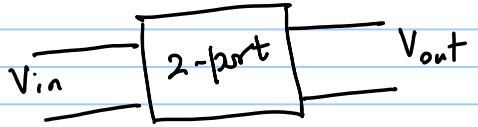
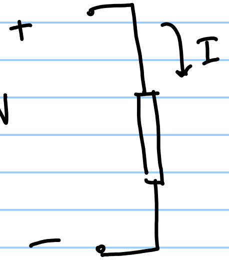
8/8/17

ANALOG ELECTRONIC CIRCUITS

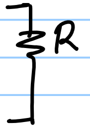
"Microelectronic Circuits"
Sedra & Smith



$\longrightarrow V_{out} \propto V_{in}$



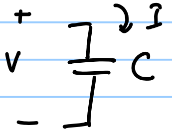
$\longrightarrow \boxed{V = kI} \longleftarrow$ Resistor



✓

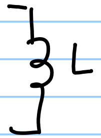
$Q = CV$

$\underbrace{I = C\frac{dV}{dt}}$



linear ✓

$V = L\frac{dI}{dt}$



✓

$Y = f(X)$

$$\left.\begin{array}{l} X_1 \rightarrow Y_1 \\ X_2 \rightarrow Y_2 \end{array}\right\} \quad \begin{array}{l} X_1 + X_2 \longrightarrow Y_1 + Y_2 \\ aX_1 + bX_2 \longrightarrow aY_1 + bY_2 \end{array}$$

"Superposition"

$Y = X + 2$ "Not linear"

$X_1 = 0 \rightarrow Y_1 = 2$ $\Big\}$ $X_1 + X_2$

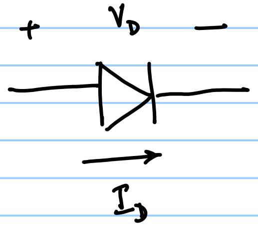
$1 \longrightarrow 3$

$X_2 = 1 \rightarrow Y_2 = 3$

For linearity : $X = 0 \Rightarrow Y = 0$

Non-linear 2-T element

pn junction diode



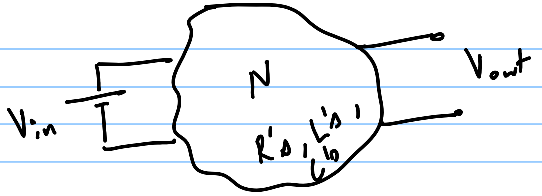
$$I_D = I_s \left[\exp\left(\frac{V_D}{V_t}\right) - 1 \right]$$

I_s: reverse saturation current

thermal voltage $V_t = \frac{kT}{q}$

$\approx 25.9mV$ @ RT

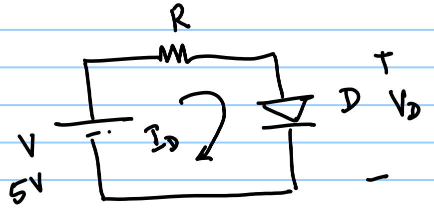
$$\approx I_s \exp\left(\frac{V_D}{V_t}\right) \text{ if } \exp\left(\frac{V_D}{V_t}\right) \gg 1$$



V across each element

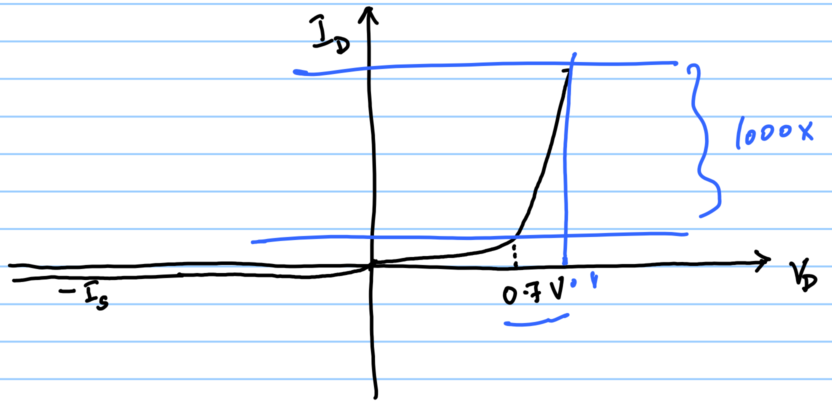
I through " "

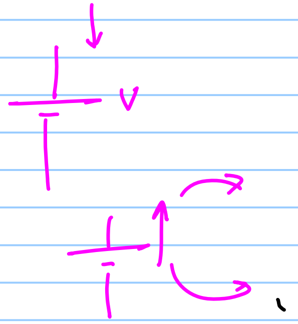
KVL, KCL + element relationships } — solve



$$V = I_D R + V_D$$

$$I_D = I_s\left[\exp\left(\frac{V_D}{V_t}\right)-1\right]$$

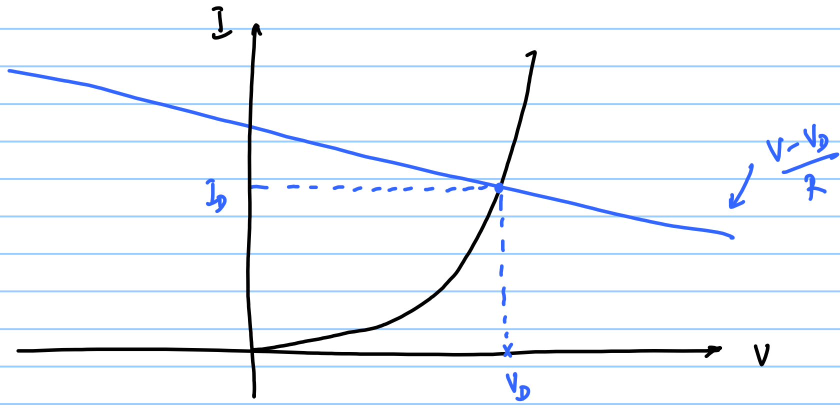


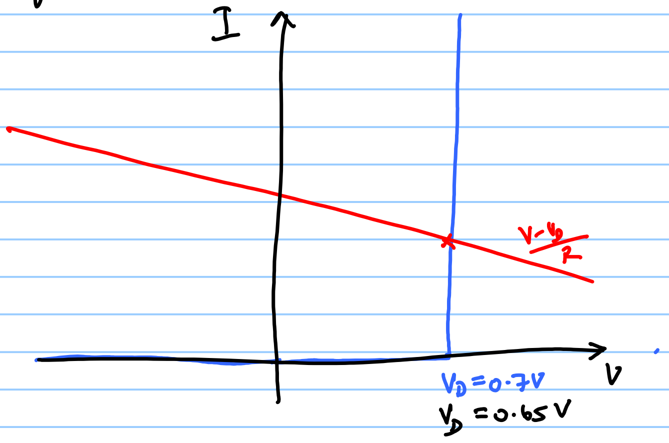


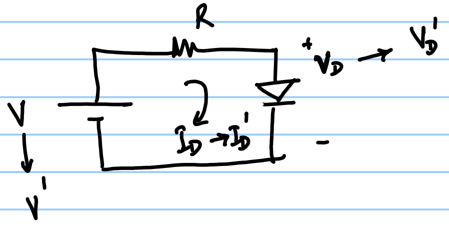
$$\boxed{\frac{V-\overset{\checkmark}{V_D}}{R} = \overset{\checkmark}{I_D}} = I_s\left[\exp\left(\frac{V_D}{V_T}\right)-1\right] \checkmark$$

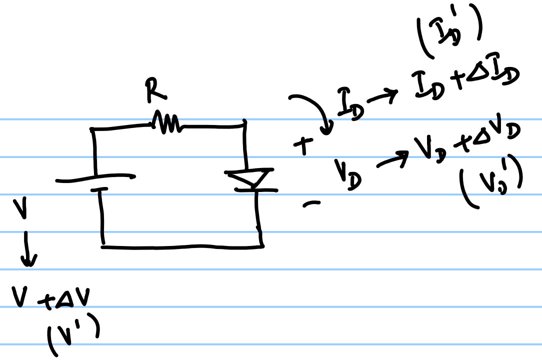
1) Iteration

2) Graphically









$$\frac{V-V_D}{R} = I_D = I_s\left[\exp\left(\frac{V_D}{V_t}\right)-1\right]$$

$$\frac{V'-V_D'}{R} = I_D' = I_s\left[\exp\left(\frac{V_D'}{V_t}\right)-1\right]$$

Taylor Series

$Y = f(x)$ $\qquad Y_0 = f(X_0) \leftarrow$ operating point

$$Y(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}\cdot(x-x_0)^n \leftarrow$$

$$= f(x_0) + f'(x_0)\cdot(x-x_0) + \frac{f''(x_0)}{2}\cdot(x-x_0)^2 + \cdots$$

$$X = \underset{\uparrow}{X_0} + \Delta X$$

$$I_D' = I_D + \Delta I_D = I_s\left[\exp\left(\frac{V_D+\Delta V_D}{V_t}\right)-1\right]$$

$$= \underbrace{I_s\left[\exp\left(\frac{V_D}{V_t}\right)-1\right]}_{f(X_0)} + \overset{f'(x_0)\cdot(x-x_0)}{\frac{I_s}{V_t}\exp\left(\frac{V_D}{V_t}\right)\cdot\Delta V_D}$$

$$+ \ldots\left(\Delta V_D^2, \Delta V_D^3 \ldots\right)$$

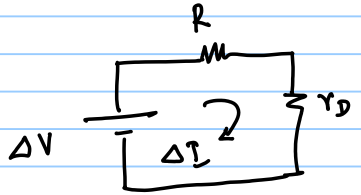
$$\frac{(V+\Delta V) - (V_D+\Delta V_D)}{R} = I_D + \Delta I_D = I_s\left[\exp\left(\frac{V_D}{V_t}\right)-1\right] + \frac{I_s}{V_t}\exp\left(\frac{V_D}{V_t}\right)\cdot \Delta V_D$$

$$\cancel{\frac{V-V_D}{R}} + \frac{\Delta V - \Delta V_D}{R} = \cancel{I_D} + \Delta I_D = \cancel{I_s\left[\exp\left(\frac{V_D}{V_t}\right)-1\right]} + \underbrace{\frac{I_s}{V_t}\exp\left(\frac{V_D}{V_t}\right)}_{\approx \frac{I_D}{V_t}}\cdot \Delta V_D$$

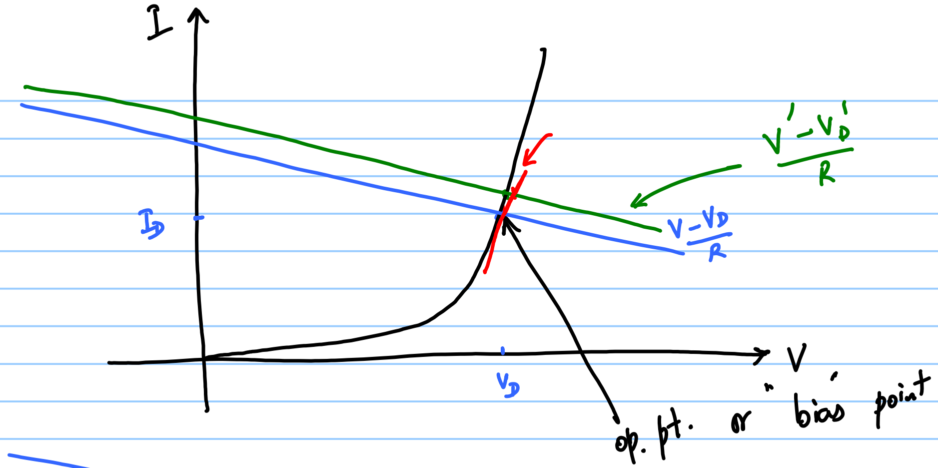
$$\frac{\Delta V - \Delta V_D}{R} = \Delta I_D = \frac{I_D}{V_t}\cdot \Delta V_D$$

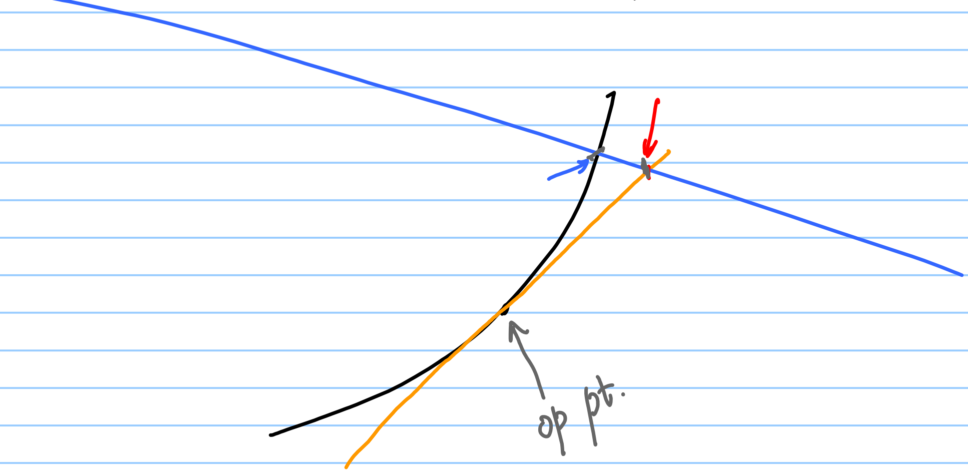
$\Delta V, \Delta V_D, \Delta I_D$ → increments

$$\frac{\Delta V_D}{\Delta I_D} = r_D = \frac{V_t}{I_D}$$



Incremental equivalent circuit


I
V
I_D
V_D
$\frac{V'-V_D'}{R}$
$\frac{V-V_D}{R}$
op. pt. or "bias" point


op pt.

www.ee.iitm.ac.in/~vlsi/teaching/start.html

↓ /~nagendra/videolectures.doku

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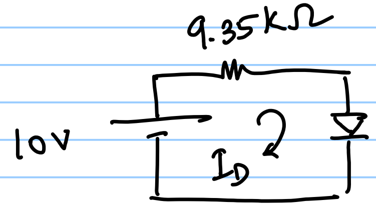
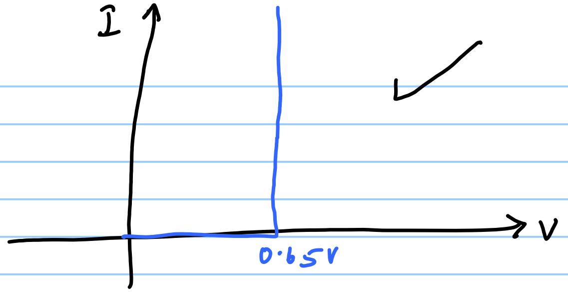
Lec 2



$V_D = 0.65V$

$I_D = 1mA$ = op pt. I_D



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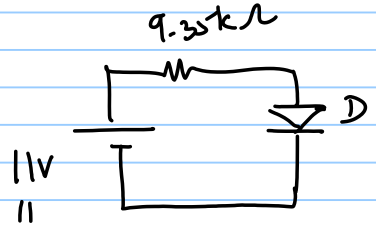


=

10 + 1

op pt.

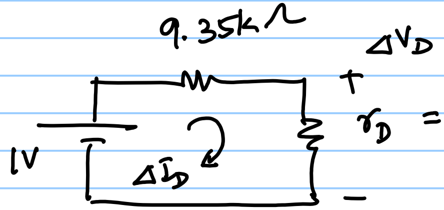
incr.

9V

$V_D'' = 0.65V - 2.7mV$

$I_D'' = 0.893mA$

→



$$r_D = \frac{V_t}{I_D} = \frac{25mV}{1mA} = 25\Omega$$

$$\Delta I_D = \frac{1V}{9.35k+25} = 107\mu A$$

$$\Delta V_D = \Delta I_D \cdot r_D = 2.7mV$$

$$V_D' = V_D + \Delta V_D = 0.65V + 2.7mV$$

$$I_D' = I_D + \Delta I_D = 1.107mA$$

$I_D' = I_s \left[\exp\left(\frac{V_D}{V_t}\right) - 1\right]$ $\qquad f(v)$

$+$

$\frac{I_D}{V_t} \cdot \Delta V_D$ $\qquad f'(v)$

$+$

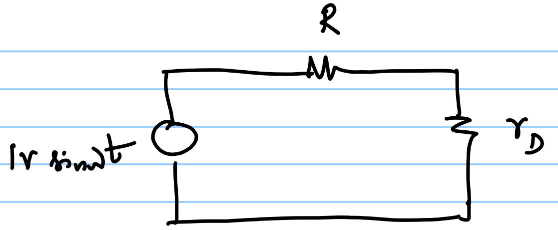
$\frac{1}{2} \cdot \frac{I_D}{V_t^2} \cdot \Delta V_D^2$ $\qquad f''(v)$

$$\frac{\frac{I_D}{2 V_t^2} \cdot \Delta V_D^2}{\frac{I_D}{V_t} \Delta V_D} << 1$$

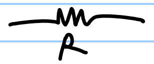
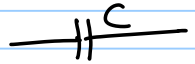
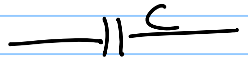
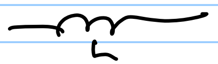
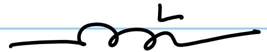
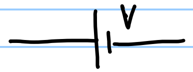
$$\boxed{\Delta V_D << 2 V_t}$$

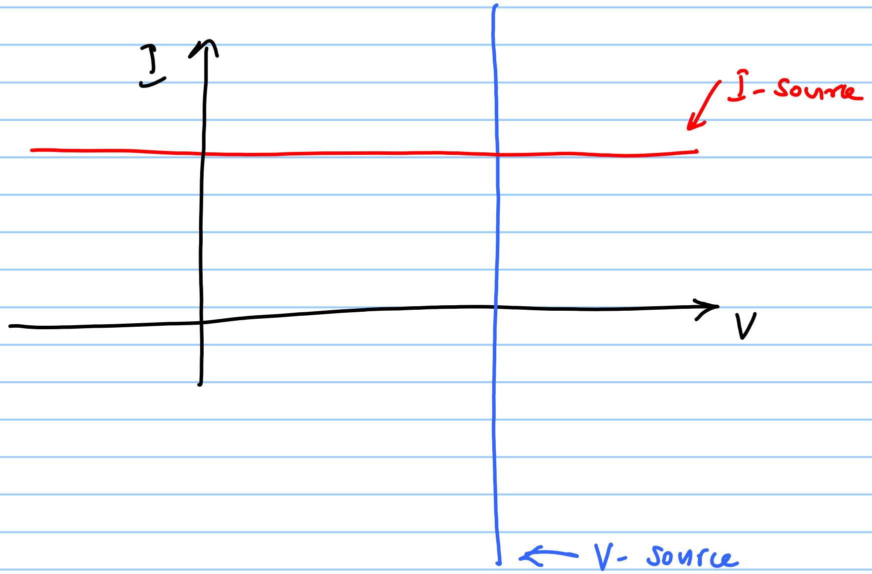
$2.7mV << 50mV \Longleftarrow$ incremental (linear) approx. is valid.

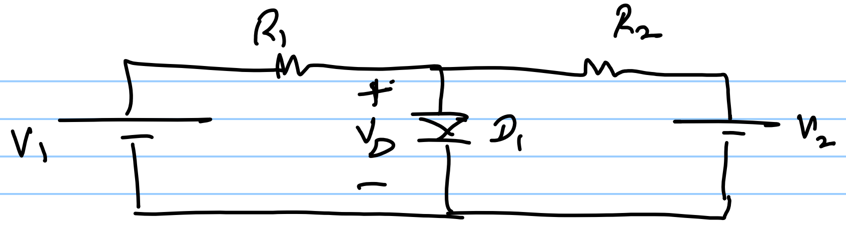
$\Longrightarrow$

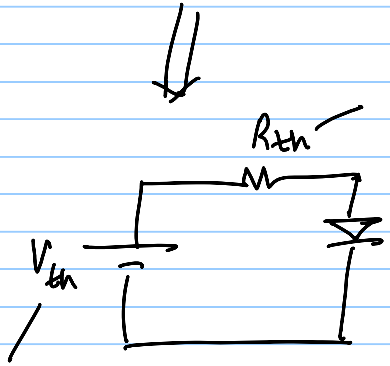


Small-signal eq. ckt

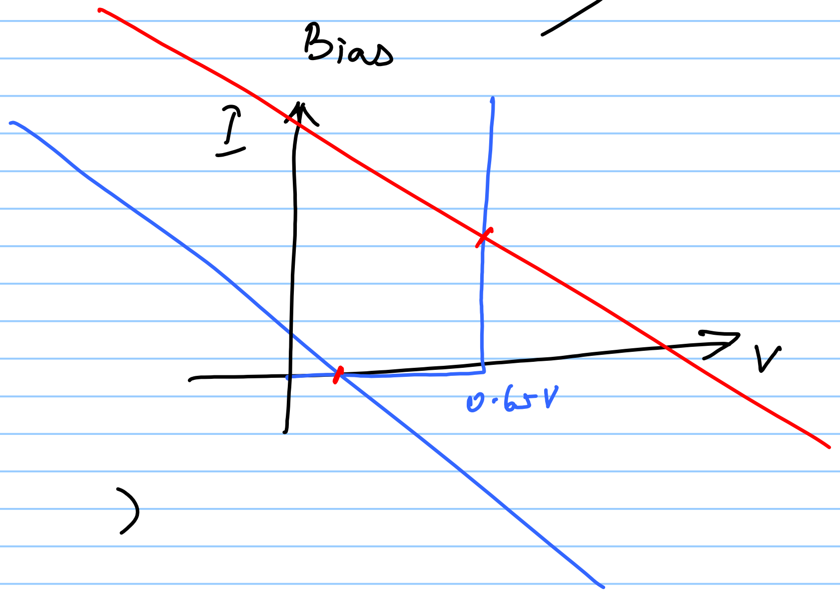
Element		Ss eq.	
			
			
			
	F.B.		$r = \frac{V_T}{I_D}$
	R.B.		O.C.
	$I = f(v)$		$\frac{1}{f'(V)}$
			S.C.
			O.C.



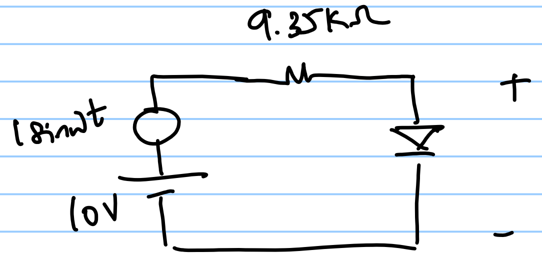




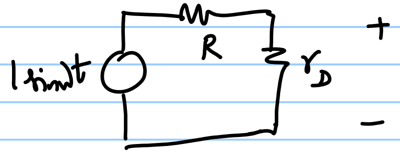
Operating
Quiescent } point
Bias



$$\frac{V_{th} - V_D}{R_{th}} = I_D = (\qquad)$$



$V_D = 0.65V$, $I_D = 1mA \Rightarrow r_D = 25\Omega$



$$v_d = \frac{r}{R+r} \cdot 1 \sin\omega t$$

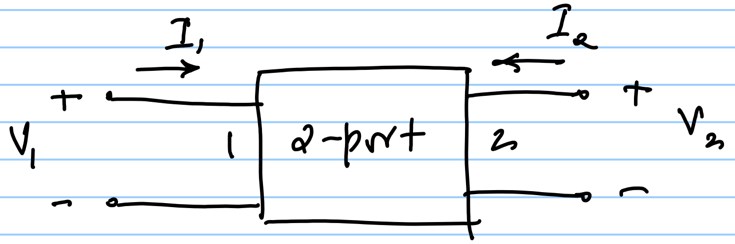
Transformer — Voltage gain ✓

Power gain X

Diode — Voltage gain possible if r is negative

$$r = \frac{1}{f'(v)} \longleftarrow \text{negative slope}$$

2-port



$[Z]$, $[Y]$, $[G]$, $[H]$, $[ABCD]$, $[A'B'..]$

$[S]$

$$\left.\begin{aligned} I_1 &= Y_{11}V_1 + Y_{12}V_2 \\ I_2 &= Y_{21}V_1 + Y_{22}V_2 \end{aligned}\right\} \Rightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

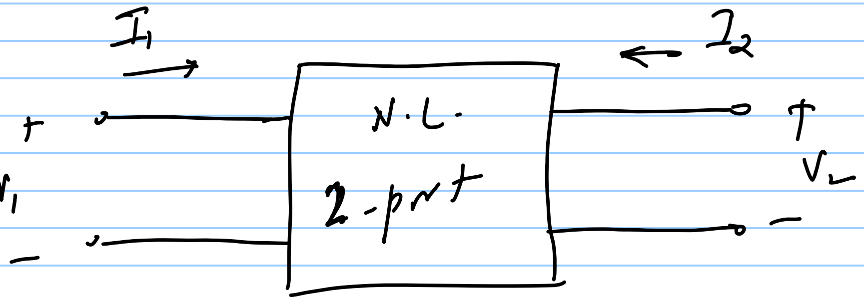
$$[I] = [Y] \cdot [V]$$

17/8/17

Lec 3

$$I_1 = f(V_1, V_2)$$

$$I_2 = g(V_1, V_2)$$



2-D Taylor Series

$$\cancel{I_1}+\Delta i_1 = I_1' = f\cancel{(V_1,V_2)}\Big|_{op\ pt.} + \frac{\partial f}{\partial V_1}\cdot\Delta V_1 + \frac{\partial f}{\partial V_2}\cdot\Delta V_2 + \cdots$$

$$\cancel{I_2}+\Delta i_2 = I_2' = g\cancel{(V_1,V_2)}\Big|_{op\ pt} + \frac{\partial g}{\partial V_1}\Delta V_1 + \frac{\partial g}{\partial V_2}\cdot\Delta V_2 + \cdots$$

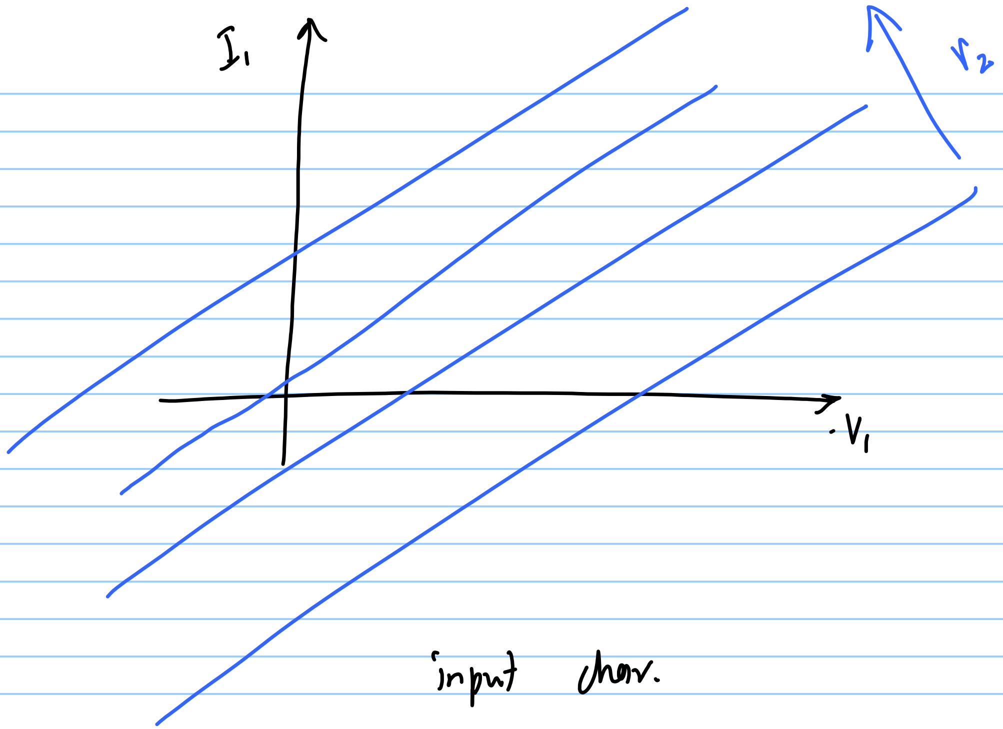
$$\Delta i_1 = \partial f/\partial v_1 \cdot \Delta v_1 + \frac{\partial f}{\partial v_2} \cdot \Delta v_2$$

$$\Delta i_2 = \partial g/\partial v_1 \cdot \Delta v_1 + \partial f/\partial v_2 \cdot \Delta v_2$$

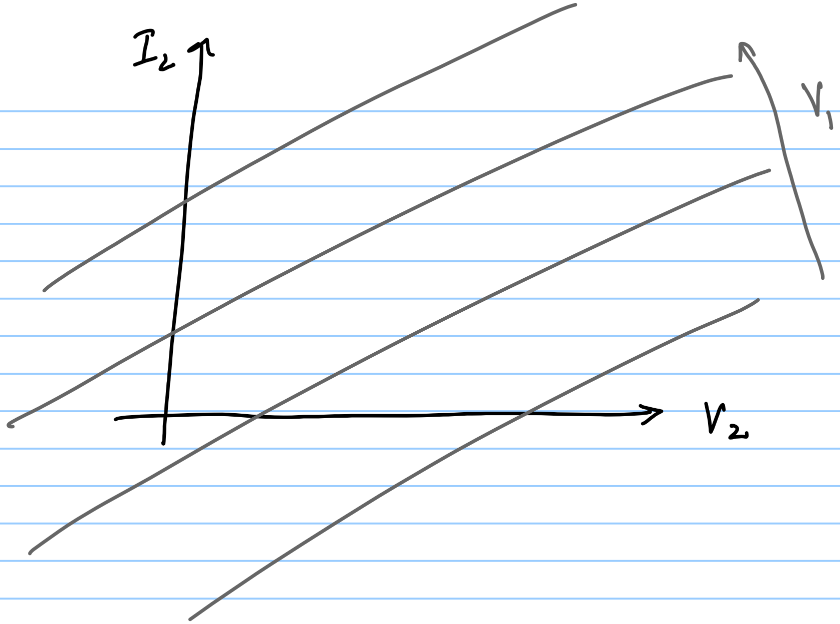
$$i_1 = \partial f/\partial v_1 \cdot v_1 + \partial f/\partial v_2 \cdot v_2$$

$$i_2 = \partial g/\partial v_1 \cdot v_1 + \partial g/\partial v_2 \cdot v_2$$

$$\left.\begin{aligned}
\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} &= \begin{bmatrix} \partial f/\partial v_1 & \partial f/\partial v_2 \\ \partial g/\partial v_1 & \partial g/\partial v_2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \\
\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} &= \underbrace{\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}}_{} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}
\end{aligned}\right\}$$



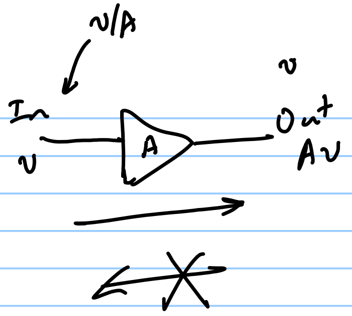
input char.

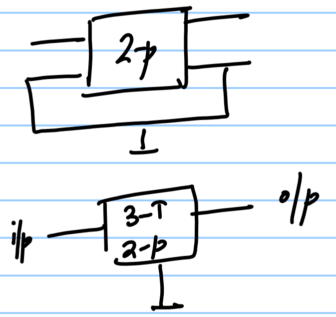


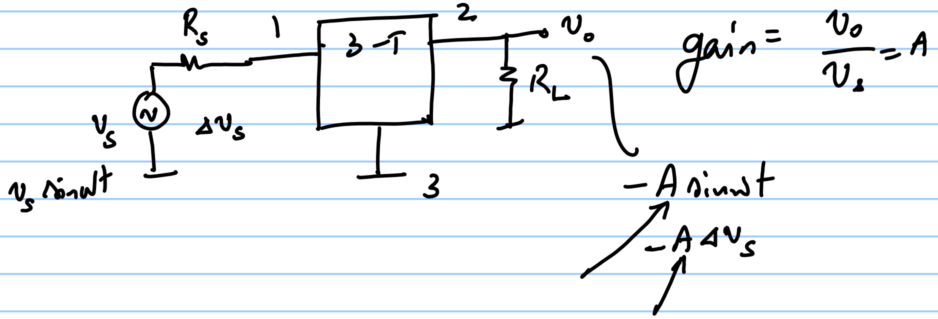
Output char.

Amplifier

1) High gain
2) gain independent of R_s
3) " " of R_L
4) "Unilateral" 2-port



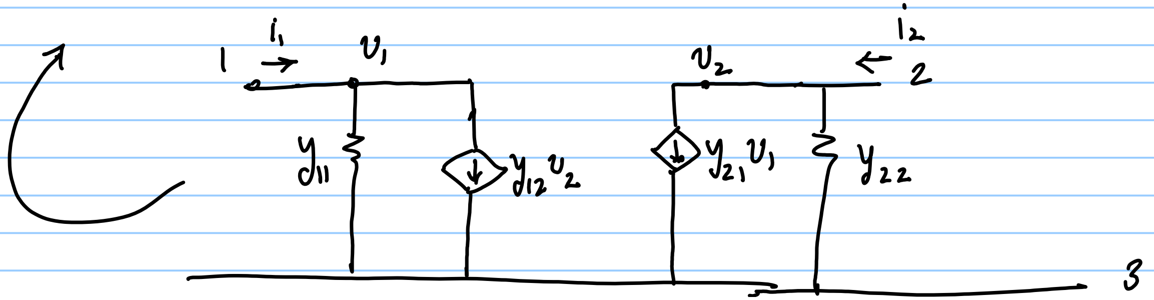


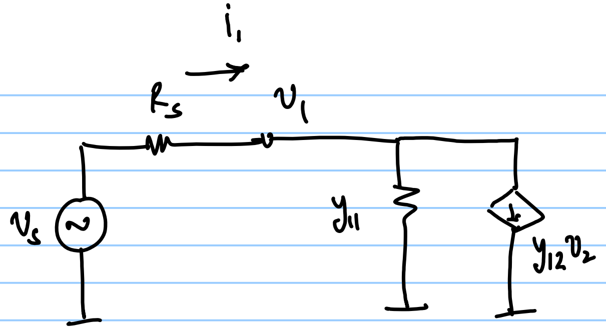


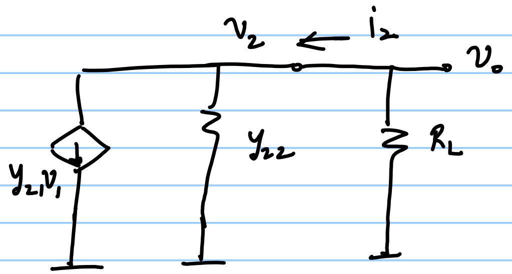
$$gain = \frac{v_o}{v_s} = A$$

$$i_1 = y_{11} v_1 + y_{12} v_2$$

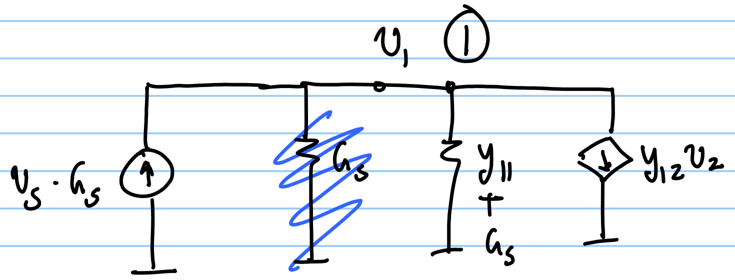
$$i_2 = y_{21} v_1 + y_{22} \cdot v_2$$

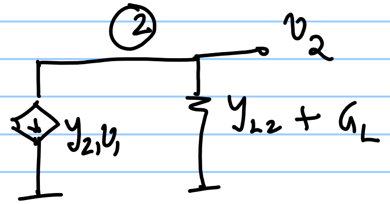






$$\frac{v_o}{v_s} = ? \left\{ = \frac{v_2}{v_s} \right\}$$





KCL @ ① ⇒ $v_s \cdot G_s = v_1 (y_{11} + G_s) + y_{12} v_2$

KCL @ ② ⇒ $y_{21} v_1 + (y_{22} + G_L) \cdot v_2 = 0$ ←

$v_1 = (v_s \cdot G_s - v_2 \cdot y_{12}) \cdot \frac{1}{y_{11} + G_s}$ ← plug back into

$$\left(\frac{y_{21}}{y_{11}+G_s}\right)\left(v_s G_s - v_2 \cdot y_{12}\right) + v_2\left(y_{22}+G_L\right) = 0$$

$$v_s \cdot \left(y_{21} \cdot G_s\right) = v_2 \cdot \left[y_{12} y_{21} - (y_{11}+G_s)(y_{22}+G_L)\right]$$

$$\boxed{\frac{v_2}{v_s} = \frac{y_{21} G_s}{y_{12} y_{21} - (y_{11}+G_s)(y_{22}+G_L)}}$$

1) High gain : ∞ gain if $y_{12} y_{21} = (y_{11}+G_s)(y_{22}+G_L)$ $\times$ Happens due to instability

4) Unilateral : $\boxed{y_{12} = 0}$

$$\frac{v_2}{v_s} = \frac{-y_{21} G_s}{(y_{11}+G_s)(y_{22}+G_L)}$$

2) Gain indep. of R_s : $\boxed{y_{11} = 0}$

$$\frac{v_2}{v_s} = \frac{-y_{21}}{y_{22}+G_L} \longrightarrow \boxed{y_{22} = 0}$$

$$\frac{v_2}{v_s} = \frac{-y_{21}}{G_L}$$

$$\boxed{y_{21} = \text{as large as possible}}$$

3) Indep of R_L ? ← gain will be a fn. of R_L

$y_{11} = 0$

$y_{12} = 0$

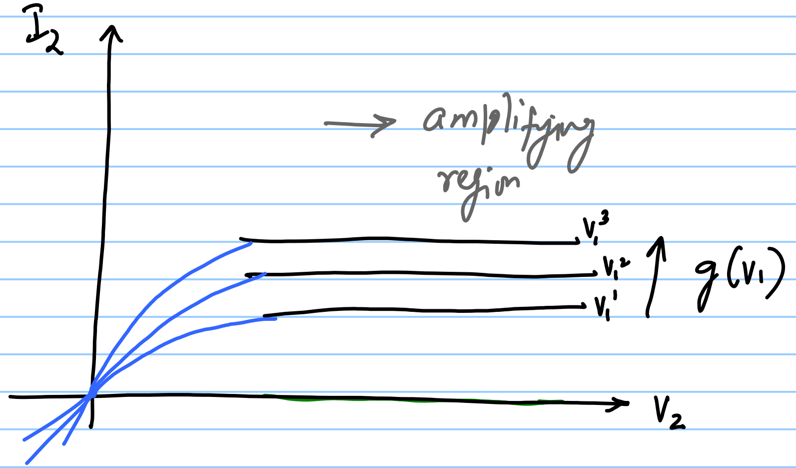
$$[y] = \begin{bmatrix} 0 & 0 \\ \text{large} & 0 \end{bmatrix}$$

$y_{21} =$ as large as possible

$y_{22} = 0$

$$y_{11} = \frac{\partial f}{\partial V_1} = 0 \quad \text{and} \quad y_{12} = \frac{\partial f}{\partial V_2} = 0 \Rightarrow \boxed{I_1 = \text{constant}}$$

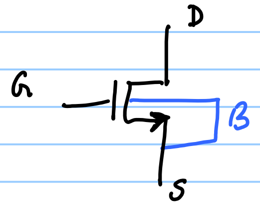
$$y_{22} = \frac{\partial g}{\partial V_2} = 0 \Rightarrow \boxed{I_2 = g(V_1)}$$



- * MOSFET : $I_1 = 0$; $I_2 = g_1(V_1)$
- * BJT, JFET : $I_1 =$ small, constant

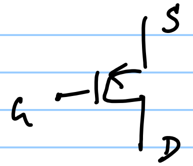
  $I_2 = g_2(V_1)$

MOSFET



n MOSFET ~ nMOS

e^- : charge carriers



(later)

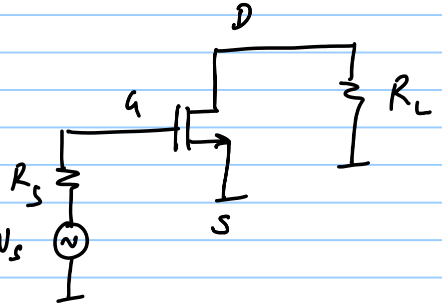
pMOS

holes : charge carriers

G : port 1

D : port 2

S : ref. terminal



$$I_G = 0 = I_1$$

$$I_2 = I_D = 0 \quad \text{if} \quad V_{GS} < V_T$$

$$V_{GS} > V_T \begin{cases} = \mu_n C_{ox} \left(\frac{W}{L}\right)\left[(V_{GS}-V_T)V_{DS} - \frac{V_{DS}^2}{2}\right] & \text{for } V_{DS} < V_{GS}-V_T \\ = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_T)^2 & \text{for } V_{DS} \geq V_{GS}-V_T \end{cases}$$

V_T : Threshold Voltage

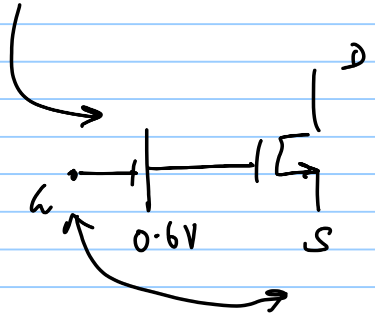
μ_n : mobility of $\bar{e}$s

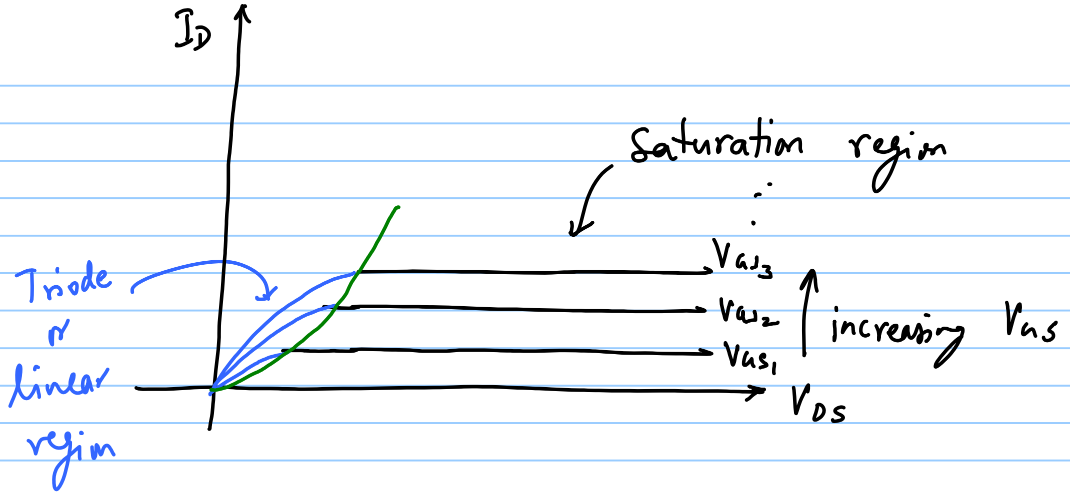
C_{ox} : Oxide capacitance (per unit area cap.)

