

$$m_2 = \frac{R_L^2 C_L}{L_2} \; ;$$

k_m = Xfmr coupling factor

$$= \frac{M}{\sqrt{L_1 L_2}}$$

$$Z_N(s) = \frac{1 + \left[\frac{1}{m_1} + \frac{k_m}{\sqrt{m_1 m_2}}\right]\frac{s}{\omega_0}}{1 + \frac{s}{\omega_0} + \left[\frac{1}{m_1} + \frac{k_c}{m_2} + \frac{2k_c k_m}{\sqrt{m_1 m_2}}\right]\frac{s^2}{\omega_0^2} + \left[\frac{k_c(1-k_c)}{m_2}\right]\frac{s^3}{\omega_0^3} + \left[\frac{k_c(1-k_c)(1-k_m^2)}{m_1 \, m_2}\right]\frac{s^4}{\omega_0^4}}$$

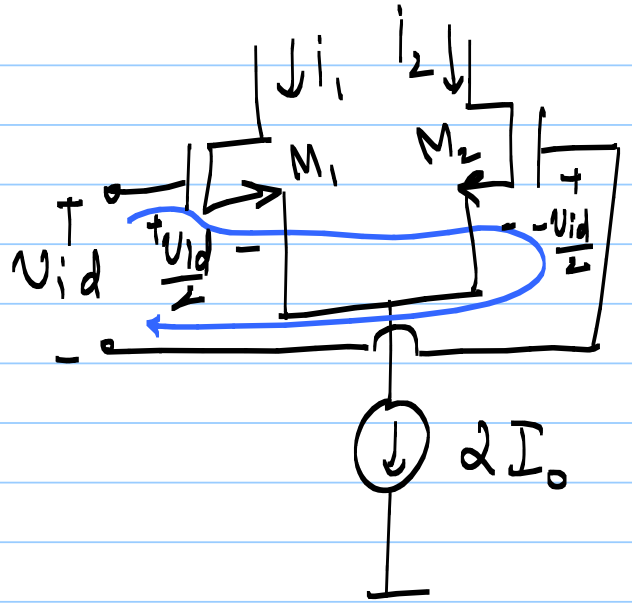
BW enhancement (f_T doublers)

MOSFET $\omega_T = \dfrac{g_m}{C_{gs} + C_{gd}} \approx \dfrac{g_m}{C_{gs}}$

$\uparrow \omega_T \Rightarrow \uparrow I_D, \downarrow W$

$(\uparrow V_{GS} - V_T)$

$\Rightarrow$ you will hit velocity sat.



$$i_d = i_1 - i_2 = g_m v_{id}$$

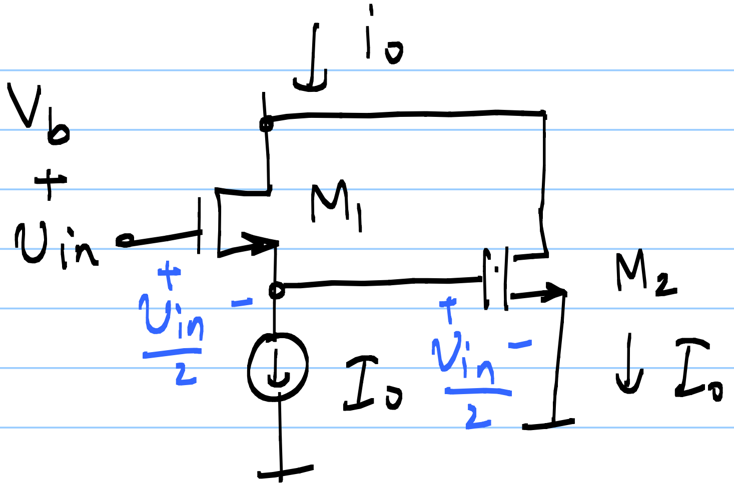
$$C_{in} = C_{gs_1} \text{ series } C_{gs_2}$$

$$= \frac{C_{gs}}{2}$$

$$\text{eq. } f_T = \frac{2 g_m}{C_{gs}}$$

* Diff. amp. acts as an f_T doubler

* Single ended In & Out ⇒ interchang G & S of M_2



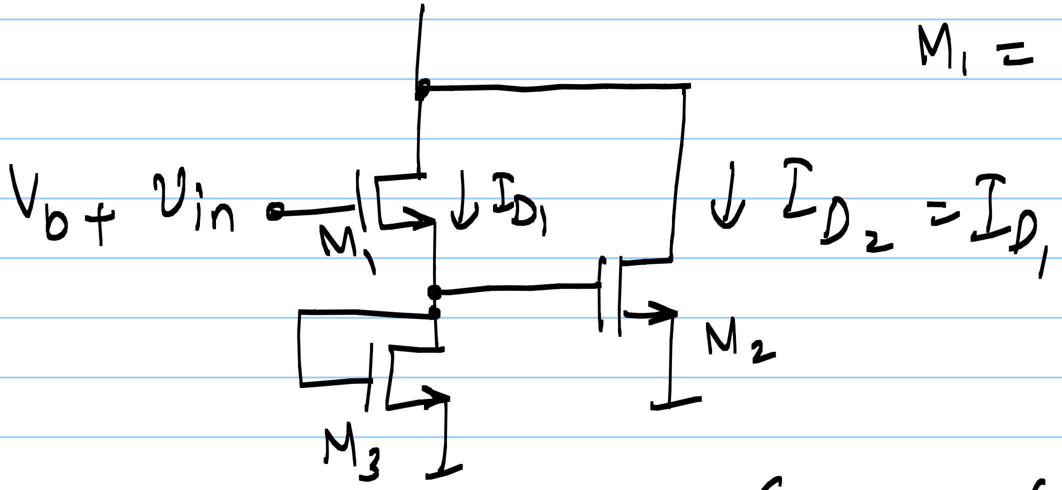
$M_1 = M_2$

$I_{D_1} = I_{D_2}$

$$\left.\begin{matrix} M_1 = M_2 \\ I_{D_1} = I_{D_2} \end{matrix}\right\} \quad \begin{matrix} C_{in} = C_{gs}/2 \\ i_1 = i_2 = g_m \frac{v_{in}}{2} \\ i_o = i_1 + i_2 \end{matrix}$$

$$f_T = \frac{2g_m}{C_{gs}}$$

Battjes f_T doubler



$M_1 = M_2 = M_3$

$f_T \approx 1.5X$ improvement due to C_{gs} & C_{db} of M_3

$$C_{in} \approx C_{gs_1} \text{ ser.} (C_{gs_2} + C_{gs_3}) \approx \frac{1}{1.5} C_{gs}$$

Cascaded Amplifiers

each stage : $H(s) = \frac{A_0}{s\tau + 1}$ $\qquad \tau = \frac{1}{\omega_0}$

$H_N(s) = \frac{1}{s\tau + 1}$ normalized TF

Cascade n v amplifiers
identical

cascaded gain (normalized) $A_N(s) = \left[\frac{1}{s\tau + 1}\right]^n$ ⇒ 3 dB BW of cascade

$|A_N(j\omega)| = \frac{1}{\sqrt{2}}$

$$\left| \frac{1}{j\omega\tau+1} \right|^n = \frac{1}{\sqrt{2}}$$

$$\left(\frac{1}{\sqrt{(\omega\tau)^2+1}} \right)^n = \frac{1}{\sqrt{2}}$$

BW shrinkage

$$\left((\omega\tau)^2+1 \right)^n = 2$$

$$\Uparrow$$

$$\omega = \frac{1}{\tau}\sqrt{2^{1/n}-1} = \omega_0\sqrt{2^{1/n}-1}$$

$$n \to \infty \quad \Rightarrow \quad \omega \to 0$$

$$2^{1/n} = \exp\left(\ln\left(2^{1/n}\right)\right) = \exp\left[\frac{1}{n}\ln 2\right]$$

If n is large,

$$\exp\left[\frac{1}{n}\ln 2\right] \approx 1 + \frac{1}{n}\ln 2$$

$$\omega = \omega_0\sqrt{2^{1/n} - 1} \approx \omega_0\sqrt{\frac{1}{n}\ln 2} \approx \frac{0.833\,\omega_0}{\sqrt{n}}$$

Optimum gain per stage

Let overall desired gain $= G$ of cascade

gain of each stage $= G^{1/n}$

$$\omega_u = G^{1/n} \cdot \omega_0$$

$$\omega_0 = \frac{\omega_u}{G^{1/n}}$$

$$\omega_{3dB} \approx \frac{\omega_0 \cdot \sqrt{\ln 2}}{\sqrt{n}} \approx \frac{\omega_u}{G^{1/n}} \frac{\sqrt{\ln 2}}{\sqrt{n}}$$

we want to maximise ω_{3dB}

$\Rightarrow$ minimise $\dfrac{1}{\omega_{3dB}}$

$$\frac{d}{dn}\left[\frac{1}{\omega_{3dB}}\right] = 0$$

$$\frac{d}{dn}\left[\frac{1}{\omega_u\sqrt{\ln 2}}\cdot\sqrt{n}\cdot G^{1/n}\right] = 0$$

$$\frac{d}{dn}\left[\sqrt{n}\; G^{1/n}\right] = 0$$

$$\Rightarrow \quad \ln\left[G^{1/n}\right] = 1/2 \quad \Rightarrow \quad \boxed{G^{1/n} = \sqrt{e}}$$ gain per stage

$$\boxed{n = 2\ln G}$$ # of stages

$$\omega_{3dB} = \omega_u\sqrt{\frac{\ln 2}{2e\ln G}} \approx \frac{0.357\,\omega_u}{\sqrt{\ln G}}$$

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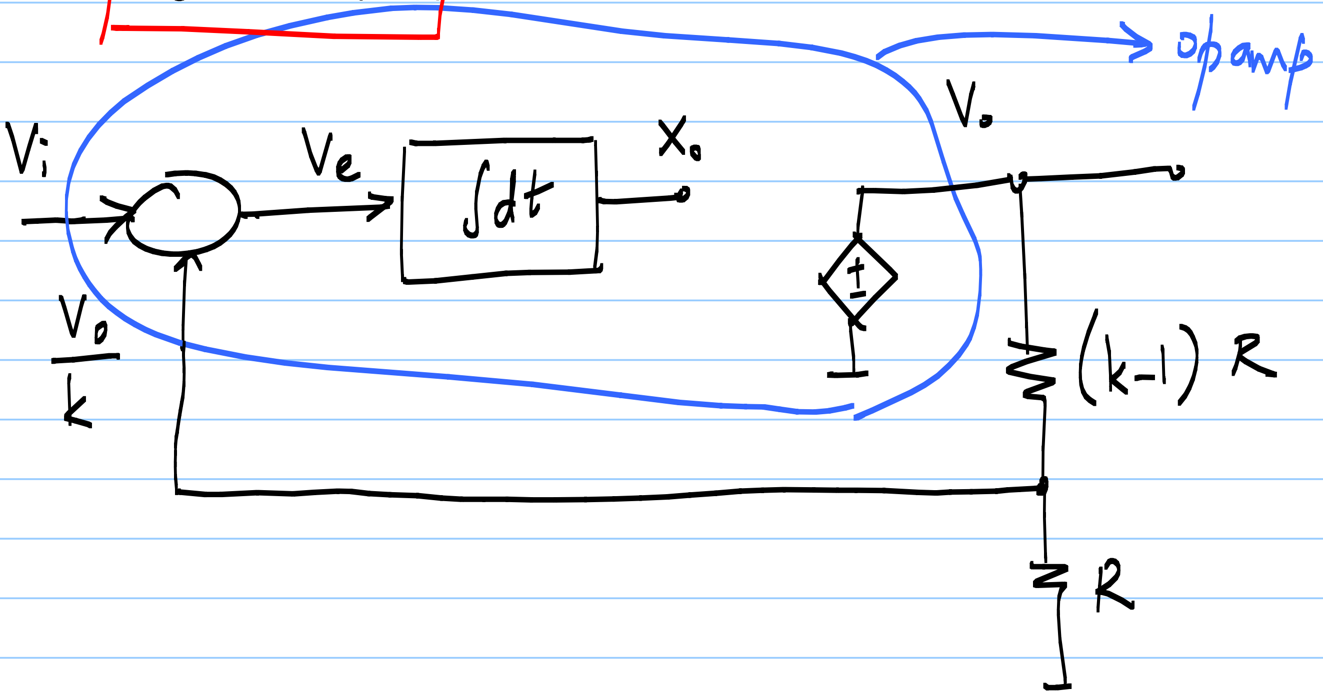
Phase locked loops (PLL)

