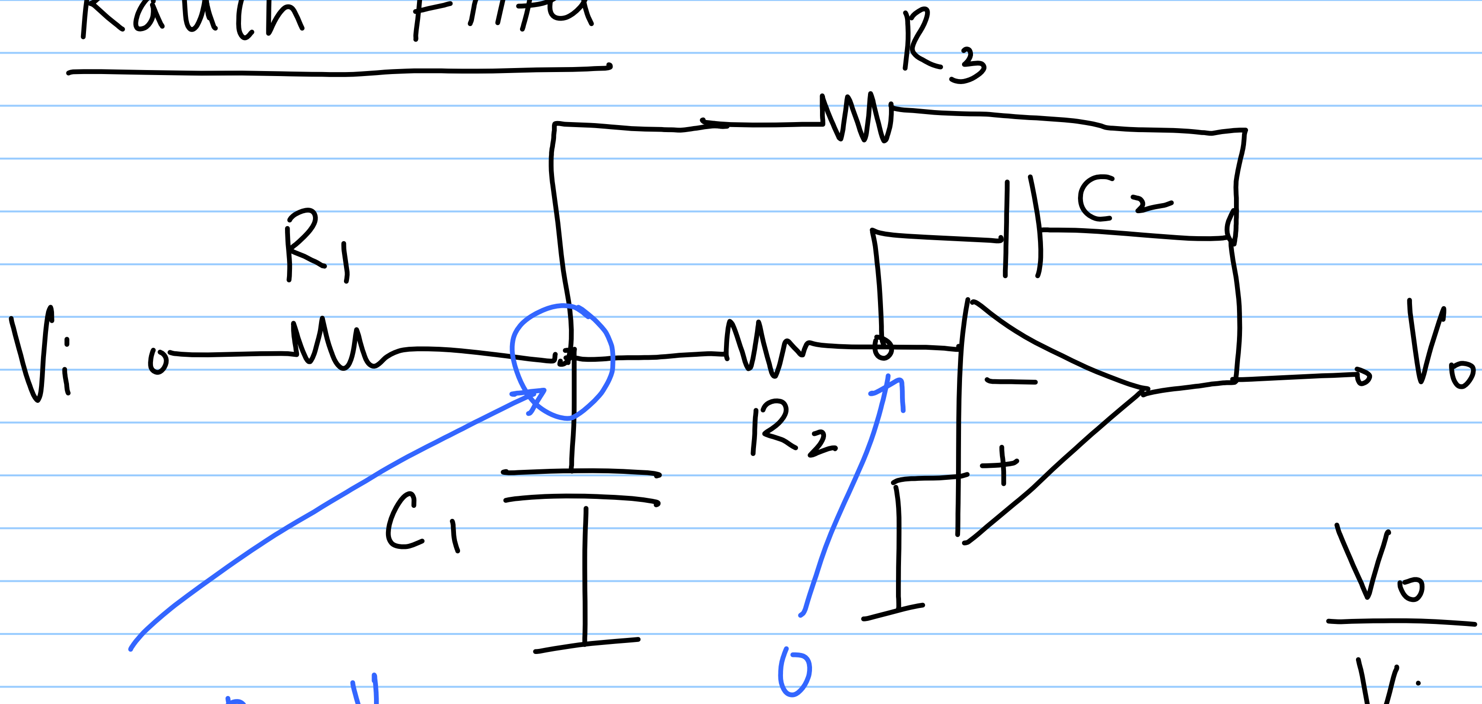
26/03/2019

1- opamp 2nd order Filter

1) Rauch Filter



$-sC_2R_2 \cdot V_o$

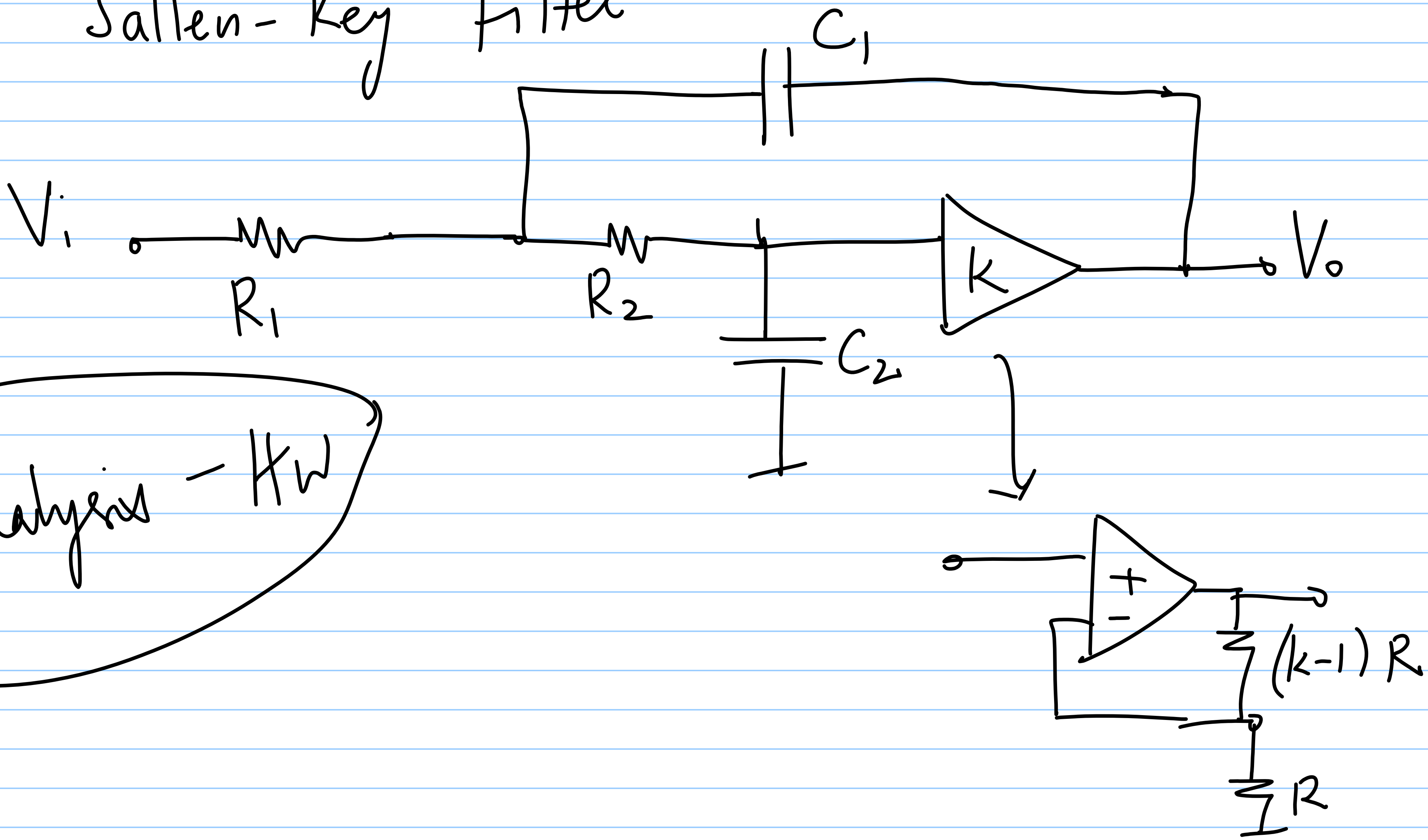
$$\frac{V_o}{V_i}(s) = ?$$

$$\left(\frac{-sC_2R_2V_o - V_i}{R_1}\right) + \left(-sC_2R_2V_o \cdot sC_1\right)$$

$$+\left(-sC_2R_2V_o \cdot \frac{1}{R_2}\right) + \left(\frac{-sC_2R_2V_o - V_o}{R_3}\right) = 0$$

$$\frac{V_o}{V_i}(s) = \frac{-\left(R_3/R_1\right)}{s^2C_1C_2R_2R_3 + sC_2\cdot\left(R_3 + R_2 + \frac{R_3R_2}{R_1}\right) + 1}$$