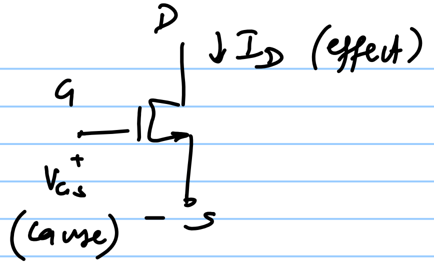
W, L : Width & length of MOSFET (channel) — geometric parameters

"enhancement" mode nMOSFET : $V_T > 0$ $\quad V_{T_e} = 0.3V$

"depletion" mode nMOSFET : $V_T < 0$ $\quad V_{T_d} = -0.3V$







In saturation:

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_T)^2$$

$$\downarrow$$

$$(V_{GS}-V_T) = \sqrt{\frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)}}$$

$y_{11} = y_{12} = 0$ bec. $I_G = 0$

$y_{22} = 0$ bec. $I_D = g(V_{GS})$ only

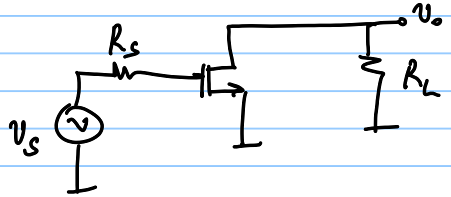
$y_{21} = \frac{\partial g}{\partial V_1} = \frac{\partial I_D}{\partial V_{GS}} =$ "transconductance" g_m

$$g_m = \mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_T)$$ at op pt. value V_{GS_0}

$$g_m = \mu_n C_{ox}\left(\frac{W}{L}\right)\sqrt{\frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)}}$$ ①

$$g_m = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) I_D}$$ ②

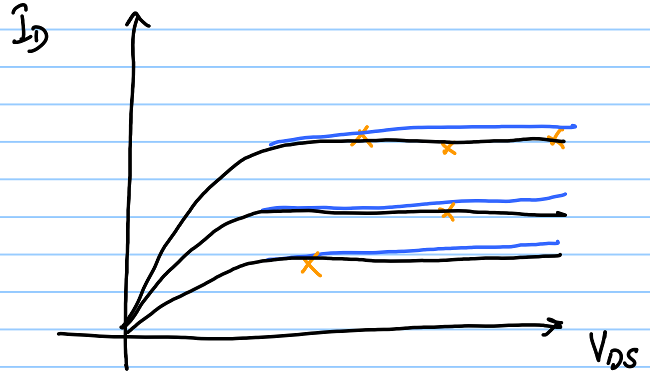
$$g_m = \frac{2I_D}{(V_{GS}-V_T)}$$ ③



$$\frac{v_o}{v_s} = \frac{-y_{21}}{g_L} = -g_m R_L$$

22/8/17

Lec 4



$y_{22} \neq 0$, but a small number

"channel length modulation" $L \rightarrow L + \Delta L$ $\swarrow$ $f(V_{DS})$

$$I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_T)^2 (1 + \lambda V_{DS})$$

λ = units of V^{-1}

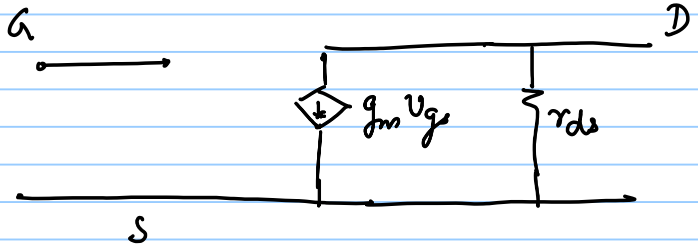
λ = v. small

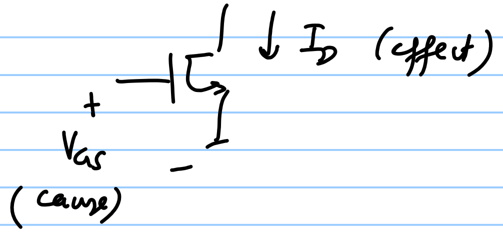
$$y_{22} = \frac{\partial I_2}{\partial V_2} = \frac{\partial I_D}{\partial V_{DS}} = \underbrace{\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_T)^2}_{I_D} \cdot \lambda$$

$y_{22} \simeq \lambda \cdot I_D$

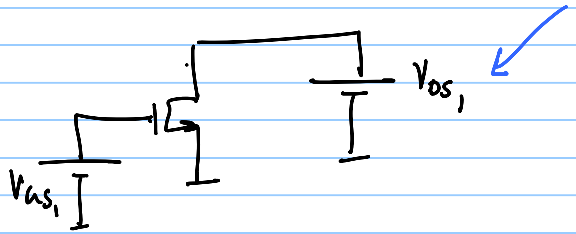
$\hookrightarrow r_{ds} = \frac{1}{\lambda I_D}$ v. large

Small-signal model of MOSFET

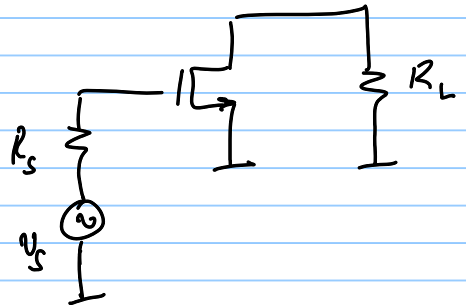


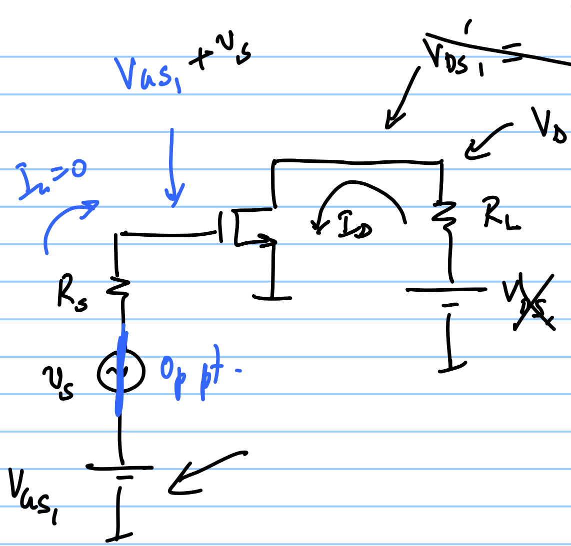


Op pt. → (V_{GS_Q}, V_{DS_Q})

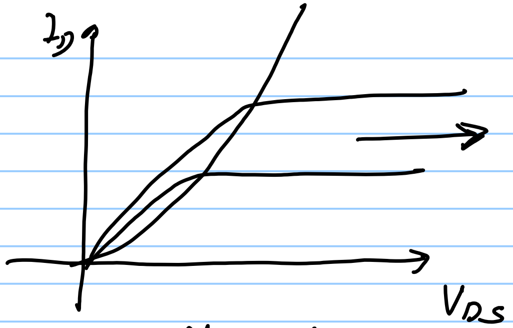


Small signal





$V_{DS_1} + I_D R_L = V_{DD} > V_{DS_1}$

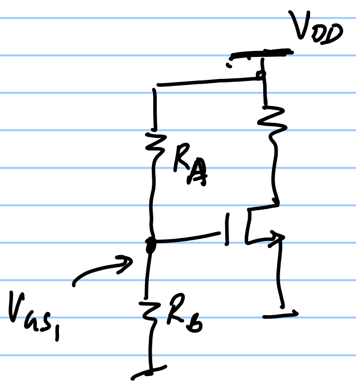


V_{DS_1} – large

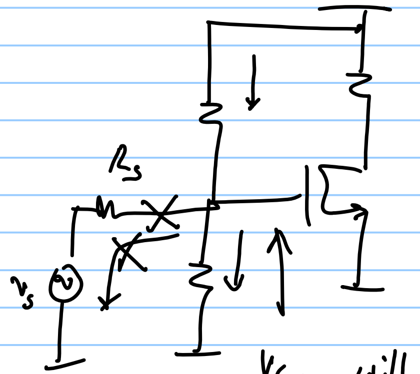
$\gg V_{GS_1} - V_T$

$> V_{GS_1}$

generate V_{GS_1} from V_{DD}

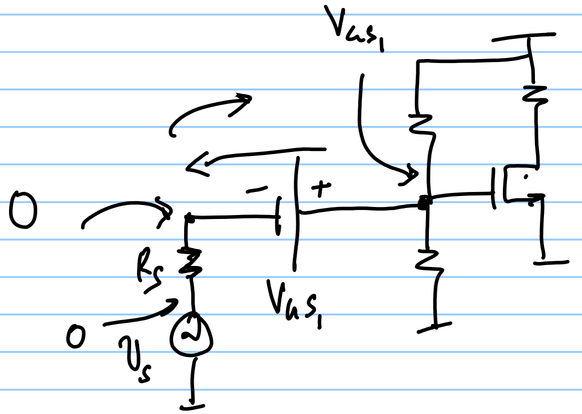


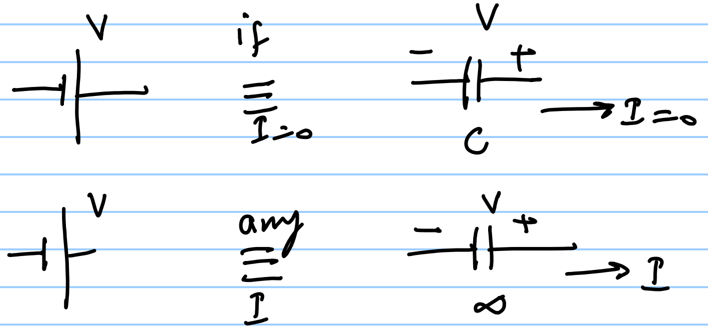
$$\boxed{V_{GS_1} = \frac{R_B}{R_A + R_B} \cdot V_{DD}}$$

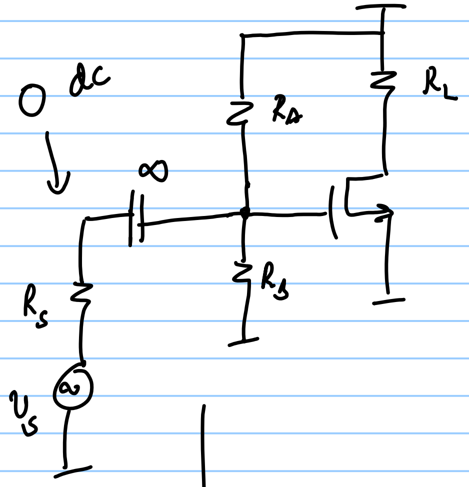


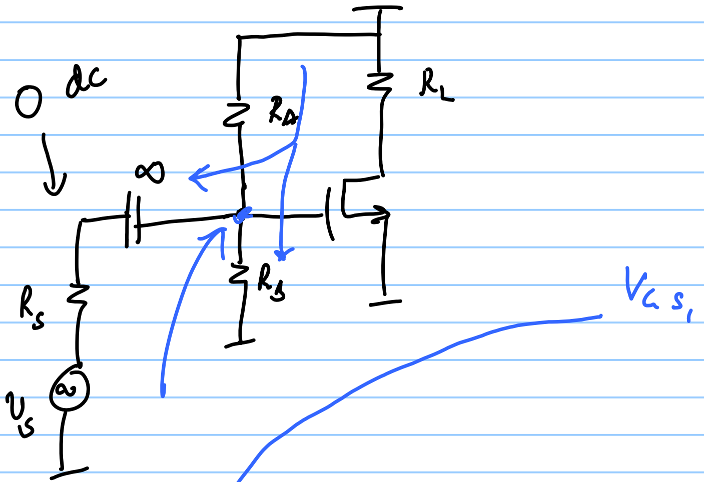
V_{GS_1} will change

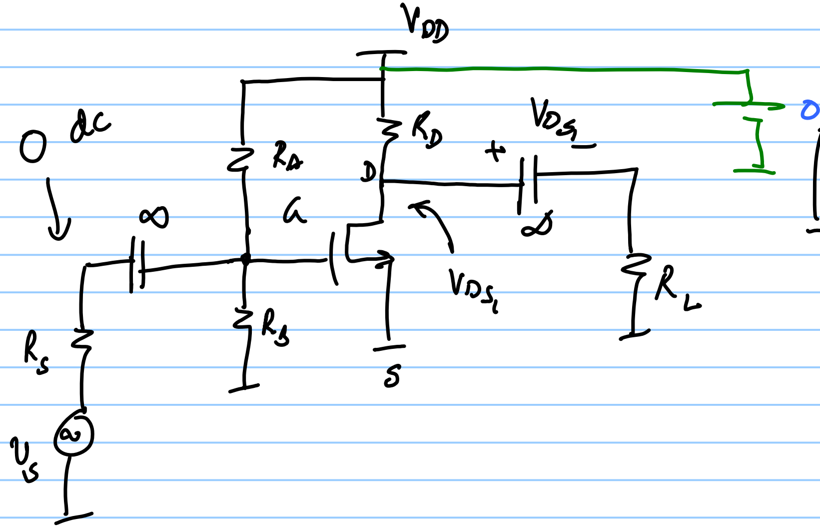
Op pt. $= f(R_s)$ X





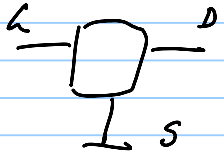




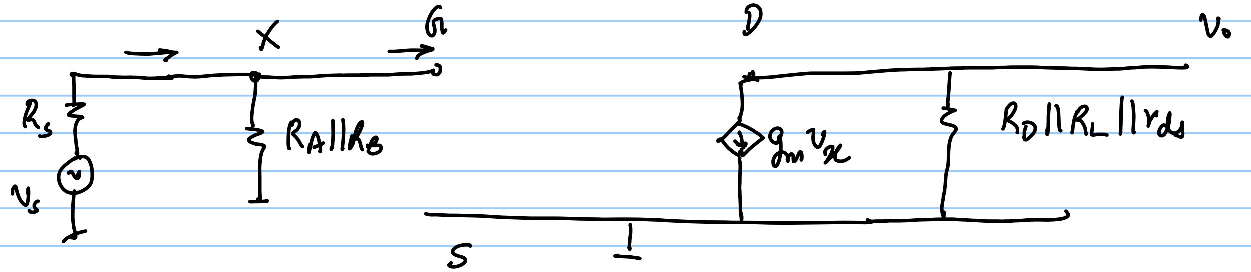


$$\boxed{V_{DD} = V_{DS_1} + I_D R_D}$$ ✓

" Common Source Amplifier "



Small Signal equivalent ciruit



$v_x = v_s \cdot \frac{R_A||R_B}{R_s + R_A||R_B}$; we want $v_x = v_s$

$v_x \simeq v_s$ if $R_A||R_B >> R_s$ $\quad$ R_A & $R_B >> R_s$

$V_{GS,} = \frac{R_B}{R_A+R_B} \cdot V_{DD}$

$v_o = -g_m (R_D||R_L||r_{ds}) \cdot v_s$

we want $v_o = -g_m R_L v_s$

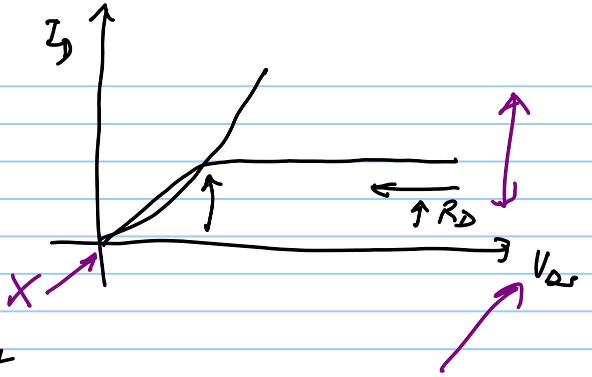
* $r_{ds} \gg R_D$ & R_L $\Rightarrow$ λ should be v. small

* $R_D \gg R_L$

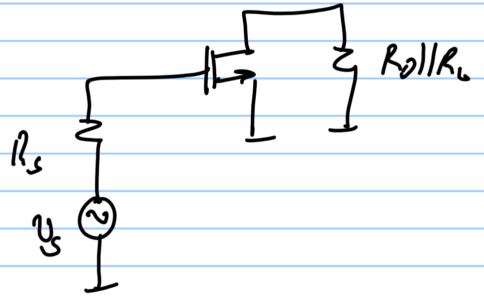
$\rightarrow$ $V_{DD} = V_{DS,} + I_D R_D$

$\uparrow$ cannot be changed by a large amount

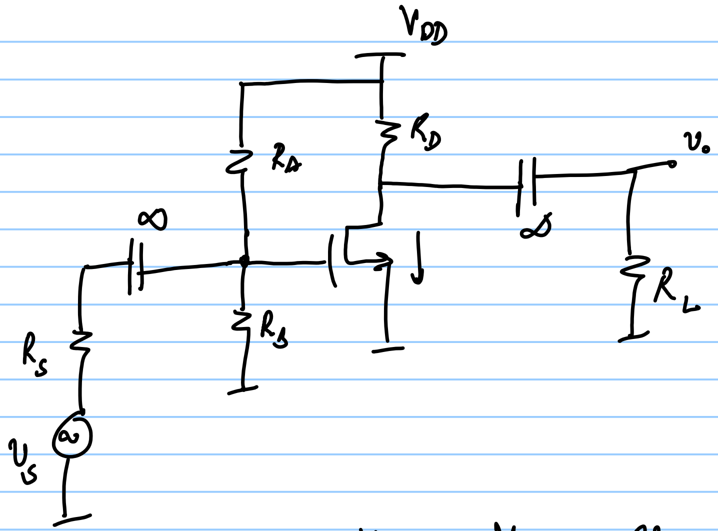
$\rightarrow$ Choose largest R_D possible



gain $\dfrac{v_o}{v_s} = -g_m (R_D || R_L)$



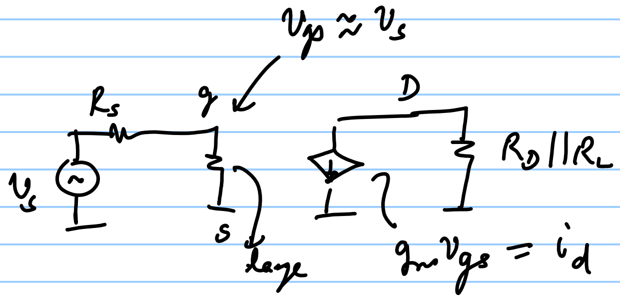
S.S. picture



$(V_{GS_1}, V_{BS_1}) \leftarrow V_{DS_1}$ large (far away from triode)

$\text{gain} \rightarrow g_m \rightarrow I_{D_1}, V_{GS_1}$

R_L



$V_{GS} = V_{GS_1} + v_{gs} \rightarrow v_{gs} = v_s$

$V_{DS} = V_{DS_1} + v_{ds} \rightarrow v_d = -g_m (R_D||R_L) \cdot v_s = -A v_s$

$I_D = I_{D_1} + i_d \rightarrow i_d = g_m \cdot v_{gs} = g_m v_s$

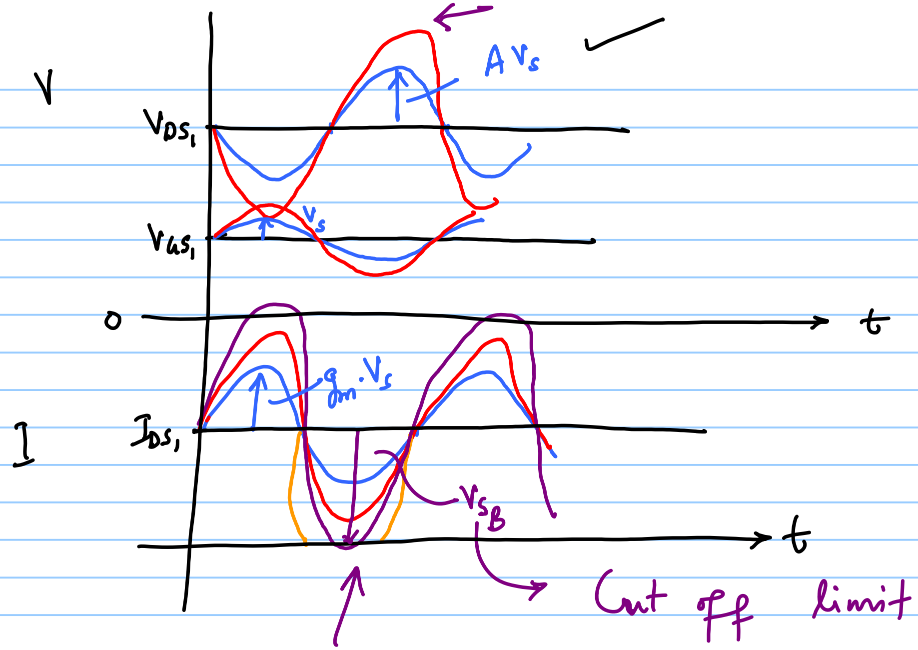
Op pt.: $V_{DS_1} > V_{GS_1} - V_T$ ✓

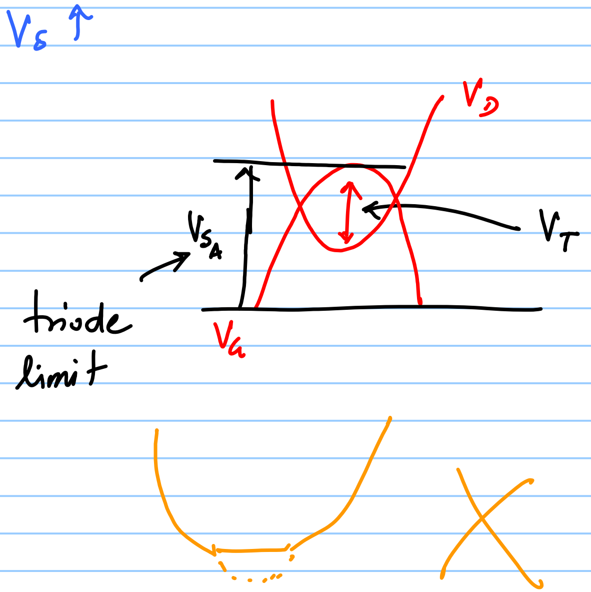
$V_{DS_1} > V_{GS_1}$

$v_s = V_s \sin\omega t$

$i_d = g_m V_s \sin\omega t$

$v_d = - A V_s \sin\omega t$





$$i_d = g_m V_s \sin\omega t$$

$$I_{D_1} + i_d = 0 \Rightarrow I_{D_1} - g_m V_{s_B} = 0 \Rightarrow \boxed{V_{s_B} = \frac{I_{D_1}}{g_m}}$$

triode limit : $V_{DS} \geq V_{GS} - V_T$

$$V_D - V_S \geq V_G - V_S - V_T$$

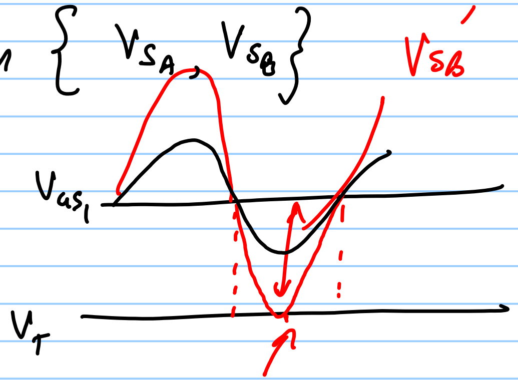
$$V_D \geq V_G - V_T$$

$$V_{DS_1} - A V_{S_A} = V_{GS_1} + V_{S_A} - V_T$$

$$\boxed{V_{S_A} = \frac{V_{DS_1} - (V_{GS_1} - V_T)}{1+A}}$$

Swing limit of C.S.A = minimum $\{ V_{S_A}, V_{S_B} \}$

$I_D = 0 \rightarrow V_{GS} < V_T$



24/8/17

Lec 5

Quiz 1 – September 19th (Tuesday)

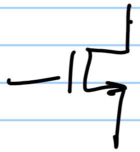
Quiz 2 – October 24th (Tue)

Quiz 3 – November 19th (Sun) –

End Semester – December 14th (Thu)

10 : 30 am – noon

Triode



$$I_D = \mu_n C_{ox}\left(\frac{W}{L}\right)\left[(V_{GS}-V_T)\cdot V_{DS} - \frac{V_{DS}^2}{2}\right]$$

$$y_{11} = y_{12} = 0$$

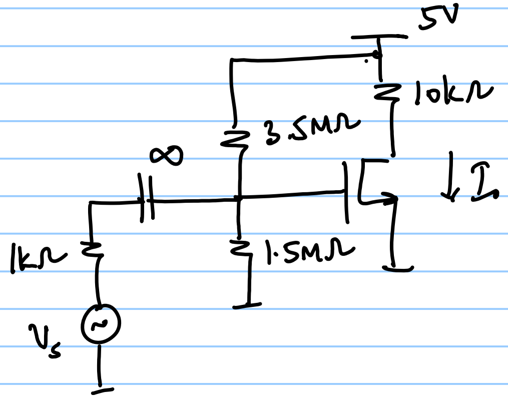
$$y_{21} = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox}\left(\frac{W}{L}\right)\cdot V_{DS} \quad ; \quad y_{22} = \frac{\partial I_D}{\partial V_{DS}} = \mu_n C_{ox}\left(\frac{W}{L}\right)\left(V_{GS}-V_T-V_{DS}\right)$$

↳ smaller than y_{21} (sat.)

↳ larger than y_{22} (sat.)

$$gain = \frac{-y_{21}}{y_{22}+G_L} \Rightarrow$$ gain drops due to larger y_{22}

gain drops due to smaller y_{21}



$\mu_n C_{ox} = 100\mu A/V^2$

$V_T = 1V$

$\left(\frac{W}{L}\right) = 10$

$V_{GS_1} = 1.5V$; $(V_{GS_1} - V_T) = 0.5V$ ✓

$I_{D_1} = \frac{1}{2} \times 100\mu \times 10 \times (0.5)^2 = 125\mu A$

$V_{DS_1} = 5 - 1.25 = 3.75V$ ✓

$$g_m = \frac{2I_{D_1}}{V_{GS_1}-V_T} = \frac{2\times 125\mu A}{0.5V} = 0.5mS$$

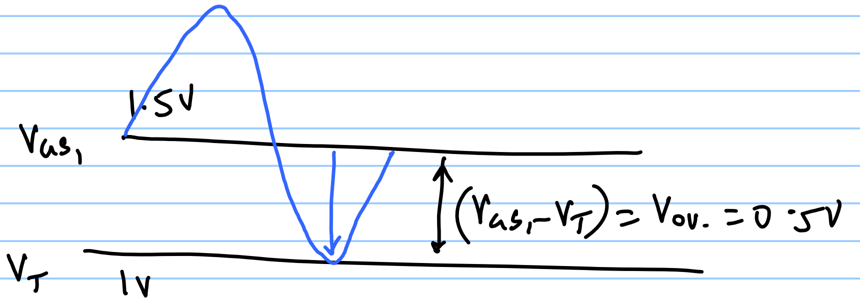
$$gain = -g_m R_L = -5$$

$$V_{S_B} = \frac{I_{D1}}{g_m} = \frac{125\mu A}{500\mu S} = 250mV$$

$$V_{SA} = \frac{V_{DS_1} - (V_{GS_1} - V_T)}{1 + g_m R_L} = \frac{3.75 - 0.5}{6} = 545mV$$

Swing limit $= 250mV$

$V_{SB}' = 0.5V$

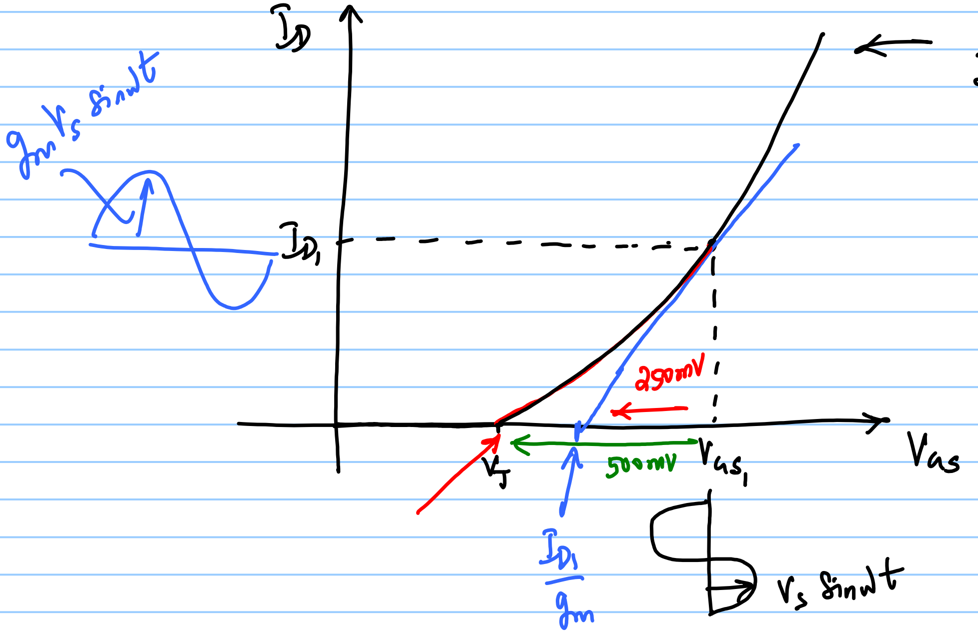


$I_D = 0 \Rightarrow V_{GS} \leq V_T$

$V_{DS} > 0$

250mV (V_{SB})

500mV (V_{SB}')



$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_T)^2$

When is SS approx. Valid ??

$$I_D = f(V_{GS}, V_{DS})$$

$$I_{D_1} + i_d = f(V_{GS_1}, V_{DS_1}) + \boxed{\frac{\partial f}{\partial V_{GS}} \cdot v_{gs}} + \cancelto{0}{\frac{\partial f}{\partial V_{DS}} \cdot v_{ds}}$$

$$+ \boxed{\frac{1}{2}\frac{\partial^2 f}{\partial V_{GS}^2} \cdot v_{gs}^2} + \frac{1}{2}\cancelto{0}{\frac{\partial^2 f}{\partial V_{DS}^2}} \cdot v_{ds}^2 + \cancelto{0}{\frac{\partial^2 f}{\partial V_{GS} \cdot \partial V_{DS}}} v_{gs} \cdot v_{ds} + \dots$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_T)^2$$

$$\frac{\partial I_D}{\partial V_{GS}} = g_m = \mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_T)$$

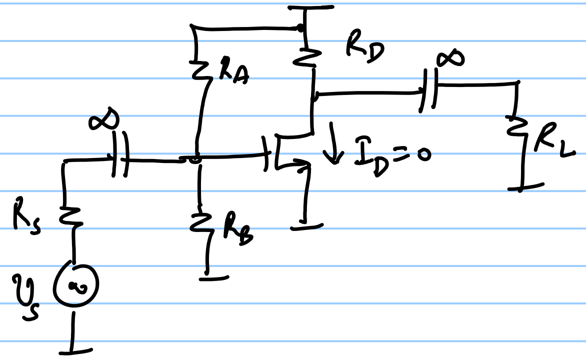
$$\frac{\partial^2 I_D}{\partial V_{GS}^2} = \mu_n C_{ox}\left(\frac{W}{L}\right)$$

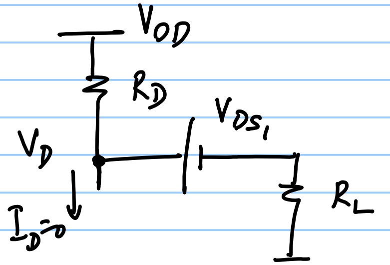
$$\frac{1}{2}\frac{\partial^2 I_D}{\partial V_{GS}^2}\cdot v_{gs}^2 << \frac{\partial I_D}{\partial V_{GS}}\cdot v_{gs}$$

$$\frac{1}{2}\cdot \mu_n C_{ox}\left(\frac{W}{L}\right)\cdot v_{gs}^2 << \mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_T)\, v_{gs}$$

$$\Rightarrow \boxed{v_{gs} << 2(V_{GS}-V_T)}$$

Not Valid when calculating r_{SB}

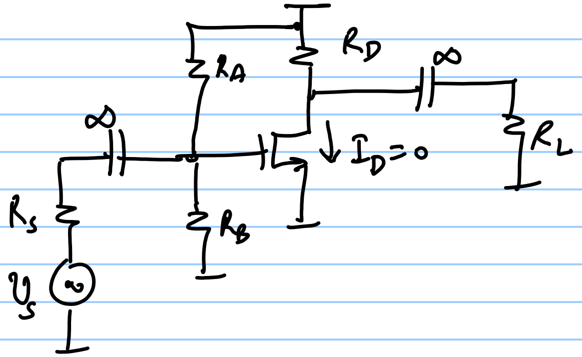




$$\frac{V_{DD}-V_D}{R_D} = \frac{V_D-V_{DS_1}}{R_L}$$

$$V_D\cdot\left(\frac{1}{R_D}+\frac{1}{R_L}\right) = \frac{V_{DD}}{R_D} + \frac{V_{DS_1}}{R_L}$$

$$\boxed{V_D = \frac{V_{DD}\cdot R_L + V_{DS_1}\cdot R_D}{R_D+R_L}}$$



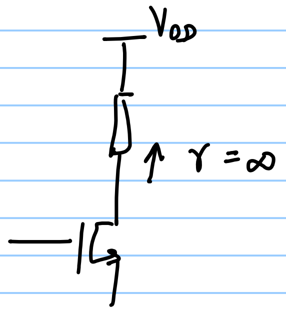
$A = -g_m(R_D \| \overset{\infty}{\cancel{R_L}} \| r_{ds})$

$\uparrow R_D \Rightarrow V_{DD} \uparrow$ or $V_{DS_1} \downarrow$

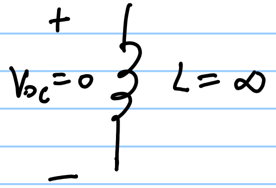
$V_{DD} = V_{DS_1} + I_D R_D$

$\frac{v_o}{v_s} \rightarrow$ depends on incremental resistance

$V_{DS_1} \rightarrow$ depends on total resistance



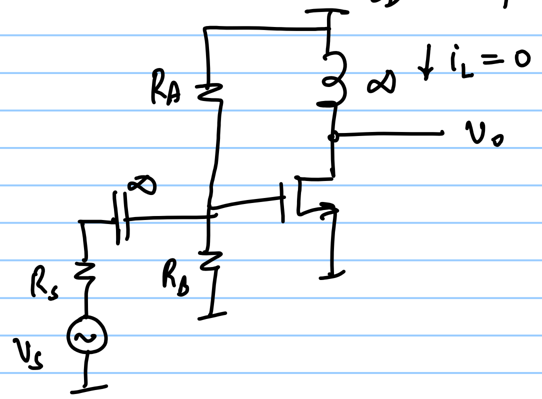
$R_{dc} =$ small



$Z_L = j\omega L$

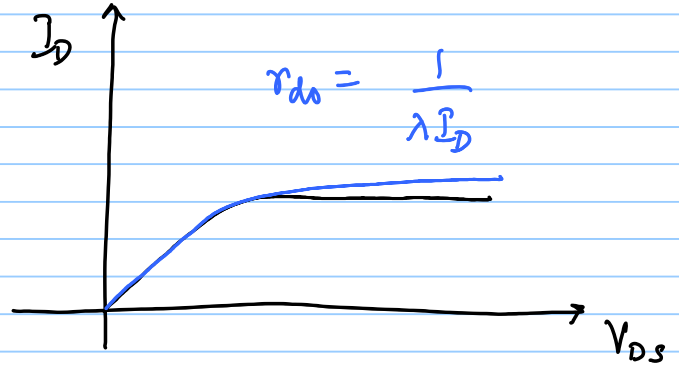
$Z_L = \infty$ if $L = \infty$

$V_{DD} = V_{DS_1}$ indep. of I_D

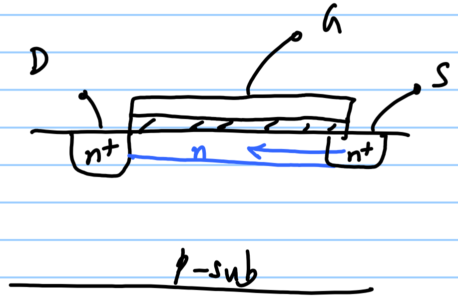


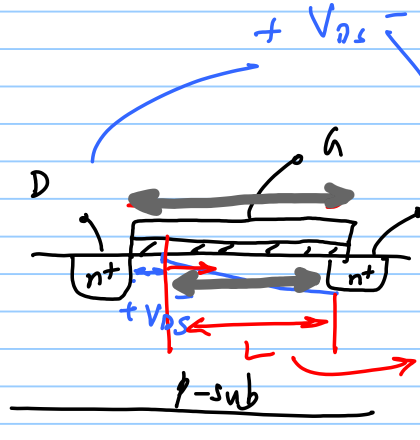
$$\boxed{\frac{v_o}{v_s} = -g_m r_{ds}}$$

Intrinsic gain of MOSFET (Maximum)



$r_{do} = \frac{1}{\lambda I_D}$



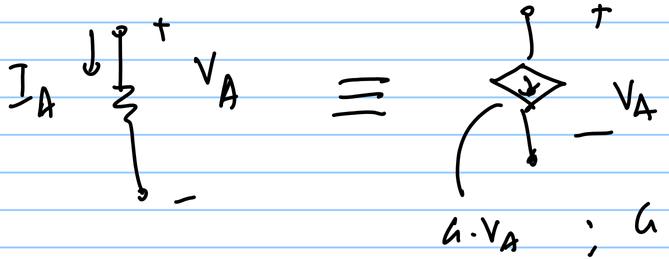


$L + \Delta L$ when $V_{DS} \rightarrow V_{DS} + \Delta V_{DS}$

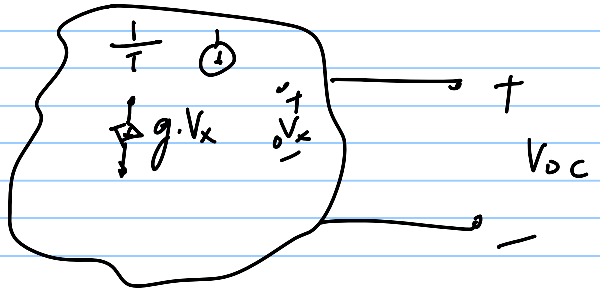
Channel length modulation

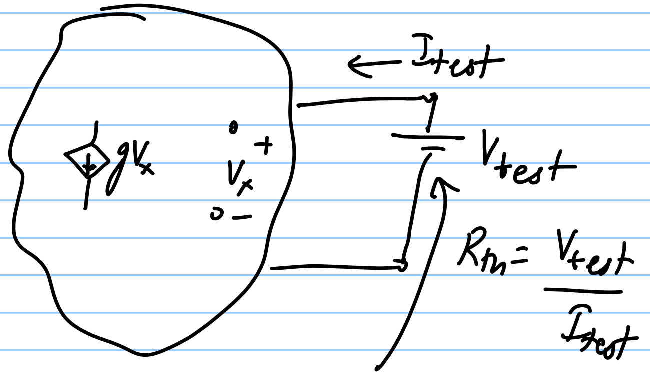
29/8/17

Lec 6

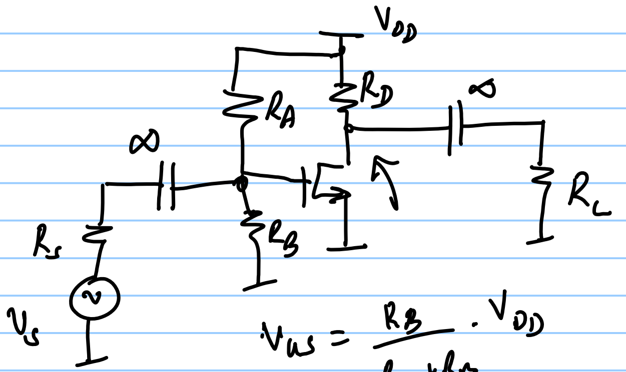


$; \quad G = \frac{1}{R}$





$$R_{th} = \frac{V_{test}}{I_{test}}$$



$$V_{GS} = \frac{R_B}{\underbrace{R_A+R_B}_{\text{ratios}}} \cdot V_{DD}$$

track well (low tolerance)