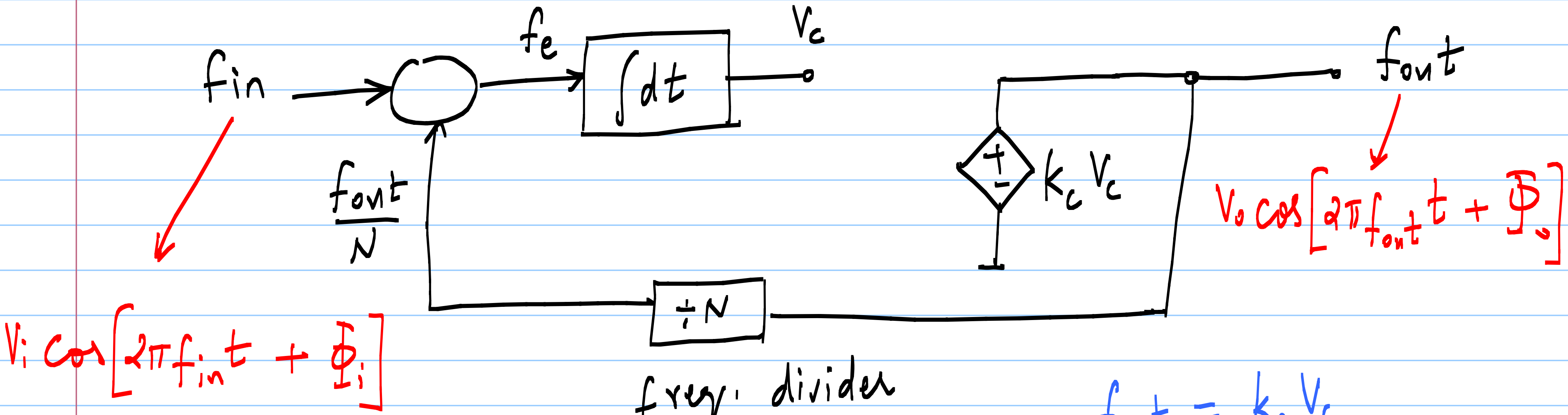
First part of the course - closed loop voltage amp.

$$\boxed{V_o = kV_i}$$



Frequency Multiplier

$f_{out} = N f_{in}$



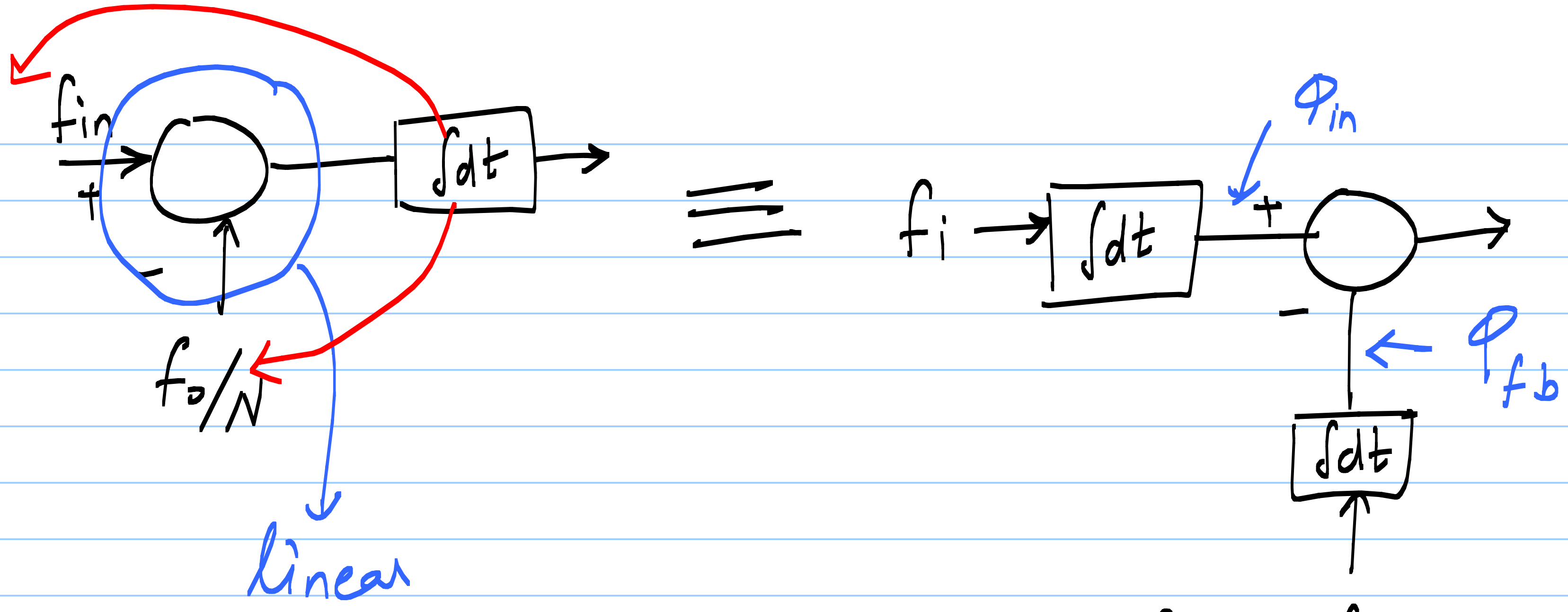
freq. divider

$\equiv$ digital counter

$N \equiv$ integer

$f_{out} = k_c V_c$

Voltage controlled oscillator (VCO)

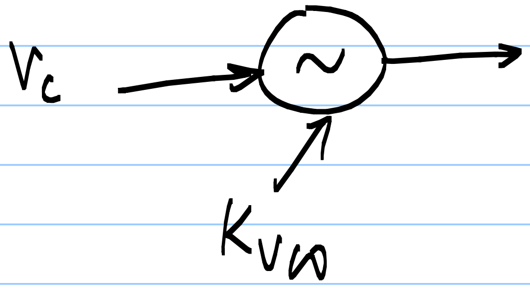


$$f_{fb} = f_{out}/N$$

"phase detector"

VCO

$$f_{out} = K_c V_c$$



$\cos(\omega_{out} t)$ (periodic @ 2π)

$$f_{out} = \underbrace{f_{free}}_{\text{constant}} + K_{vco} \cdot V_c$$

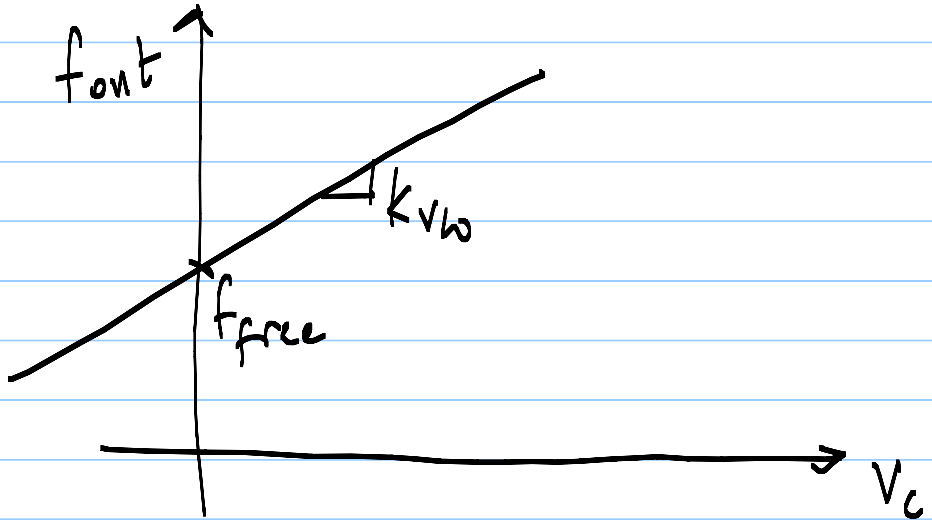
units of Hz/V

$$\phi_{out} = 2\pi \int f_{out}\; dt$$

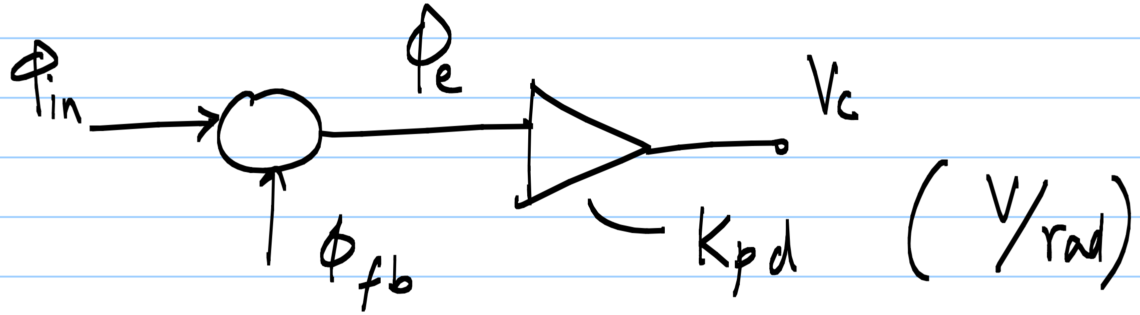
$$= 2\pi \int \left(f_{free} + K_{vco} \cdot V_c \right) dt$$