2) Sallen-Key Filter



Analysis - HW

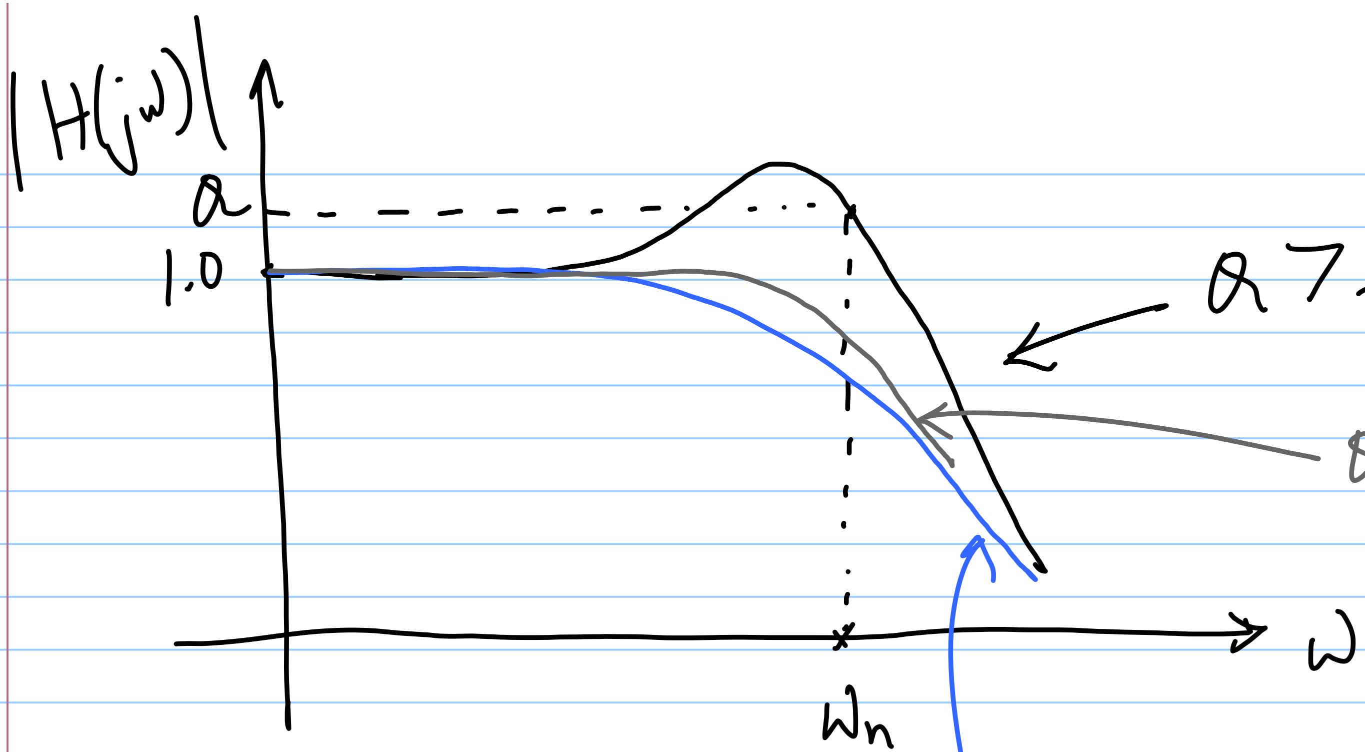
2nd order LPF

$$H(s) = \frac{1}{1 + \frac{s}{\omega_n Q} + \frac{s^2}{\omega_n^2}}$$

$$|H(j\omega)|^2 = \frac{1}{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \frac{\omega^2}{(Q\omega_n)^2}}$$

$$= \frac{1}{1 + (\omega/\omega_n)^2\left[-2 + \frac{1}{Q^2}\right] + \left(\frac{\omega}{\omega_n}\right)^4}$$