μ_n, C_{ox}, $\left(\frac{W}{L}\right)$, V_T } g_m

vary with T, process

R → ±20% ← C

devices are nominally identical on an IC

Bias Stabilization

g_m indep. of V_T

$$g_m = \mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_T)$$

V_{GS} should track V_T

$V_{GS}-V_T$ = constant $\Longrightarrow$ I_D = constant

$V_{GS} >> V_T \Rightarrow g_m$ large ✗

$V_{DS} >> V_{GS}-V_T$

Negative Feedback

Desired quantity D

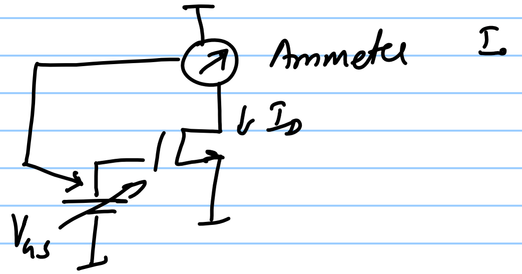
Actual quantity A

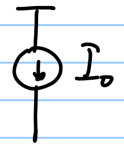
- Sense D & A
- Compare A with D
- Drive A towards D

desired drain current = I_o ✓

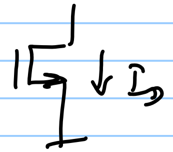
actual " " $= I_D$

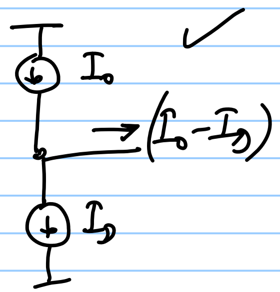
A−D or D−A) — sign & magnitude

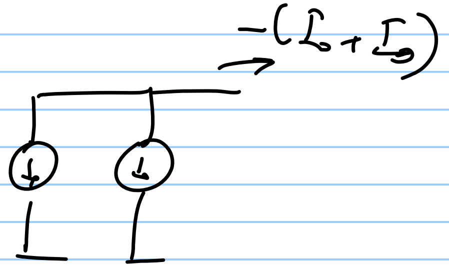


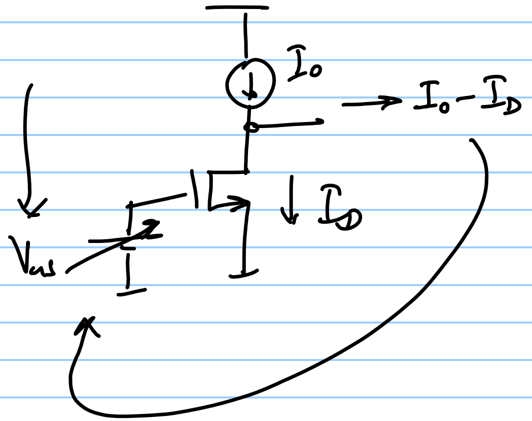


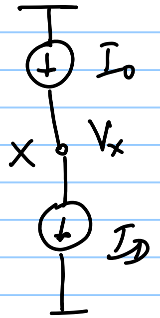
$(I_o - I_D)$





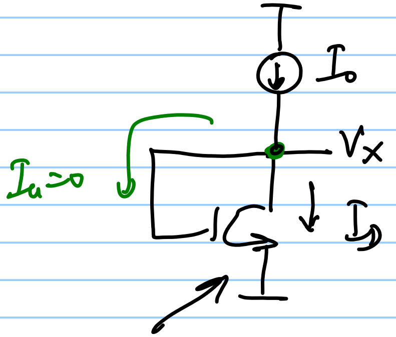




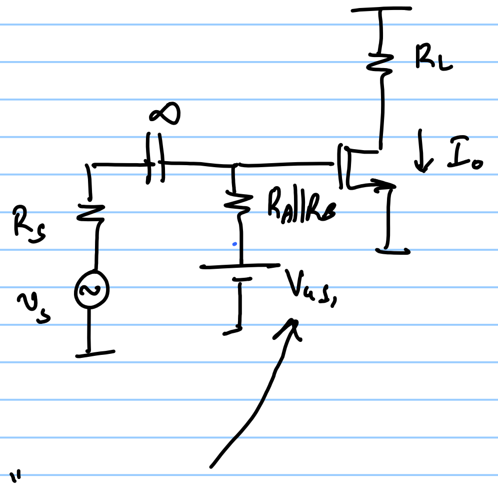


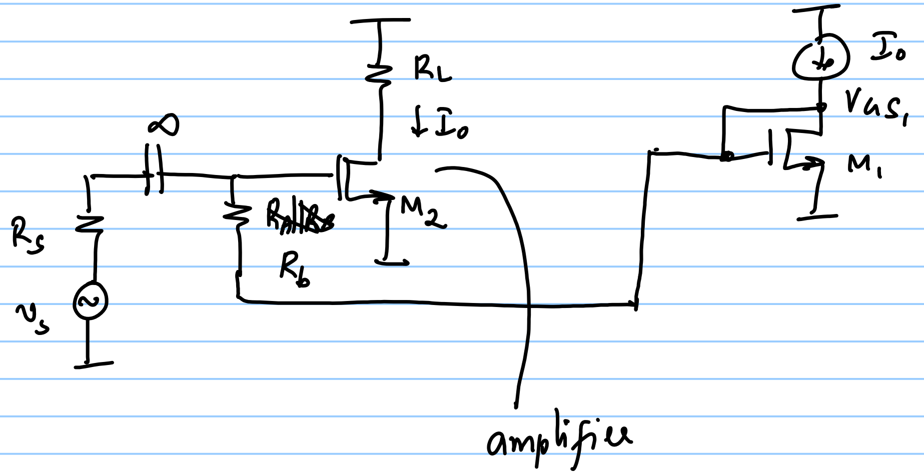
If $I_o > I_D$, V_x ↑ (we want to ↑ V_{GS})

If $I_o < I_D$, V_x ↓ (we want to ↓ V_{GS})

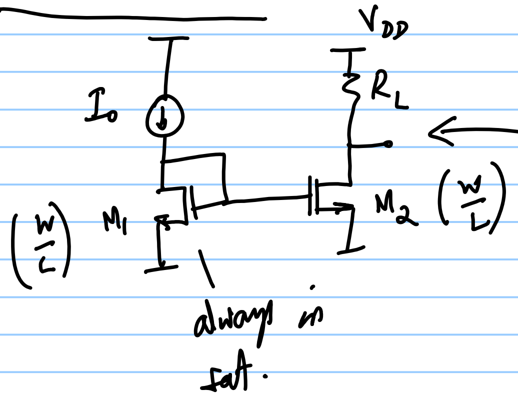


$V_x = V_{GS}$ for a drain current $I_D = I_o$



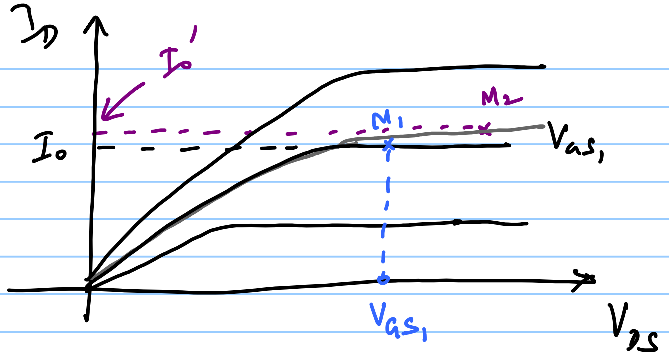


"Current Mirror"



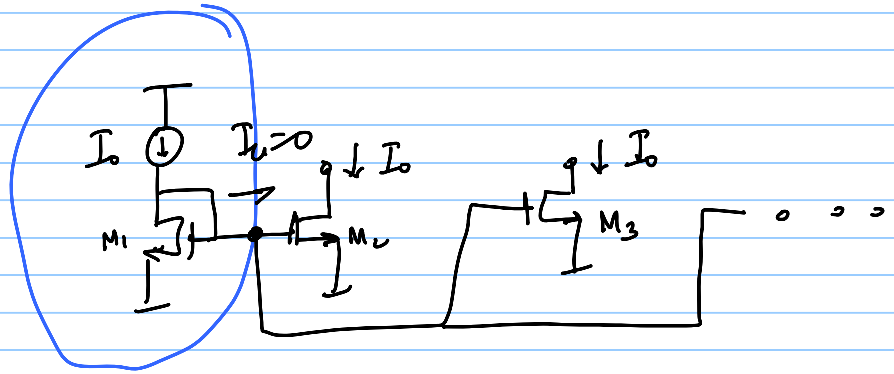
$V_{DS_2} = V_{DD} - I_o R_L$

M_1 & M_2 → both in saturation

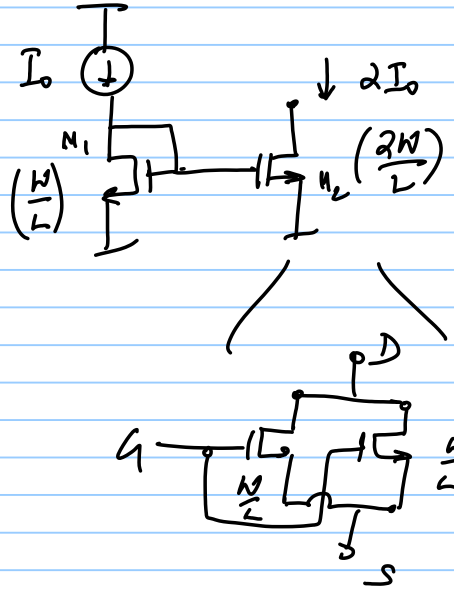


Ensure r_{ds} is large

1)



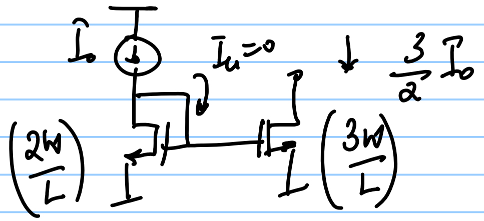
2)

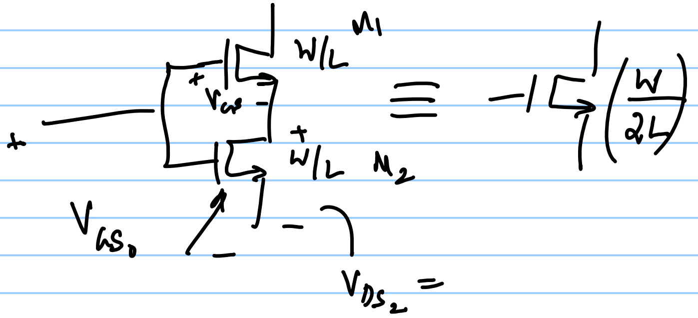


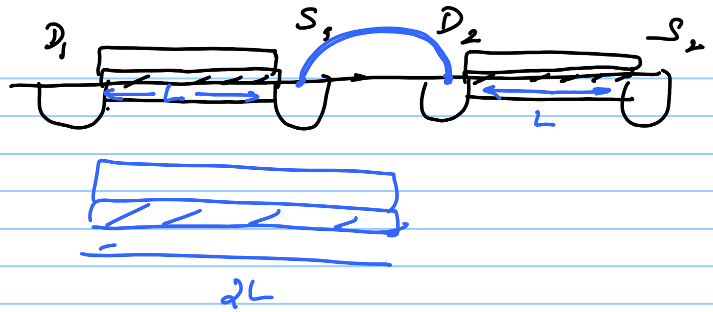
$L_1 = L_2$

✓

3)

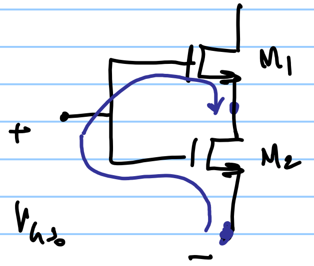






M_2 : $V_{GS_2} = V_{GS_0}$

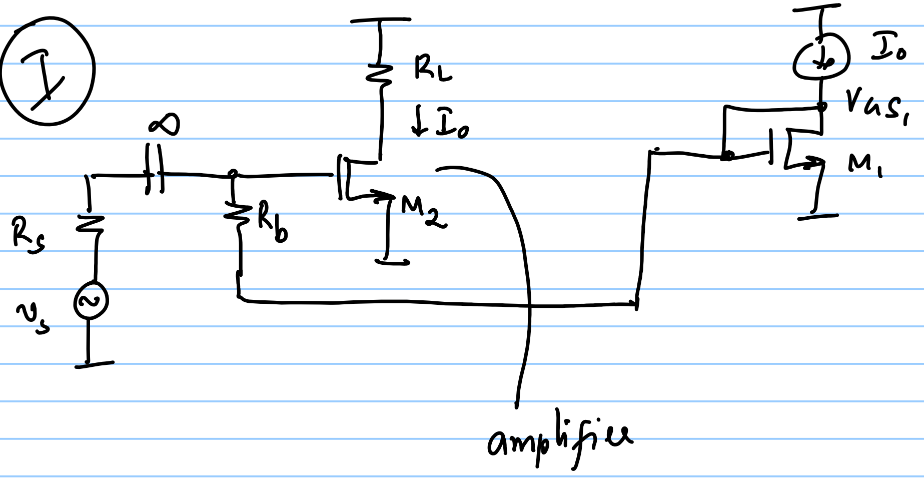
$V_{DS_2} = V_{GS_0} - V_{GS_1}$



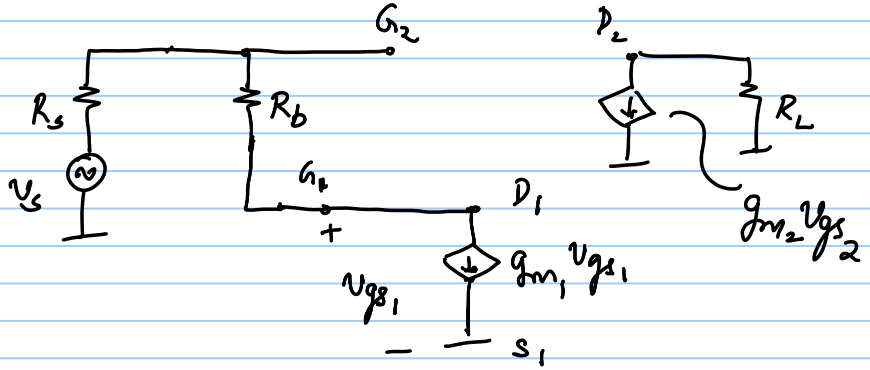
$V_{DS_2} > V_{GS_2} - V_T$

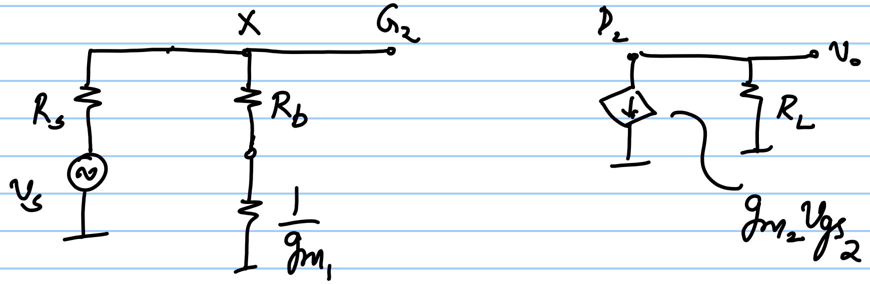
$V_{GS_0} - V_{GS_1} > V_{GS_0} - V_T \Rightarrow V_{GS_1} < V_T$??

Ⅰ



$\xrightarrow[\text{eq. ckt.}]{\text{ss}}$





$$v_x = \frac{R_b + 1/g_{m_1}}{R_s + R_b + 1/g_{m_1}} \cdot v_s$$

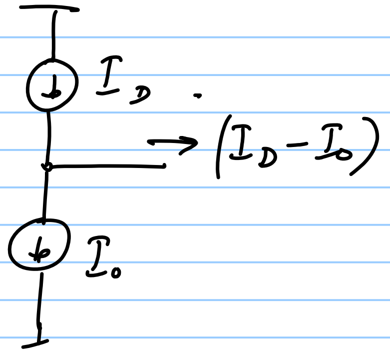
We want $v_x \approx v_s$

$\Rightarrow R_b + 1/g_{m_1} >> R_s$

i.e. $\boxed{R_b >> R_s}$

$$v_o = -g_{m_2} R_L \cdot v_s$$

$A - D \Rightarrow (I_D - I_o)$



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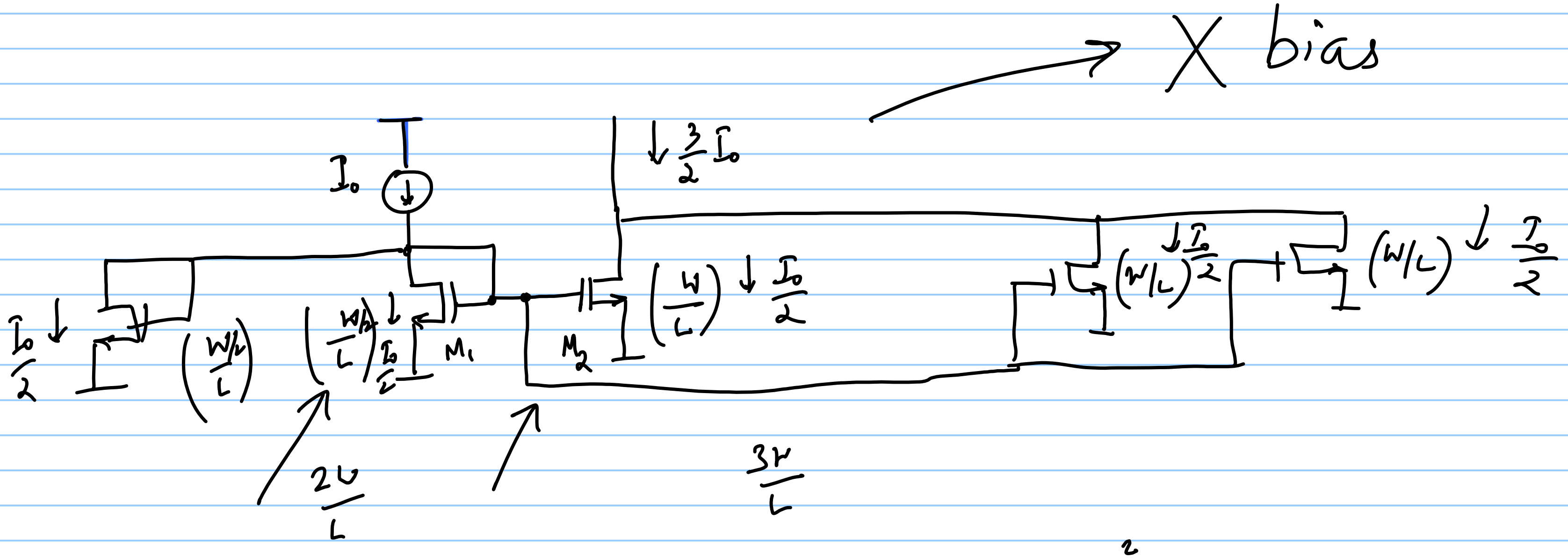
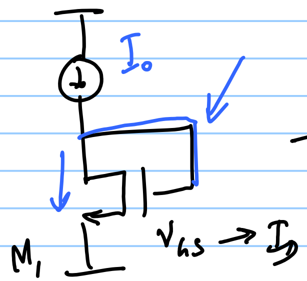
Lec 7



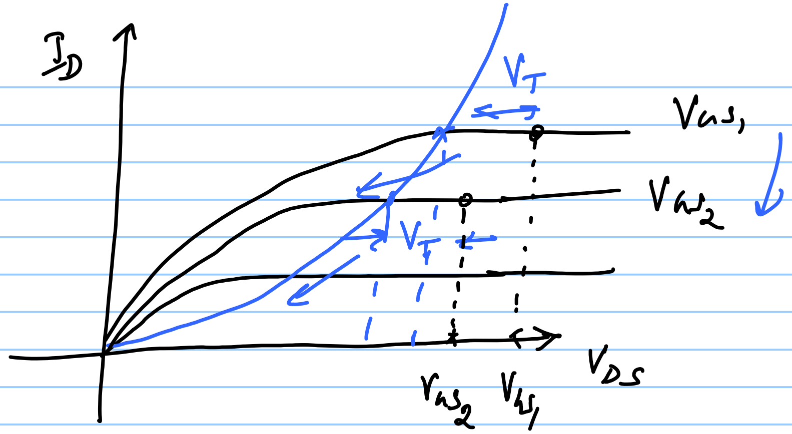
$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\underbrace{(V_{GS}-V_T)}^{2}$$

$\frac{1}{2} \times$ $\qquad$ $\frac{1}{2}\times\left(\frac{W}{L}\right)$



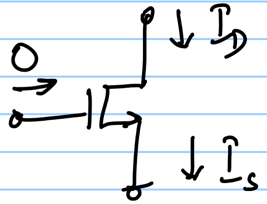
→ always in sat.

$$\boxed{V_{GS} = V_{DS}}$$

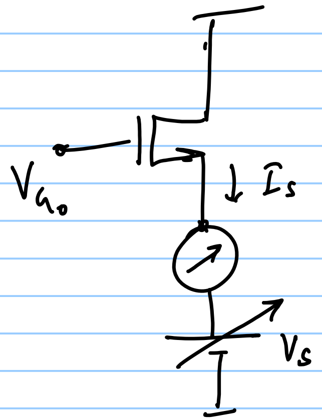


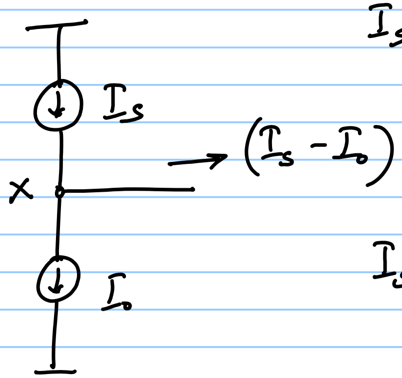
$(I_D - I_0)$

$I_G = 0 \Rightarrow I_D = I_S$



$\longrightarrow (I_S - I_0)$

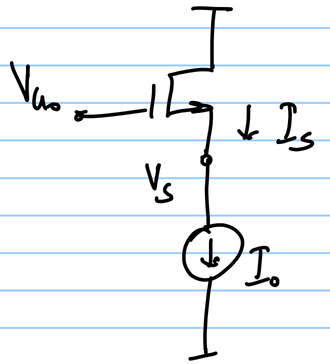




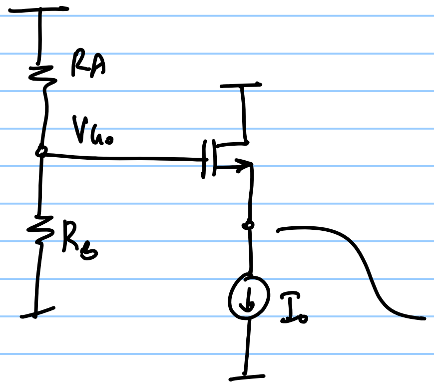
$I_s > I_0 \Rightarrow V_X \uparrow$ $\left\{ \text{we want } \underset{\overset{\parallel}{(V_{G0}-V_s)}}{V_{GS}} \downarrow \Rightarrow V_s \uparrow \right\}$

$I_s < I_0 \Rightarrow V_X \downarrow$ $\left\{ \text{we want } V_{GS} \uparrow \Rightarrow V_s \downarrow, \right\}$

$\Rightarrow V_X = V_s$



$$\boxed{I_s = I_0}$$

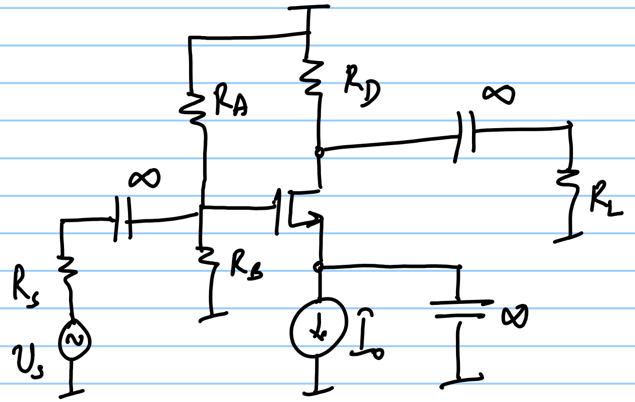


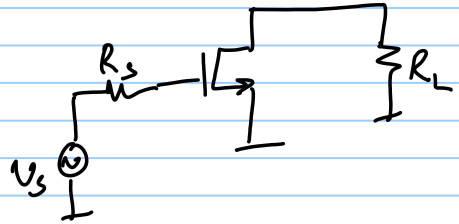
$$I_o = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\left(V_{GS_o}-V_T\right)^2$$

$$V_S = V_{G_o} - V_{GS_o}$$

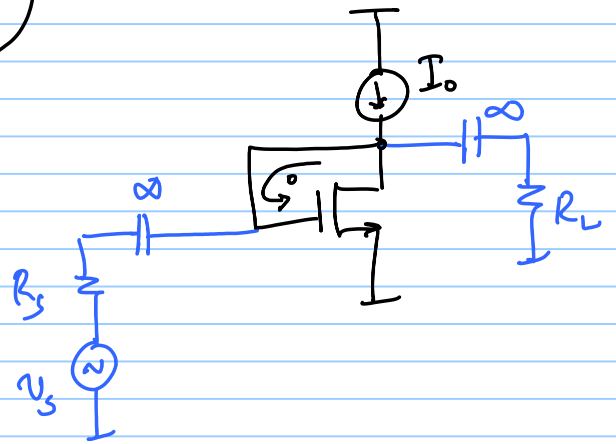
$$= \frac{V_{DD}\cdot R_B}{R_A+R_B} - \left(V_T + \sqrt{\frac{2I_o}{\mu_n C_{ox}\left(\frac{W}{L}\right)}}\right)$$

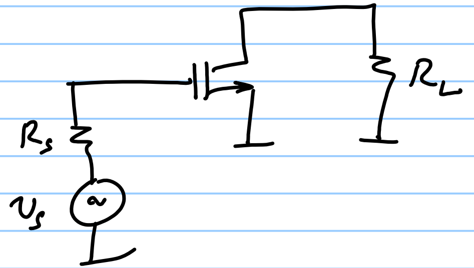
II



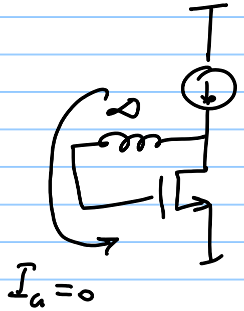


1.5



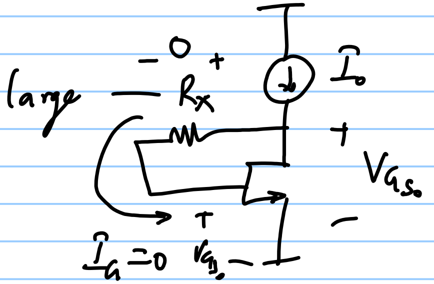


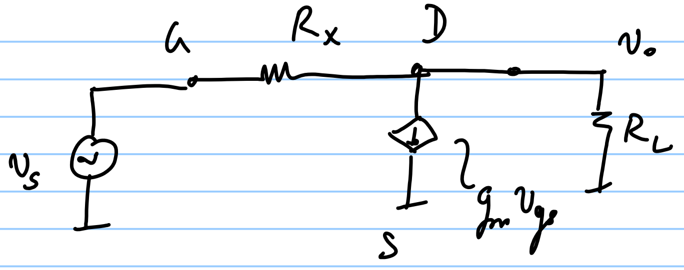
$\rightsquigarrow$ gain $= \frac{-g_m}{G_L}$



✓

L – large ✗





KCL @ D

$$(v_s - v_o)\cdot G_X = g_m v_s + v_o \cdot G_L$$

$$v_s\,(G_X - g_m) = v_o\,(G_X + G_L)$$

$$\frac{v_o}{v_s} = \frac{G_X - g_m}{G_X + G_L} \approx \left(-\frac{g_m}{G_L}\right)\underbrace{\left(\frac{1-\frac{G_X}{g_m}}{1+\frac{G_X}{G_L}}\right)}_{\approx 1}$$

$\Rightarrow$ $G_X \ll g_m$ & $G_X \ll G_L$

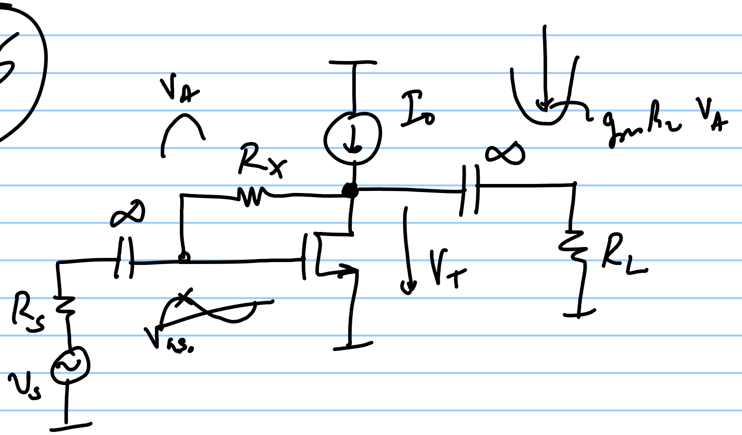
$R_X \gg 1/g_m$ & $R_X \gg R_L$

$|\text{gain}| = g_m R_L \gg 1$

$\Rightarrow R_L \gg 1/g_m$

$$\boxed{R_X \gg R_L}$$

I.5



Swing limits?

(V_{GS_0}, V_{GS_0})

Cut off limit $= \frac{I_o}{g_m}$

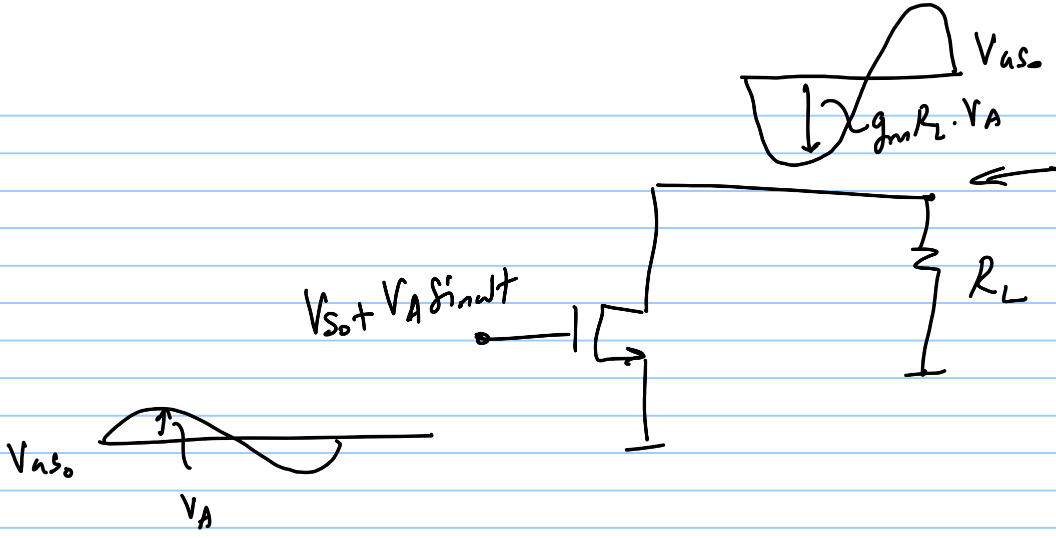
Triode limit: $\frac{V_T}{(1+g_m R_L)}$

I

Swing limits will be the same as original amplifier (V_{GS_0}, V_{DS_0}) has not changed.

Cut off limit $= \frac{I_o}{g_m}$

Triode limit $= \frac{V_{DS_0} - (V_{GS_0} - V_T)}{1 + g_m R_L}$ }

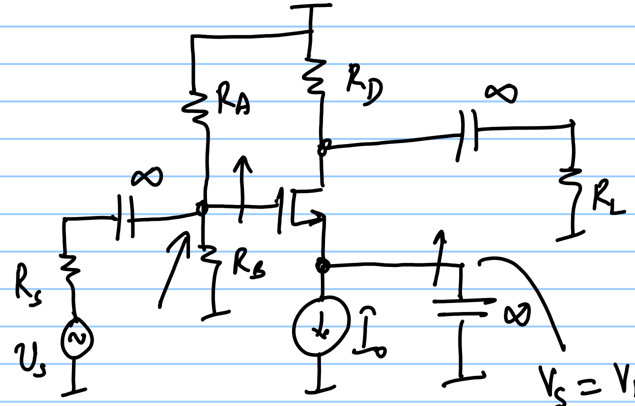


$V_{GS_0} - g_m R_L V_A \sin\omega t$

$$- \cancel{V_{GS_0}} - g_m R_L V_A = \cancel{V_{GS_0}} + V_A - V_T$$

$$\boxed{V_A = \frac{V_T}{(1+g_m R_L)}}$$

Ⅱ



Swing limits

Cut off limit $= \dfrac{I_o}{g_m}$

Triode limit : $V_D = V_G - V_T$

$$V_{S_0} = \frac{V_{DD} \cdot R_B}{R_A + R_B} - \left(V_T + \sqrt{\frac{2I_D}{\mu_n C_{ox}\frac{W}{L}}}\right)$$

$$\left\{ \begin{aligned} V_D - V_S &= V_G - V_S - V_T \\ V_{DS} &= V_{GS} - V_T \end{aligned} \right\}$$

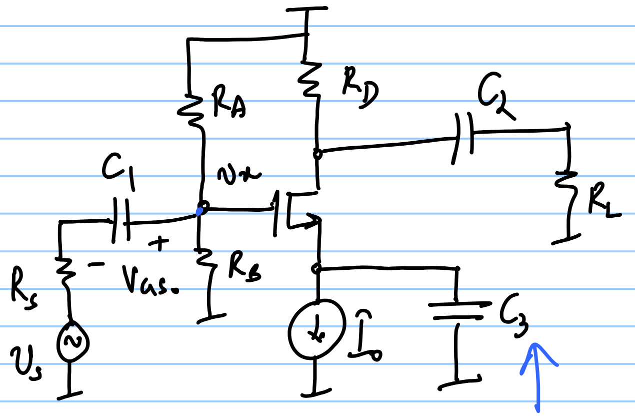
$$\underbrace{\frac{V_{DD}\cdot R_B}{R_A+R_B}}_{V_{G0}} + \underbrace{V_A \cancelto{1}{\sin\omega t}}_{v_s} - V_T = \left(V_{DD} - I_0 R_D\right) - g_m\left(R_D \| R_L\right)\cdot V_A \cancelto{1}{\sin\omega t}$$

$$V_A\left(1 + g_m\left(R_D\|R_L\right)\right) = \frac{V_{DD}\cdot R_A}{R_A+R_B} - I_0 R_D + V_T$$

$$\boxed{V_A = \frac{V_T + \left(\frac{V_{DD}\cdot R_A}{R_A+R_B} - I_0 R_D\right)}{1 + g_m\left(R_D\|R_L\right)}}$$

lower than case I

Case I : $$V_A = \frac{\left(V_{DD} - I_0 R_D\right) - \left(\sqrt{\frac{2I_0}{\mu_n C_{ox}\left(\frac{W}{L}\right)}}\right)}{1 + g_m\left(R_D\|R_L\right)}$$

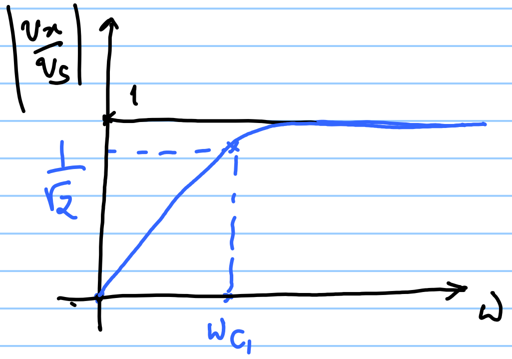


C_1: How large?

freq. ω

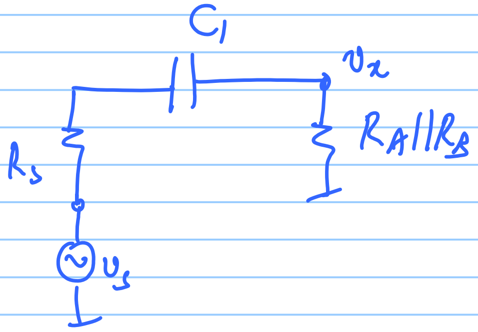
$$Z_{C_1} = \frac{1}{j\omega C_1}$$

$$\boxed{\omega_{C_1} = \frac{1}{C_1\left(R_S + R_A\|R_B\right)}}$$



$\omega_{C_1} \ll$ lowest frequency content of v_s

$$\boxed{C_{1\,min}}$$



$$\frac{v_x}{v_s} = \frac{R_A\|R_B}{R_S + R_A\|R_B + \frac{1}{j\omega C_1}}$$

5/9/17

Lec 8

- * Quiz 3 – Nov. 19th 10:30am – noon
- * Tutorial 2 – Due on 15th Sep. 2017
- * Tutorial 1 discussion session with TAs – Friday 8th Sep. 4-5pm.
- * Tutorial 2 " " " " – Friday 15th Sep. 5-6pm

Feedback Bias Stabilisation

Sensed : I_D , I_S

Controlled : V_G , V_S

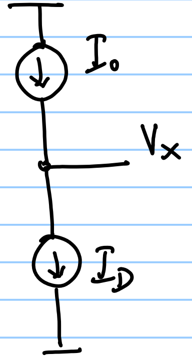
} 4 ways of bias stab.