$$= 2\pi f_{free} \cdot t + 2\pi K_{vco} \int_{-\infty}^{t} V_c\, dt + \phi_0$$

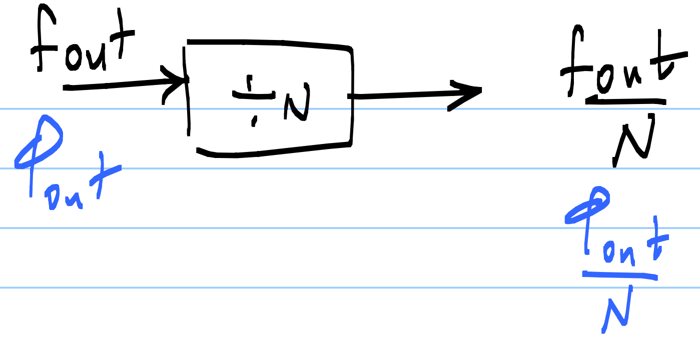
VCO



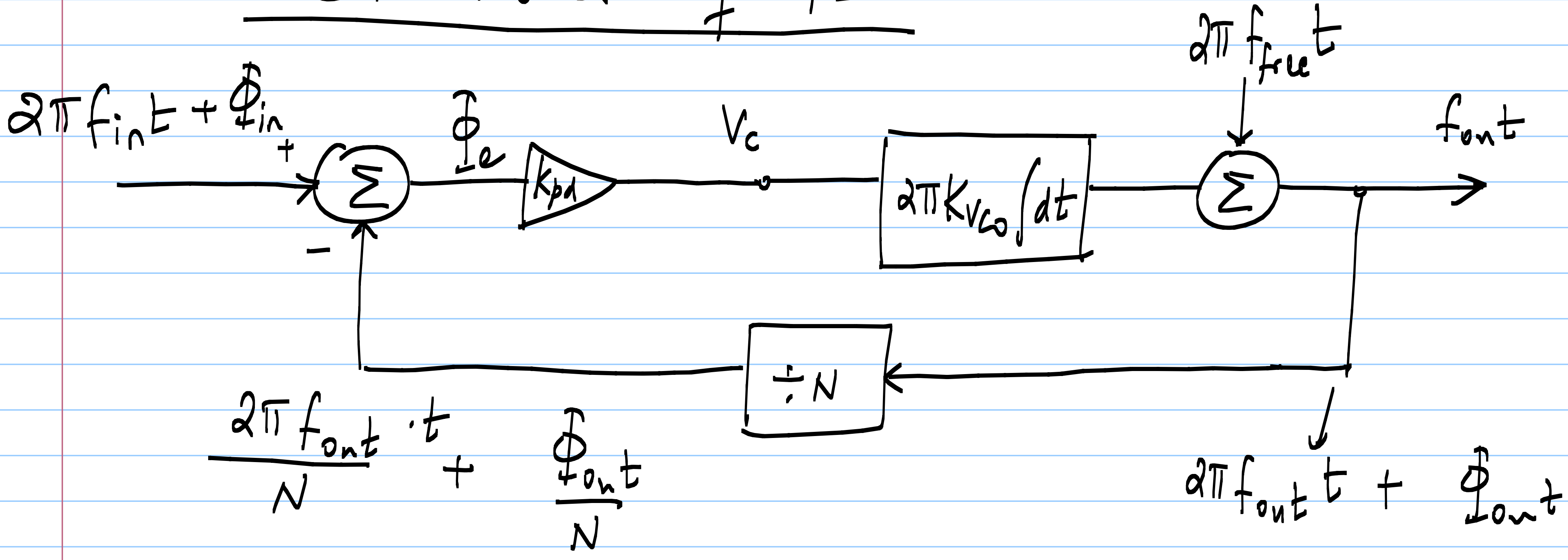
PD



Freq. divider

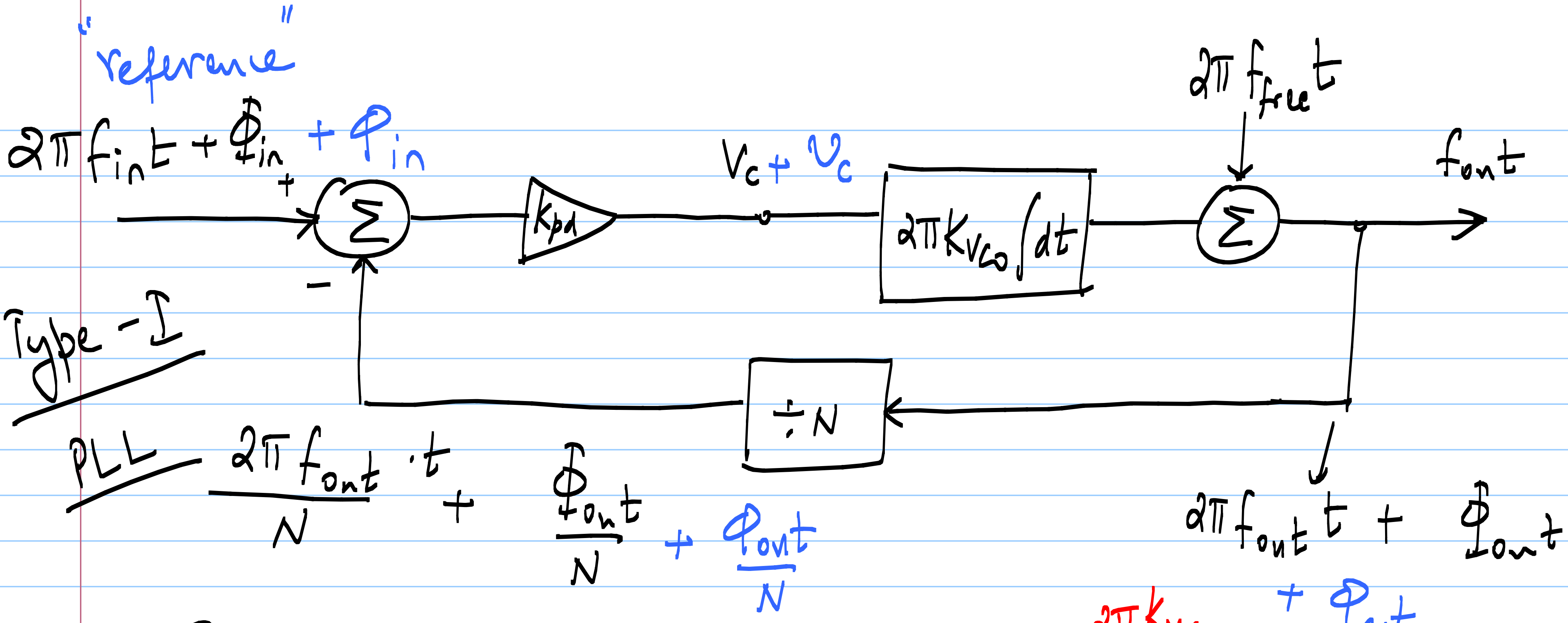


Linear model of PLL

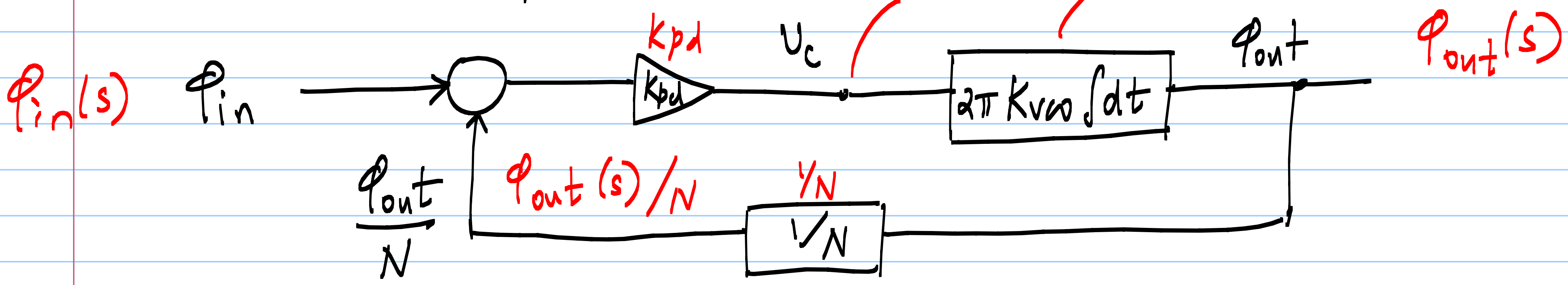


In steady state (lock) : $f_{in} = \frac{f_{out}}{N}$

Type-I PLL



Incremental picture



Loop gain $L(s) = \dfrac{2\pi K_{pd} K_{vco}}{N s}$

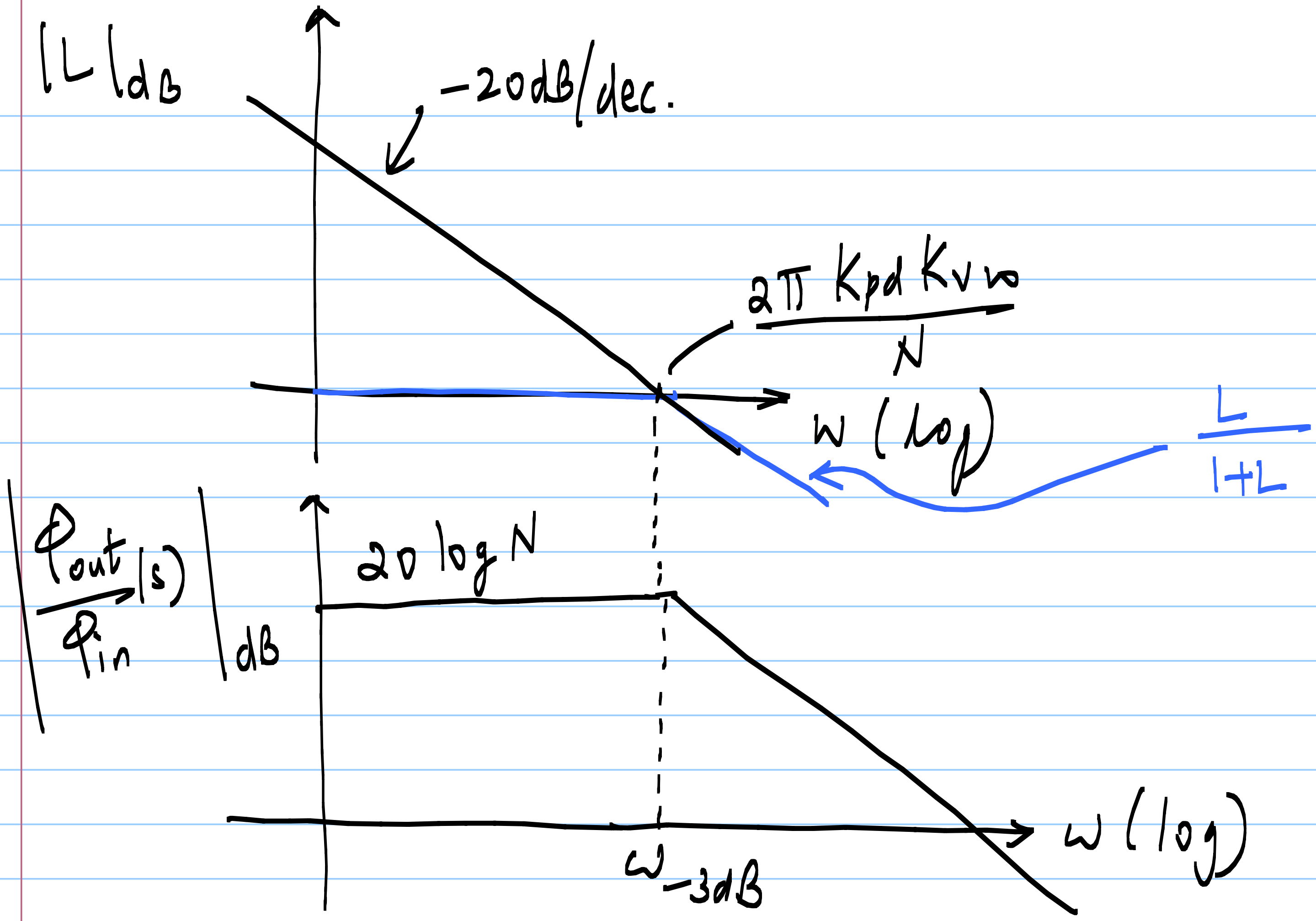
closed loop gain $\dfrac{\phi_{out}(s)}{\phi_{in}(s)} = N \cdot \dfrac{L(s)}{1 + L(s)}$

$$= N \cdot \frac{1}{1 + \frac{1}{L(s)}}$$

$$\frac{\phi_{out}}{\phi_{in}}(s) = N \cdot \frac{1}{1 + \frac{N}{2\pi K_{pd} K_{vco}} \cdot s}$$

closed loop $-3\,dB$ BW $f_{-3dB} = \dfrac{K_{pd} K_{vco}}{N}$

$$\omega_{-3dB} = \frac{2\pi K_{pd} K_{vco}}{N}$$


|L|dB
-20dB/dec.
2π Kpd Kvco / N
ω (log)
L / 1+L
|Φout / Φin (s)|dB
20 log N
ω−3dB
ω (log)

