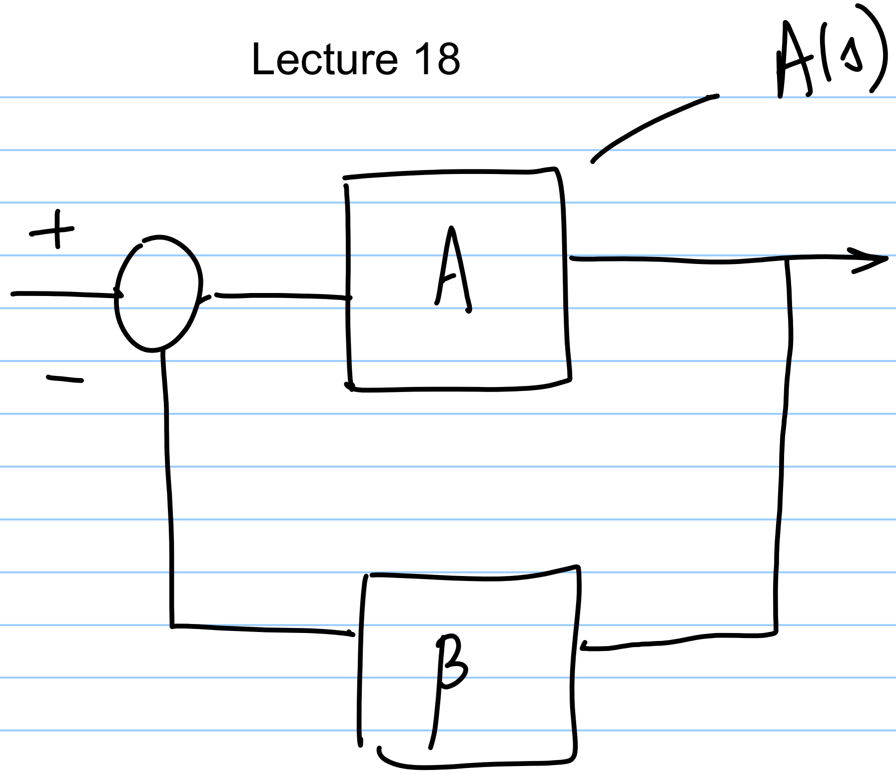
20/02/2019

Lecture 18



$$\text{loop gain} = A\beta$$

$$CLG = \frac{1}{\beta} \cdot \frac{A\beta}{1+A\beta}$$

$$LG(s) = \beta A(s) = \beta \cdot \frac{A_0}{\left(1+\frac{s}{\omega_p}\right)^3}$$

$$CLG(s) = \frac{1}{\beta} \cdot \frac{A_0\beta}{1+A_0\beta} \cdot \frac{1}{1+\left[\frac{3s}{\omega_p} + \frac{3s^2}{\omega_p^2} + \frac{s^3}{\omega_p^3}\right]\left(\frac{1}{1+\beta A_0}\right)}$$

$$D(s) = 1 + \frac{3s}{\omega_p(1+A_0\beta)} + \frac{3s^2}{\omega_p^2(1+A_0\beta)} + \frac{s^3}{\omega_p^3(1+\beta A_0)}$$

roots of $D(s) = 0$

Roots of $D\left({}^{s}/_{\omega_p} \right)$

$$\left(1 + A_0\beta\right) + 3s + 3s^2 + s^3 = 0$$

$$\left(1+s\right)^3 + A_0\beta = 0$$

$$s = -1 + \left(-A_0\beta\right)^{1/3}$$

e.g. $A_0\beta = 8$

$$s_1 = -1 - 2 = -3 ;$$

$$s_2 = -1 - 2e^{-j2\pi/3} \quad ; \quad s_3 = -1 - 2e^{+j2\pi/3}$$

$$\left.\begin{aligned} s_1 &= -3\omega_p \\ s_2 &= +j\sqrt{3}\,\omega_p \\ s_3 &= -j\sqrt{3}\,\omega_p \end{aligned}\right\}\quad \begin{array}{c} \text{If } A_0\beta > 8 \\ \Downarrow \\ \text{poles in RHP} \end{array}$$

We wanted large $A_0\beta$

Solutions :

1) Pole – zero cancellation :

$$\frac{A_0}{\left(1+ s/\omega_p\right)^3} \times \frac{\left(1+ s/\omega_z\right)}{\left(1+ s/\omega_p'\right)}$$