Sense I_D → Drive V_u → Case I

Sense I_S → Drive V_S → Case II

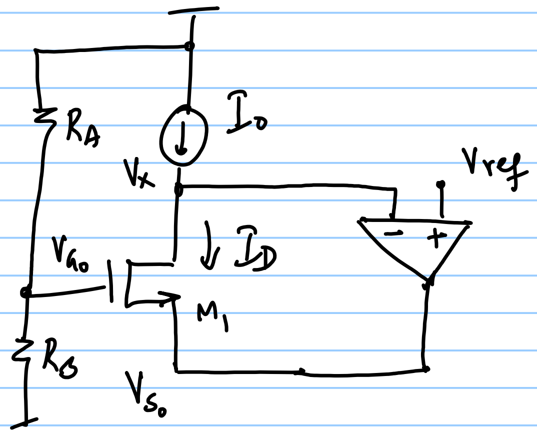
Case III

Sense I_D → Drive V_s



If $I_D > I_0$; $V_x \downarrow$ (we want $V_S \uparrow$)

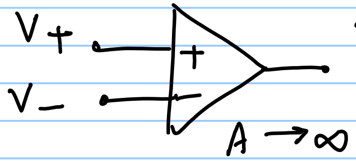
If $I_D < I_0$; $V_x \uparrow$ (we want $V_S \downarrow$)



V_{ref} chosen such that M_1 is well into the saturation region

$$V_{G_0} = \frac{V_{DD} \cdot R_B}{R_A + R_B} \quad ; \quad I_D = I_0$$

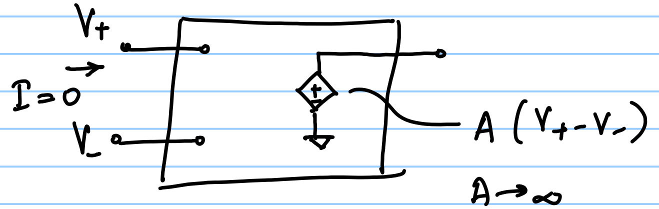
$$V_{GS_0} = V_T + \sqrt{\frac{2I_0}{\mu_n C_{ox}\left(\frac{W}{L}\right)}}$$



$V_o = A(V_+ - V_-)$

$Z_{in} = \infty$

$Z_{out} = 0$



"Virtual Short" between + & − terminals

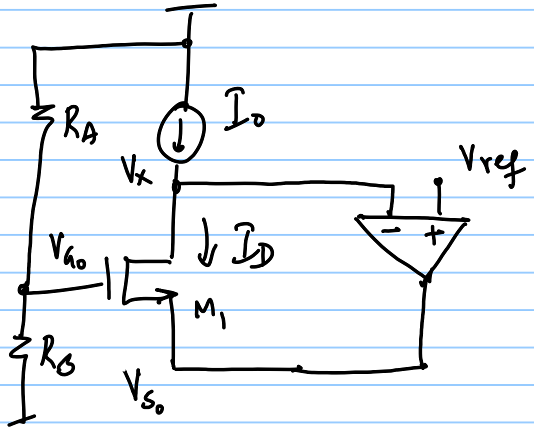
$I_{in} = 0$
but $V_+ = V_-$) Virtual Short

only when opamp is placed in negative feedback

$$\boxed{V_{Dsat} = \sqrt{\frac{2 I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} = V_{GS} - V_T = V_{ov}}$$

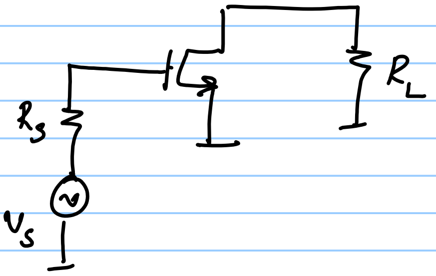
$$V_{S_0} = V_{G_0} - V_{GS_0} = V_{G_0} - V_T - V_{Dsat}$$

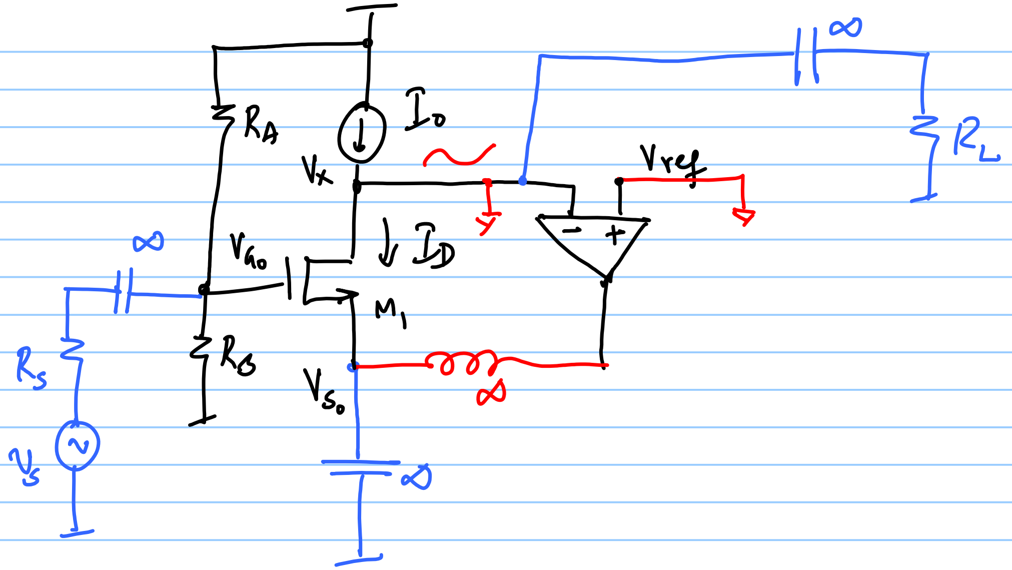
$$V_{D_0} = V_X = V_{ref}$$



→

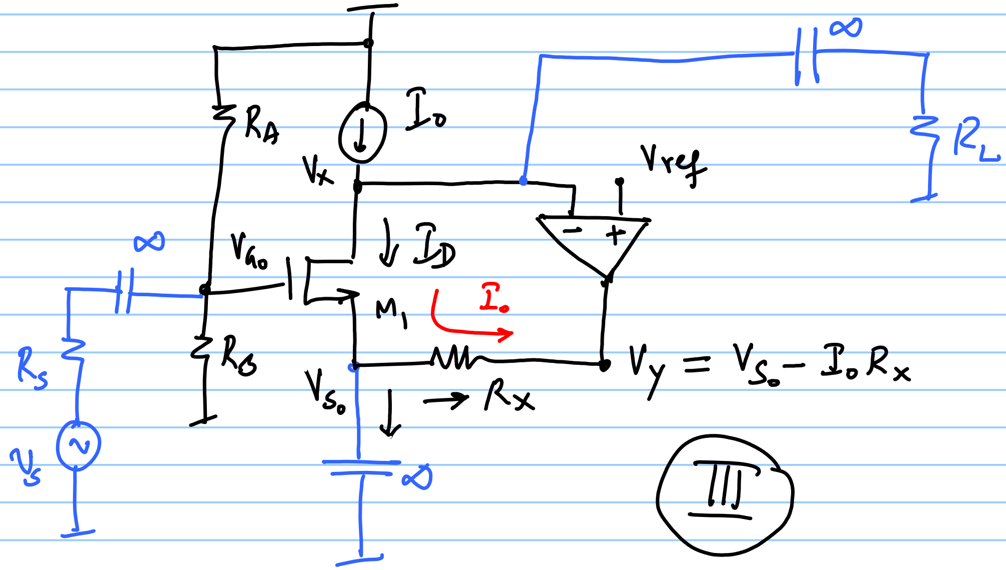
Small Signal picture





1) $L = \infty$ between Opamp output & V_{S_o}

2) Use large R_x



$V_y = V_{S_o} - I_o R_x$

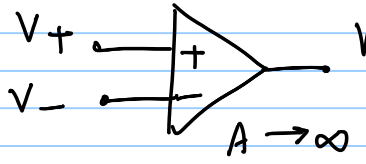
III

Case IV - HW

Swing limits for Cases III & IV - HW

Case IV - Sense I_S, Drive V_G

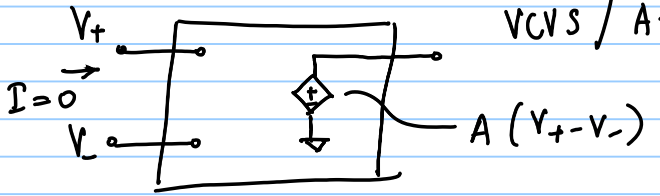
Negative feedback for AC



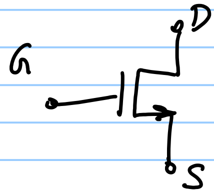
$V_o = A(V_+ - V_-)$

$Z_{in} = \infty$

$Z_{out} = 0$



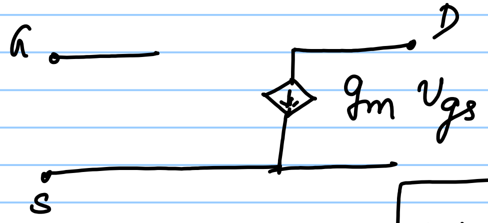
Ideal VCVS with $A \to \infty$



Ideal VCCS

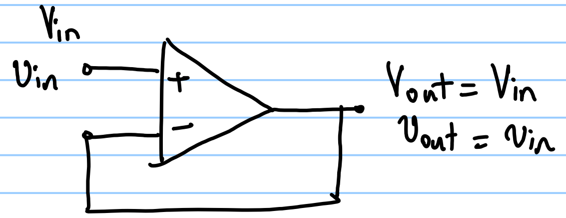
$Z_{in} = \infty$

$Z_{out} = \infty$



$g_m \to \infty$ $\quad g_m = \sqrt{2 \mu_n C_{ox} \left(\frac{W}{L}\right) \cdot I_D}$

VCVS of gain = 1



$Z_{in} = \infty$

$Z_{out} = 0$

$V_{out} = V_{in}$

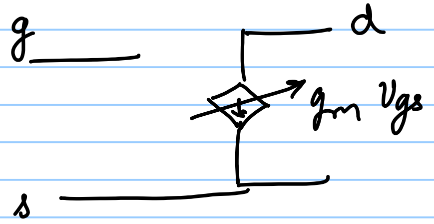
$v_{out} = v_{in}$

MOSFET-based VCVS of gain = 1

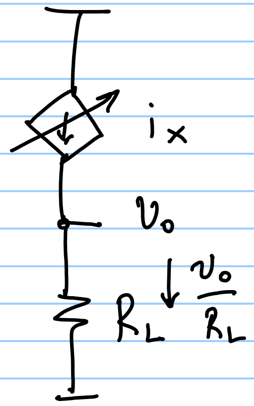
Negative Feedback

- * Sense D & A values
- * Compare A with D
- * Drive A → D

$D = v_{in}$; $A = v_{out}$



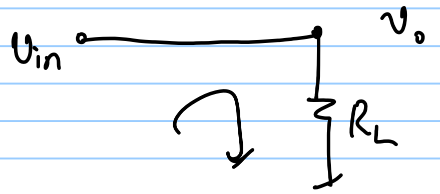
→ i_d can be controlled through v_{gs}



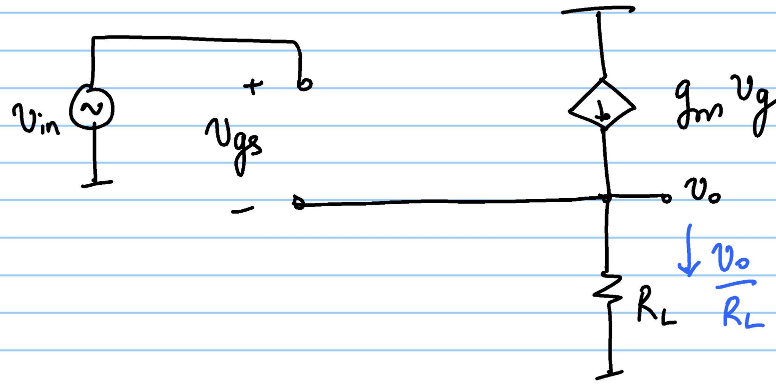
$v_o \nearrow = i_x \nearrow \cdot R_L$

$D = \frac{v_{in}}{R_L}$

$A = \frac{v_o}{R_L}$



X $Z_{in} = R_L$ not ∞



$g_m v_{gs} = g_m(v_{in} - v_o)$

If $\frac{v_o}{R_L} > g_m v_{gs} \Rightarrow v_o \downarrow \Rightarrow g_m v_{gs} \uparrow$

If $\frac{v_o}{R_L} < g_m v_{gs} \Rightarrow v_o \uparrow \Rightarrow g_m v_{gs} \downarrow$

$$g_m(v_{in} - v_o) = \frac{v_o}{R_L}$$

$$\boxed{v_{in} = v_o + \frac{v_o}{g_m R_L}}$$

for $v_o = v_{in} \Rightarrow g_m \to \infty$

$$v_{in} = v_o\left(\frac{1 + g_m R_L}{g_m R_L}\right)$$

We actually want $g_m R_L >> 1$ i.e. $\boxed{g_m >> \frac{1}{R_L}}$

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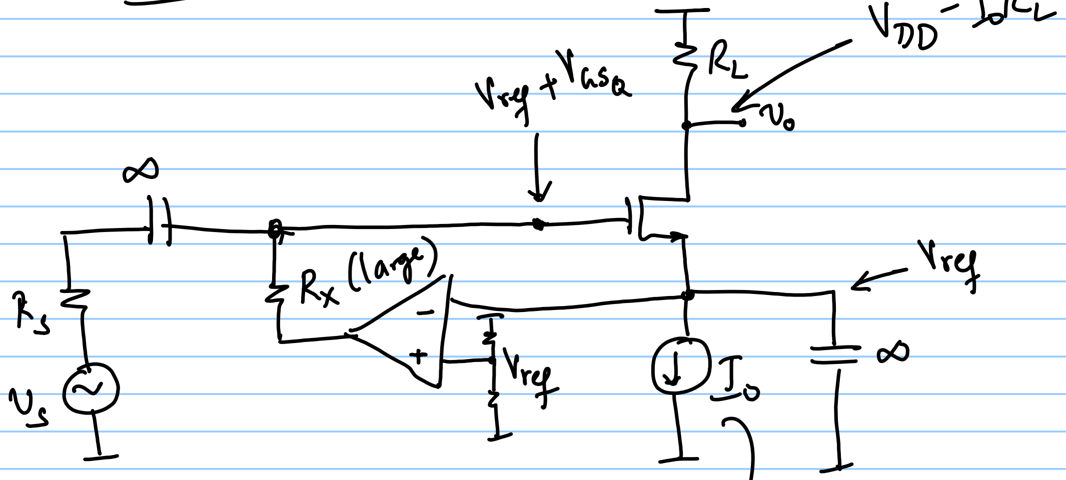
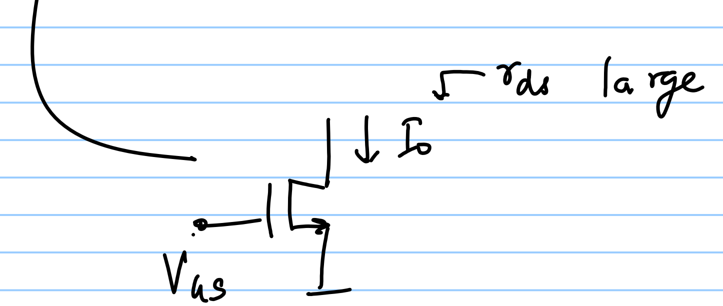
Lec 9

Case IV

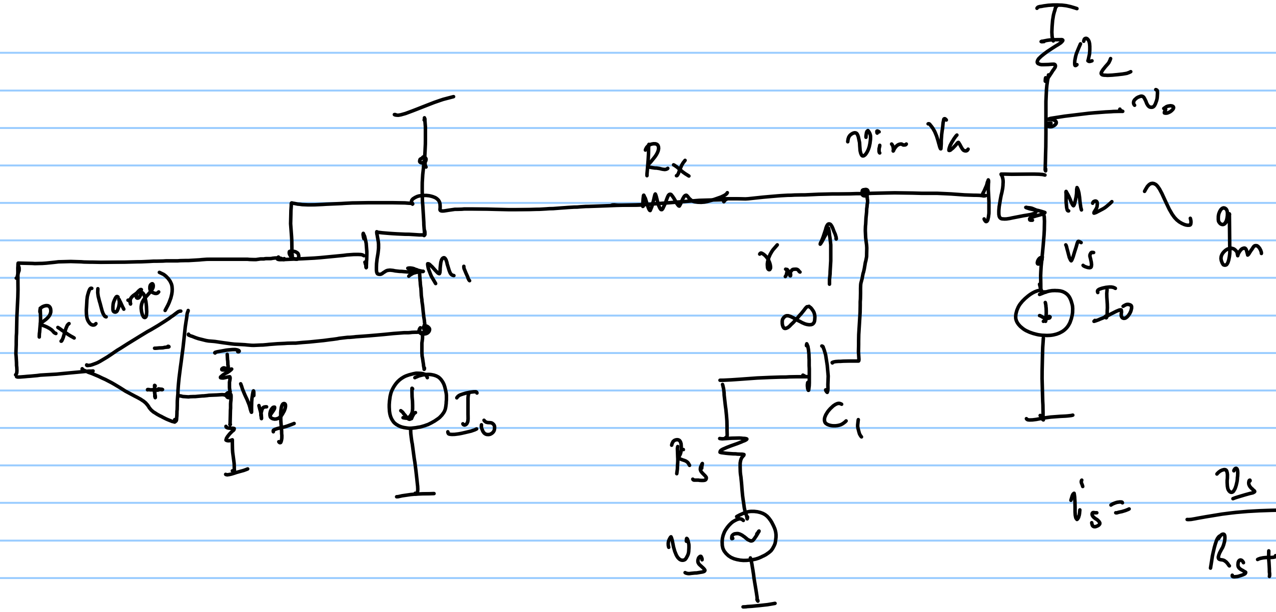


$$V_{GSQ} = V_T + \underbrace{\sqrt{\frac{2I_0}{\mu_n C_{ox}\left(\frac{W}{L}\right)}}}_{V_{Dsat,Q}}$$

$$\text{gain} = -g_m R_L$$



Replica Biasing



$M_1 \equiv M_2$

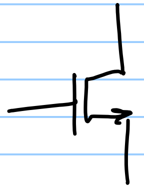
$$i_s = \frac{v_s}{R_s + \frac{1}{j\omega C_1} + r_{in}}$$ ← SS input res.

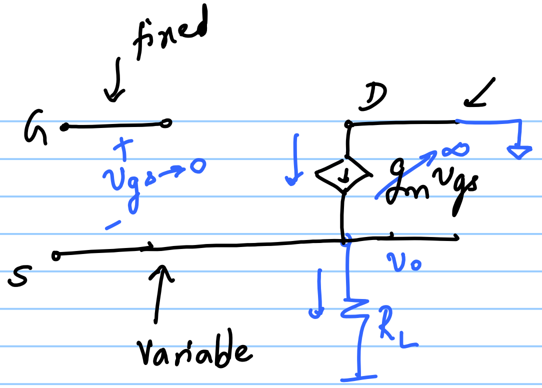
$i_s \times R_{in} = v_{in}$

$v_{in} \approx 0.99 v_s$

$$V_D = V_a - V_T$$

$$V_{D_0} + v_d = (V_{DD} - I_0 R_L) + (-g_m R_L)(V_A \sin\omega t)$$





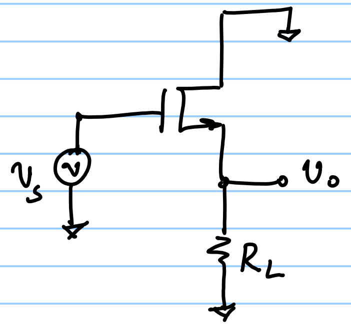
If $g_m \to \infty \Rightarrow v_{gs} \to 0$
with finite i_d
when in -ve f.b.

VCVCs of gain = 1

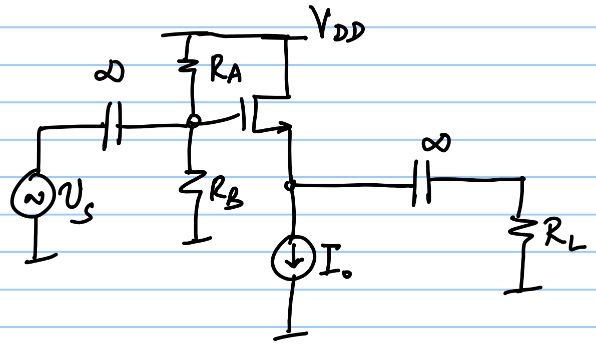
$v_o = v_{in}$

(A) (D)

S G



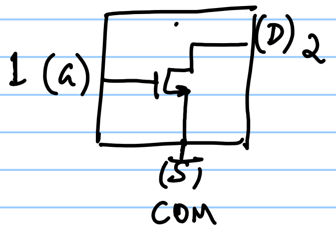
signal picture only!



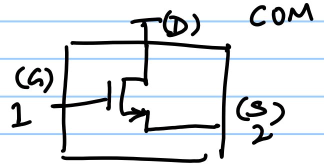
DC + AC

"Common Drain Amplifier" or "Source follower"

CSA



CDA



$$V_{GQ} = \frac{V_{DD} \cdot R_B}{R_A + R_B} \quad ; \quad V_{DQ} = V_{DD} \quad ;$$

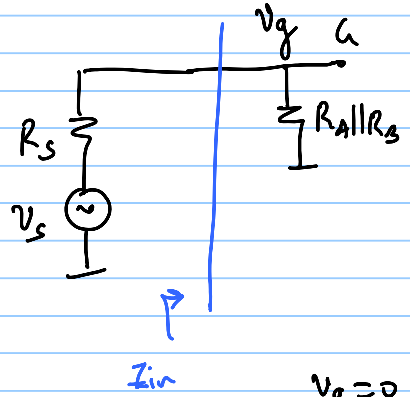
$$V_{S,Q} = V_{GQ} - V_{GSQ} = V_{GQ} - V_T - V_{DSAT}\Big|_{I_o} \quad ; \quad I_D = I_o$$

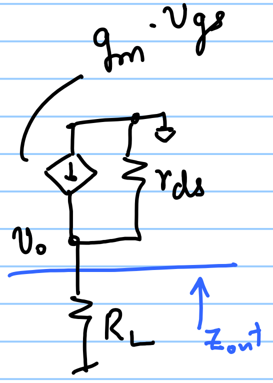
$$\boxed{V_G = V_S + V_{GS}}$$

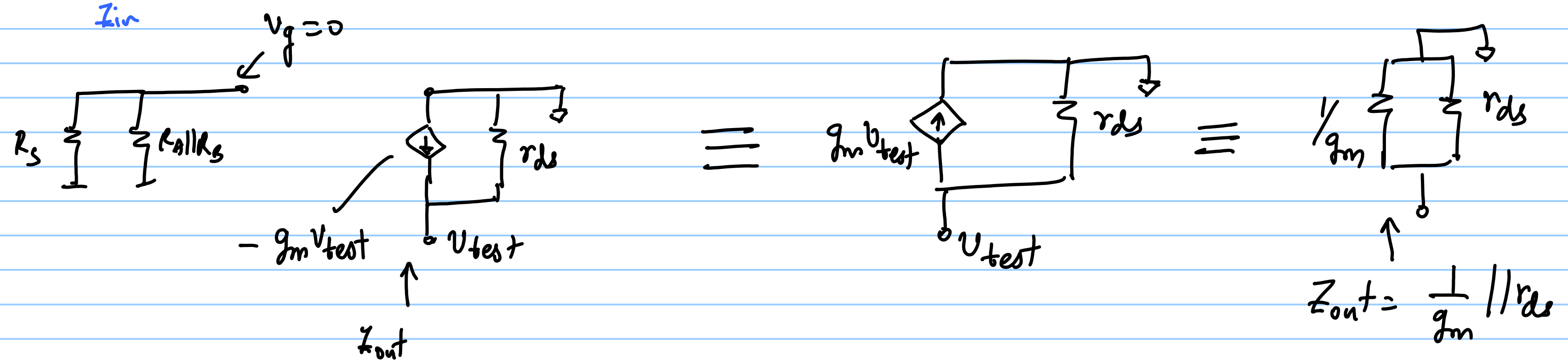
$Z_{in} = R_A || R_B$ should be much larger than R_s

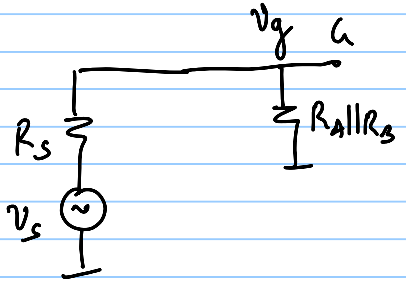
$Z_{out} = \frac{1}{g_m} || r_{ds} \to 0$ if $g_m \to \infty$

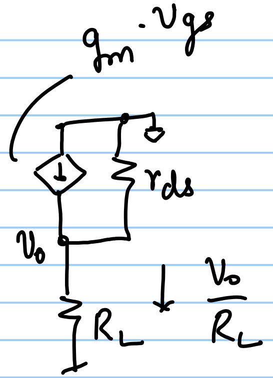
$\frac{v_o}{v_s} =$











$v_g \simeq v_s$

KCL @ source:

$$g_m \cdot v_{gs} = (G_L + g_{ds}) \cdot v_o$$

$$g_m (v_s - v_o) = (G_L + g_{ds}) \cdot v_o$$

$$g_m v_s = (g_m + G_L + g_{ds}) \cdot v_o$$

$$\Rightarrow \boxed{\frac{v_o}{v_s} = \frac{g_m}{g_m + G_L + g_{ds}}}$$

$\frac{v_o}{v_s} \to 1$ if $g_m \to \infty$

$\frac{v_o}{v_s}$ slightly less than 1 in practice

$$\frac{v_o}{v_s} = \frac{g_m}{g_m + G_L} = \frac{g_m R_L}{1 + g_m R_L} \to 1 \quad \text{if} \quad g_m R_L \gg 1$$

Swing limits

$v_s = V_A \sin\omega t$

Cut off

$$I_D = \underset{\substack{\downarrow \\ I_o}}{I_q} + i_d$$

$$i_d \rightarrow \frac{v_o}{R_L} = \frac{1}{R_L} \cdot \frac{g_m R_L}{1 + g_m R_L} \cdot v_s$$

$$I_D = 0 \Rightarrow I_o = \frac{1}{R_L} \cdot \frac{g_m R_L}{1 + g_m R_L} \cdot V_{A_1}$$

$$\Rightarrow \boxed{V_{A_1} = I_o R_L \left(1 + \frac{1}{g_m R_L}\right)}$$

Triode

$$V_D = V_a - V_T$$

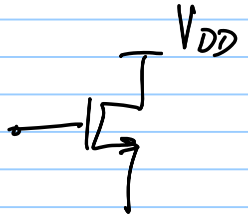
$$V_{DD} = \frac{V_{DD} \cdot R_B}{R_A + R_B} + V_{A_2} \cancelto{1}{\sin\omega t} - V_T$$

$$\boxed{V_{A_2} = \frac{V_{DD}\cdot R_A}{R_A+R_B} + V_T}$$

$$V_G = V_{G,Q} + v_g$$

total, instantaneous

$$V_{A,max} = \min.\left\{V_{A_1}, V_{A_2}\right\}$$



$V_{GS,Q} \longrightarrow$

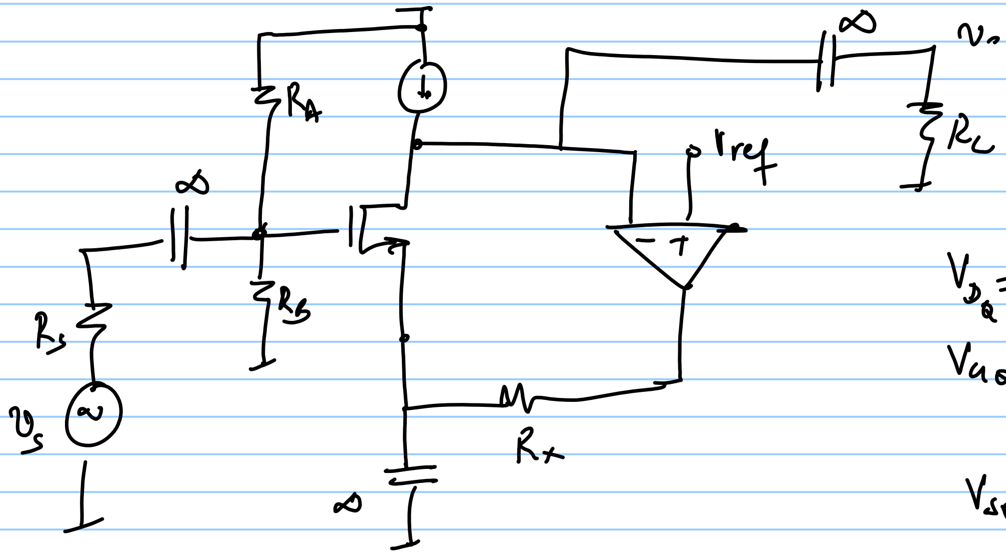
$$V_{G_{max}} = V_{DD} + V_T$$

* VCCS ?

$$V_{GS,Q} + V_{A_2}\sin\omega t$$

R_A, R_B

$R_B R_A$



$V_{DQ} = V_{ref}$

$V_{aQ} = \frac{R_B}{R_A + R_B} \cdot V_{DD}$

$V_{sQ} = V_{aQ} - V_{asQ}$

Triode :

$V_D = V_a - V_T$

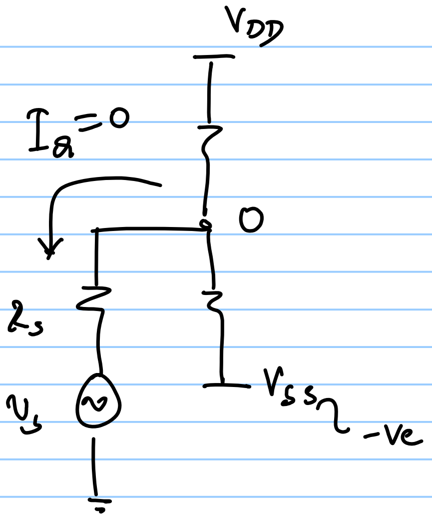
$V_{ref} - g_m R_L V_{A_1} = V_{aQ} + V_{A_1} - V_T$

$$V_{A_1} = \frac{V_{ref} - V_{a,Q} + V_T}{1 + g_m R_L}$$

Cut off

$I_D = I_Q + i_d = 0$

$$V_{A_2} = \frac{I_Q}{g_m}$$


V_DD
I_G = 0
0
R_s
v_s
V_SS
-ve

12/9/17

Lec 10

large I_D

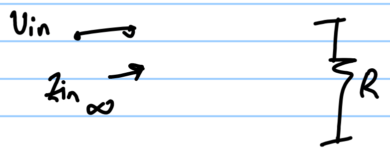
$(V_{GS} - V_T)$ small ← large $\left(\frac{W}{L}\right)$

$$g_m = \sqrt{2 \mu_n C_{ox} \underline{\left(\frac{W}{L}\right)} \underline{I_D}}$$

VCCS of gain $\frac{1}{R}$ $\boxed{i_{out} = \frac{V_{in}}{R}}$

$Z_{in} = \infty$

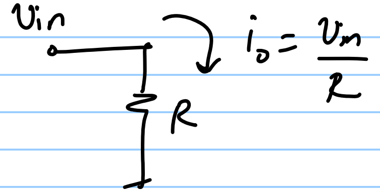
$Z_{out} = \infty$



$\downarrow i_{out} = \frac{V_{in}}{R} \leftarrow R$

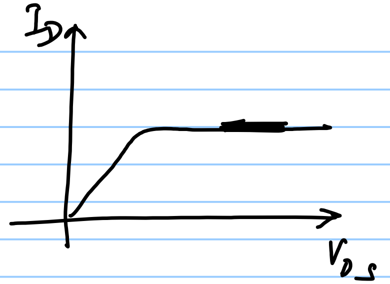
$r_{ds} = \frac{1}{\lambda I_D}$

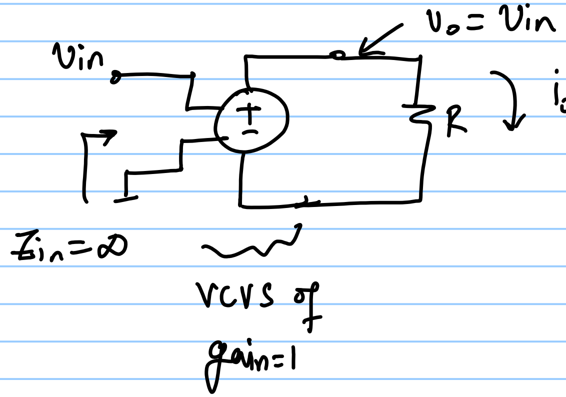
$r_{ds} \neq \frac{V_{DSQ}}{I_{DQ}}$



$i_o = \frac{v_{in}}{R}$

$+ \quad Z_{in} = \infty$



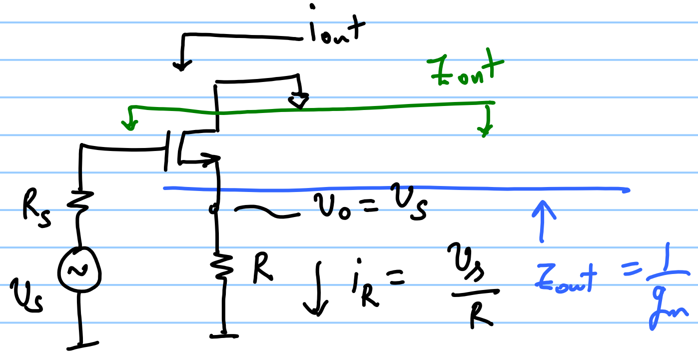


$v_o = v_{in}$

$i_o = \frac{v_{in}}{R}$

$+ \quad \boxed{Z_{out} = \infty}$

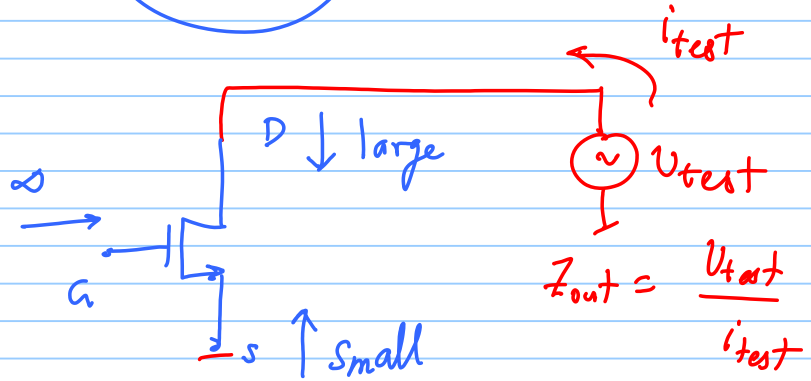
$\frac{v_d}{i_d}$ large



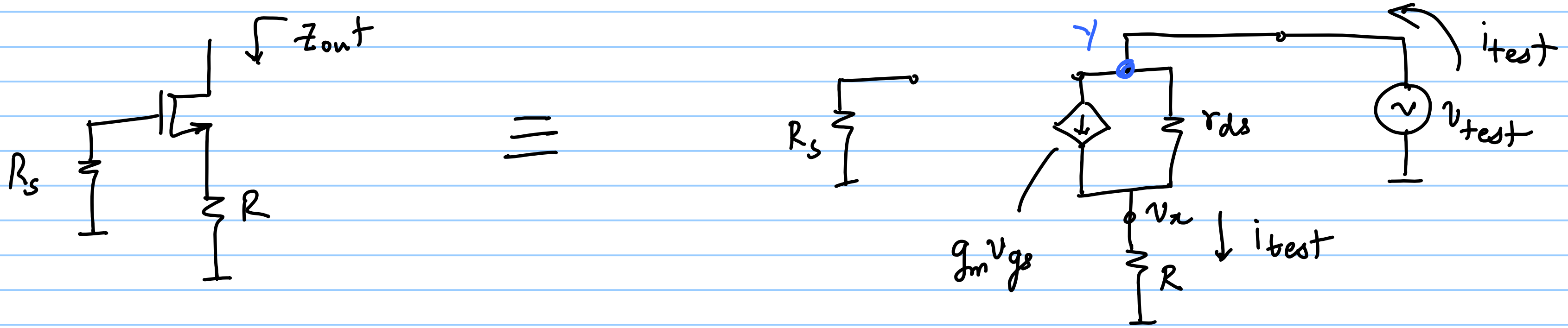
$v_o = v_s$

$i_R = \frac{v_s}{R}$

$Z_{out} = \frac{1}{g_m}$



$Z_{out} = \frac{v_{test}}{i_{test}}$



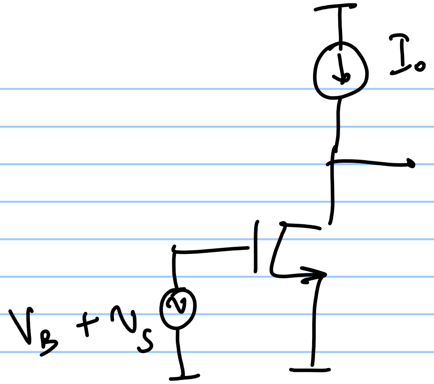
$v_x = R \cdot i_{test}$; $v_{gs} = -v_x$

$g_m v_{gs} + \frac{(v_{test} - v_x)}{r_{ds}} = i_{test}$ KCL @ node Y

$-g_m R i_{test} + g_{ds} v_{test} - g_{ds} R \, i_{test} = i_{test}$

$g_{ds} \cdot v_{test} = i_{test} \left(1 + g_m R + g_{ds} R \right)$

$$\frac{v_{test}}{i_{test}} = \underline{\underline{r_{ds} + g_m R \cdot r_{ds} + R}} = Z_{out}$$

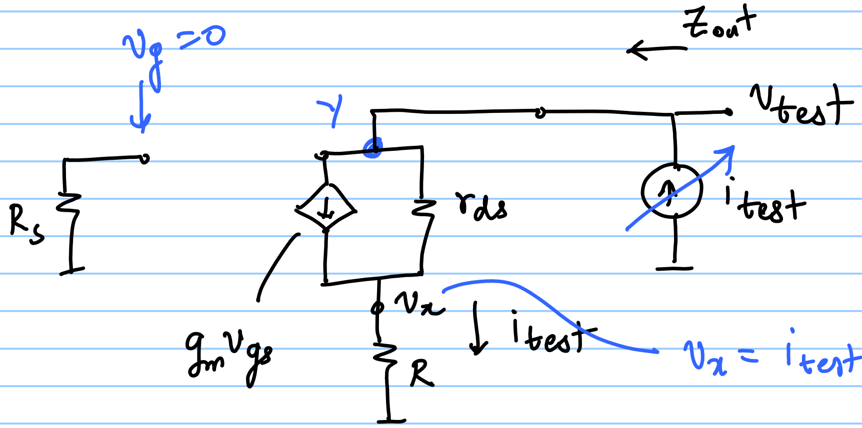


$$v_o = v_s \cdot (\underbrace{-g_m r_{ds}}_{\text{"intrinsic gain"}})$$

$Z_{out} = r_{ds}$

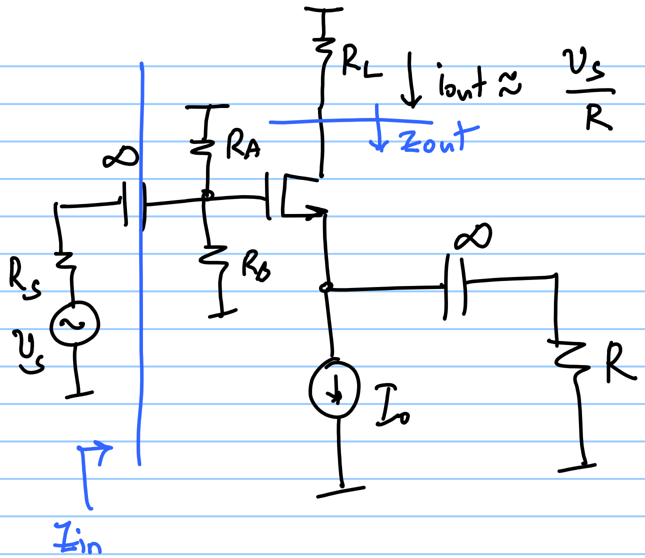
w/ -ve f.b: $Z_{out} = (g_m R)\, r_{ds}$

$y_{22} = \lambda I_D$; $r_{ds} = \frac{1}{y_{22}} = \frac{1}{\lambda I_D}$

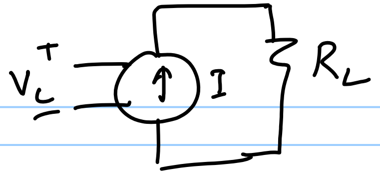


$v_x = i_{test} \cdot R \Rightarrow v_{gs} \downarrow \Rightarrow i_d \downarrow$ (negative f.b.)