In steady state

$$f_{in} = \frac{f_{out}}{N} \qquad \overset{\text{in general,}}{\left\{ f_{out} > f_{free} \right\}}$$

i.e. $V_c > 0$

$$V_c = \frac{(N f_{in} - f_{free})}{K_{vco}}$$

in lock $\quad f_{out} = f_{free} + K_{vco} \cdot V_c$

We also know that

$$V_c = K_{pd} \left[\Phi_{in} - \frac{\Phi_{out}}{N} \right]$$

In general, $\quad f_{out} \neq f_{free} \quad ; \quad V_c > 0$

$$\Phi_{in} \neq \frac{\Phi_{out}}{N} \quad \left\{ \text{though } f_{in} = \frac{f_{out}}{N} \right\}$$

$\Rightarrow$ Type-I PLL

$f_e = 0$ in steady state

but $\phi_e \neq 0$

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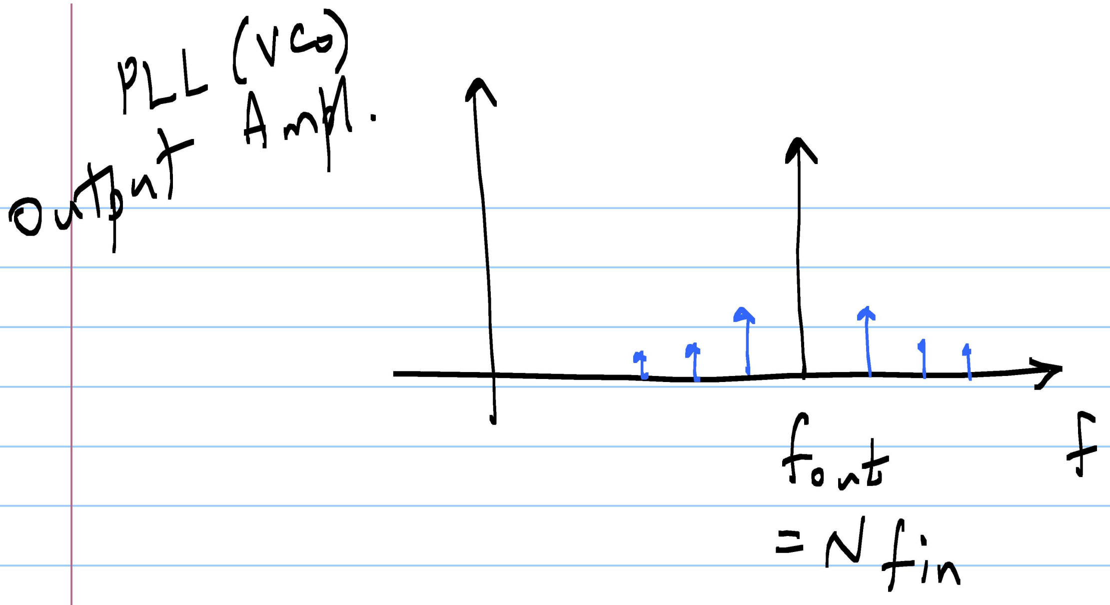
$$V_c = \frac{N f_{in} - f_{free}}{K_{vco}}$$ in steady state

input phase error $$\Phi_e = \frac{V_c}{K_{pd}} = \frac{N f_{in} - f_{free}}{K_{pd} K_{vco}} \neq 0$$

* All real PD also have a periodic term in addition to Φ_e

$\Rightarrow$ V_c will also have a periodic component

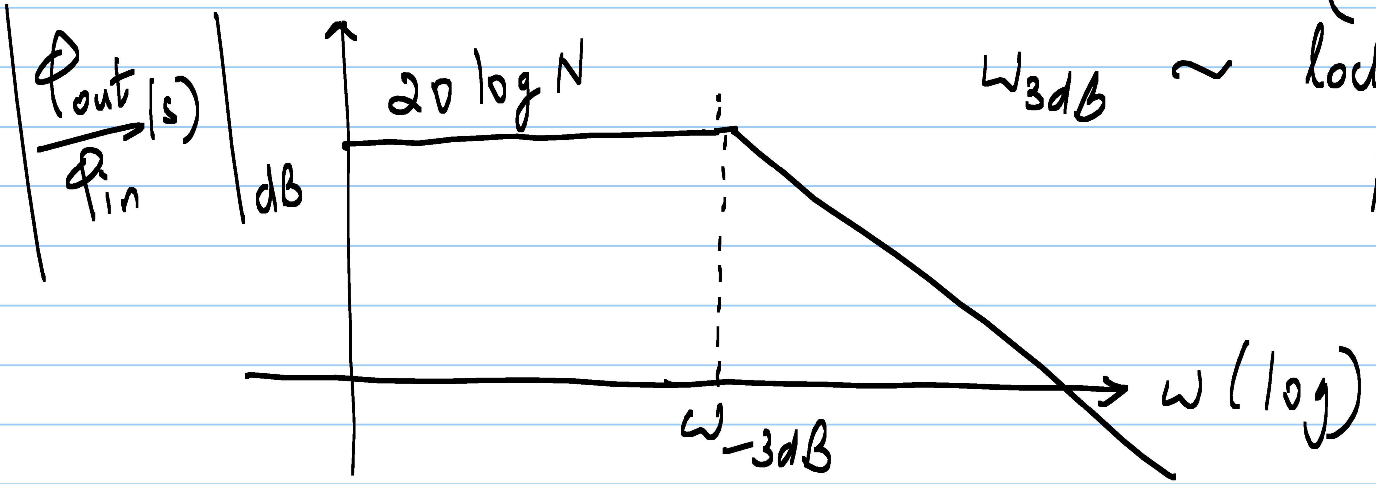


"spurious" tones @

$N f_{in} \pm k f_{in}$

≡ "reference feedthrough"

$f_{3dB} << f_{in}$



$$= \frac{2\pi K_{pd} K_{vco}}{N}$$

(related to)