If $Q > \frac{1}{\sqrt{2}}$, peaking in response



$Q > \frac{1}{\sqrt{2}}$

"Butterworth" response

$Q = \frac{1}{\sqrt{2}}$ — Maximally Flat response

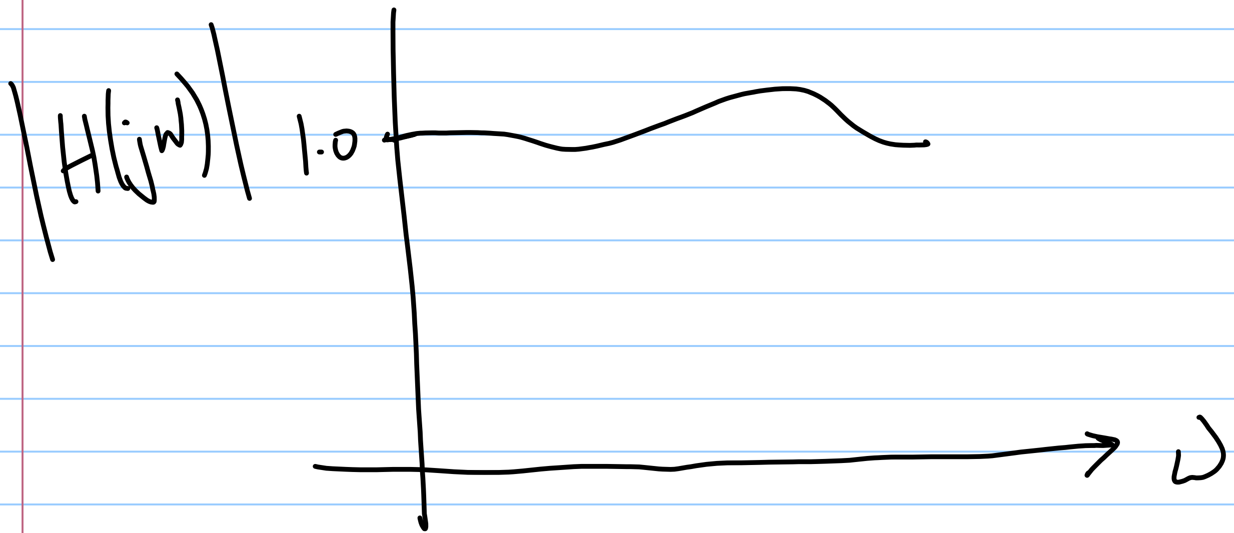
$Q < \frac{1}{\sqrt{2}}$

$\omega_{3dB} = \omega_n$

$$|H(j\omega)|^2 = \frac{1}{1 + \left(\frac{\omega}{\omega_n}\right)^4}$$

$$|H(j\omega)|^2 = \frac{1}{1 + C_2\left(\frac{\omega}{\omega_n}\right)^2 + C_4\left(\frac{\omega}{\omega_n}\right)^4 + \dots + C_{2N}\cdot\left(\frac{\omega}{\omega_n}\right)^{2N}}$$

Nth order filter



Maximally flat response:

$$c_2 = c_4 = \cdots c_{2N-2} = 0$$

$$|H(j\omega)|^2 = \frac{1}{1 + \left(\frac{\omega}{\omega_n}\right)^{2N}}$$

— N^{th} order Butterworth Filter magnitude response

$$|H(j\omega)|^2 = H(j\omega) \cdot H^*(j\omega)$$

$$H(s) = \frac{1}{1 + a_1\left(\frac{s}{\omega_n}\right) + a_2\left(\frac{s}{\omega_n}\right)^2 + \dots + a_n\left(\frac{s}{\omega_n}\right)^n}$$