$\omega_z = \omega_p$

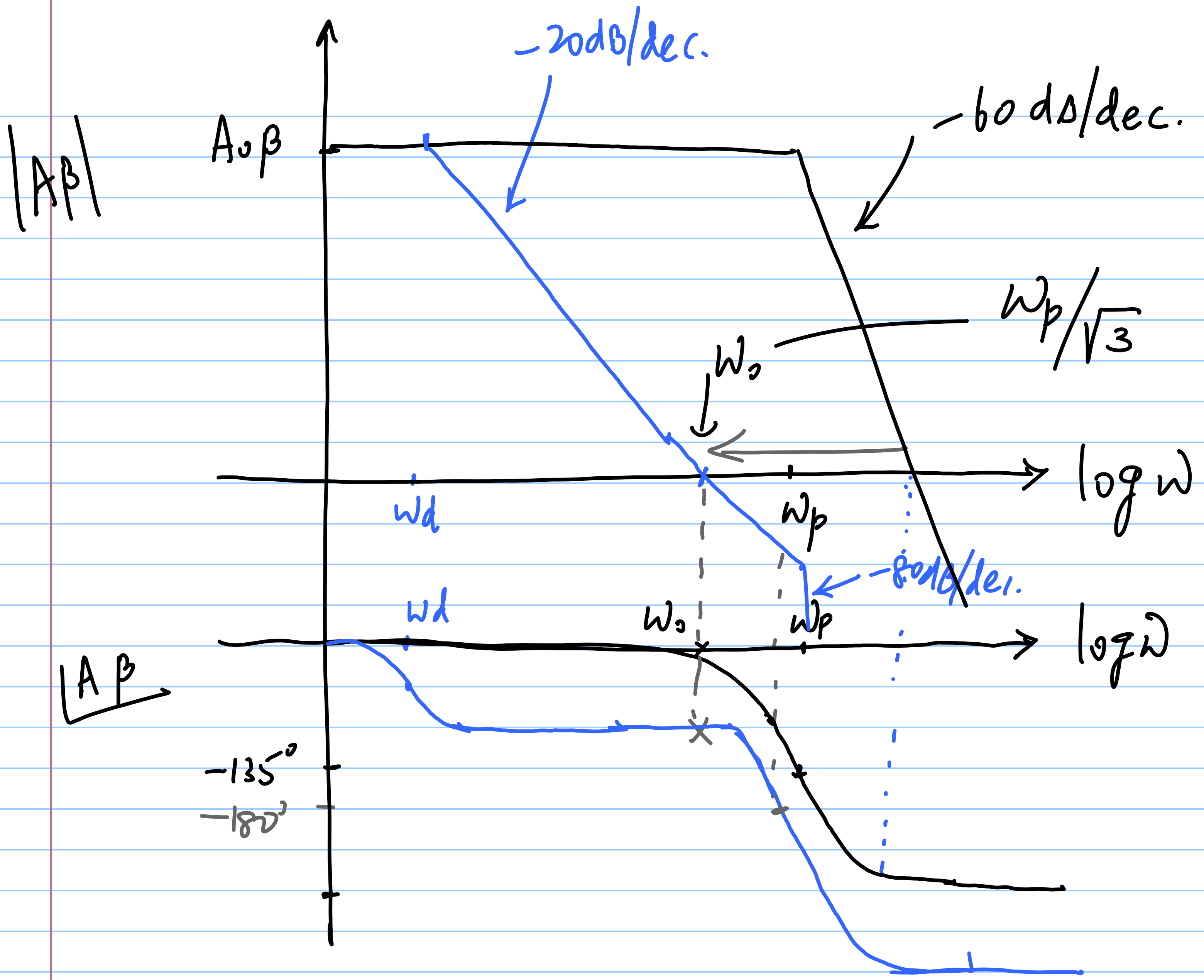
$\omega_p' > \omega_p$

2) Make 3rd order system look like 1st order system

e.g. add pole @ ω_d

$$A(s) = \frac{A_0}{\left(1 + s/\omega_d\right)\left(1 + s/\omega_p\right)^3}$$

$$\omega_d = \frac{\omega_p}{1000}$$



disadvantage;

* lower BW

<u>advantage.</u>

* unconditionally stable
* Very low steady state error.

$$\omega_d = \frac{\omega_p}{1000} \qquad \omega_0 \gg \omega_d$$

$$|L_a(j\omega_0)| = 1$$

$$\angle G(j\omega_0) = -180^\circ$$

$$0 - 3\tan^{-1}\left(\frac{\omega_0}{\omega_p}\right) - \underbrace{\tan^{-1}\left(\frac{1000\,\omega_0}{\omega_p}\right)}_{-\pi/2} = -\pi$$

$$-3\tan^{-1}\left(\frac{w}{w_p}\right) = -\pi/2$$

$$\tan^{-1}\left(\frac{w}{w_p}\right) = +\pi/6$$

$$\frac{w}{w_p} = \frac{+1}{\sqrt{3}}$$

$$\left| LG(j\omega_0) \right| = 1$$

$$\left| \frac{A_0 \beta}{\left(1+\frac{j}{\sqrt{3}}\right)^3 \cdot \left(1+\frac{1000j}{\sqrt{3}}\right)} \right| = 1$$

$$\frac{A_0 \beta}{\left(\sqrt{1+1/3}\right)^3 \left(\frac{1000}{\sqrt{3}}\right)} = 1 \Rightarrow A_0\beta = 888.88$$

much larger original limit of $A_0\beta = 8$

$\omega_d \equiv$ "Dominant Pole"

Technique $\equiv$ "Dominant pole compensation"