ω_{3dB} ~ lock range of the PLL

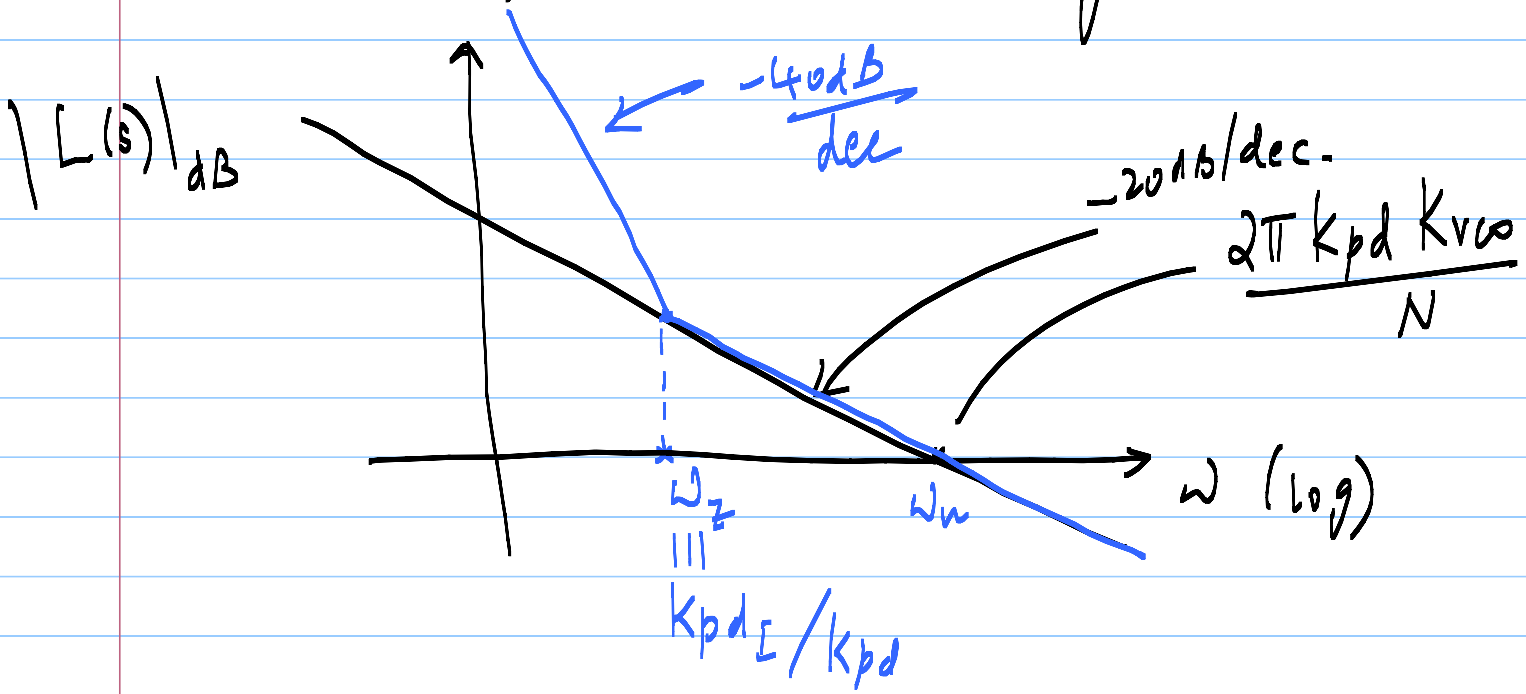
$$N = \frac{f_{out}}{f_{in}}$$

↑ lock range ⟷ you have to ↑ $K_{pd} K_{vco}$

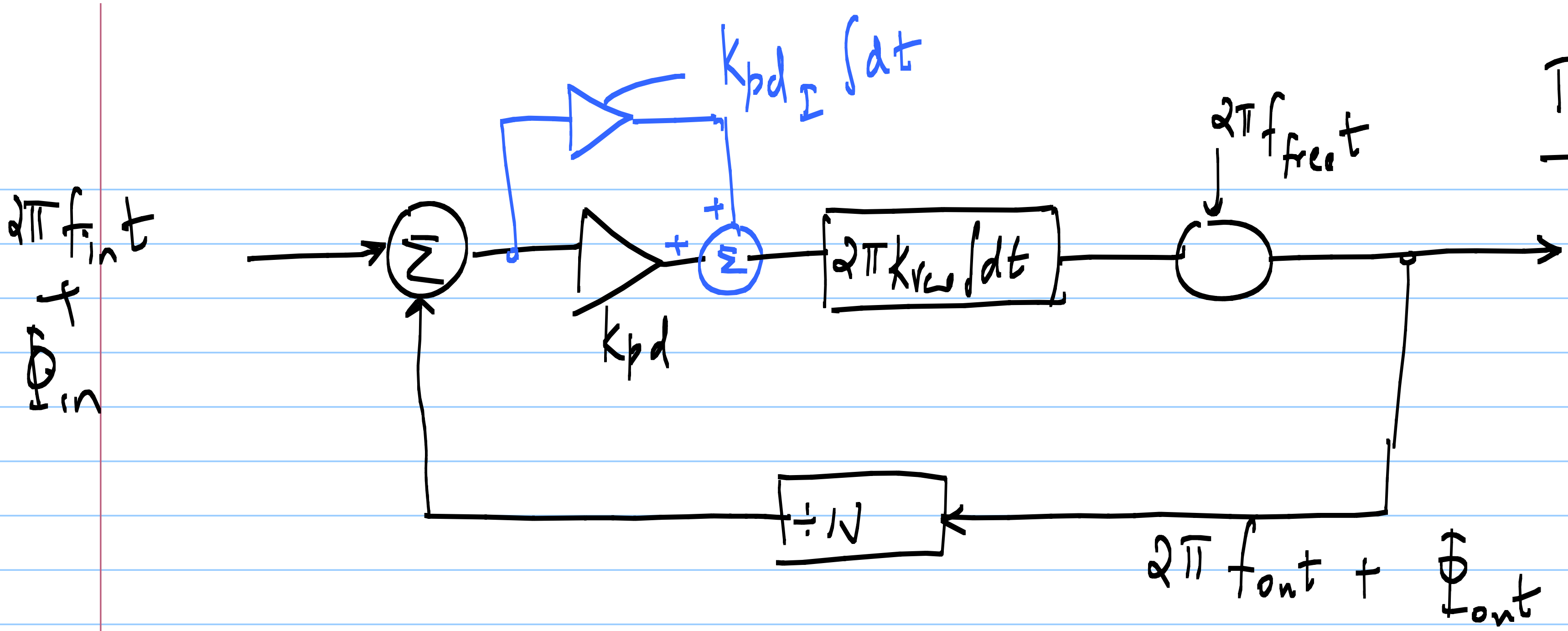
↑ f_{-3dB} → worse reference feedthrough

* Ideally, we want to ↑ $K_{pd} K_{vco}$ only @ dc

⇒ use an integrator (in the limit)



Type - II PLL



$$L(s) = \frac{2\pi K_{vco}}{N s}\left[K_{pd} + \frac{K_{pd_I}}{s}\right]$$

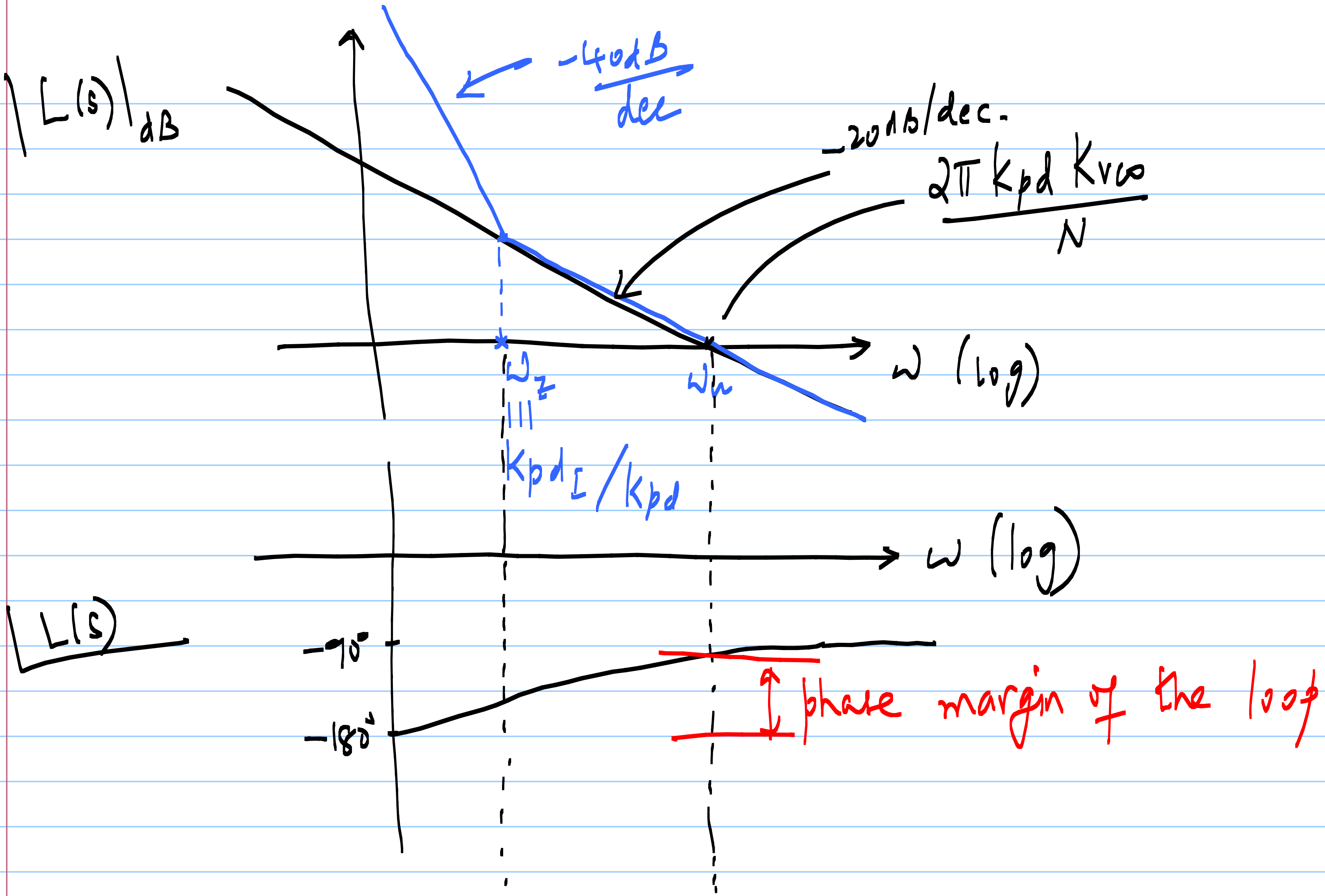
$$= \frac{2\pi K_{pd_I} \cdot K_{vco}}{N s^2}\left[1 + \frac{K_{pd} \cdot s}{K_{pd_I}}\right]$$

ω_z is set using K_{pd_I}

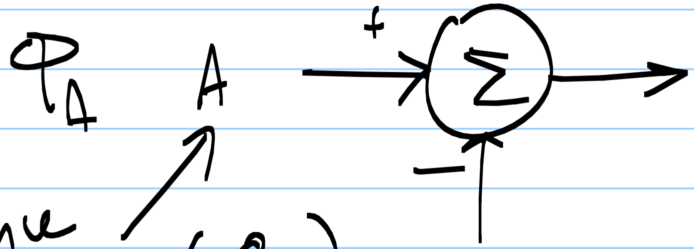
$$\frac{\varphi_{out}}{\varphi_{in}}(s) = N \frac{1 + s \cdot \left(\frac{K_{pd}}{K_{pd_I}}\right)}{1 + s \cdot \frac{K_{pd}}{K_{pd_I}} + s^2 \cdot \frac{N}{2\pi K_{pd_I} K_{vco}}}$$

$$\omega_n = \sqrt{\frac{2\pi K_{pd_I} K_{vco}}{N}} \quad ; \quad Q = \frac{\sqrt{(N K_{pd_I}) / (2\pi K_{vco})}}{K_{pd}}$$

$$\zeta = \frac{1}{2Q}$$


|L(s)|dB
-40dB/dec
-20dB/dec.
2π Kpd Kvco / N
ω (log)
ωz
|||
KpdI/Kpd
ωu
ω (log)
∠L(s)
-90°
-180°
phase margin of the loop

Phase detector

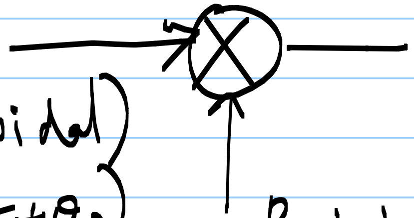


$$V_{out} = K_{pd} \cdot (\varphi_A - \varphi_B)$$

reference input (φ_{in})

B φ_B (φ_{fb}) from divider

Multiplier PD



$$V_{out} = V_A(t) \times V_B(t)$$

A input (sinusoidal)

$$= V_A \sin(\omega_A t + \theta_A)$$

B input (square wave) $= V_B \left[\frac{4}{\pi} \cos(\omega_B t + \theta_B) + \frac{4}{3\pi} \cos(3\omega_B t + \theta_B) + \dots \right]$

$$V_{out}(t) = V_A V_B \cdot \frac{4}{\pi} \sin(\omega_A t + \theta_A) \cdot \cos(\omega_B t + \theta_B)$$

$$+ V_A V_B \cdot \frac{4}{3\pi} \sin(\omega_A t + \theta_A) \cdot \cos(\omega_B t + \theta_B)$$

$$+ \dots$$

$$= \frac{2}{\pi} V_A V_B \Big[\sin\left((\omega_A + \omega_B)t + \theta_A + \theta_B\right)$$

$$+ \sin\Big(\underbrace{(\omega_A - \omega_B)t}_{\text{desired output of } \Delta f} + \underbrace{\theta_A - \theta_B}_{\text{desired output } \Delta\varphi}\Big)$$

$$+ \dots \Big]$$

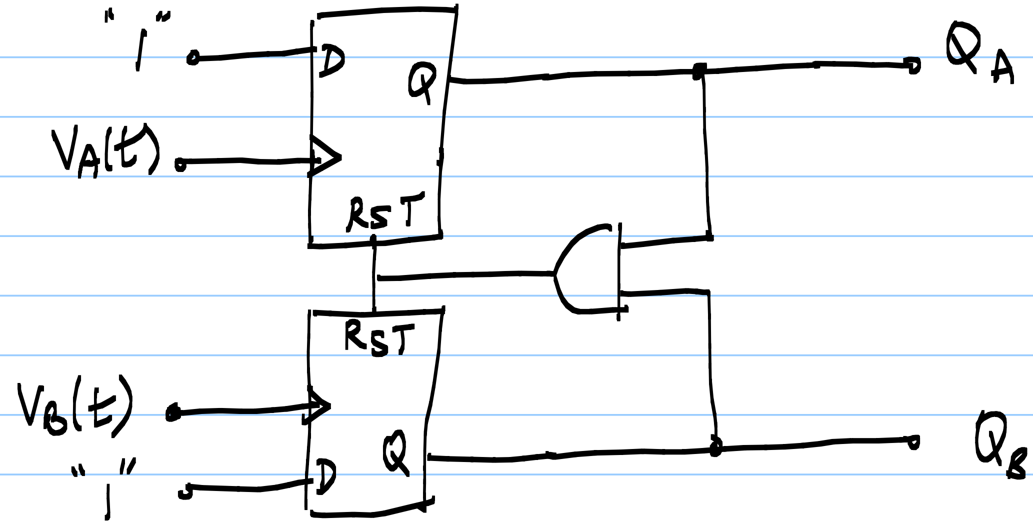
* $\omega_A + \omega_B$; $\omega_A \pm k\omega_B$ for $k > 1$ are undesired periodic terms @ PD output