$Z_{out} = (g_m R)\, r_{ds}$ (very large) ; $Z_{in} = R_A || R_B$ (large)



$$\downarrow i_{out} \approx \frac{v_s}{R}$$

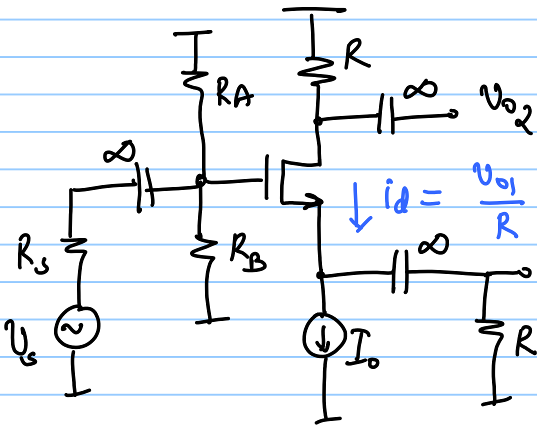


$$i_{out} = \frac{g_m}{1+g_mR} \cdot v_s$$

$$\approx \frac{1}{R} \cdot v_s \text{ if } g_mR \gg 1$$

" Transadmittance Amplifier "

$$\boxed{\text{Swing limits — H.W.}}$$



$$v_{o_2} = -\frac{g_mR}{1+g_mR} \cdot v_s$$

$$\downarrow i_d = \frac{v_{o1}}{R} = \frac{g_m}{1+g_mR} \cdot v_s$$

$$v_{o_1} = \frac{g_mR}{1+g_mR} \cdot v_s$$

" phase splitter "

$$v_{o_1} = -v_{o_2}$$

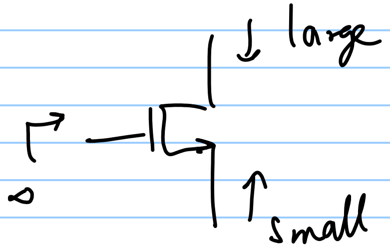
$$|v_{o_1}| = |v_{o_2}| \approx v_s$$

CCCS of gain 1

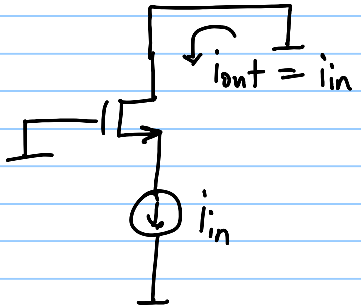
$i_{out} = i_{in}$

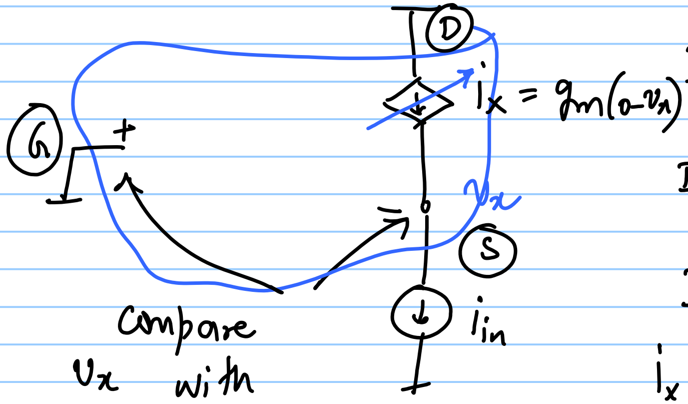
$Z_{in} = 0$

$Z_{out} = \infty$



If $g_m \to \infty \Rightarrow v_x \to 0$





compare v_x with signal ground

small signal quantities

If $i_x > i_n \Rightarrow v_x \uparrow$

If $i_x < i_{in} \Rightarrow v_x \downarrow$

If $i_x = i_{in} \Rightarrow v_x = 0$

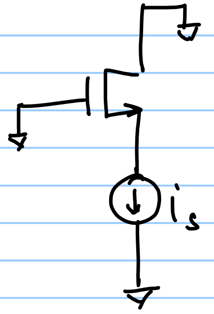
$i_x = i_{out}$

" Common gate amplifier"

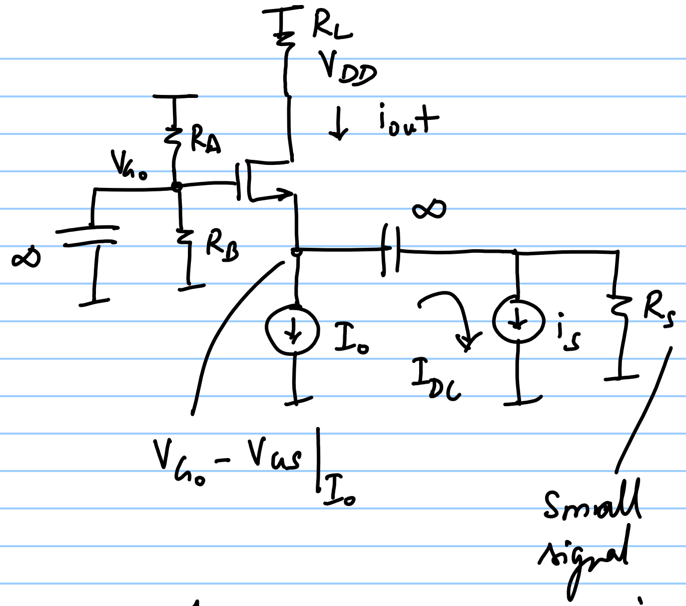
$i_{out} = i_{in}$

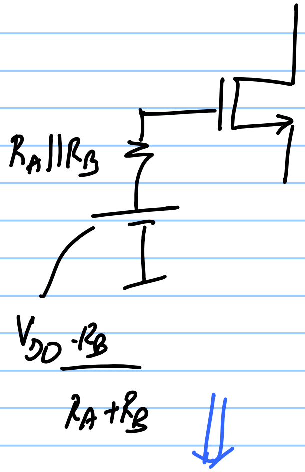
14/9/17

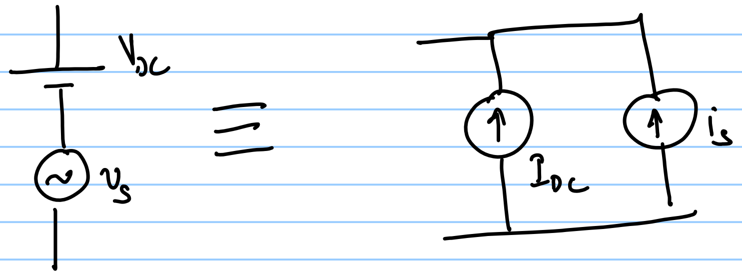
Lec 11

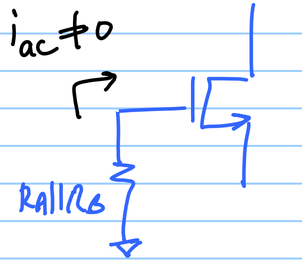


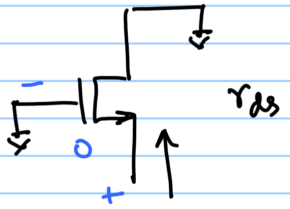
+ DC ⇒











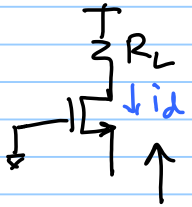
$i_d = i_s$

$i_g = 0$

$Z_{in} = \frac{1}{g_m} \| r_{ds}$

$\downarrow$

0 if $g_m \to \infty$

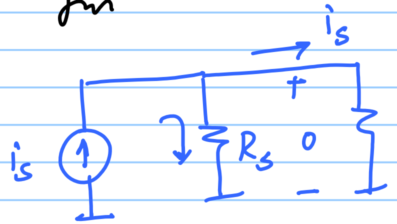


$\frac{1}{g_m}$ if r_{ds} is ∞

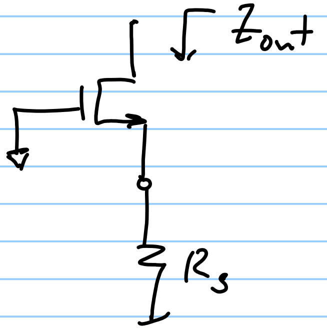
H.W.

analyse and determine Z_{in} if 1) $r_{ds} = \infty$

2) r_{ds} is finite

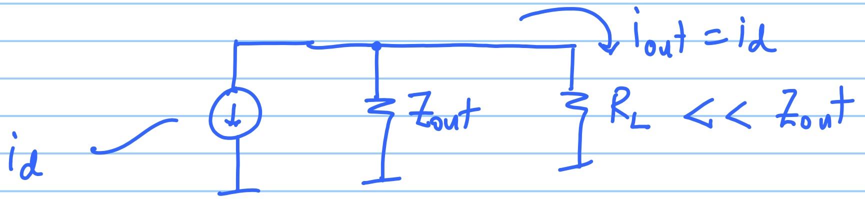


$Z_{in} = 1/g_m$



* $Z_{in} = \frac{1}{g_m} \to 0$ if $g_m \to \infty$

* $Z_{out} = R_s + r_{ds} + g_m r_{ds} R_s \to \infty$ if $g_m \to \infty$

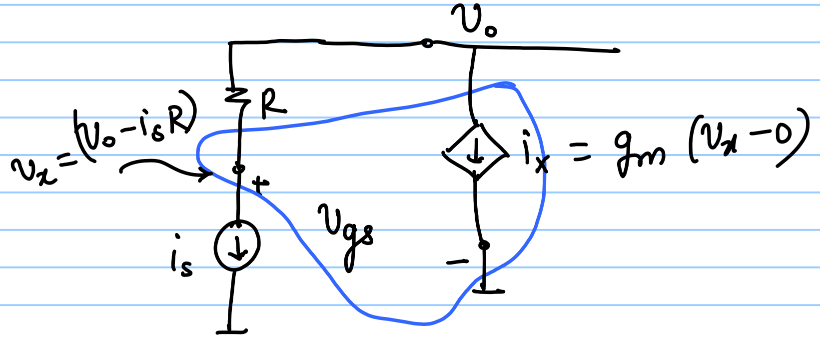


$$\boxed{\text{If } g_m \to \infty, \quad i_{out} = i_{in}}$$

CCVS

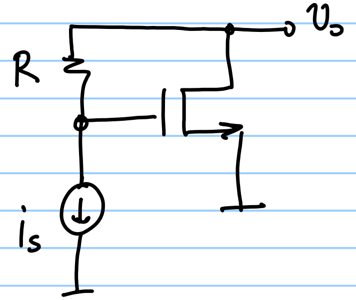
$v_{out} = i_s \cdot R$; $Z_{in} = 0$; $Z_{out} = 0$

$(v_o - i_s R) = 0$

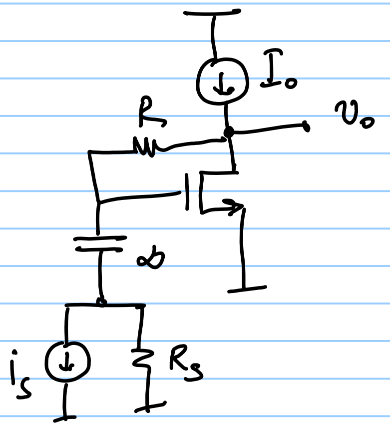


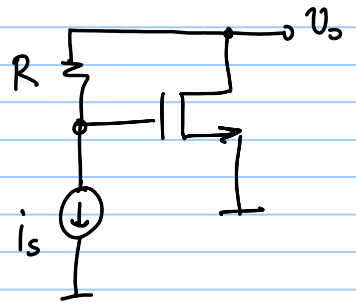
If $g_m \to \infty \Rightarrow v_{gs} \to 0$ (−ve f.b.)

$\Rightarrow v_x - 0 = 0 \Rightarrow v_o - i_s R = 0 \Rightarrow \boxed{v_o = i_s R}$



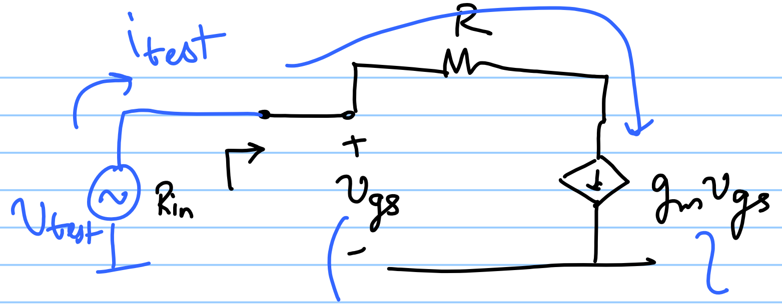
+ DC $\Rightarrow$





$$Z_{in} = \frac{1}{g_m}$$

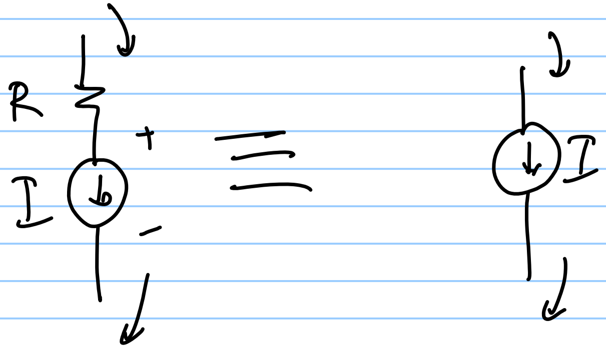
$$Z_{out} = \frac{1}{g_m}$$

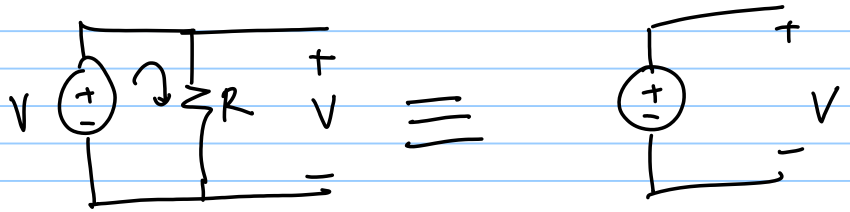


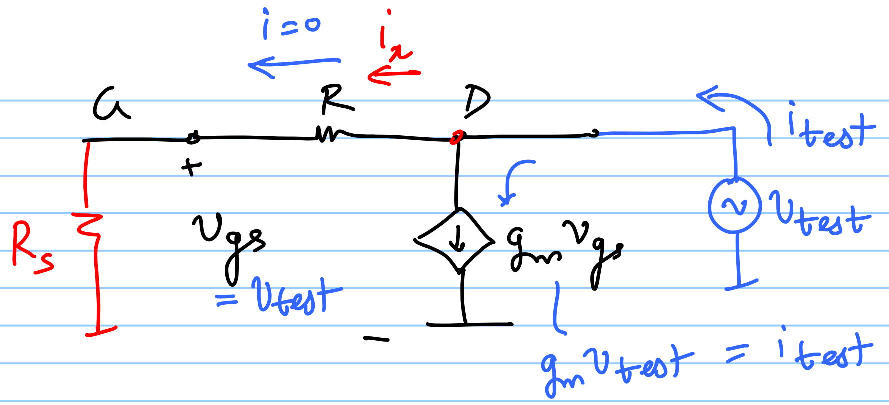
$= v_{test}$

$$g_m \cdot v_{test} = i_{test}$$

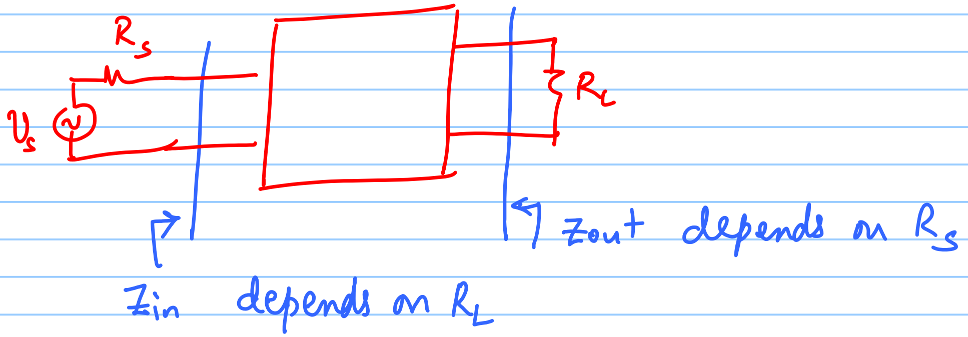
$$\Rightarrow \frac{v_{test}}{i_{test}} = \frac{1}{g_m}$$







$$Z_{out} = \frac{1}{g_m}$$



$$v_{gs} = i_x \cdot R_s$$

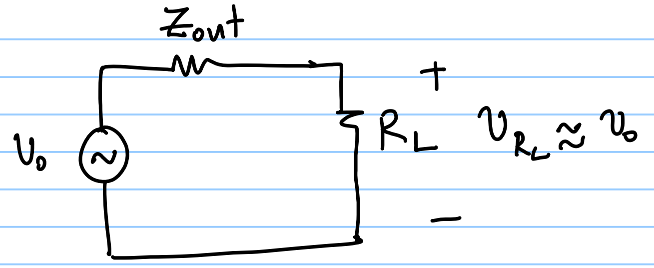
$$i_{test} = i_x + g_m v_{gs} = i_x (1 + g_m R_s)$$

$$i_x = \frac{1}{1+g_m R_s} \cdot i_{test}$$

$$v_{test} = i_x (R + R_s)$$

$$= \frac{R + R_s}{1 + g_m R_s} \cdot i_{test}$$

$$Z_{out} = \frac{v_{test}}{i_{test}} = \frac{R + R_s}{1 + g_m R_s}$$

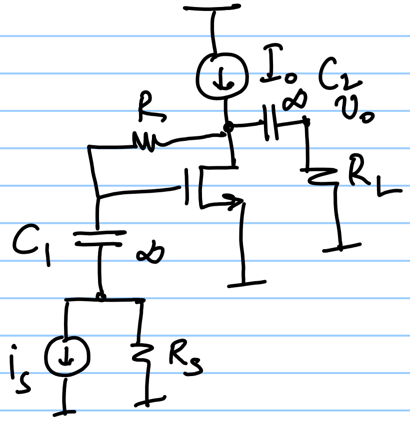


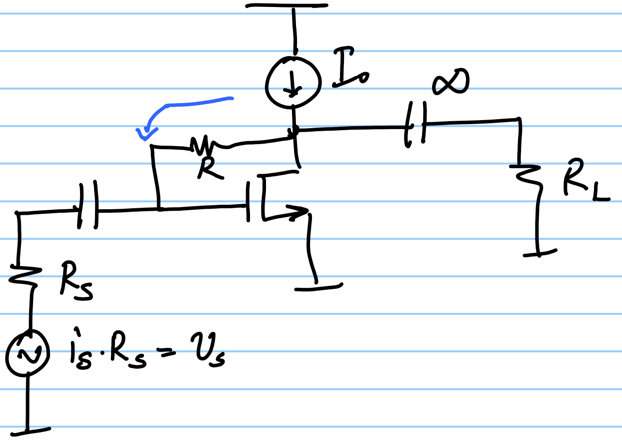
Choose g_m to be large enough such that

$$\frac{R + R_s}{1 + g_m R_s} << R_L$$

$\boxed{Z_{in} \text{ with } R_L \text{ present} - \text{H.W. exercise}}$

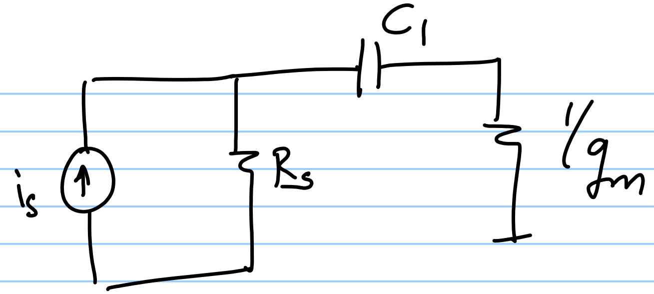
Swing limits

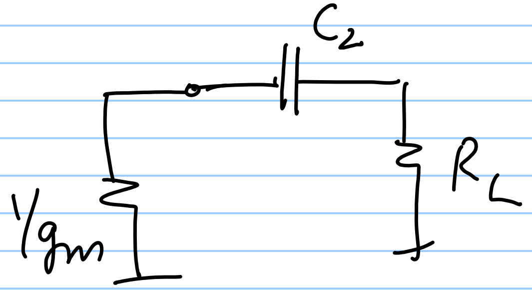




case 1.5

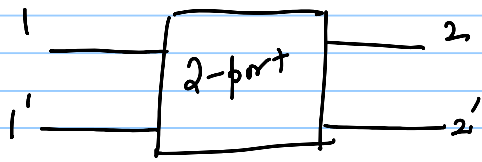
(V_{GS_0}, V_{GS_0}) is op. pt.



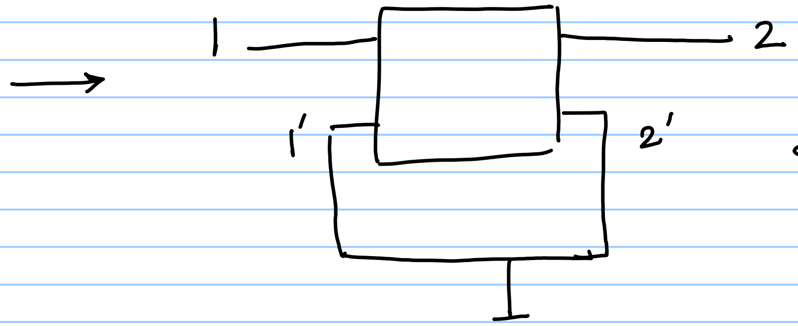


or

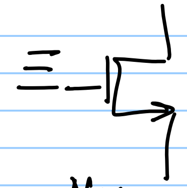
$$\frac{R + R_s}{1 + g_m R_s}$$



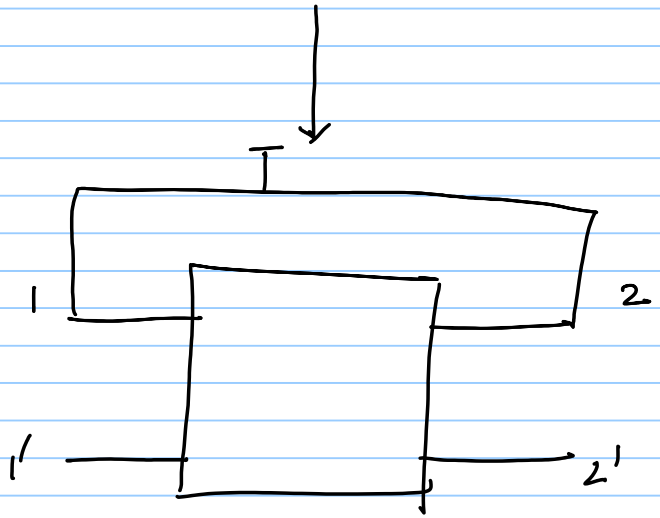
4T 2-port



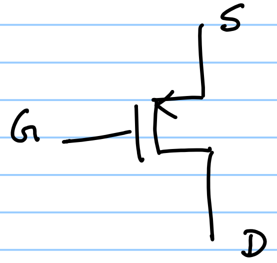
3T 2-port ≡



NMOS transistor

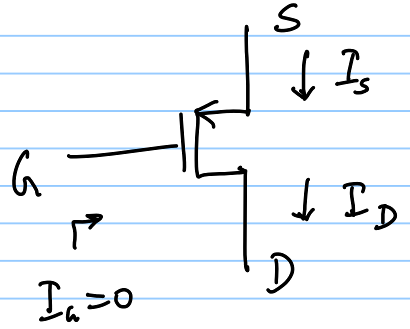


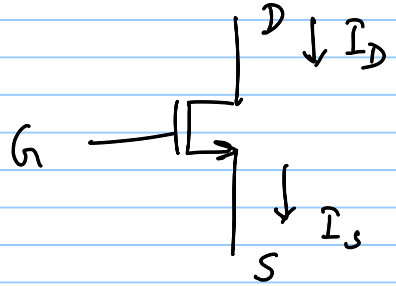
2nd 3T 2-port ≡



PMOS transistor

holes are charge carriers



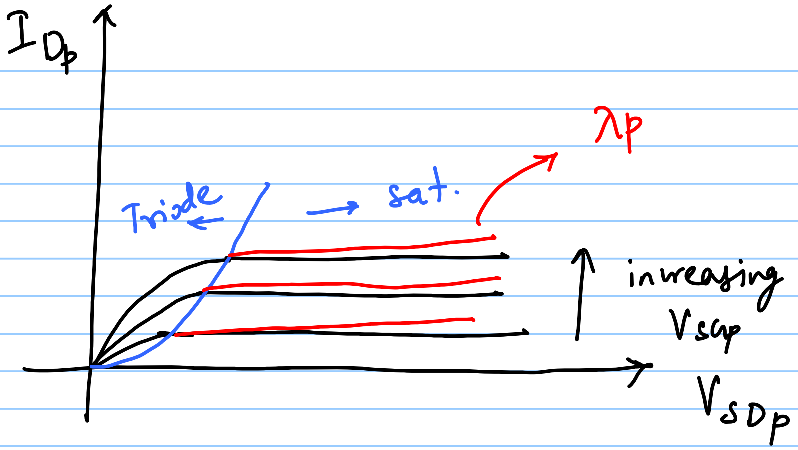


* current inside PMOS flows from S → D

* V_{SG}, V_{SD}, $V_{Tp} > 0$

* $V_{SG} > V_{Tp}$ for device to conduct

$$I_D = \begin{cases} 0 & \text{if } V_{SGp} < V_{Tp} \\ \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_p \left(V_{SGp}-V_{Tp}\right)^2 \cdot \left(1+\lambda_p V_{SDp}\right) & \text{if } V_{SGp} \geq V_{Tp} \text{ and } V_{Dp} \leq V_{Gp}-V_{Tp} \\ \mu_p C_{ox}\left(\frac{W}{L}\right)_p \left[\left(V_{SGp}-V_{Tp}\right)V_{SDp} - \frac{V_{SDp}^2}{2}\right] & \text{if } V_{SGp} \geq V_{Tp} \text{ and } V_{Dp} > V_{Gp}-V_T \end{cases}$$



$$g_m = \mu_p C_{ox}\left(\frac{W}{L}\right)_p (V_{SGp} - V_{Tp})$$

$$= \sqrt{2\mu_p C_{ox}\left(\frac{W}{L}\right)_p \cdot I_{Dp}}$$

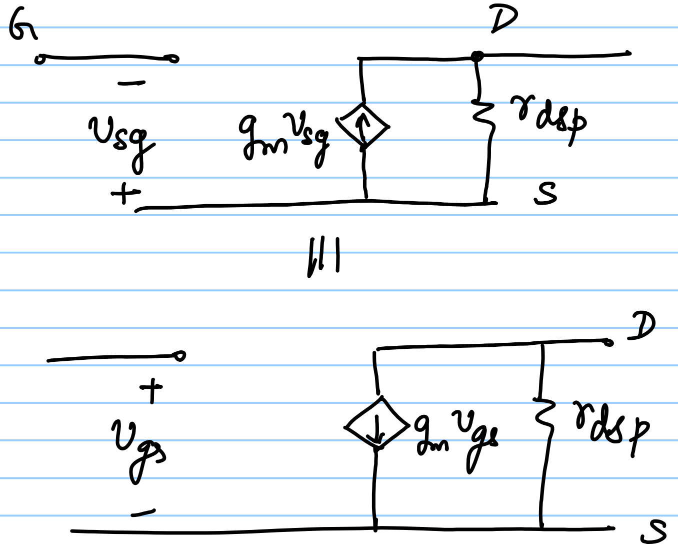
$$= \frac{2I_{Dp}}{(V_{SGp} - V_{Tp})}$$

Small- signal eq. ckt.

$$y_{11} = y_{12} = 0$$

$$y_{22} = \lambda_p \cdot I_{Dp} = \frac{1}{r_{dsp}}$$

$$y_{21} = \left.\frac{\partial I_{Dp}}{\partial V_{SGp}}\right|_{op\ pt.} = g_m$$

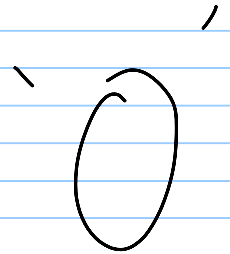


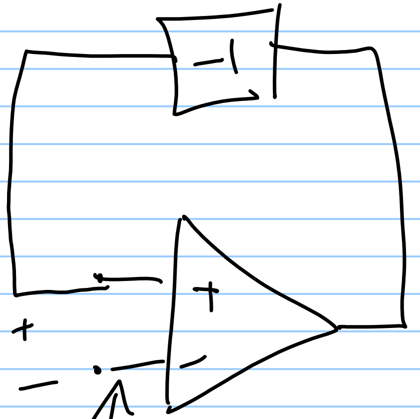
Same as that for NMOS transistor!

21/9/17

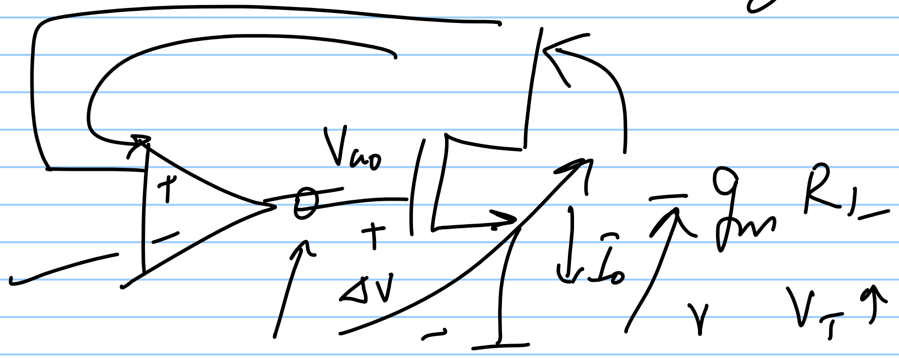
Lec 12

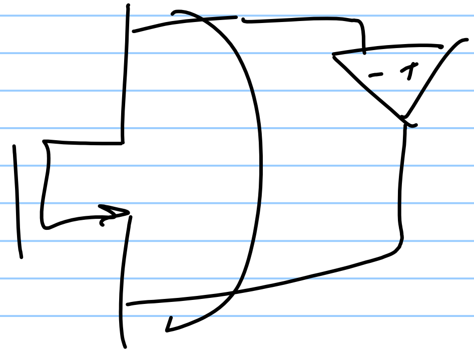
'D C' feedbo





$A(V_+ - V_-) = V_o$

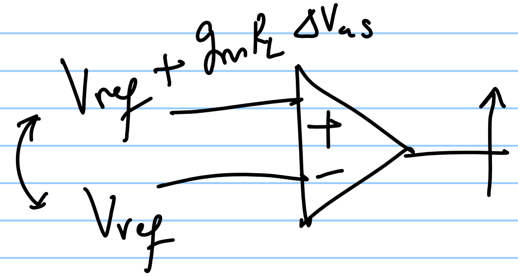




$I_D = \frac{\beta n}{2}\left(V_{GS_0} - V_{T_0}\right)^2$ $V_{T_0} \longrightarrow V_{T_1}$

$-\Delta V_{GS} \longrightarrow -g_m \Delta V_{GS} \longrightarrow +g_m R_L \cdot \Delta V_{GS}$

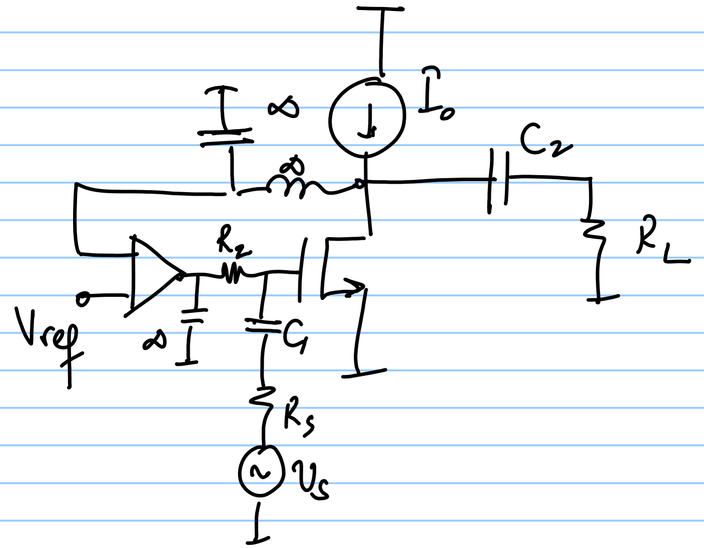
(V_{GS}) (I_D) (V_o)



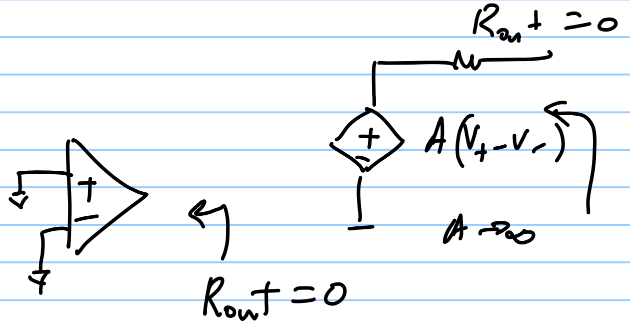
$V_{GS_1} \rightarrow V_{GS_0} + \Delta V_{GS}$

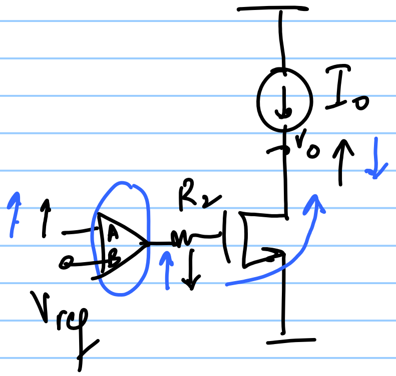
Quiz 1
Quiz 2
Quiz 3 } scaled to 50%.

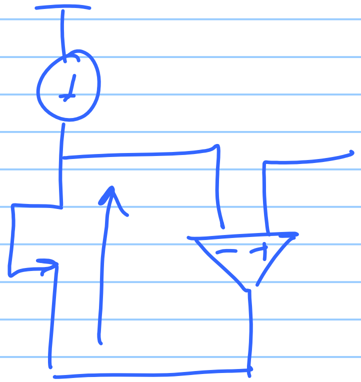
Tutorials — 10%.
Finals — 40%.



short all ind., ac V sources
open all caps, ac I sources





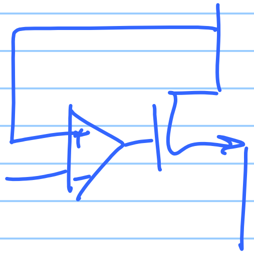


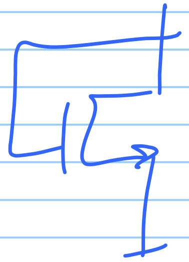
A = -ve
B = +ve

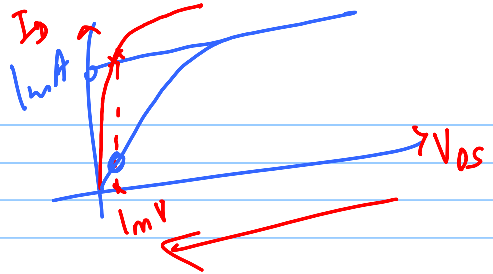
+ve fb.

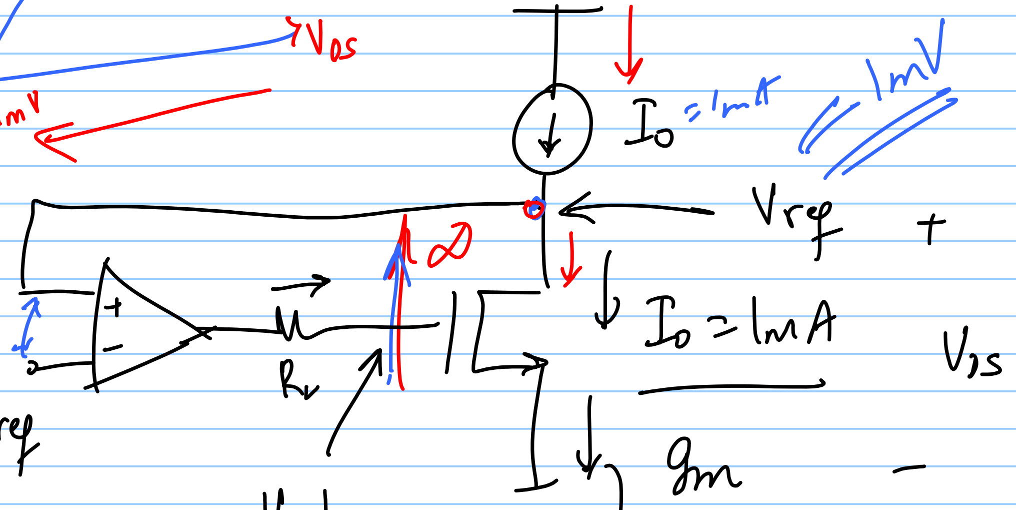
A = +ve
B = -ve

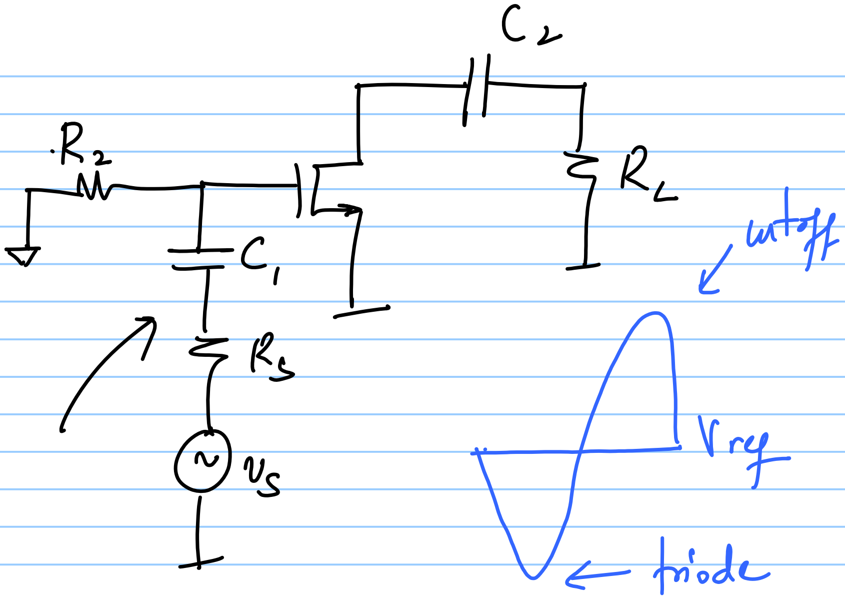
✓











$V_{GS}|_{1mA}$

$i_d = g_m v_g \times \frac{R_2}{R_s+R_2}$

$-g_m (r_{do}||R_L) \times \frac{R_2}{R_s+R_2}$

$|i_d| = I_o$

$V_{A_1} = \frac{I_D}{g_m\left(\frac{R_L}{R_s+R_L}\right)}$ cut off limit = Triode limit

$$V_{ref} - \underset{\checkmark}{g_m \left(r_{ds} \| R_L\right)} \cdot \frac{R_2}{R_S + R_L} \underset{V_{A_1}}{\underset{\|}{V_{A_2}}} \sin \cancelto{1}{\omega t} = \cancel{V_{GS_0}} + \overset{V_{A_1}}{\overset{\|}{V_{A_2}}} \sin \cancelto{1}{\omega t} \cdot \frac{R_2}{R_S + R_2}$$

$$- V_T$$

$$r_{ds} = \frac{1}{\lambda \underset{\uparrow}{I_D}}$$

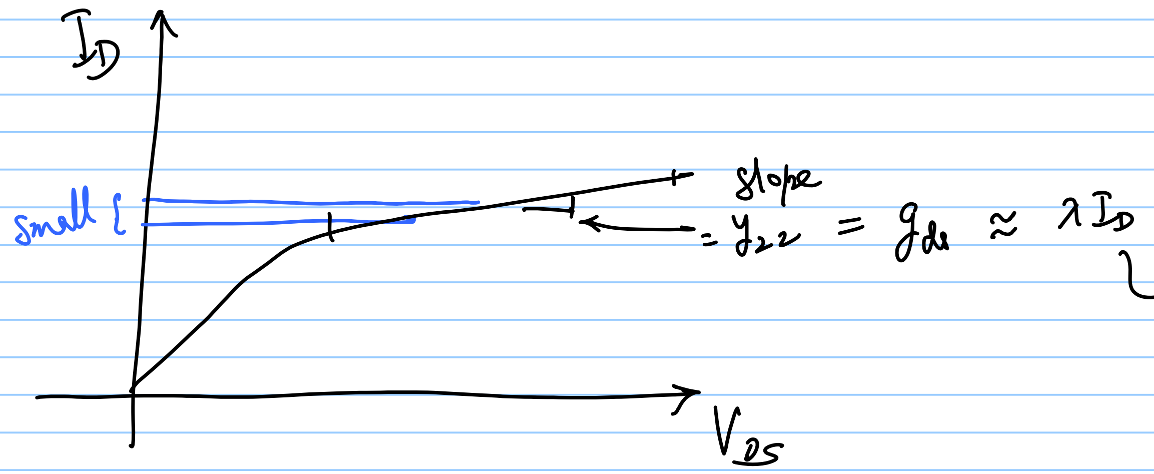
$$V_{GS} = V_{GS_0} + v_{gs}$$

$$= V_{GS_0} + \underset{\underset{V_A \sin \omega t}{\nearrow}}{v_s} \cdot \frac{R_2}{R_S + R_2}$$

Channel length modulation

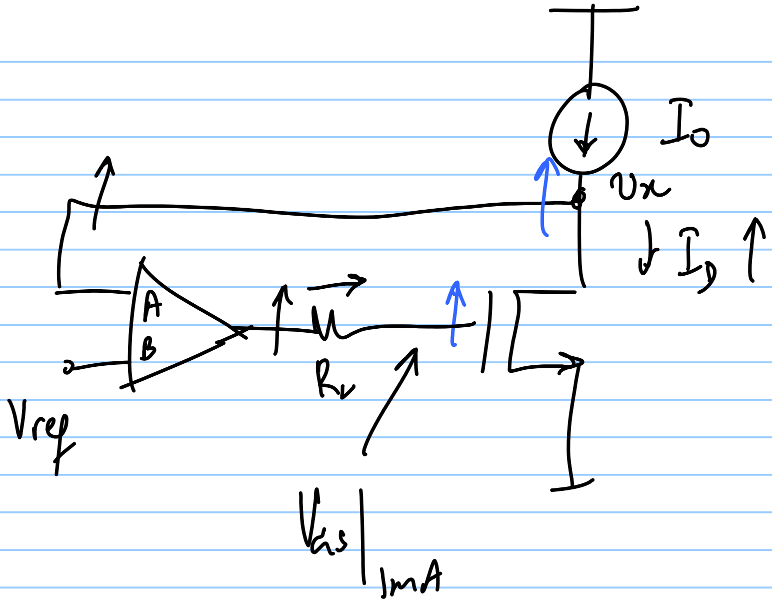
$\lambda \rightarrow$ do not use for op pt. calculations