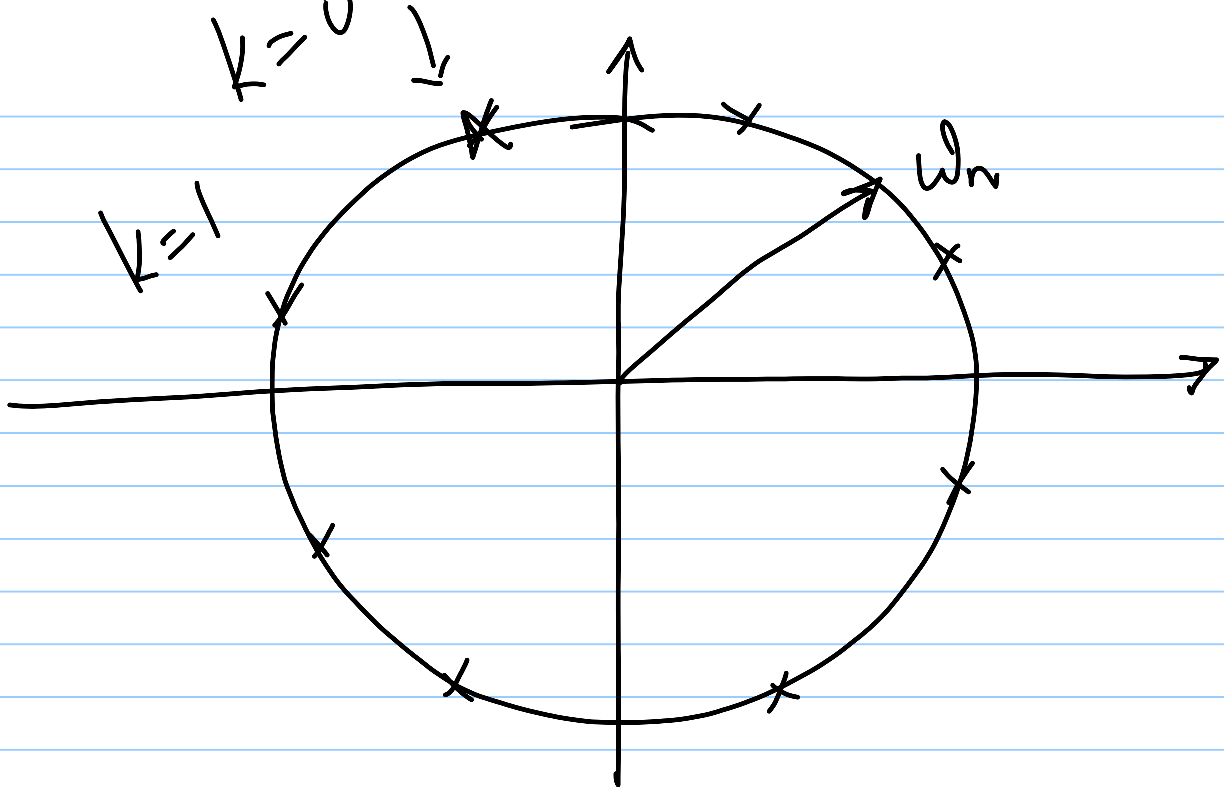
$$|H(j\omega)|^2 = H(j\omega) \cdot H(-j\omega)$$

$$= H(s) \cdot H(-s) \Big|_{s=j\omega} \quad \text{or} \quad \omega = \frac{s}{j}$$

$$H(s) \cdot H(-s) = \frac{1}{1 + \left(\frac{s}{j\omega_n}\right)^{2N}}$$

$$\left(\frac{s}{j\omega_n}\right) = (-1)^{1/2N}$$

$$\left(\frac{s}{j\omega_n}\right)^{2N} = \exp\left\{j(2k+1)\pi\right\}$$



$k = 0, 1, \ldots (2N-1)$

$$s = j\omega_n \exp\left\{ j\frac{(2k+1)\pi}{2N} \right\}$$

$$= \omega_n \cdot \exp\left\{ j\frac{(2k+1)\pi}{2N} + j\pi/2 \right\}$$

7^{th} order filter $\longrightarrow$ Cascade 1 1^{st} order

3 2^{nd} order

Realise as:

1^{st} order followed by increasing value of Q