* LPF the PD output to reduce magnitude of undesired terms

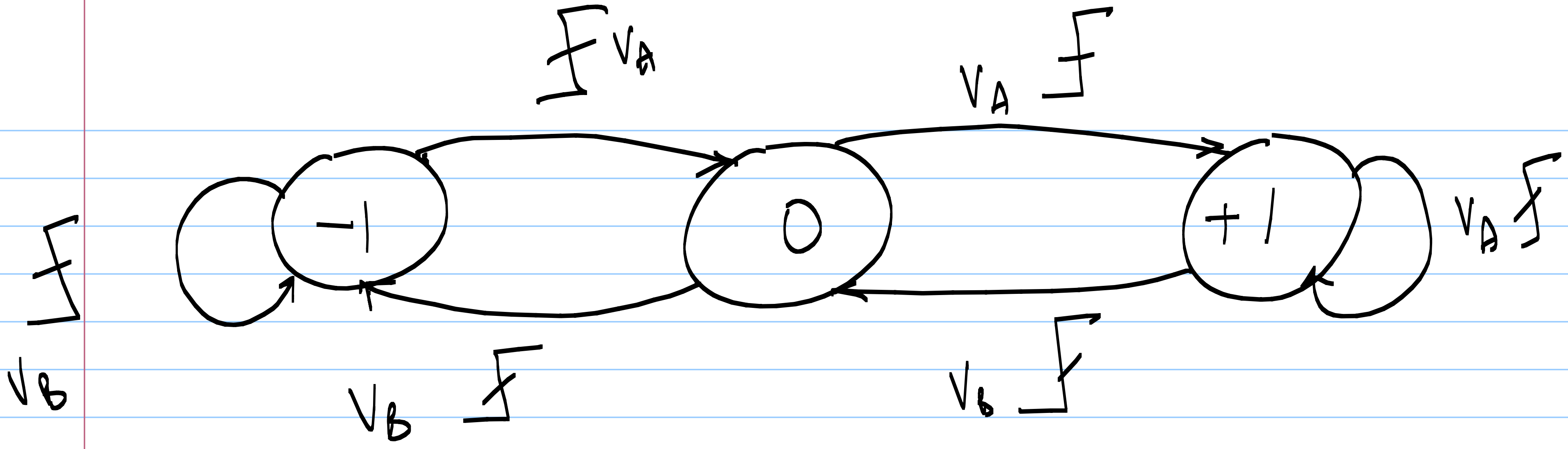
- XOR gate can also act as a PD
- Divider output is a square wave
- reference input can be converted into a square wave easily

} Can you use a digital ckt PD ?

Tristate Phase-frequency detector (PFD)



Q_A	Q_B	state
0	0	0
0	1	−1
1	0	+1
1	1	Prohibited (Reset)


-1
0
+1
V_A
V_A
V_A
V_B
V_B
V_B

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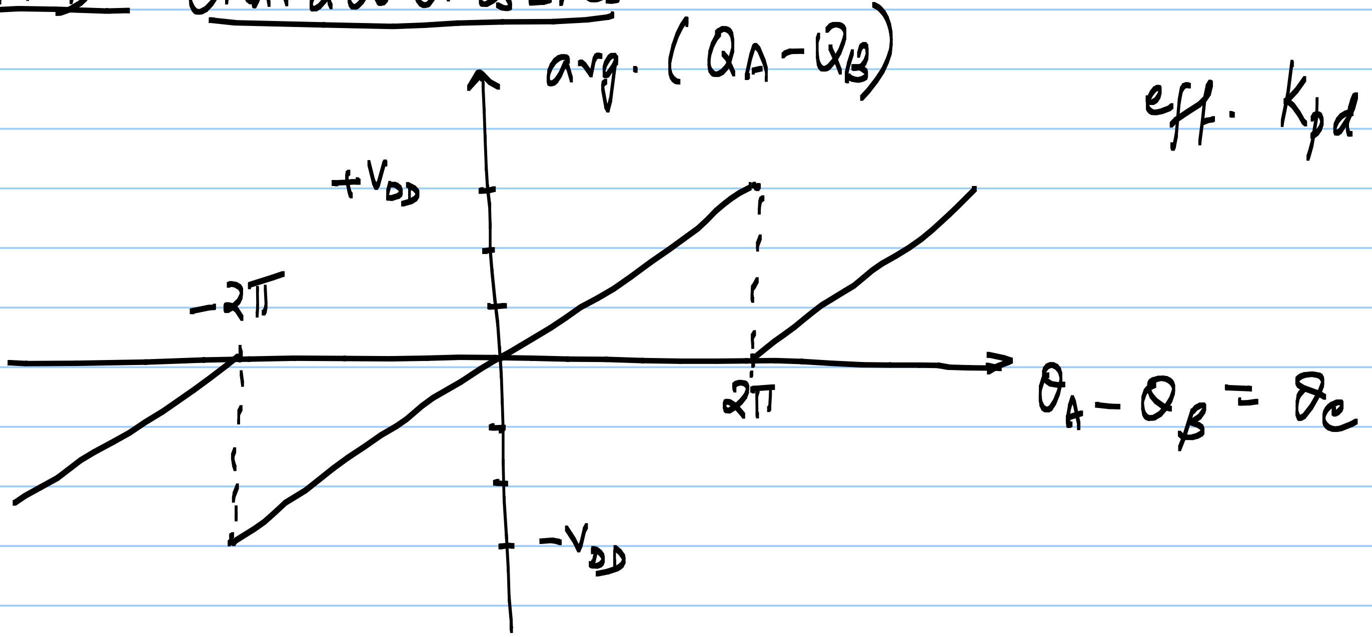
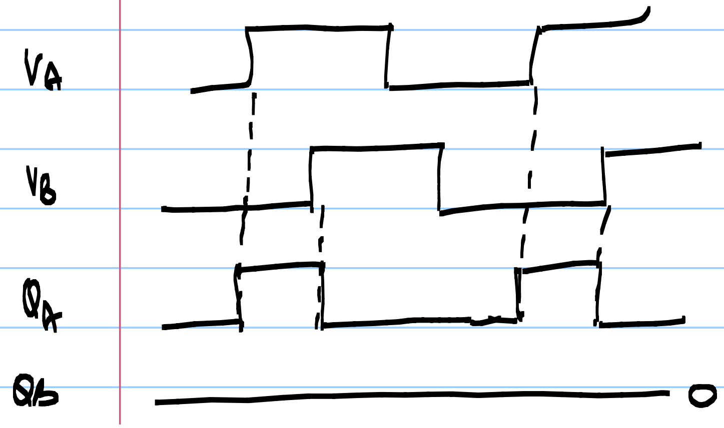
PFD characteristics



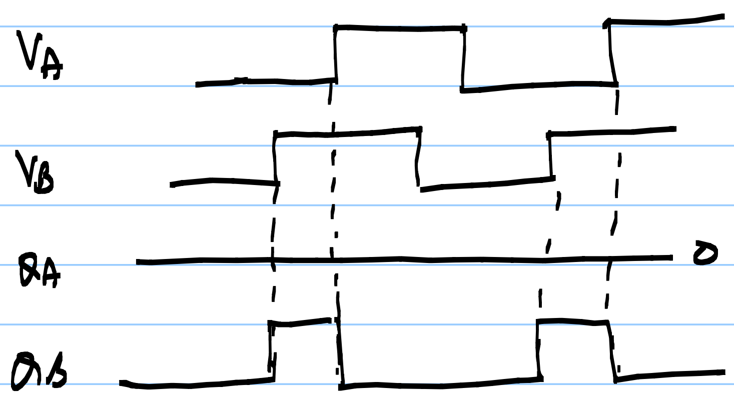
$$\text{eff. } K_{pd} = \frac{2V_{DD}}{4\pi}$$

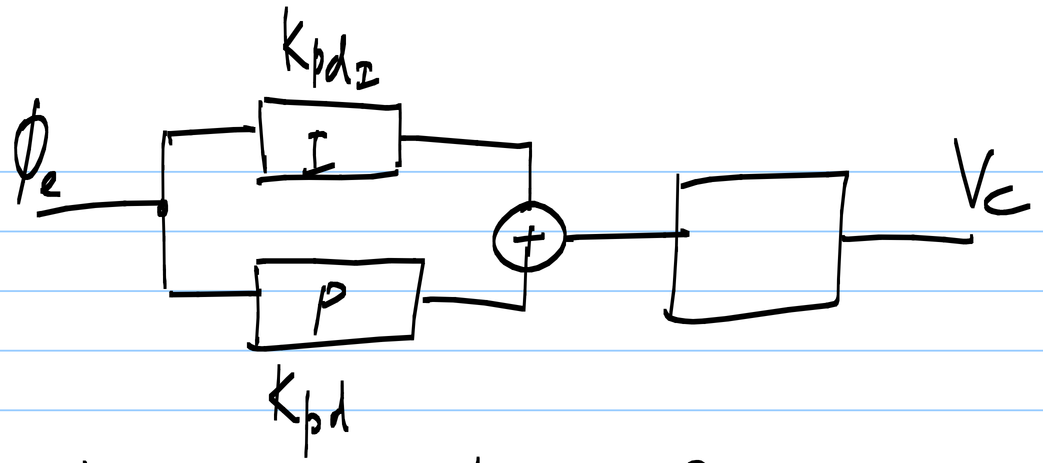
$$= \frac{V_{DD}}{2\pi}$$

$\theta_e > 0$



$\theta_e < 0$





* We have 2 outputs Q_A & Q_B

we need avg$(Q_A - Q_B)$

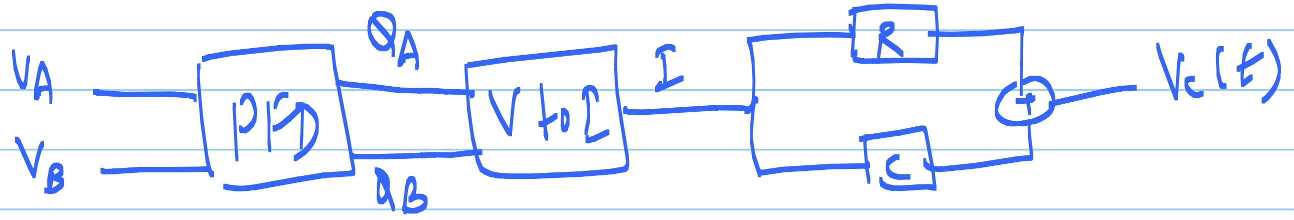
* Kpd & Kpd_I are required — pass the same I through R & C