$\lambda \rightarrow$ use only to determine r_{ds}



slope $= y_{22} = g_{ds} \approx \lambda I_D$

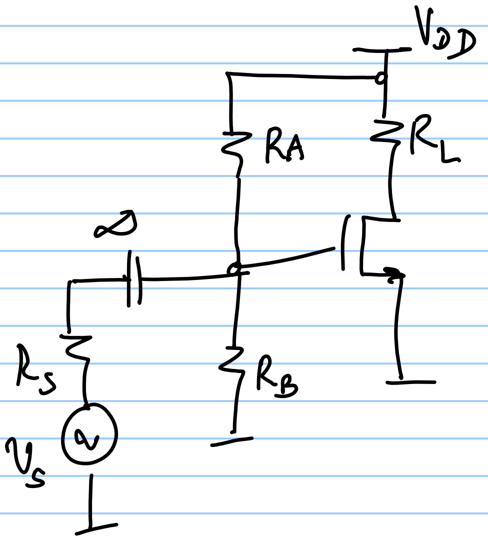
↳ I_D expression without λ

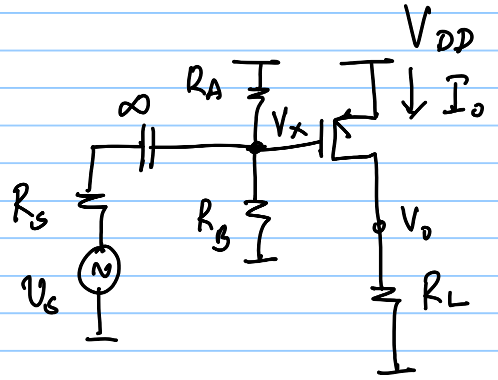


let $I_D < I_o \Rightarrow v_x \uparrow$

we want $v_g \uparrow \Rightarrow A = +ve$

PMOS Common Source Amplifier





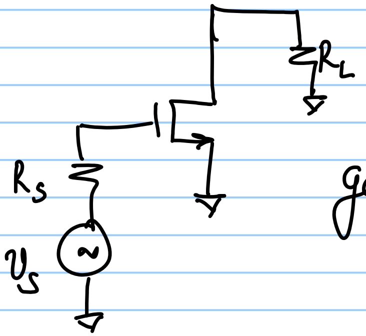
op pt.

$$V_x = \frac{R_B}{R_A+R_B} \cdot V_{DD}$$

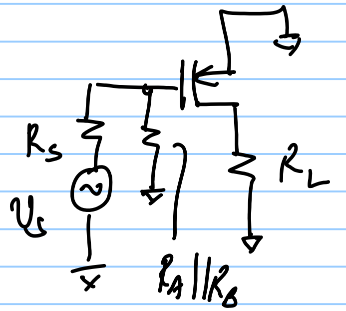
$$V_{SG_0} = V_{DD} - V_x = \frac{R_A}{R_A+R_B} V_{DD}$$

$$I_o = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_p \left(V_{SG_0} - V_{T_p}\right)^2$$

$$V_o = I_o \cdot R_L$$



$$\text{gain} = -g_m \left(r_{ds} \| R_L\right)$$



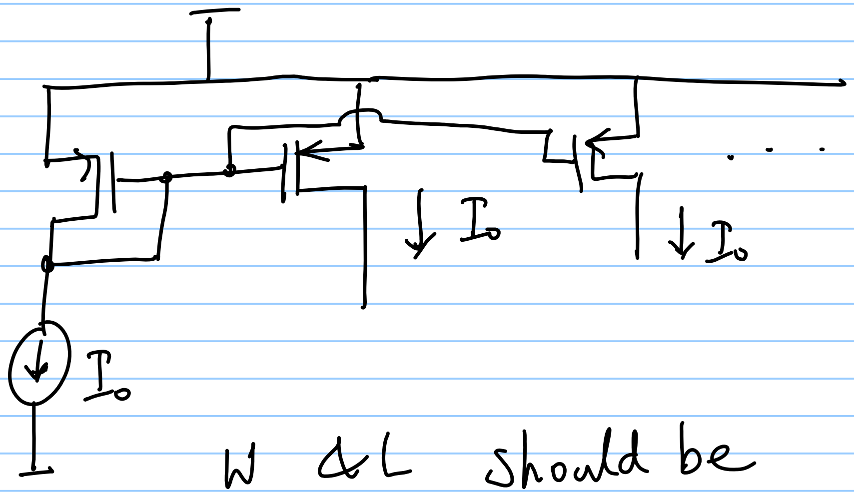
$$R_A \| R_B >> R_s$$

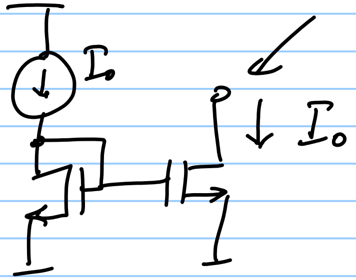
$$v_g \approx v_s$$

$$\text{gain} = -g_{m_p} \left(r_{ds_p} \| R_L\right)$$

Bias Stabilization

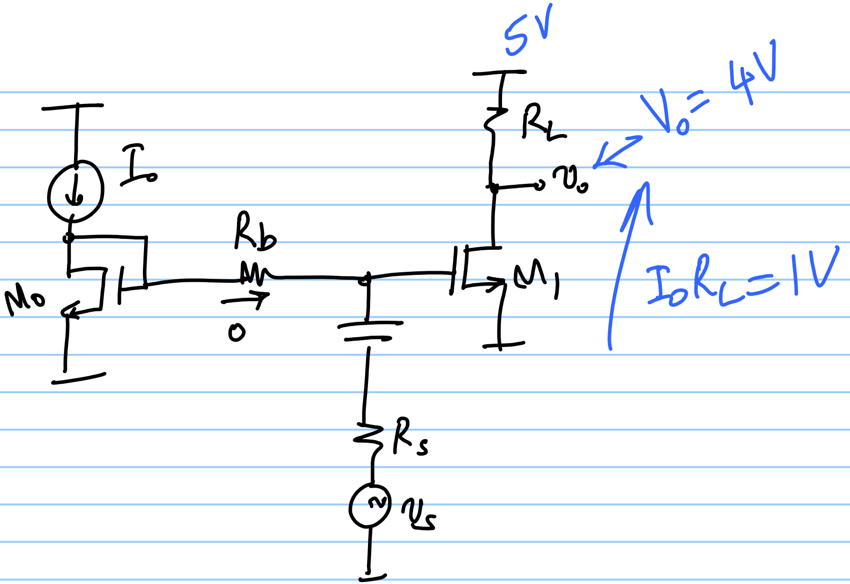
Current Mirror

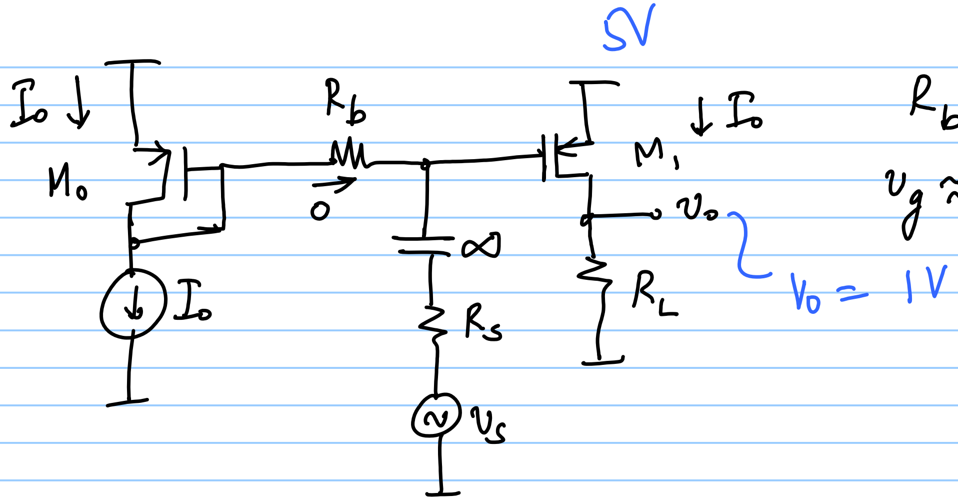




W & L should be equal

1)

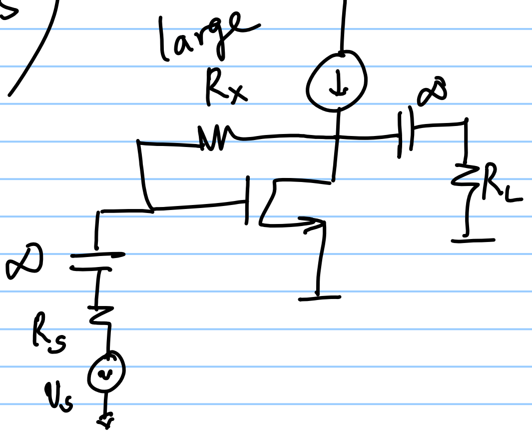


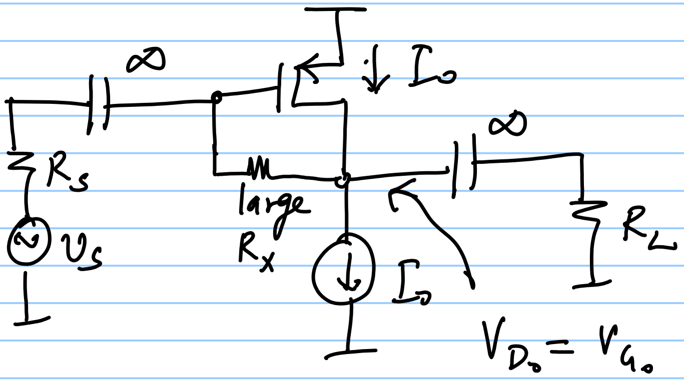


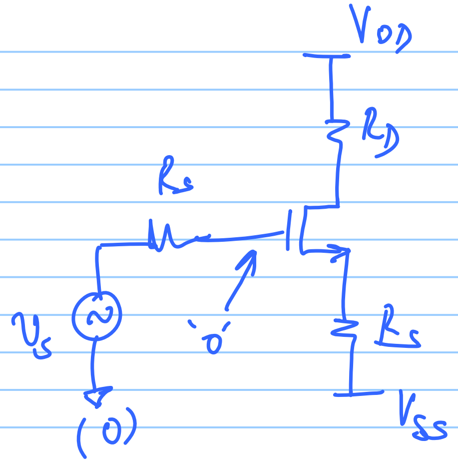
$R_b >> R_s$

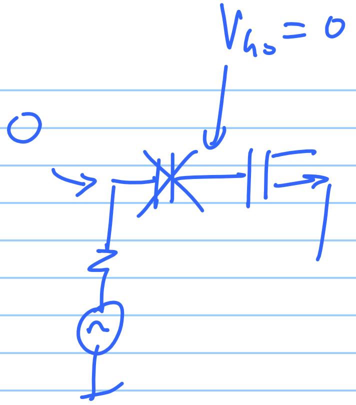
$v_g \approx v_s$

1.5)

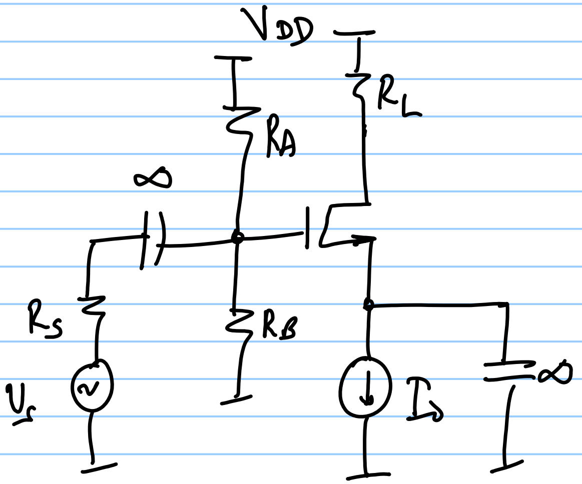


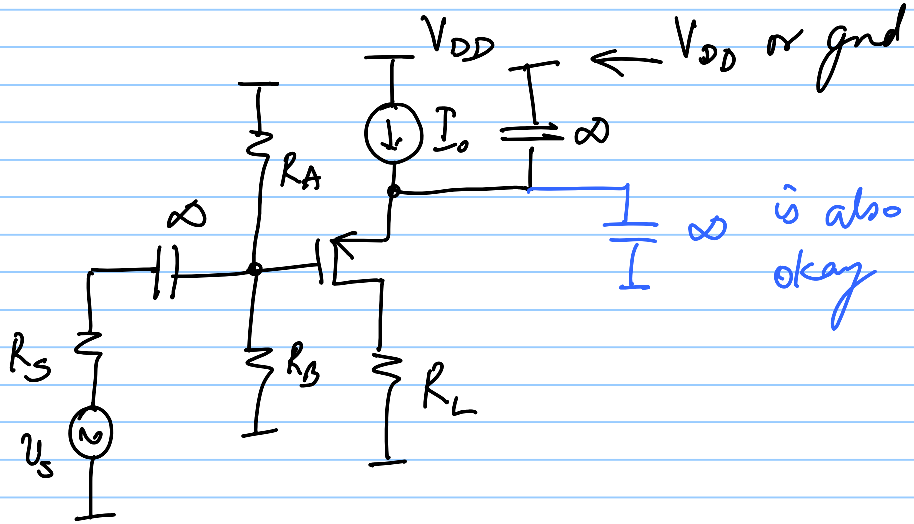



V_DD
R_D
R_s
v_s
'0'
(0)
R_s
V_SS

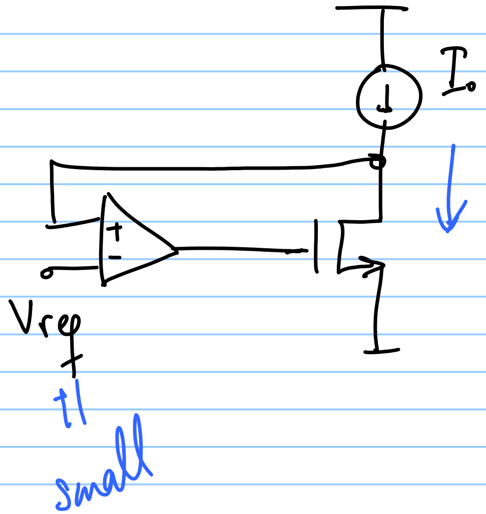

V_gs = 0
0

2)


V_DD
R_L
R_A
∞
R_B
R_s
v_s
I_o
∞


V_DD
V_DD or gnd
R_A
I_o
∞
∞
∞
is also okay
R_S
v_s
R_B
R_L

Case 3 & 4 — HW exercises

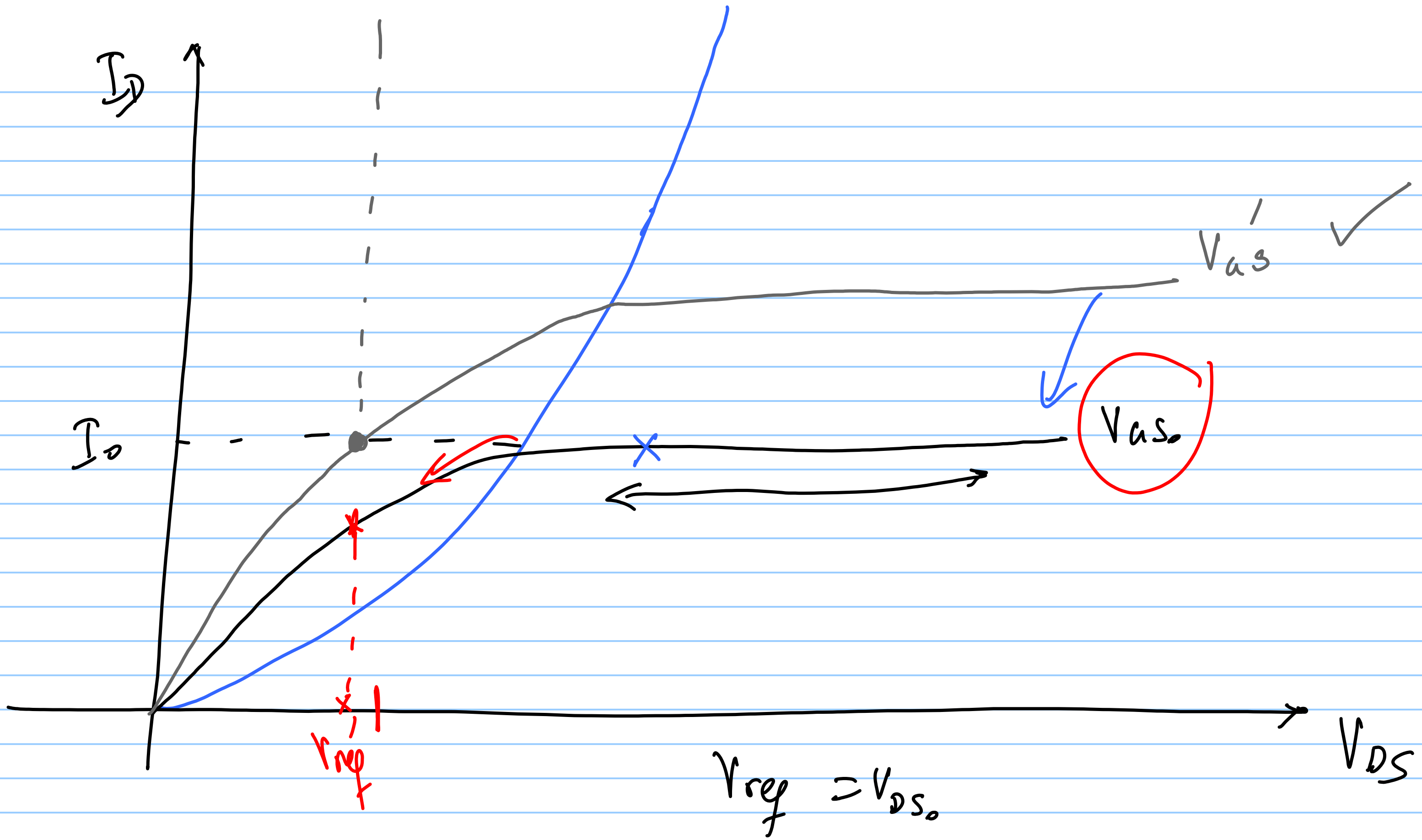


V_{ref} — design parameter

I_o — "

$\frac{W}{L}$ — "

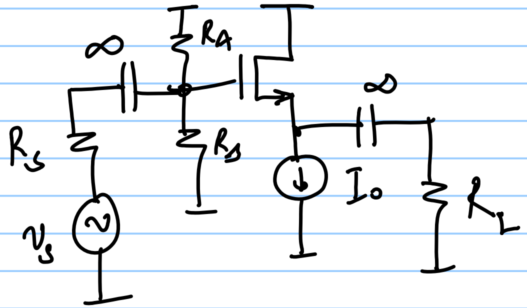
V_{DD} —

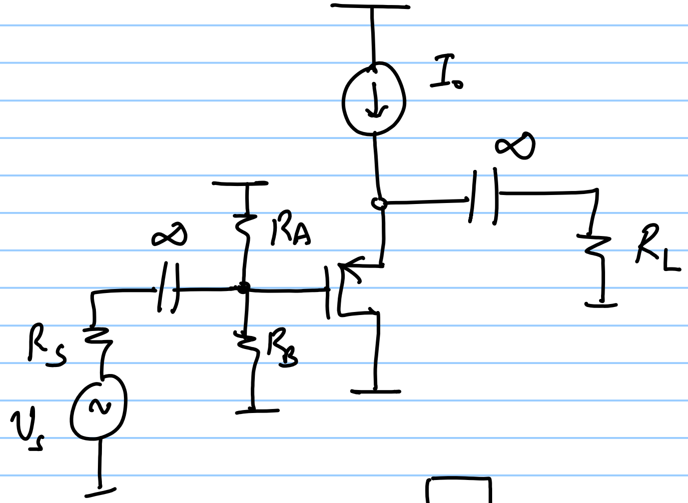

I_D
I_0
V_{GS}'
✓
V_{GS_0}
V_{DS}
V_{ref}
$V_{ref} = V_{DS_0}$

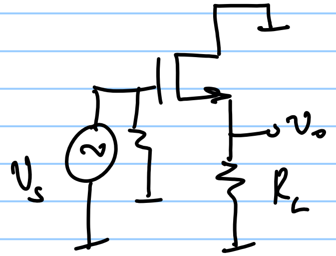
26/9/17

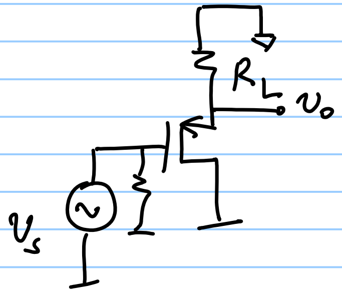
Lec 13

1) VCVS gain = 1 $Z_{in} = \infty$ $Z_{out} = 0$

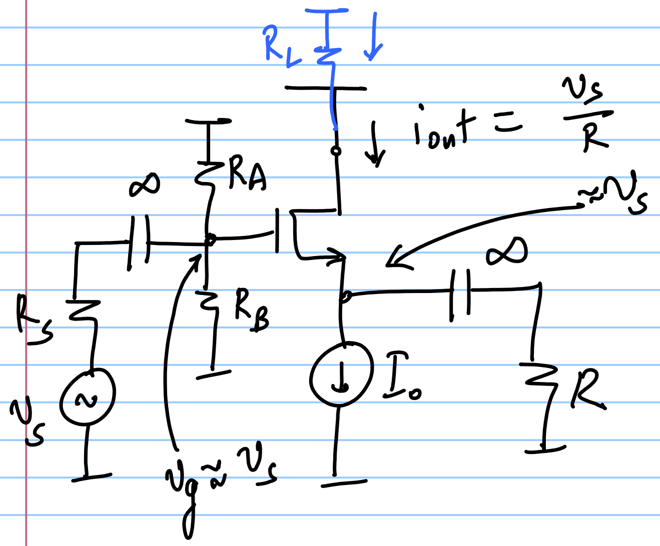


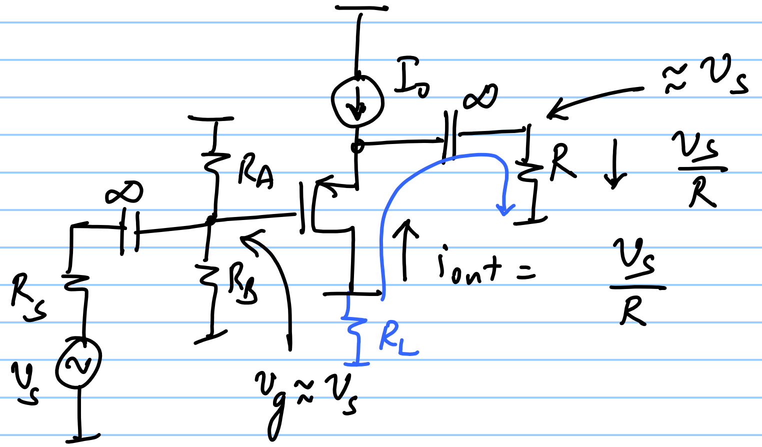






2) VCCS gain $\frac{1}{R}$ $Z_{in} = \infty$, $Z_{out} = \infty$



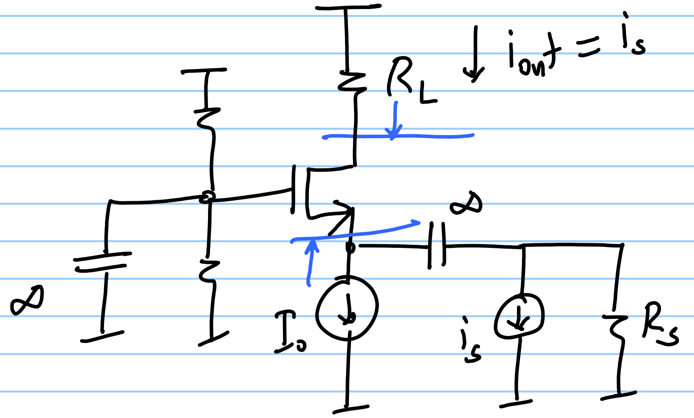


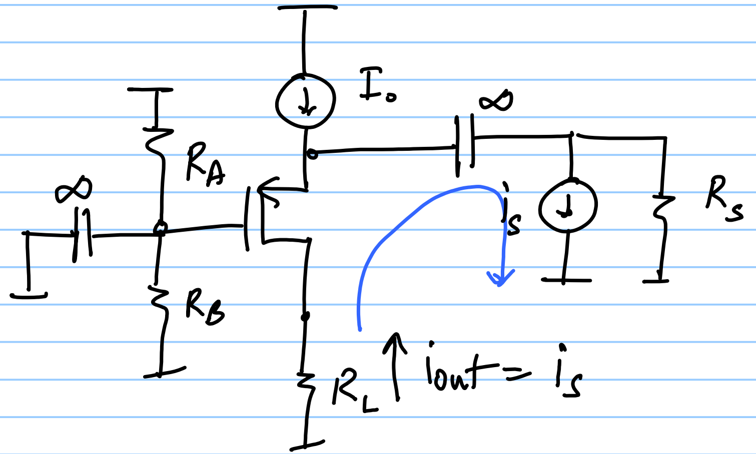
$R_{out} = R + r_{ds} + g_m R r_{ds}$

$R_{out} = R + r_{ds_p} + g_{m_p} r_{ds_p} R$

" PMOS Phase Splitter" → H.W.

3) CCCS gain $=1$ $Z_{in}=0$, $Z_{out}=\infty$

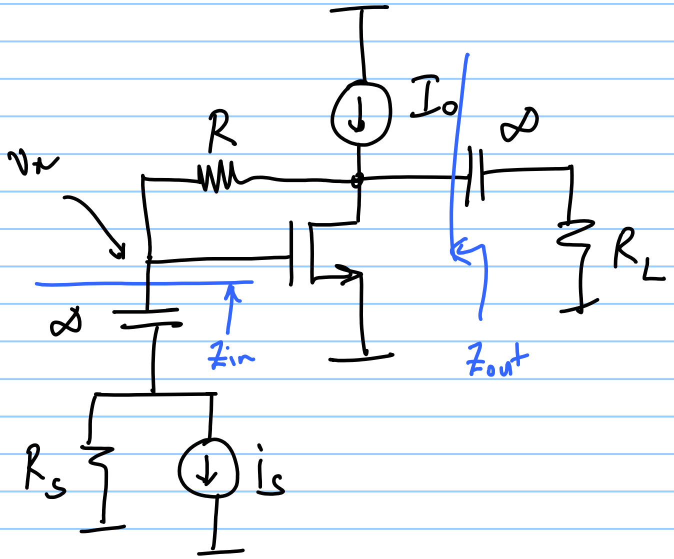


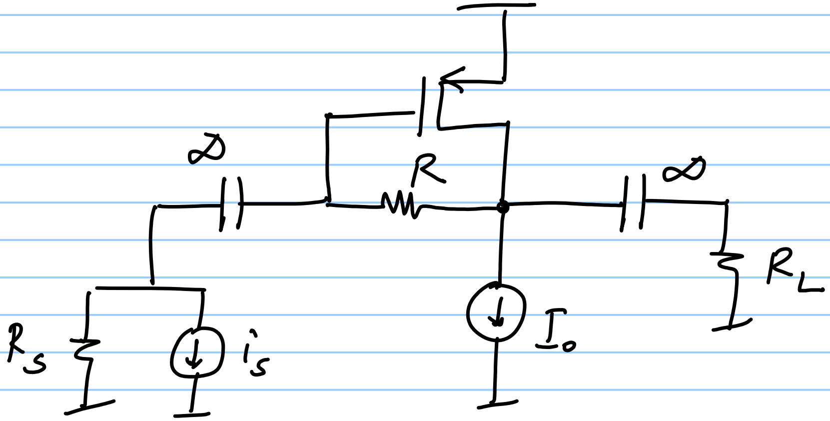


$Z_{in} \approx \frac{1}{g_{m_p}}$ small if $g_{m_p} \to \infty$

$Z_{out} = g_{m_p} r_{ds_p} \cdot R_s$ large if $g_{m_p} r_{ds_p} \to \infty$

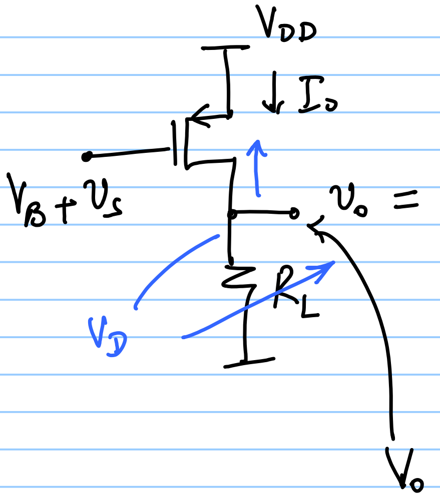
4) <u>CCVS</u> gain R $Z_{in} = 0$; $Z_{out} = 0$





$$v_o = R \cdot i_s$$

$$\underbrace{\left(v_o - R i_s\right)}_{v_x} = 0$$



$v_o = -(g_m R_L)\cdot v_s$

gain $|A| = g_m R_L$ * I want large A

↑ g_m

↑ I_0 ①

$V_D = I_0 \cdot R_L$

PMOS moves closer to triode

↑ $\left(\frac{W}{L}\right)$ ②

a) transistor gets slower

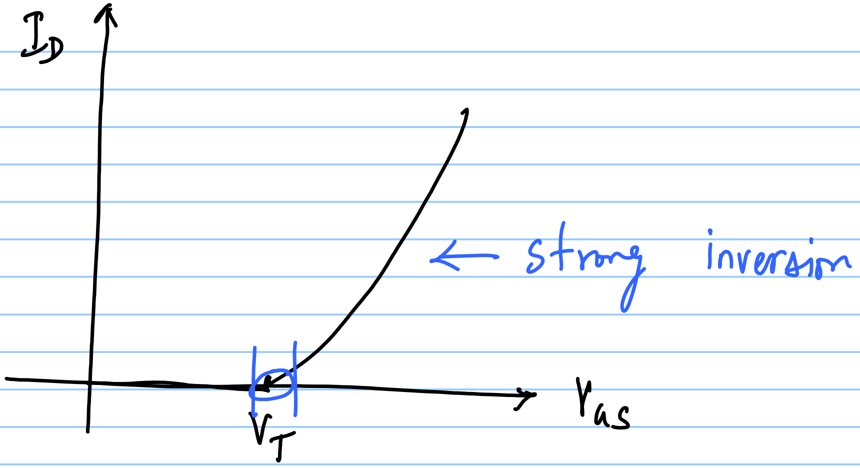
b) $(V_{GS} - V_T)$ ↓ } Square law operation changes

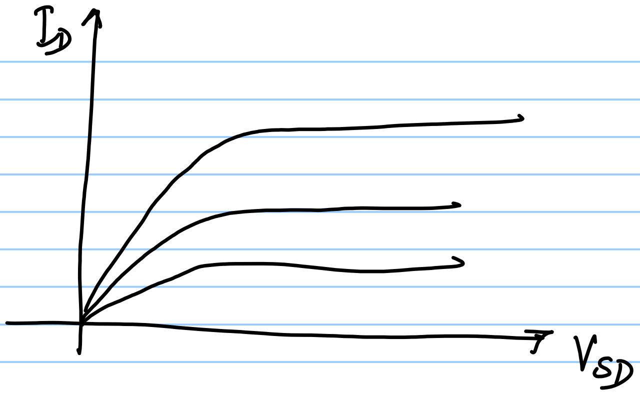
$$= \sqrt{\frac{2 I_D}{\mu C_{ox}\left(\frac{W}{L}\right)}}$$

↑ R_L ③

$V_D = I_0 \cdot R_L$

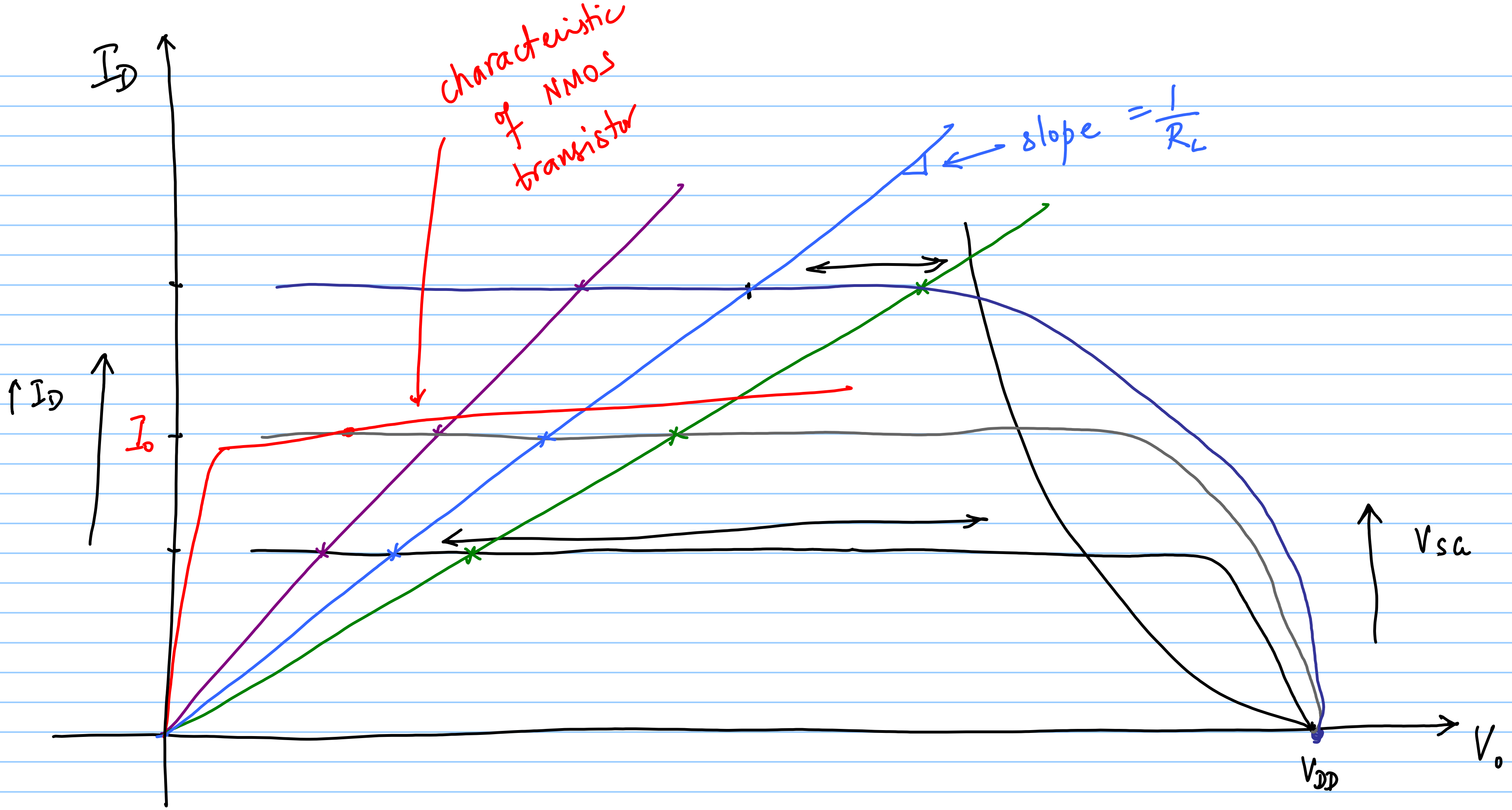
PMOS moves closer to triode

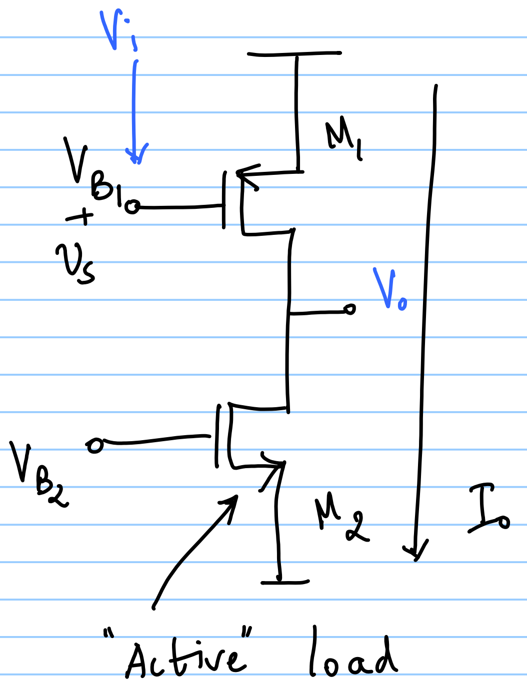


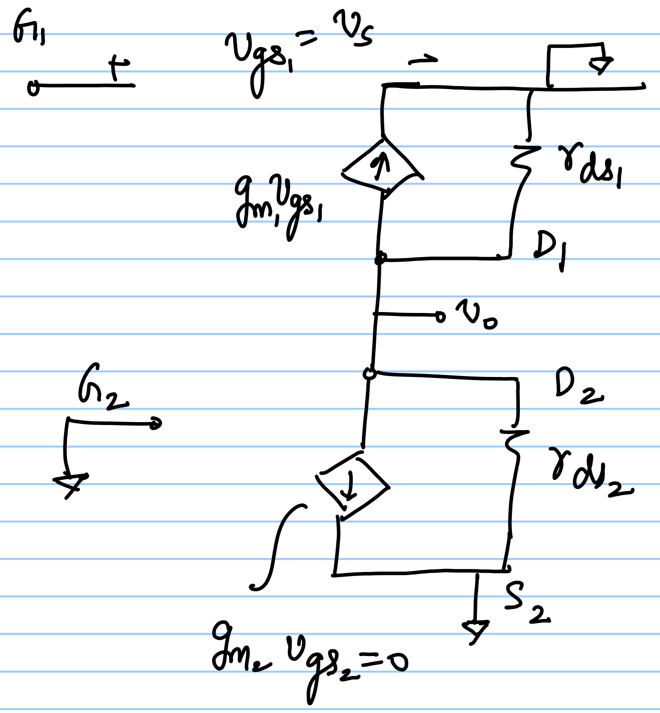


$V_o = I_D \cdot R_L$ ← Curve for R_L

$V_{SD} = V_{DD} - V_o$ ← Curve for PMOS


characteristic of NMOS transistor
slope = $\frac{1}{R_L}$
I_D
$\uparrow I_D$
I_o
V_{SG}
V_o
V_{DD}

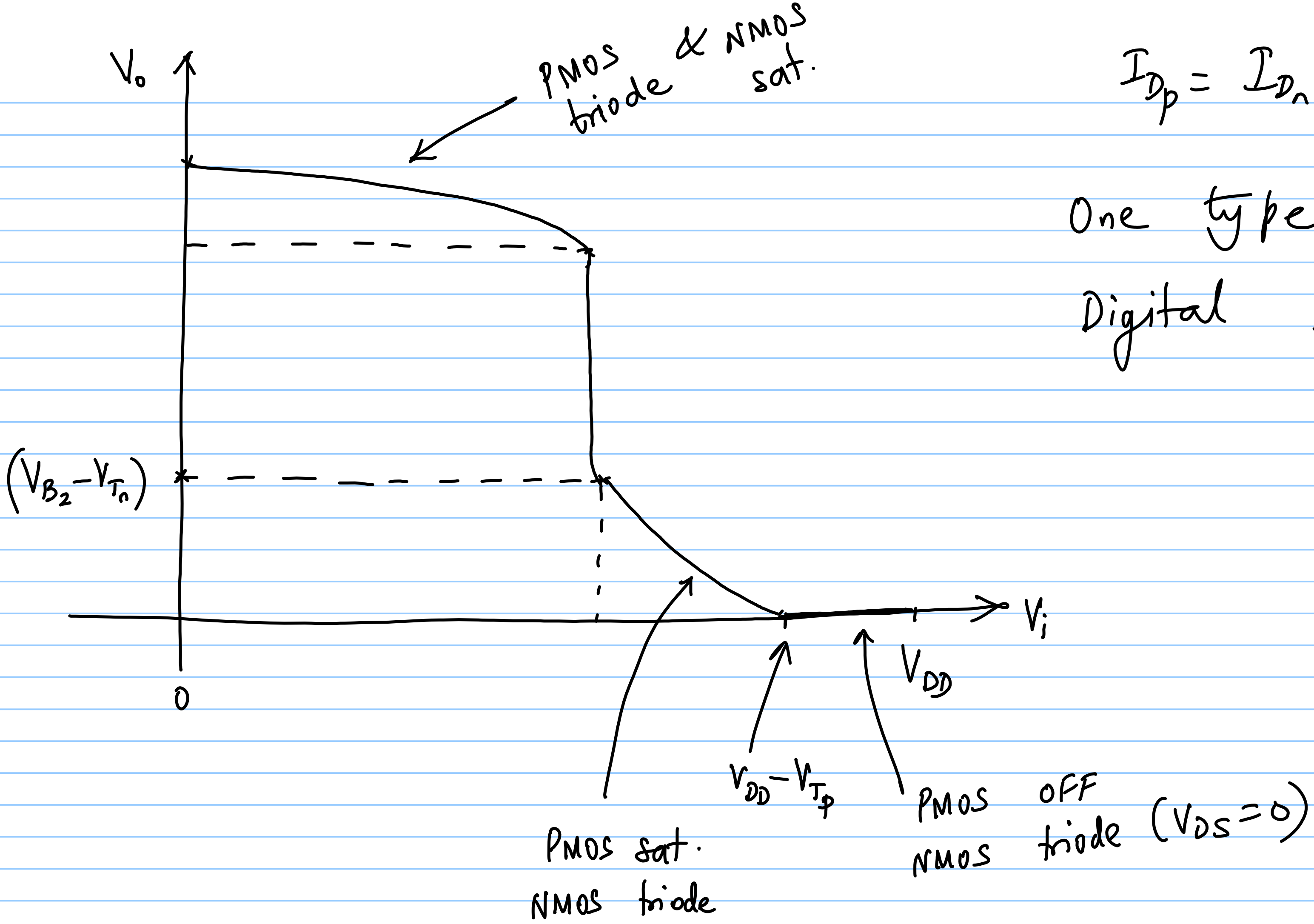




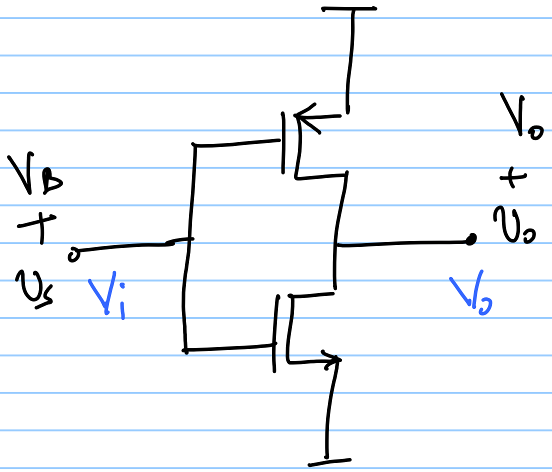
$$\frac{v_o}{v_s} = -g_{m_1}\left(r_{ds_1}||r_{ds_2}\right) = -g_{m_p}\left(r_{ds_p}||r_{ds_n}\right)$$

$I_{D_p} = I_{D_n}$

One type of Digital Inverter



$\frac{dV_o}{dV_i}$ = gain



$$\frac{v_o}{v_s} = -\left(g_{m_n} + g_{m_p}\right)\left(r_{ds_n} \| r_{ds_p}\right)$$

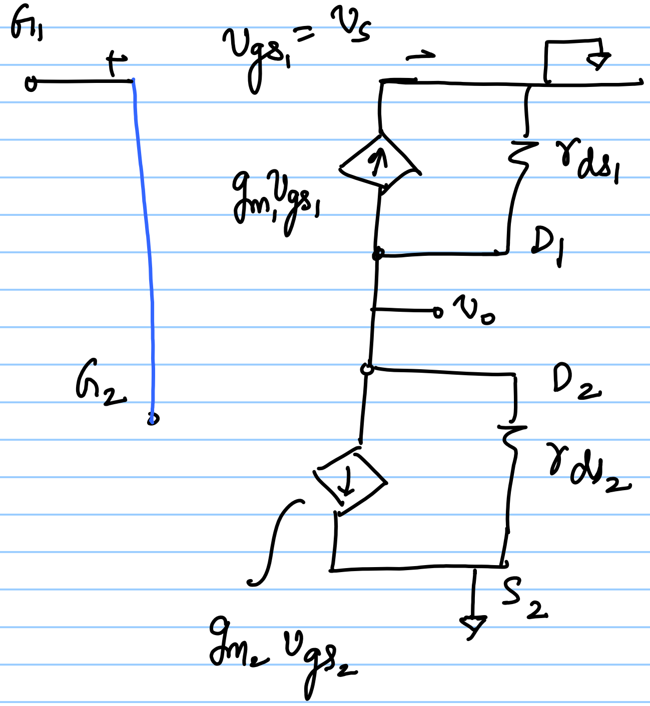
HW : Draw V_o vs. V_i

(DC characteristics)

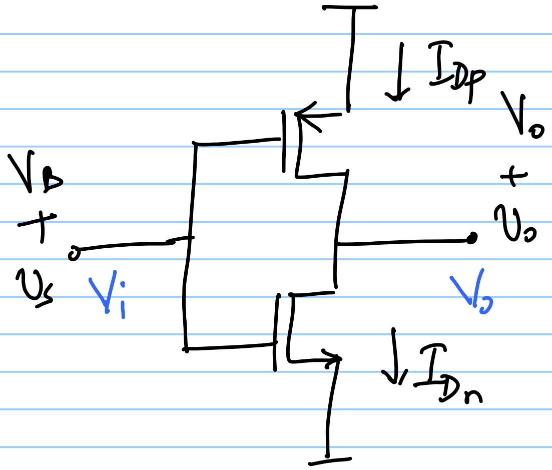
Classical CMOS Inverter

3/10/2017

Lec 14



$\Longleftrightarrow$

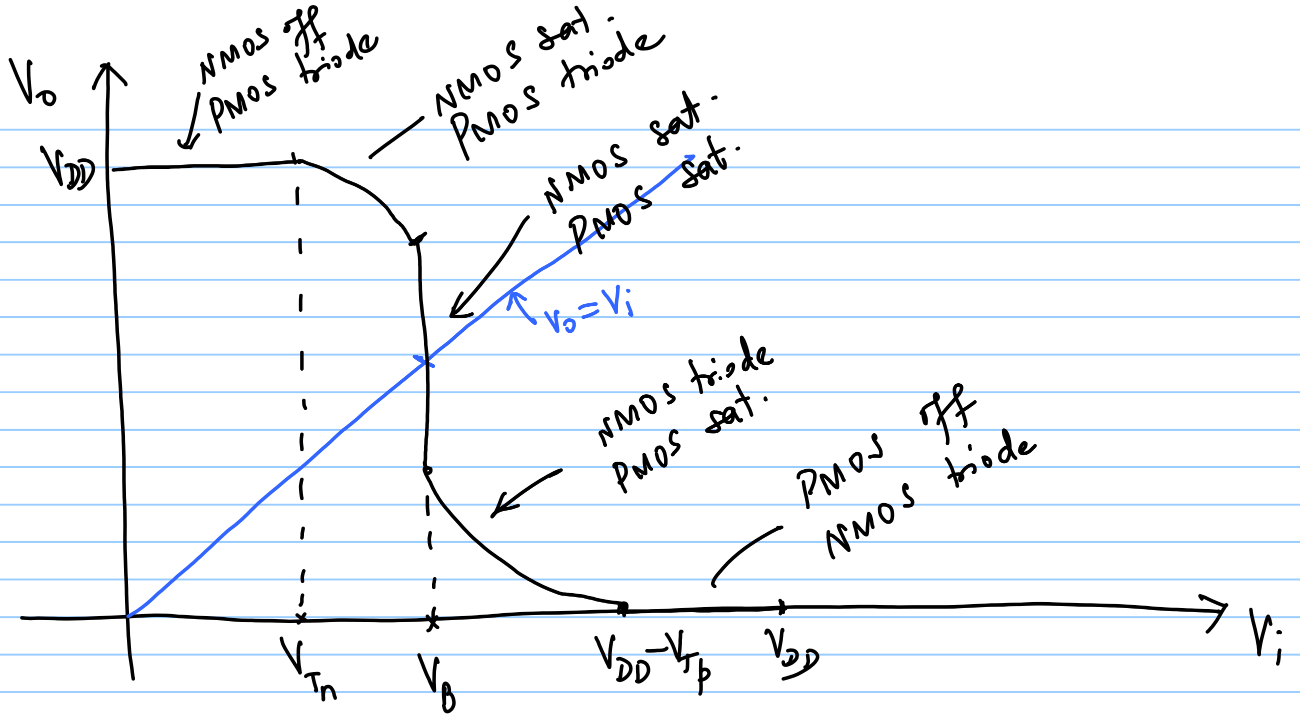


$$\frac{v_o}{v_i} = -(g_{m_n} + g_{m_p})\left(r_{ds_p} \| r_{ds_n}\right)$$

$$= - \frac{g_{m_n} + g_{m_p}}{g_{ds_n} + g_{ds_p}}$$

$g_{ds_n} = \frac{1}{r_{ds_n}}$

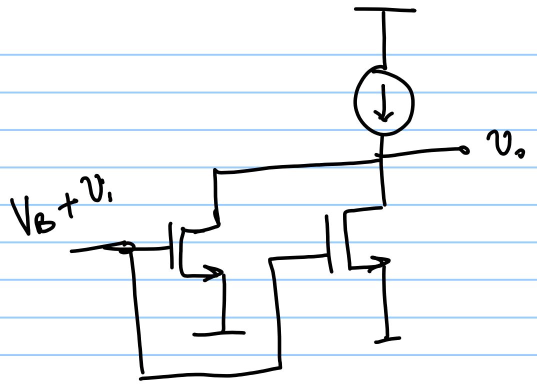
$g_{ds_p} = \frac{1}{r_{ds_p}}$

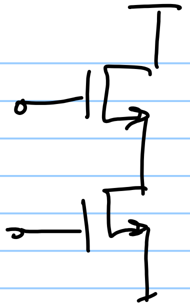


$$I_{D_p} = I_{D_n}$$

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_p \left[V_{DD} - V_B - V_{T_p}\right]^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n \left[V_B - V_{T_n}\right]^2$$

$$k = \mu_n C_{ox}\left(\frac{W}{L}\right)_n \Big/ \mu_p C_{ox}\left(\frac{W}{L}\right)_p$$





$$\left[V_{DD} - V_B - V_{Tp}\right]^2 = k\left[V_B - V_{Tn}\right]^2$$

$$V_{DD} - V_B - V_{Tp} = \sqrt{k}\cdot V_B - \sqrt{k}\cdot V_{Tn}$$

$$V_B\left(1+\sqrt{k}\right) = V_{DD} - V_{Tp} + \sqrt{k}\,V_{Tn}$$

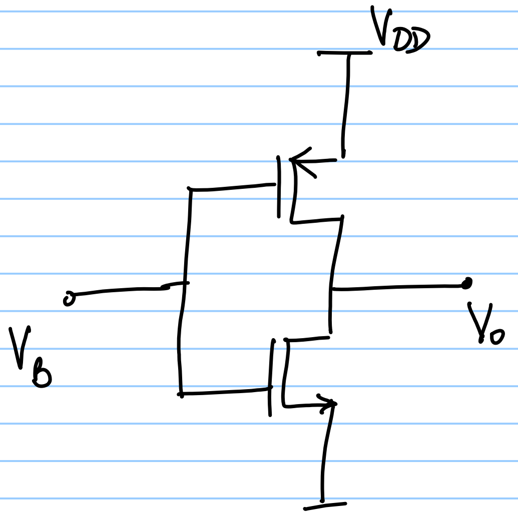
$$V_B = \frac{V_{DD} - V_{Tp} + \sqrt{k}\,V_{Tn}}{1+\sqrt{k}}$$

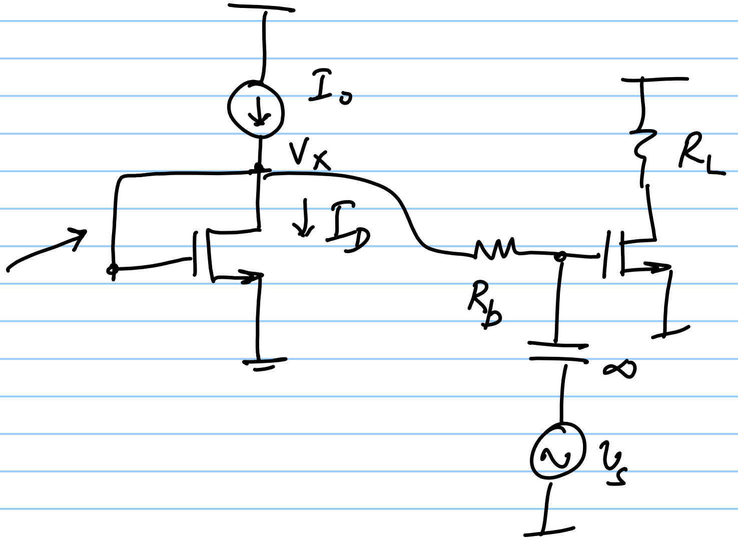
1) $V_B = \frac{V_{DD}}{2}$ if $k=1$, $V_{Tn} = V_{Tp}$

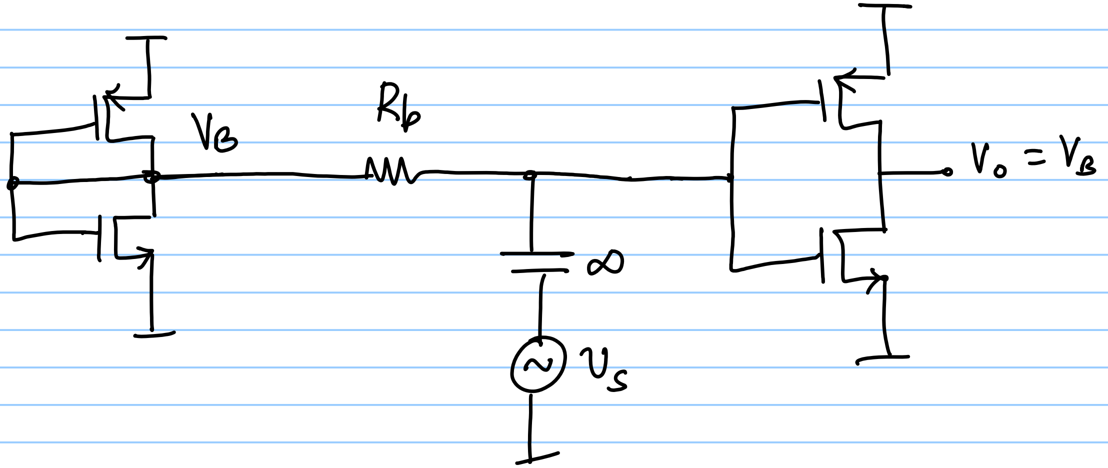
2) $k \gg 1$: $V_B \simeq V_{Tn}$

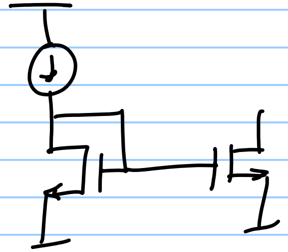
3) $k \ll 1$: $V_B \simeq V_{DD} - V_{Tp}$

Ⓘ

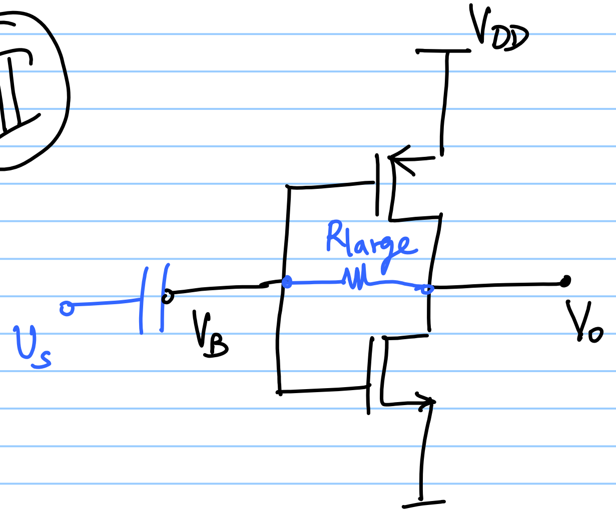


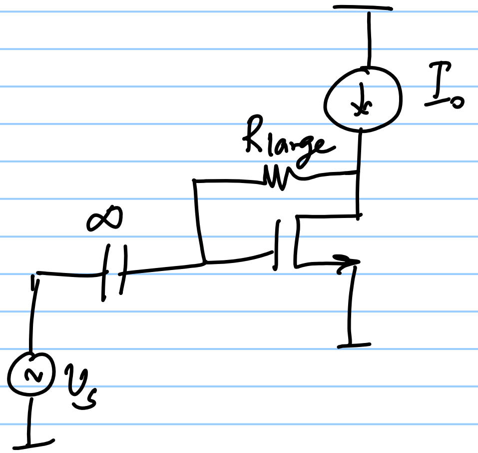




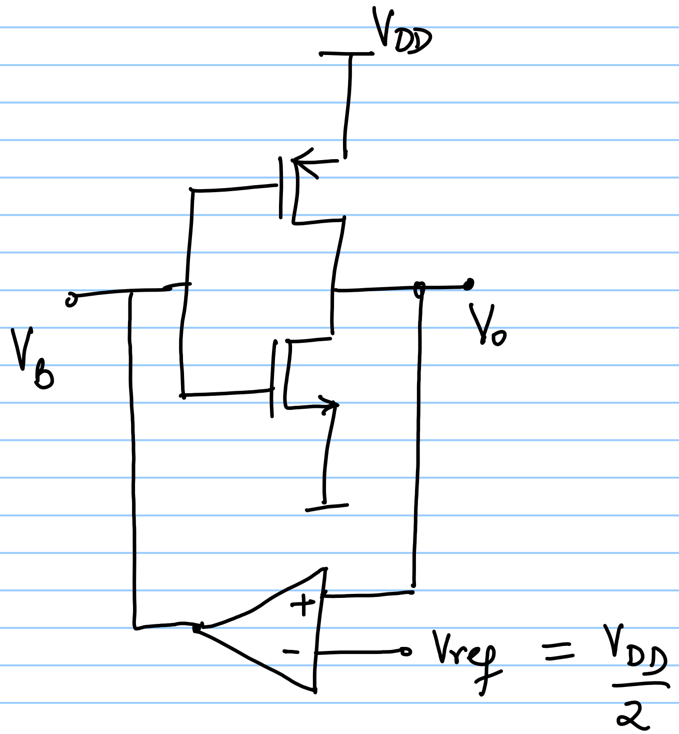


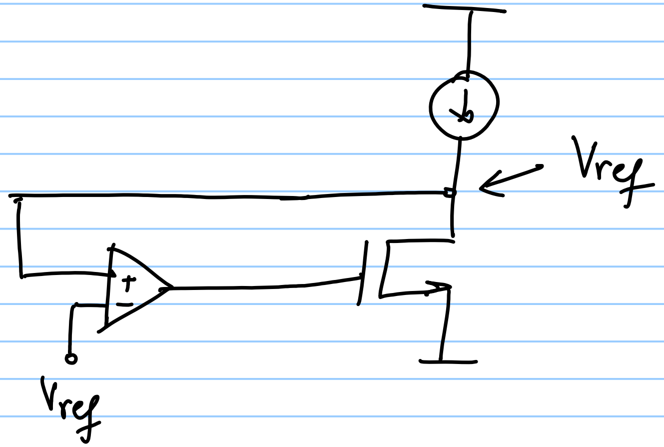
II

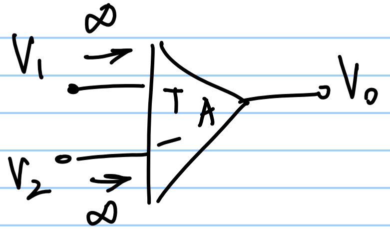




III







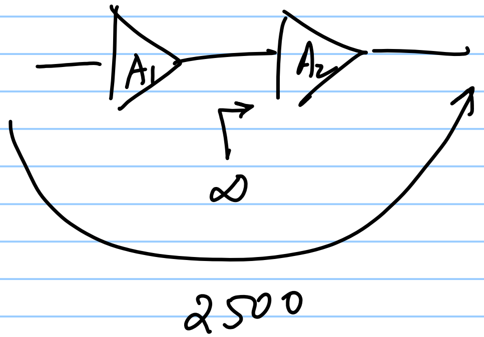
CMOS op amp

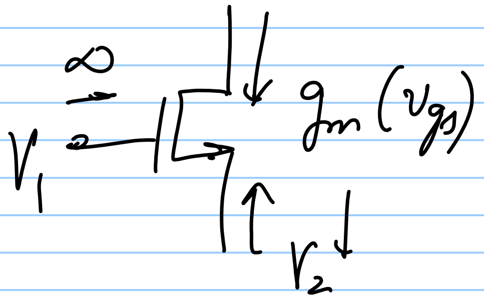
1) large gain
2) Differential input
3) Single output
4) $Z_{in} = \infty$

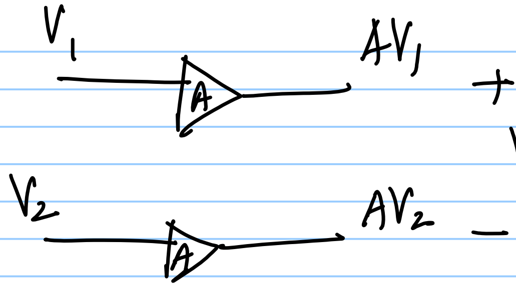
$$V_o = A\,(V_1 - V_2)$$

V_1	V_2	V_o
1	2	$-A(1)$
5	6	$-A(1)$

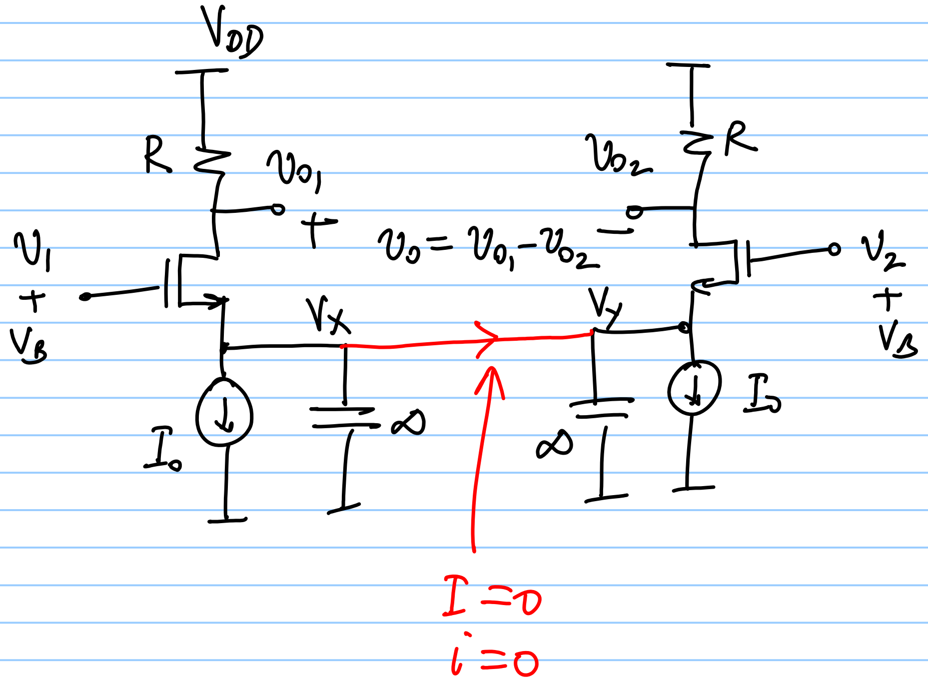
$$g_m\, r_{ds} \approx 50$$







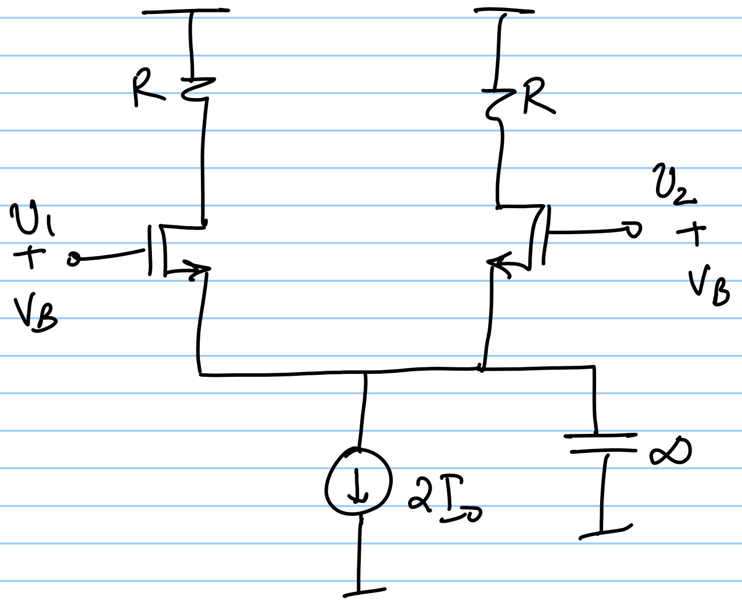
$V_o = A(V_1 - V_2)$



$V_X = V_B - V_{GS_1}$

$V_Y = V_B - V_{GS_2} = V_X$

$v_x = v_y = 0$



$$\underline{V_1 \ \& \ V_2}$$
$$\downarrow$$

$$V_1 = \underbrace{\frac{V_1 + V_2}{2}}_{\text{average or common mode voltage}} + \underbrace{\frac{V_1 - V_2}{2}}_{\text{differential mode voltage}}$$

$$V_2 = \frac{V_1 + V_2}{2} - \frac{V_1 - V_2}{2}$$

average or common mode voltage

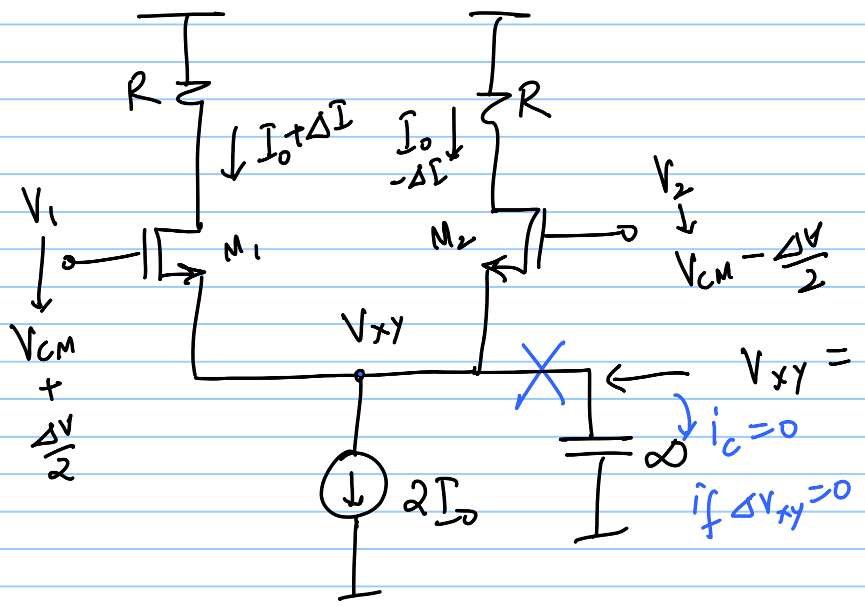
differential mode voltage

$$V_1 = V_{CM} + \frac{\Delta V}{2} \leftarrow V_{DM}$$

$$V_2 = V_{CM} - \frac{\Delta V}{2}$$

17/10/17

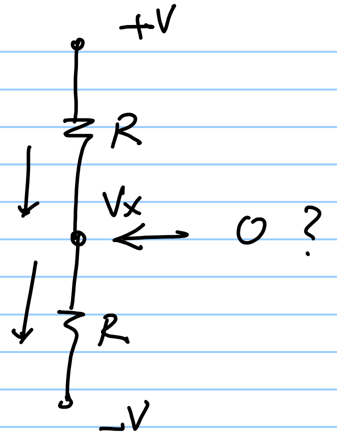
Lec 15



$V_{xy} = V_{CM} - V_{GS_{1,2}}$

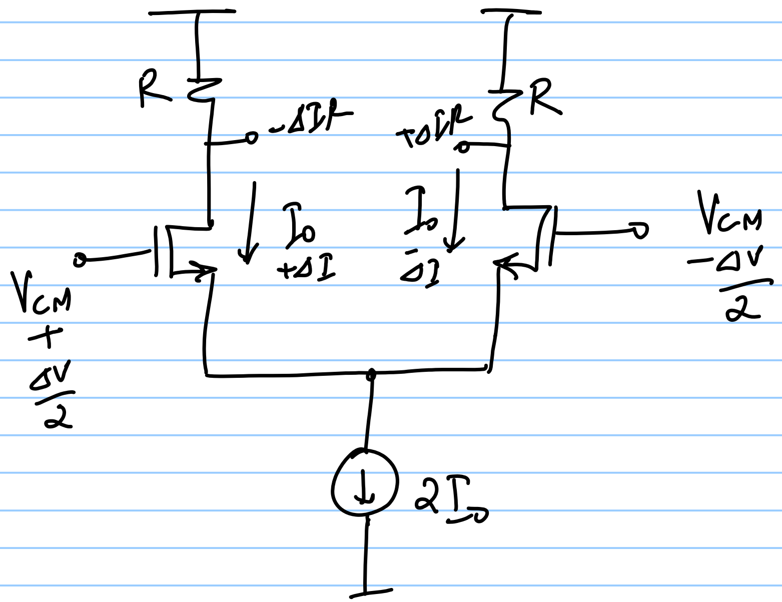
if $\Delta V = 0$

$$\left.\begin{aligned} \Delta I &= g_m\left(\frac{\Delta V}{2} - \Delta V_{xy}\right) \\ -\Delta I &= g_m\left(-\frac{\Delta V}{2} - \Delta V_{xy}\right) \end{aligned}\right) \quad \Delta V_{xy} = 0$$



$$\frac{\cancel{V} - V_x}{R} = \frac{V_x - \cancel{(-V)}}{R}$$

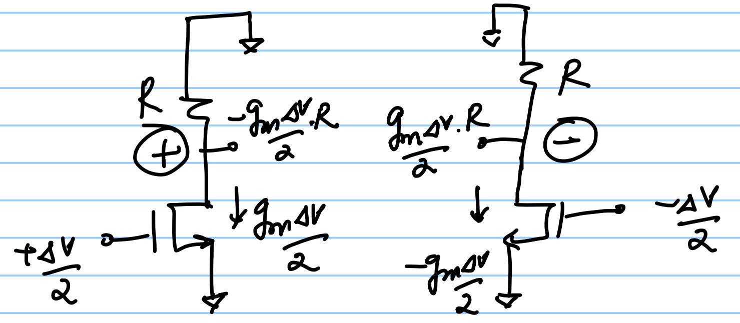
$$\frac{2V_x}{R} = 0 \Rightarrow V_x = 0$$



CM analysis ← same voltages and currents on both sides

DM analysis ← V_s & I_s on either side are equal in magnitude & opposite in direction

Half circuit analysis

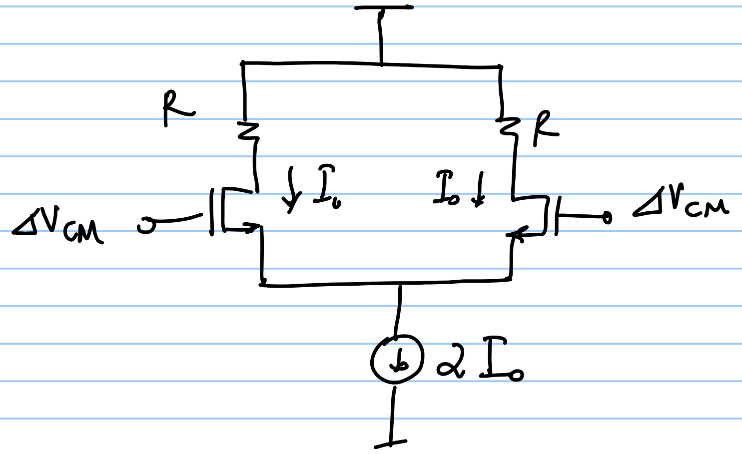


$g_m R >> 1$

$g_m >> \frac{1}{R}$

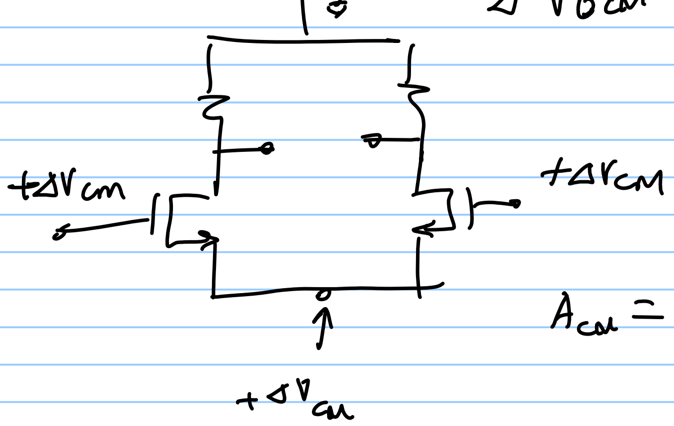
$\Delta V_o = -g_m R \Delta V$

$A_{DM} = \frac{\Delta V_o}{\Delta V} = -g_m R$



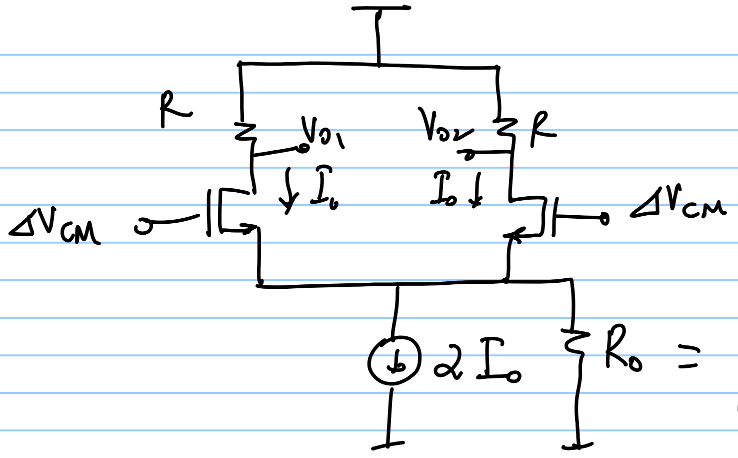
→

$\Delta V_{ocm} = 0$

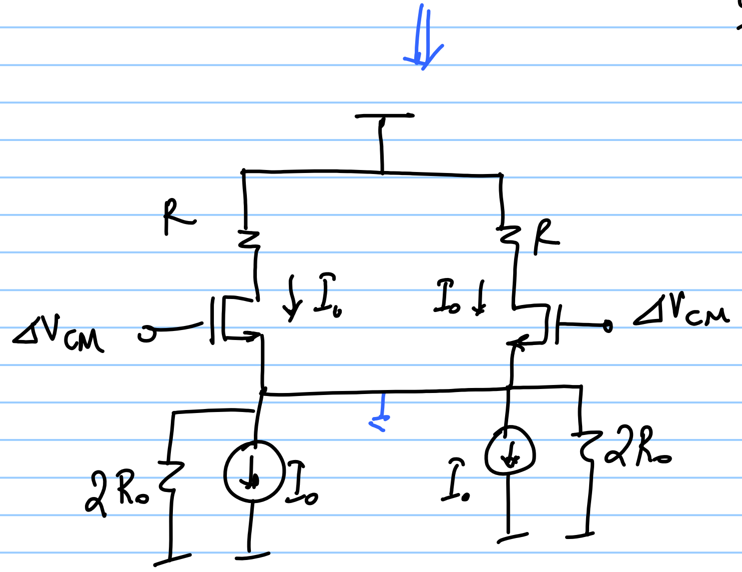


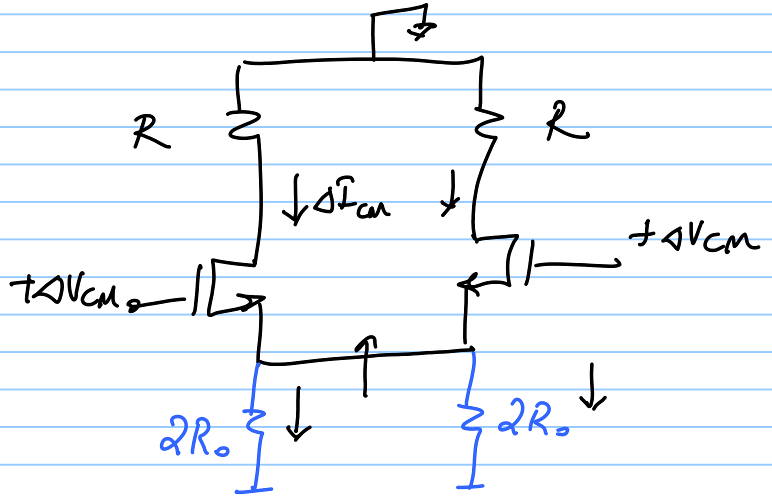
$A_{cM} = 0$

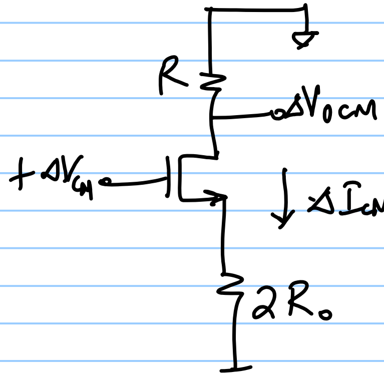
CMRR = Commm-mode rejection ratio = $\left| \frac{A_{DM}}{A_{CM}} \right| = \infty$ (ideally) for this circuit



R_o = incremental output resistance of $2I_o$ C.S.