Resistor

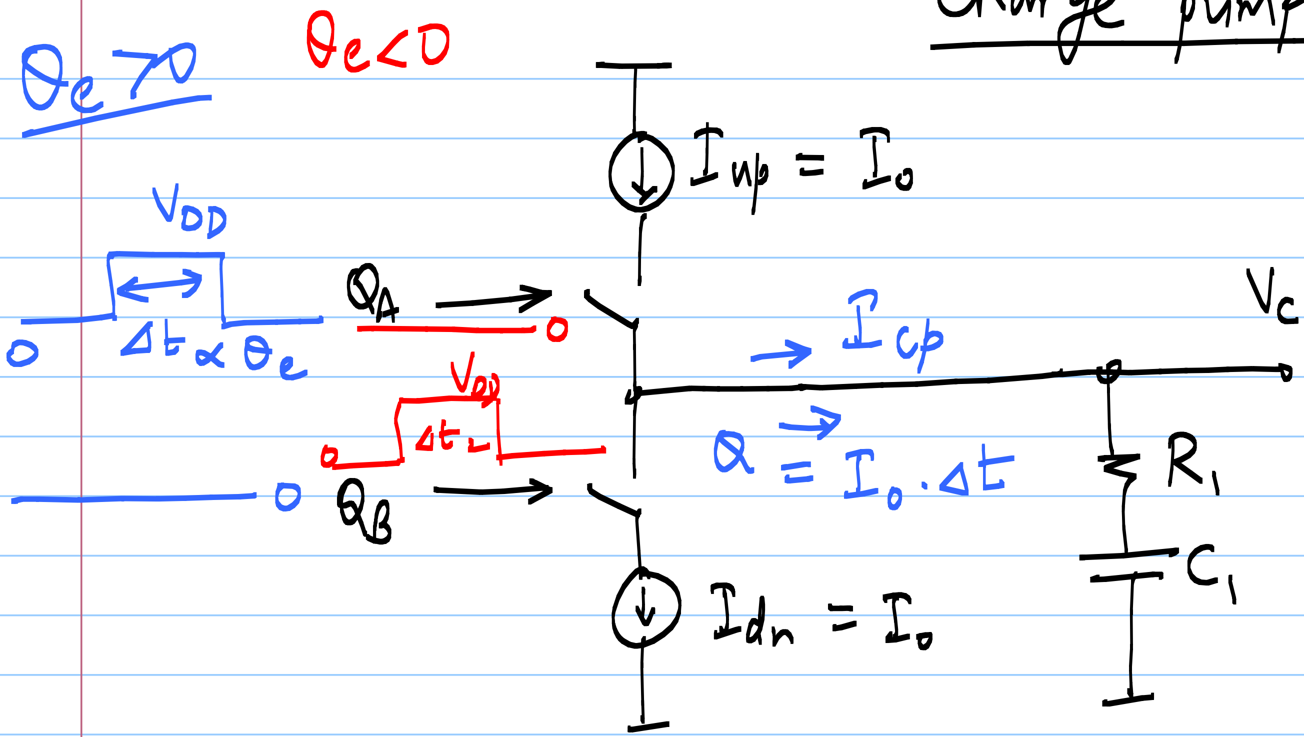
$V = IR$

capacitor

$V = \frac{1}{c}\int I dt$

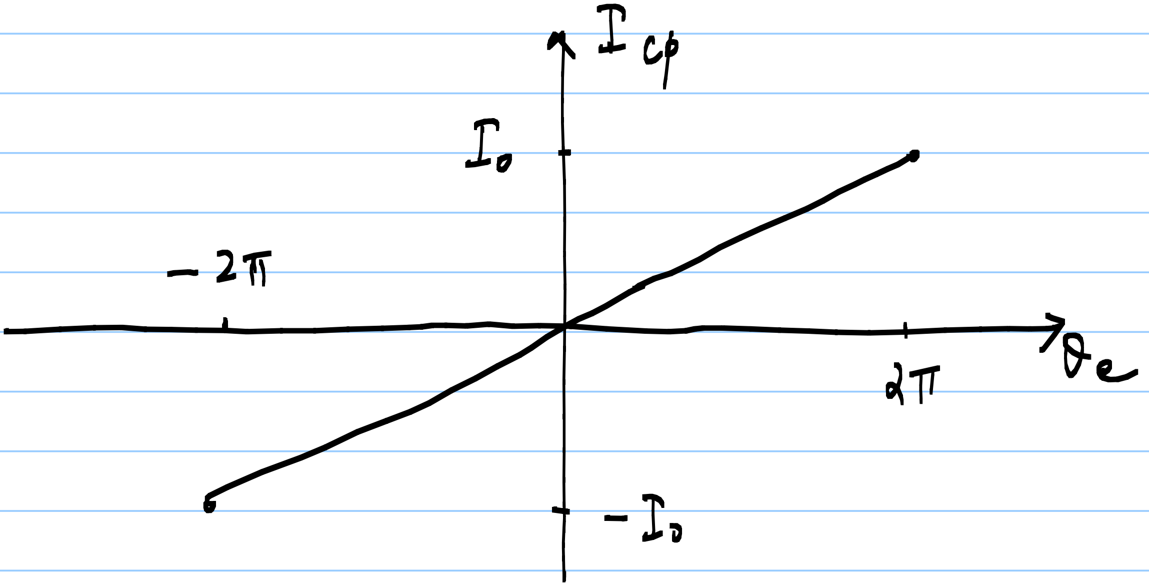


Charge pump

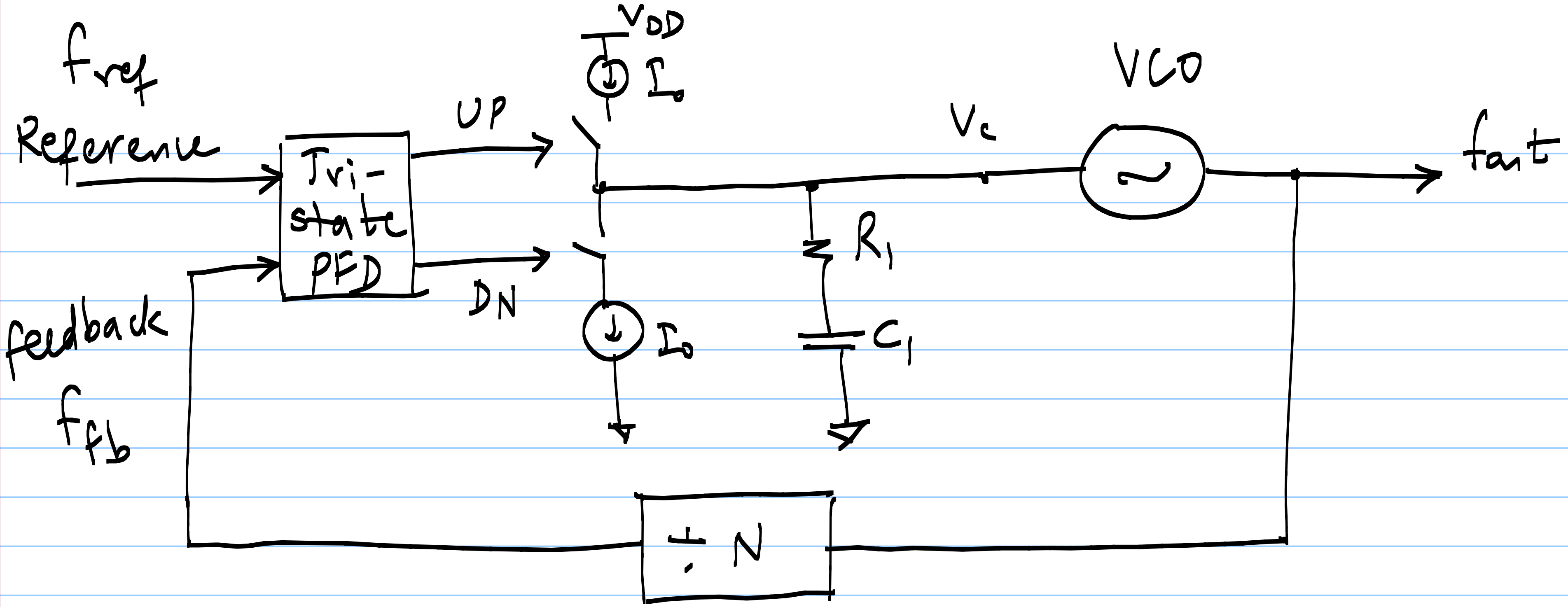


* I_{cp} is a PWM current with avg. value $\propto \theta_e$

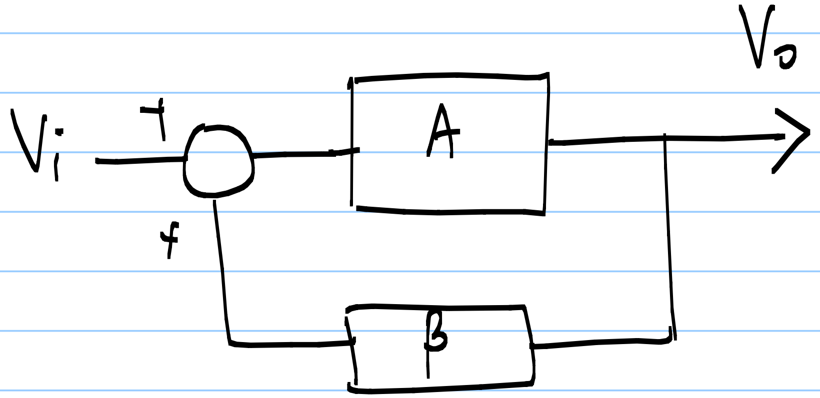
PFD + CP characteristic



$$gain = \frac{I_o}{2\pi}$$

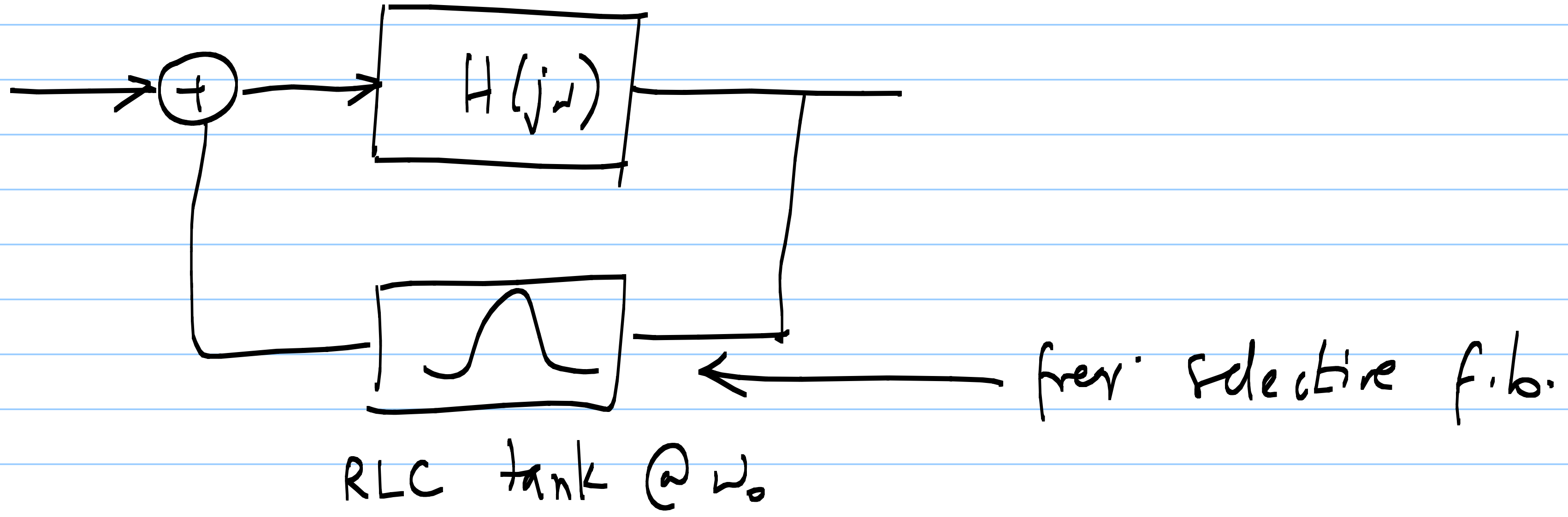


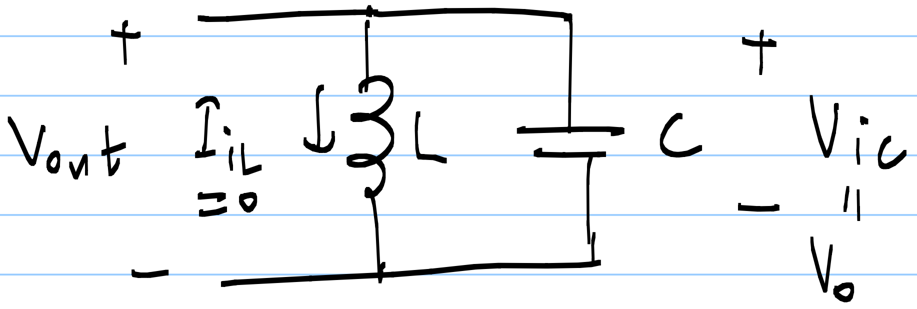
LC-VCO

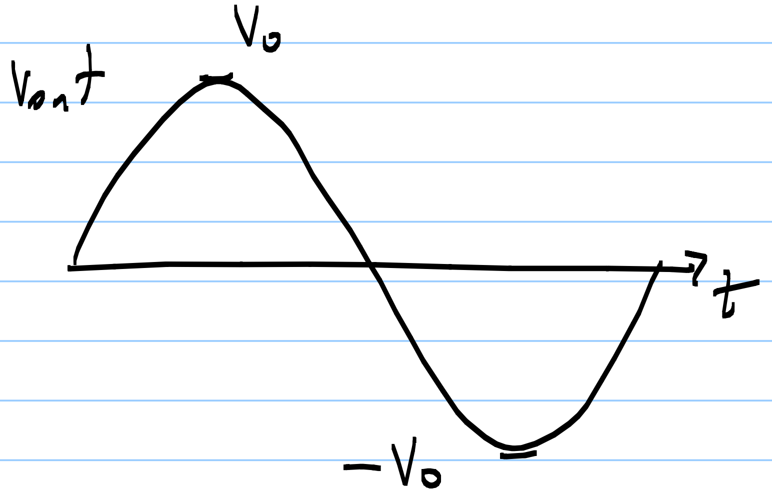


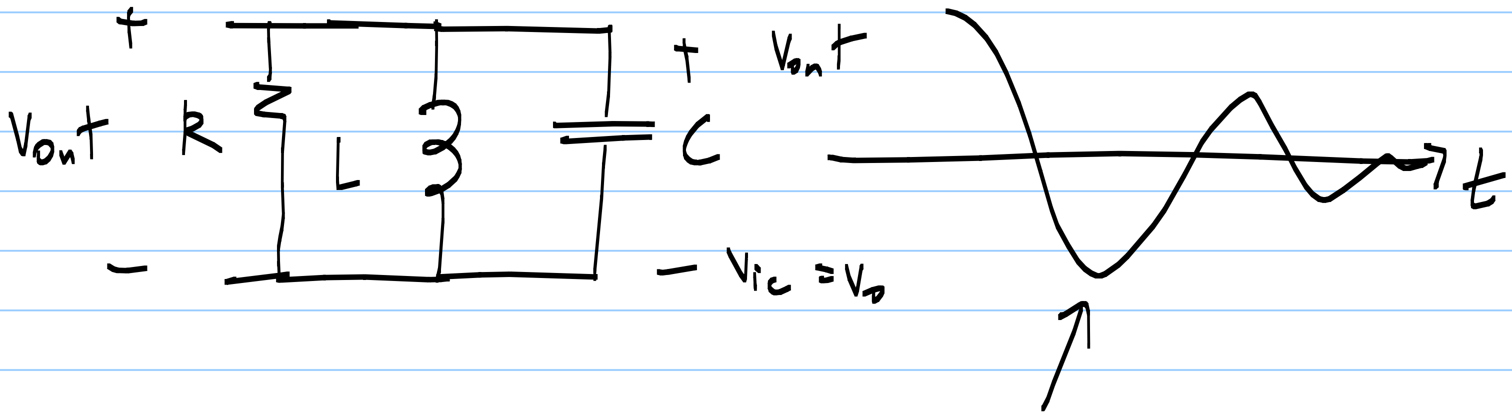
$$\left.\begin{array}{l} |LG(j\omega)| \geq 1 \\ \angle LG(j\omega) = 360^\circ \end{array}\right\} \text{instability}$$

for stable osc: $|LG(j\omega_o)| = 1$, $\angle LG(j\omega_o) = 360^\circ$ @ ω_o

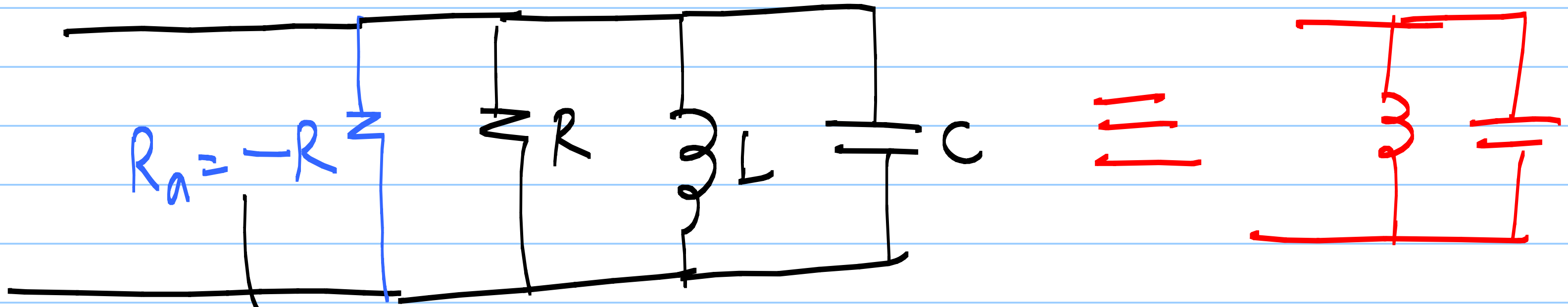

+
H(jω)
freq. selective f.b.
RLC tank @ ω₀


+
V_out
−
I_iL
=0
L
C
+
V_ic
−
||
V_0

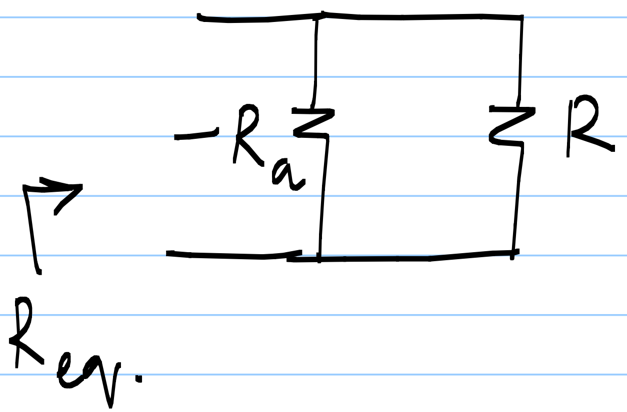

V_0
V_out
t
−V_0



dying sinusoid



−ve resistance from active circuit



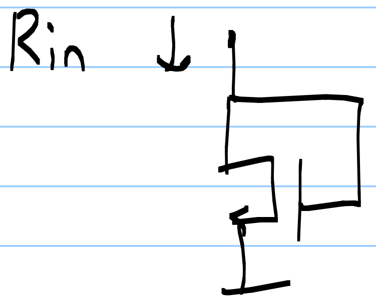
$$R_{eq.} = \frac{-RR_a}{R-R_a}$$

$$= \frac{RR_a}{R_a-R}$$

1) $R_a = R \Rightarrow R_{eq} = \infty$ (ideal LC tank)

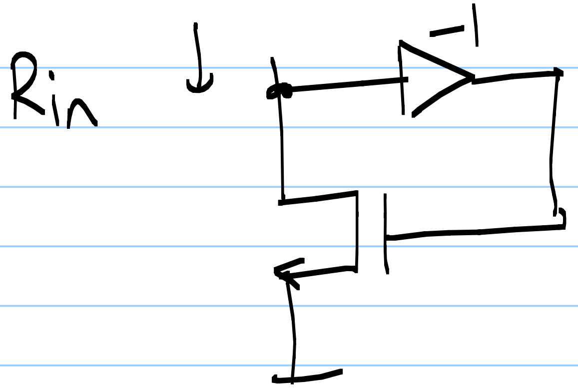
2) $R_a > R \Rightarrow R_{eq.} > 0$ (dying oscillations)

3) $R_a < R \Rightarrow R_{eq} < 0$ (growing oscillations)



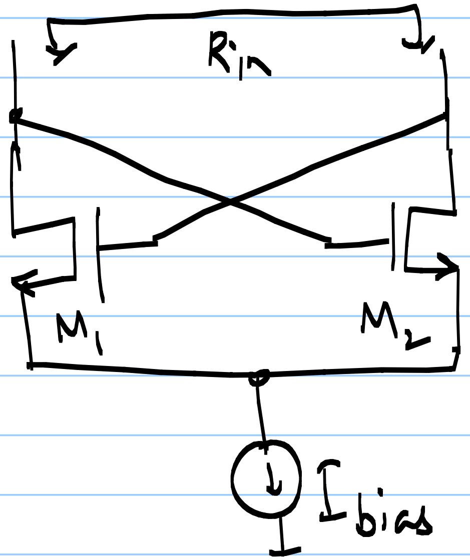
$$R_{in} = \frac{1}{g_m}$$

(-ve f.b.)



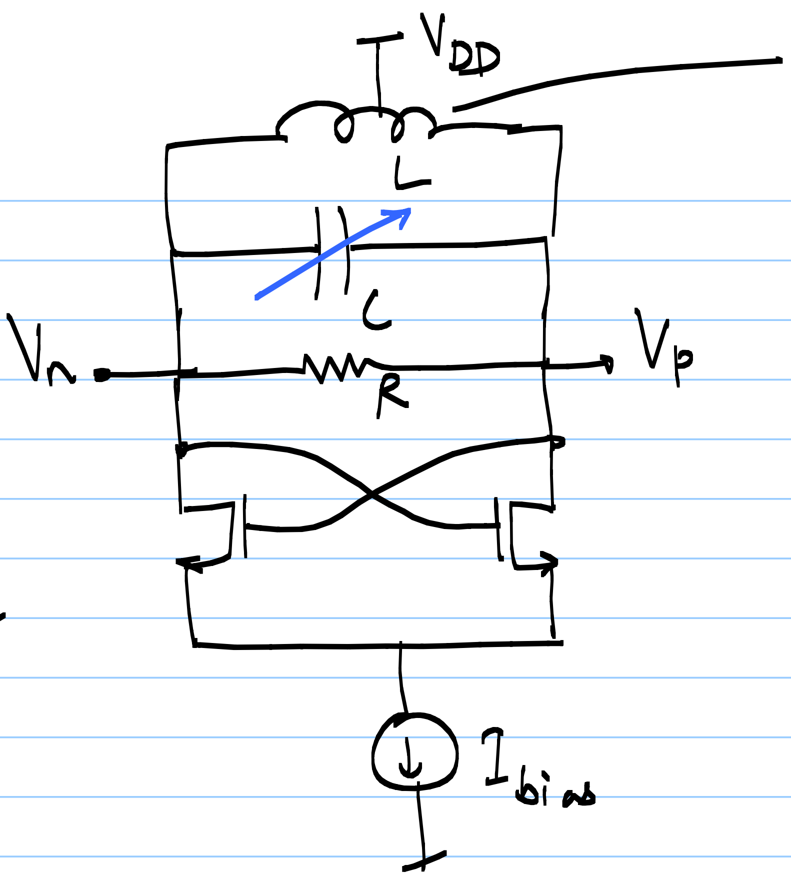
+ve f.b.

$$R_{in} = -\frac{1}{g_m} \quad \{HW\}$$



$$R_{in} = -\frac{2}{g_m}$$

Cross-coupled LC oscillator



In steady state,

$$|R| = \left| \frac{-2}{g_m} \right|$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

tuning C through V_c
(varactor)