$$\downarrow \Delta I_{CM} = \frac{g_m}{1+2g_mR_o}\cdot\Delta V_{CM}$$

$$\Delta V_{OCM} = -\Delta I\cdot R$$

$$= \underbrace{\frac{-g_mR}{1+2g_mR_o}}_{A_{CM}}\cdot\Delta V_{CM}$$

$$CMRR = (1+2g_mR_o)$$

$$V_1 = V_{CM} + \frac{\Delta V}{2} + \Delta V_{CM}$$

$$V_2 = V_{CM} - \frac{\Delta V}{2} + \Delta V_{CM}$$

$$V_{OCM} = \frac{V_{o1}+V_{o2}}{2}$$

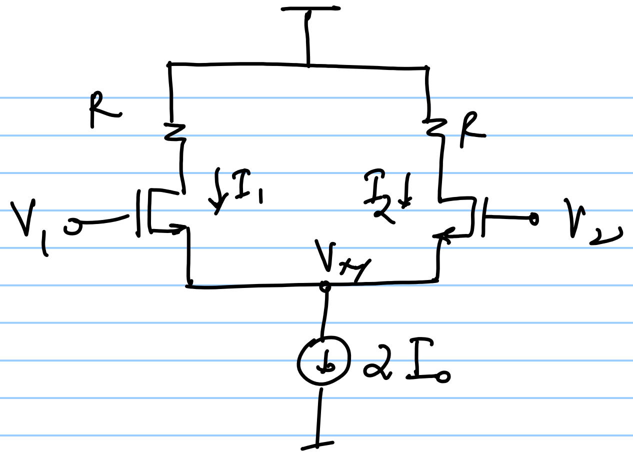
$$\frac{\Delta V_o}{2} = \frac{V_{o1}-V_{o2}}{2}$$

$$V_1 = 5V + 1 \sin\omega t$$

$$V_2 = 6V + 2 \sin\omega t$$

$$V_{CM} = 5.5V + 1.5 \sin\omega t$$

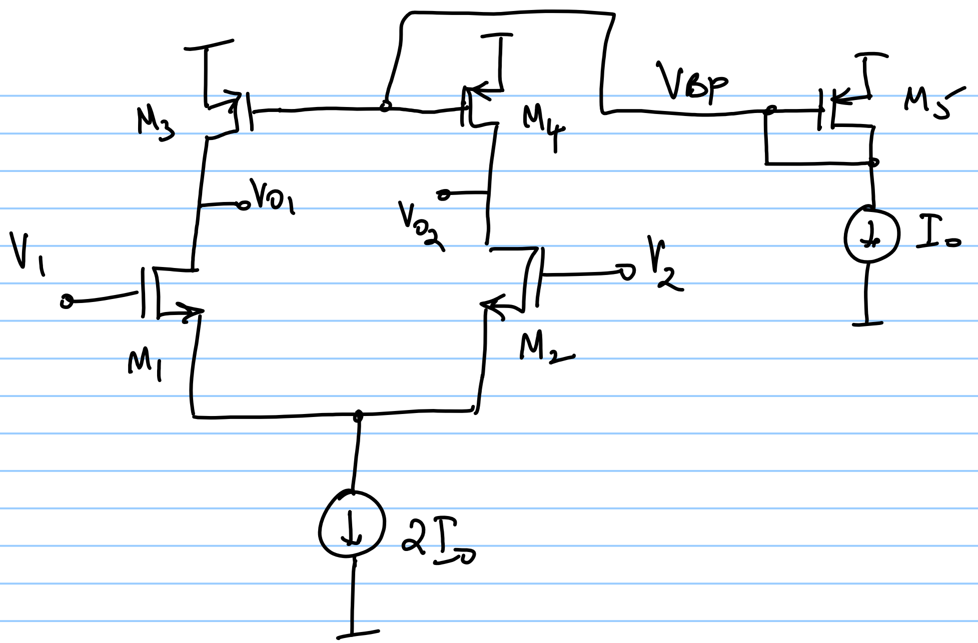
$$V_{DM} = -0.5V - 0.5 \sin\omega t$$



$$I_1 + I_2 = 2I_0$$

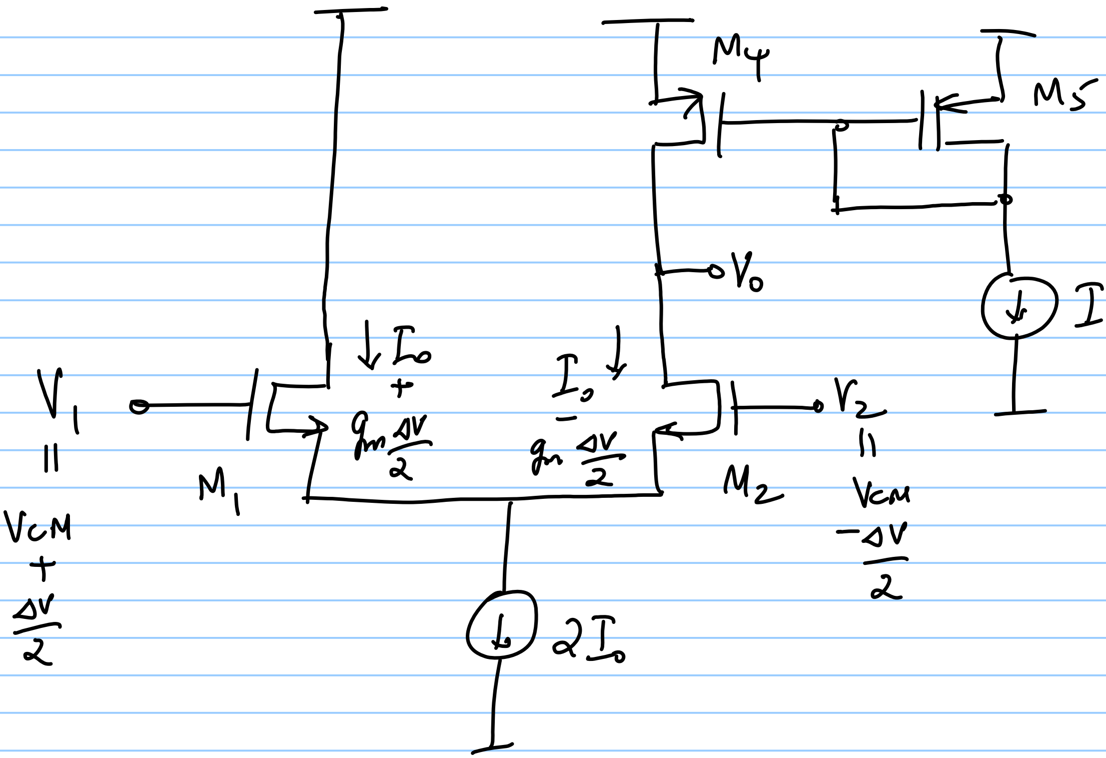
$$I_1 = \frac{1}{2} k' \left(V_1 - V_{xy}\right)^2$$

$$I_2 = \frac{1}{2} k' \left(V_2 - V_{xy}\right)^2$$



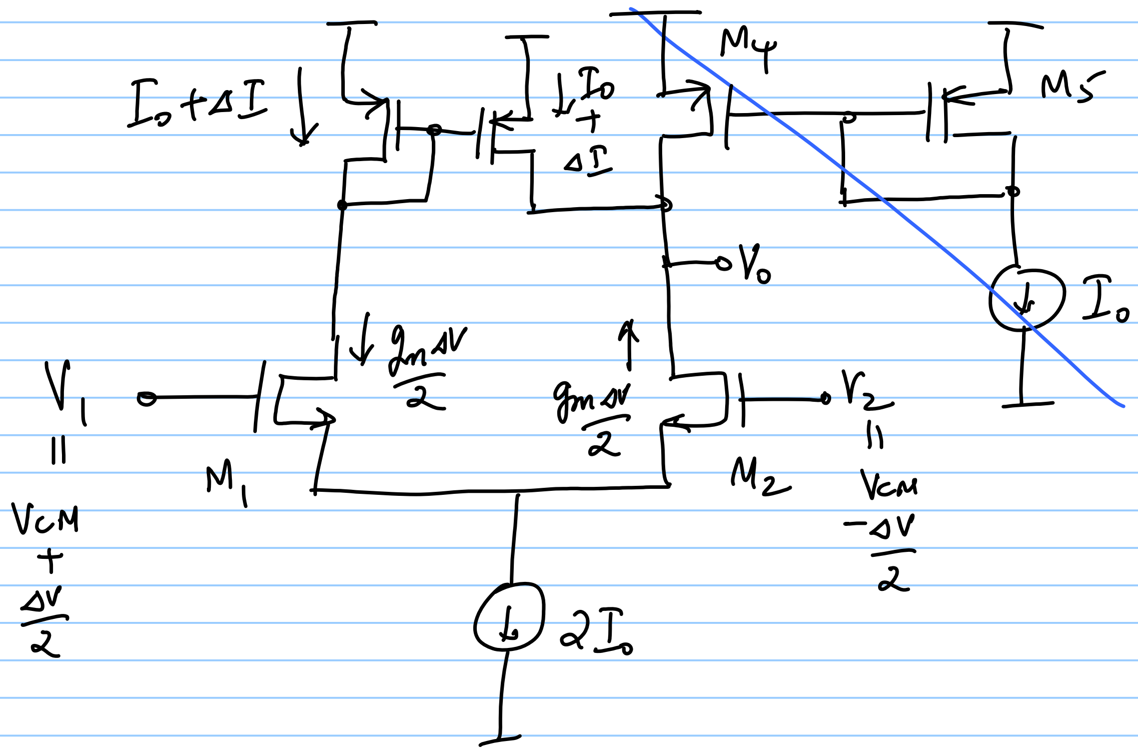
$A_{DM} = -g_m (r_{ds_1} || r_{ds_3})$

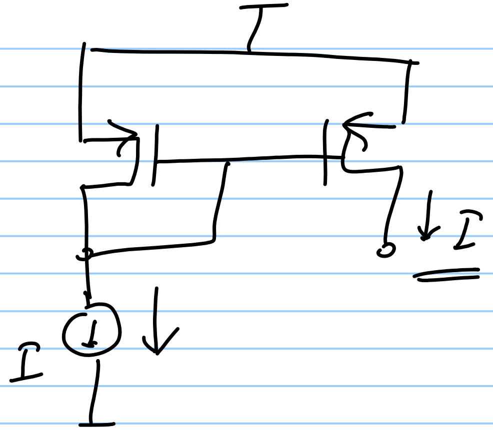
large gain



$$\Delta V_o = g_m \left(r_{ds_2} || r_{ds_4} \right) \frac{\Delta V}{2}$$

$$A_{DM} = \frac{g_m \left(r_{ds_2} || r_{ds_4} \right)}{2}$$


I₀+ΔI
I₀ + ΔI
M₄
M₅
V₀
I₀
V₁ = V_CM + ΔV/2
M₁
gm ΔV/2
gm ΔV/2
M₂
V₂ = V_CM − ΔV/2
2I₀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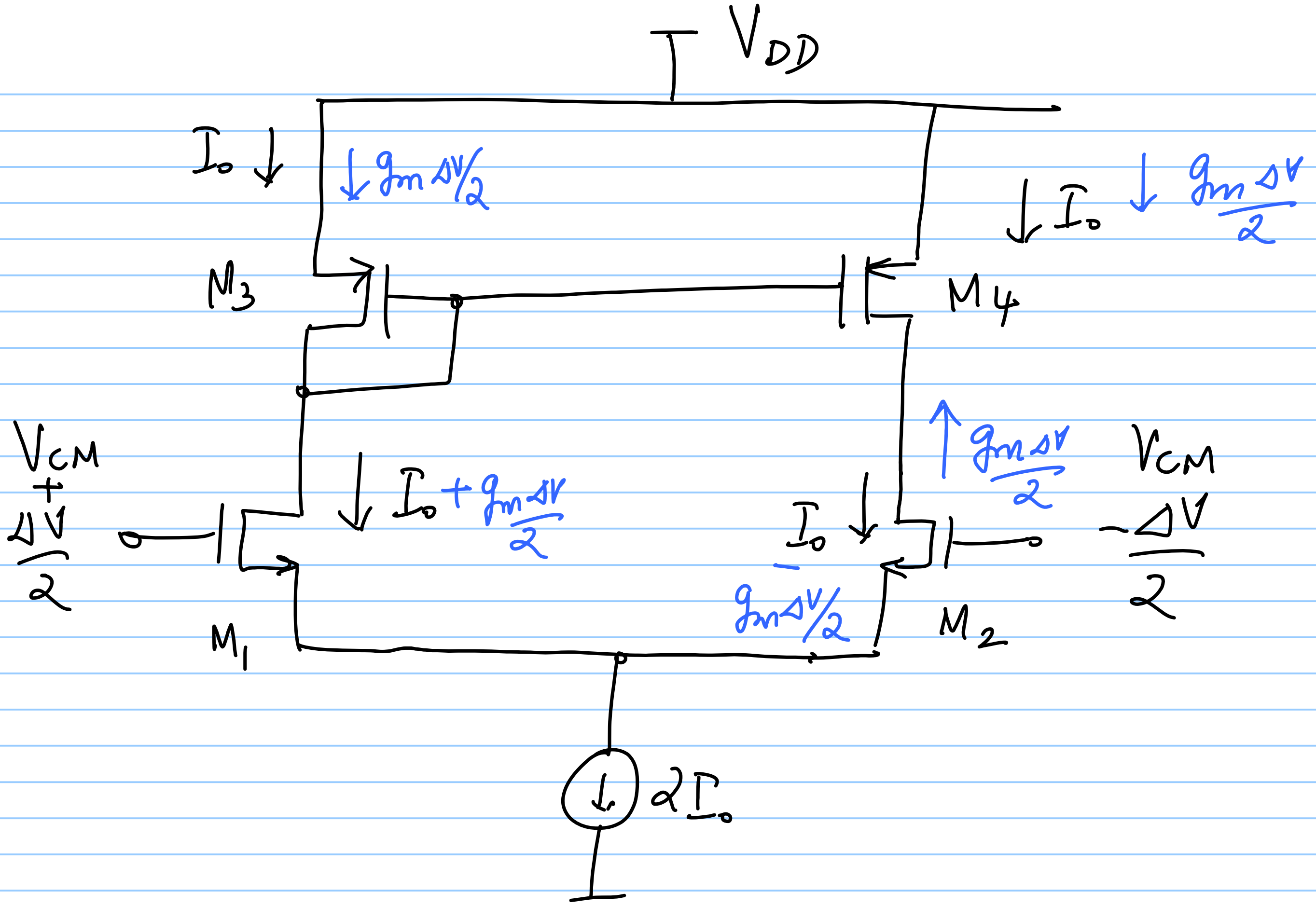


Incremental analysis

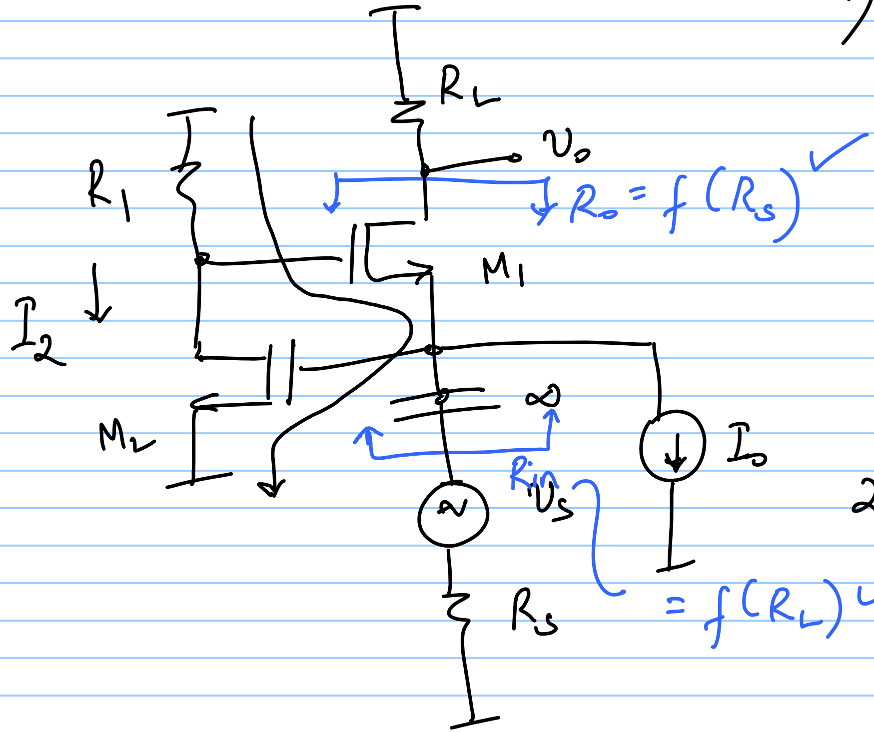
— H.W.

$A_{DM} = g_m (r_{ds_2} \| r_{ds_4})$

Single Stage Opamp

25/10/17

Quiz 2 Discussion



1) V_{GS_1} & V_{GS_2}

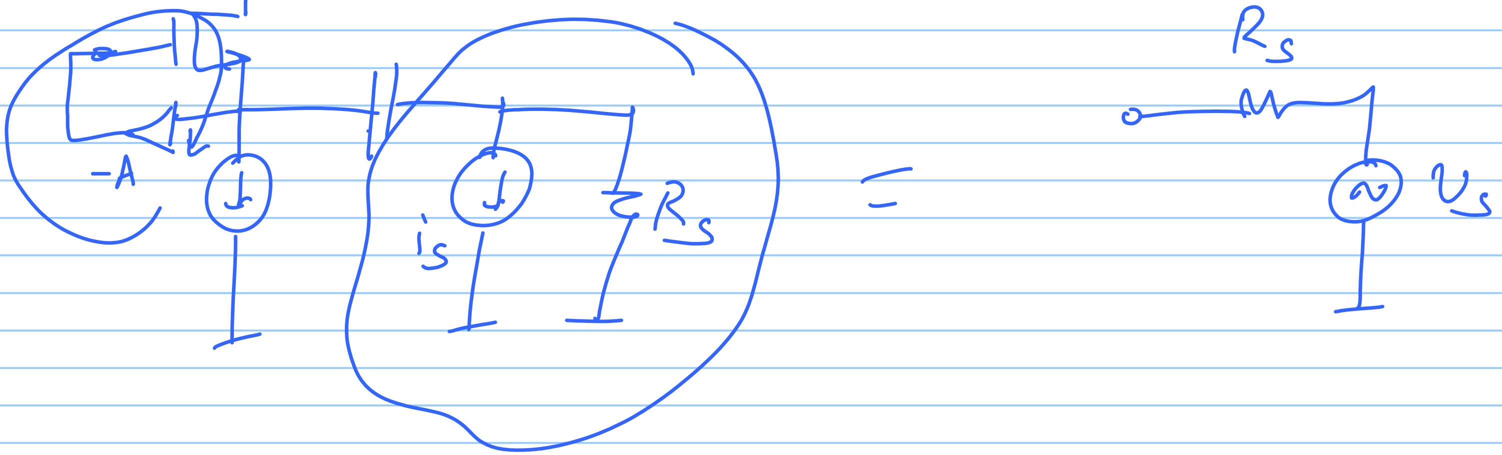
$$\downarrow \qquad \downarrow$$

$$V_{DD} = I_2 R_1 + V_{GS_1} + V_{GS_2}$$

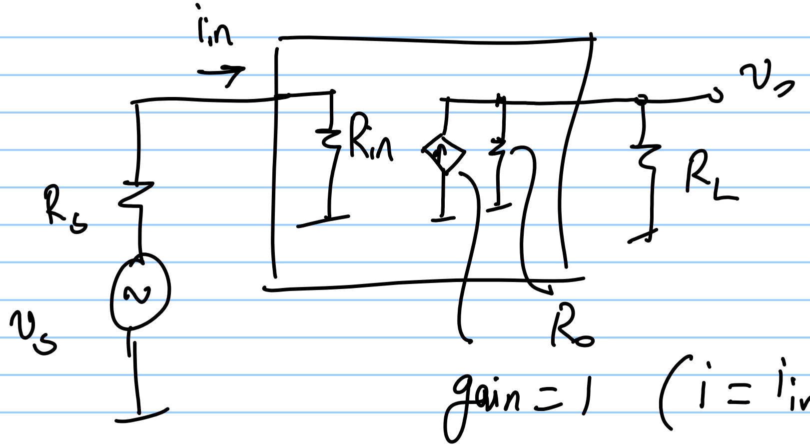
$$I_2 = \frac{1}{2} \mu C_{ox} \left(\frac{W}{L}\right) \left(V_{GS_2} - V_T\right)^2$$

2) R_{in} & R_{out}

$$R_o = (g_m r_{ds})\, R_s \longrightarrow (g_m r_{ds})\, R_s \underbrace{(\qquad)}_{\text{loop gain}}$$



$$\frac{1}{g_m} \longrightarrow \frac{1}{g_m} \cdot \frac{1}{\underbrace{(\qquad)}_{\text{loop gain}}}$$



gain = 1 $(i = i_{in})$

g_m

r_{ds}

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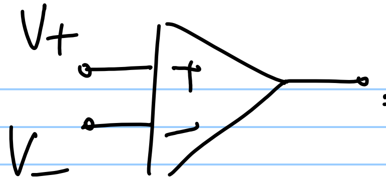
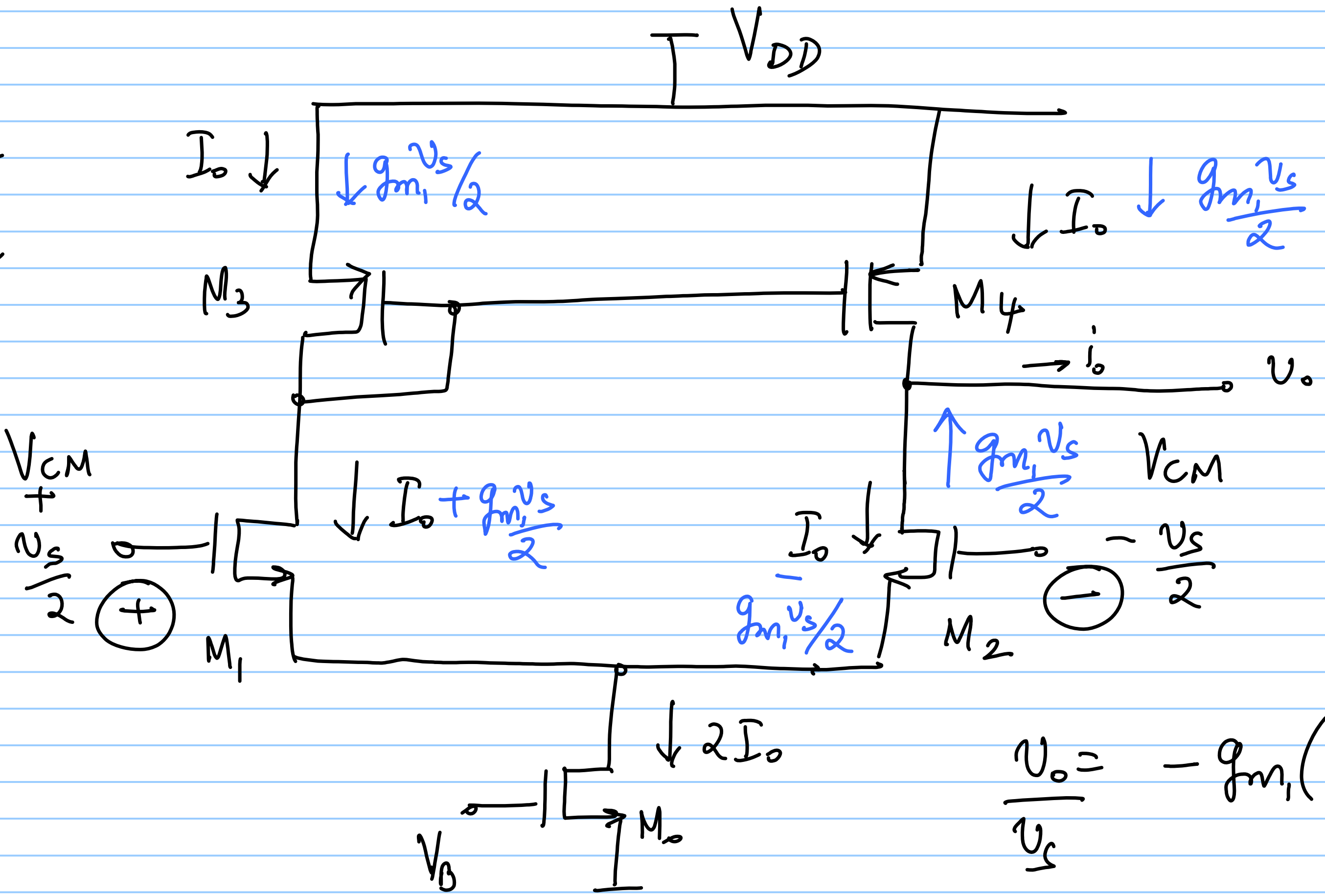


$A(V_+ - V_-)$ =

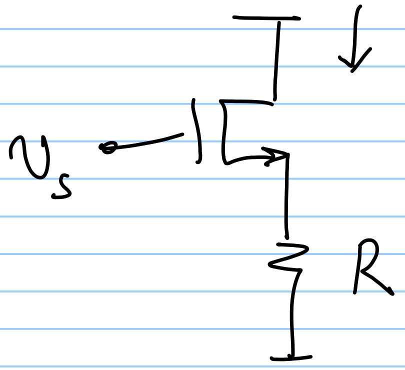
One Stage Opamp



$i_o = g_{m_1} v_s$ flows through r_{ds_2} & r_{ds_4} (parallel combo)

$$\frac{v_o}{v_s} = -g_{m_1}(r_{ds_2} \| r_{ds_4})$$

DC or low-freq. gain of opamp



$$i_o = \frac{v_s}{R}$$

$R = 1M\Omega$

$= 1k\Omega$

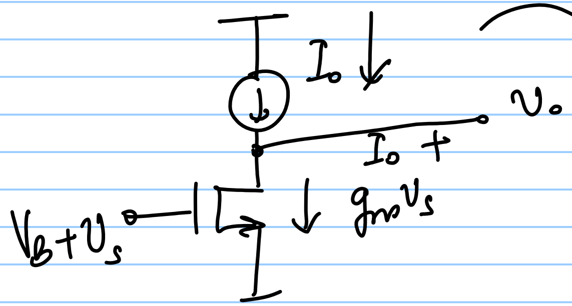
1Ω

$$i_o = \frac{g_m}{1 + g_m R} \cdot v_s$$

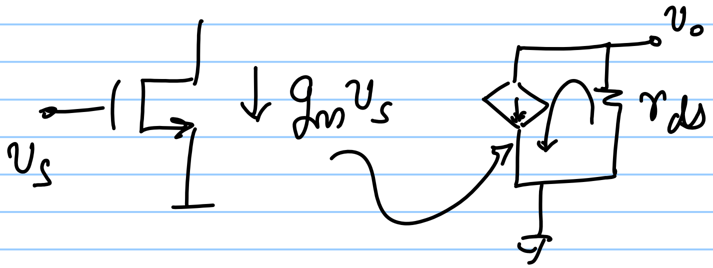
$g_m \to \infty$

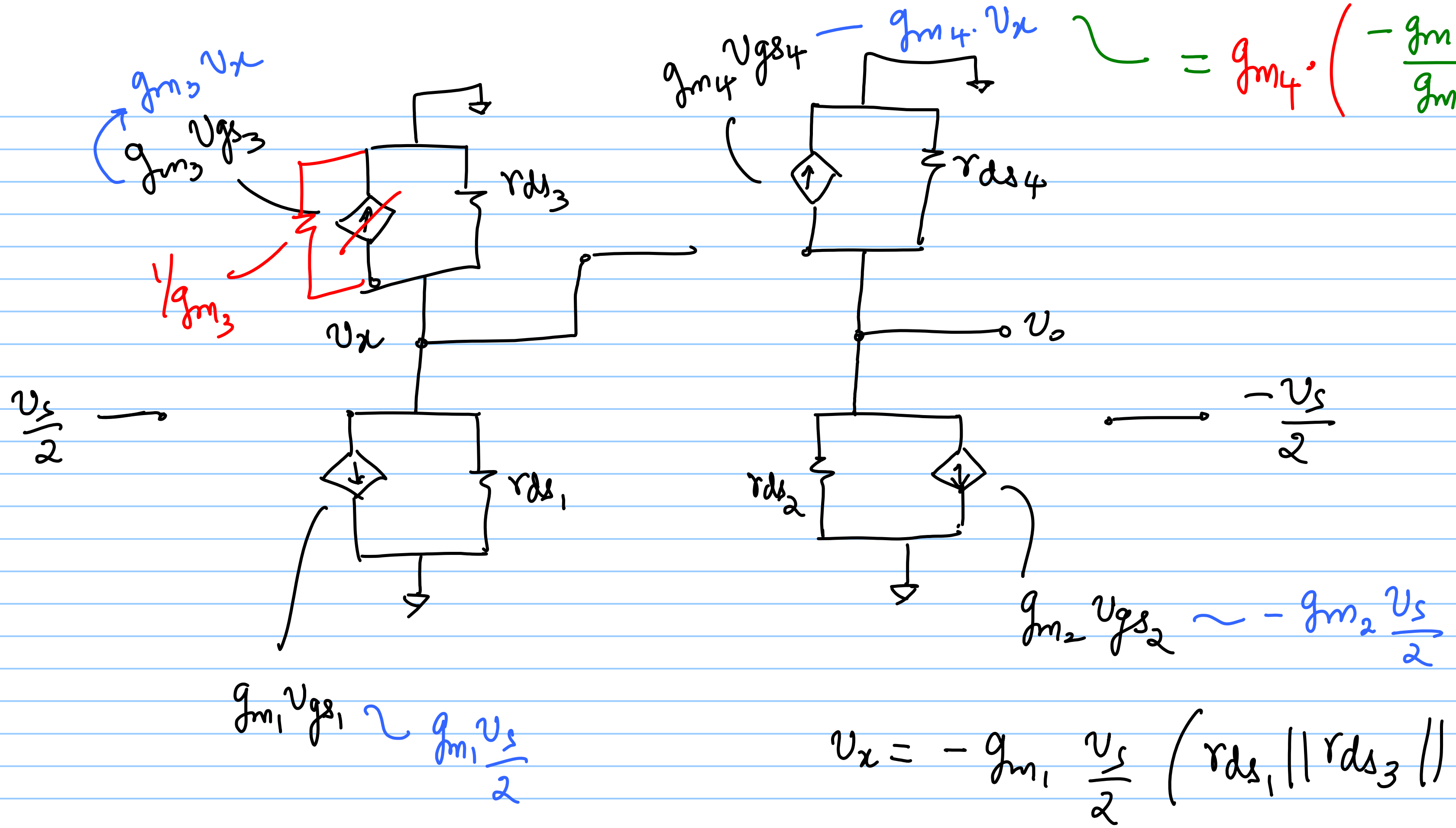
$g_m R \to \infty$

g_m



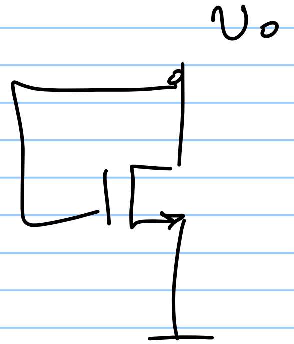
$\rightsquigarrow -g_m r_{ds}$

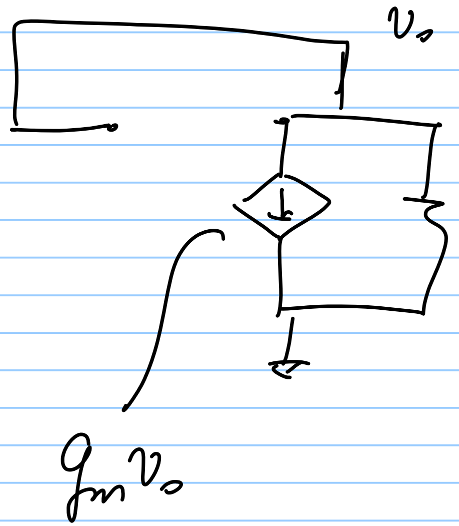


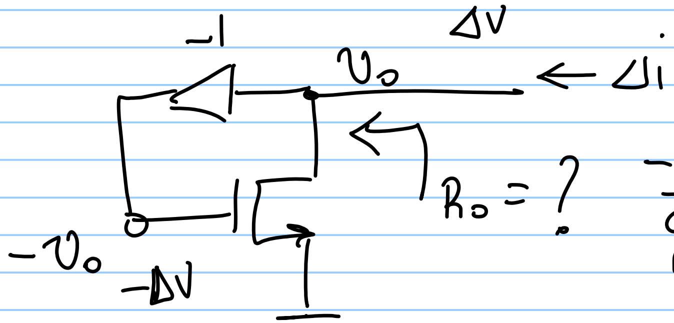


$$\sim = g_{m_4}\cdot\left(-\frac{g_{m_1}}{g_{m_3}}\cdot\frac{v_s}{2}\right) = -\frac{g_{m_1}v_s}{2}$$

$$v_x = -g_{m_1}\frac{v_s}{2}\left(r_{ds_1}\,||\,r_{ds_3}\,||\,\frac{1}{g_{m_3}}\right) \approx -\frac{g_{m_1}}{g_{m_3}}\cdot\frac{v_s}{2}$$







$R_o = ?$ $\frac{-1}{g_m}$

$i = g_m \cdot v_o \Rightarrow R_o = \frac{1}{g_m}$

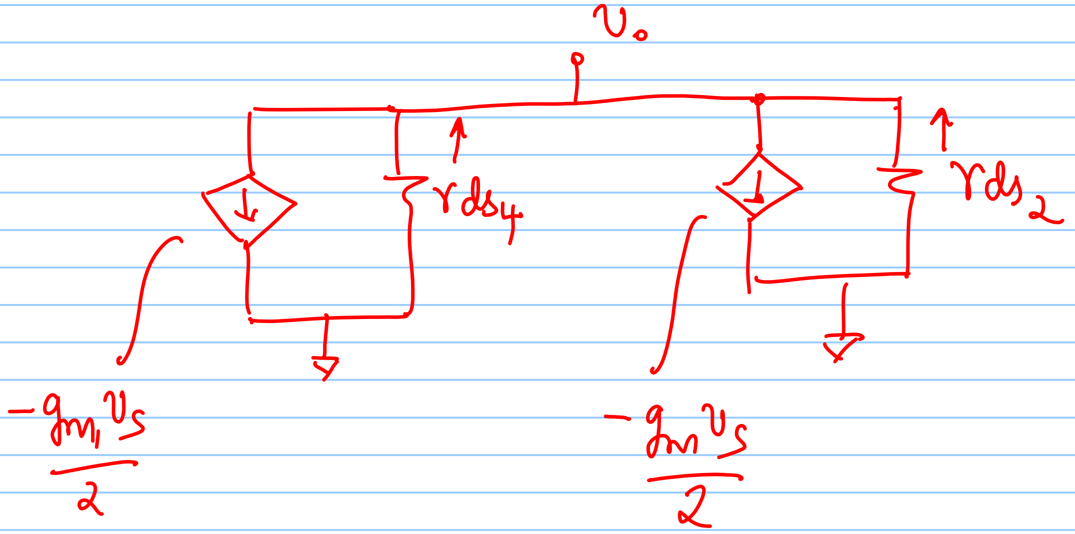
$\frac{\Delta V}{\Delta i} = R_o$

$$v_o = g_{m_1} v_s \left(r_{ds_2} \| r_{ds_4} \right)$$

$M_1 = M_2$

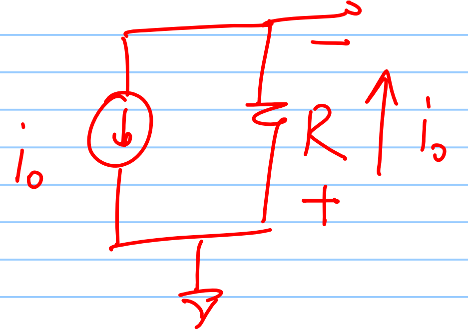
$M_3 = M_4$

$$\frac{v_o}{v_s} = g_{m_1} \left(r_{ds_2} \| r_{ds_4} \right)$$



$$v_o = \cancel{-}\left(\cancel{-} g_m v_s\right)\left(r_{ds_2} \| r_{ds_4}\right)$$

$$v_o = -i_o R$$



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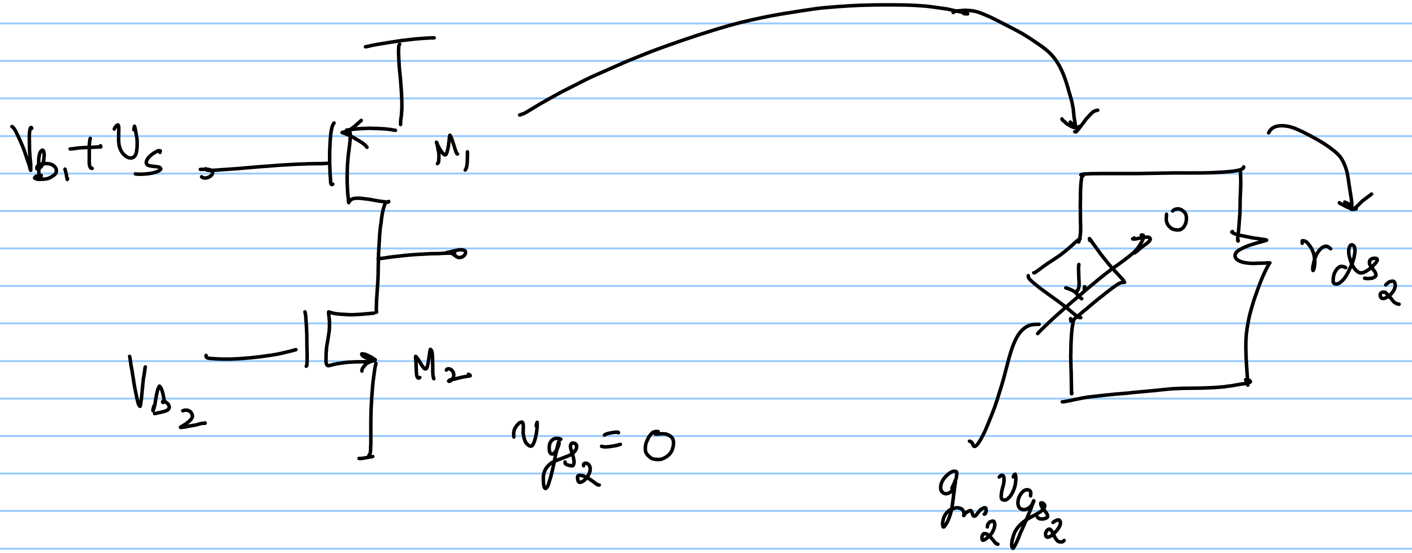
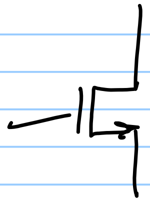
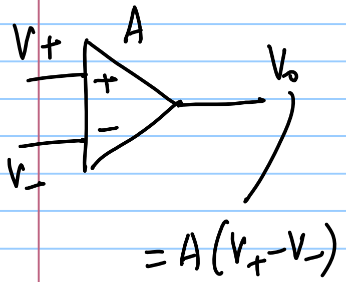
Lec 17

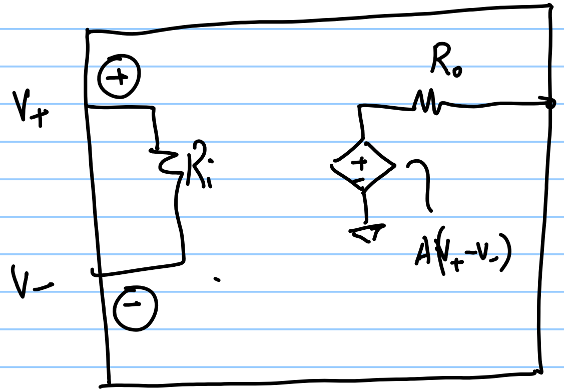
CM analysis of 1-stage opamp – H.W. exercise



1) Frequency behaviour of opamp → affects freq. response of closed loop amplifier





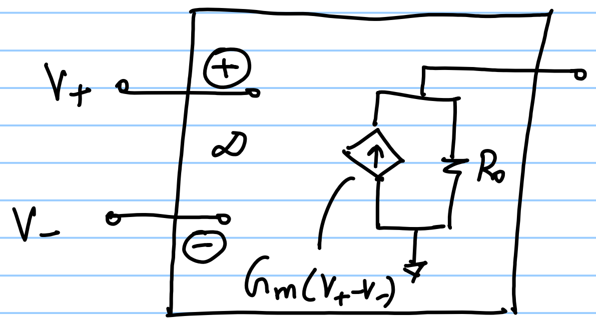


$R_i = \infty$

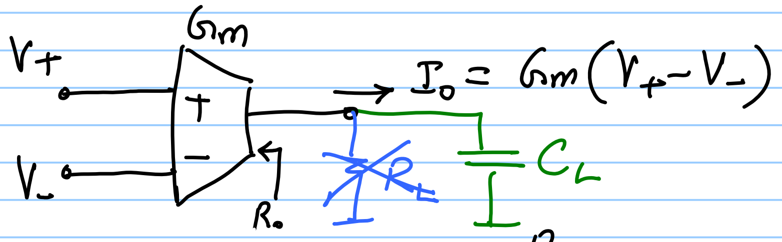
$A = g_{m_1}(r_{ds_2} \| r_{ds_4})$

$R_o = r_{ds_2} \| r_{ds_4}$ (large)

CMOS opamps – Operational Transconductance Amplifiers (OTA)

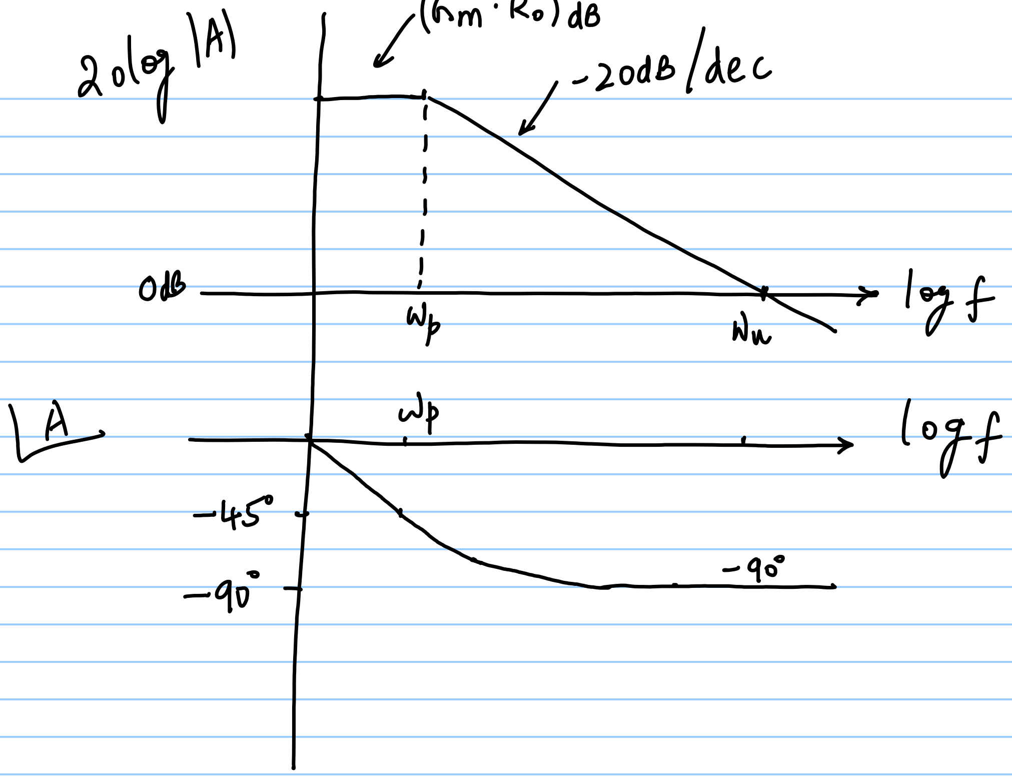


$\equiv$



ideally $R_i = \infty$; $R_o = \infty$

- * for 1-stage opamp, $G_m = g_{m_{1-2}}$
- * Do not drive resistive loads
- * Can drive capacitive loads



$\omega_u =$ unity gain frequeny $= g_m R_o \cdot \omega_p$

$\omega_p = \dfrac{1}{R_o C_L}$

$$A(s) = \frac{A_o}{1 + s/\omega_p} = \frac{g_m R_o}{1 + s/\omega_p}$$

$20\log|A(s)| = 0dB$ @ ω_u

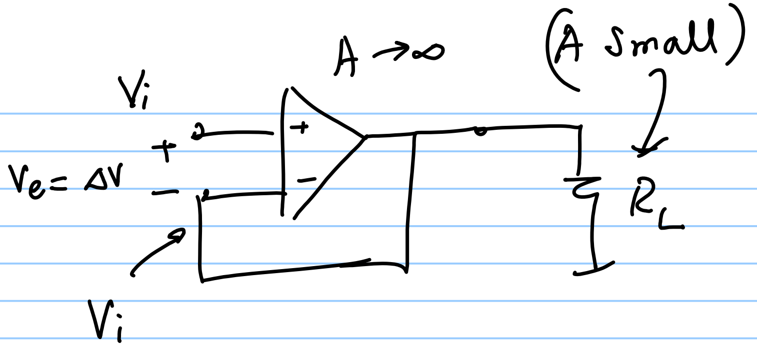
or $|A(s)| = 1$ @ ω_u

$\omega_u \gg \omega_p \Rightarrow \omega_u = g_m R_o \cdot \omega_p$

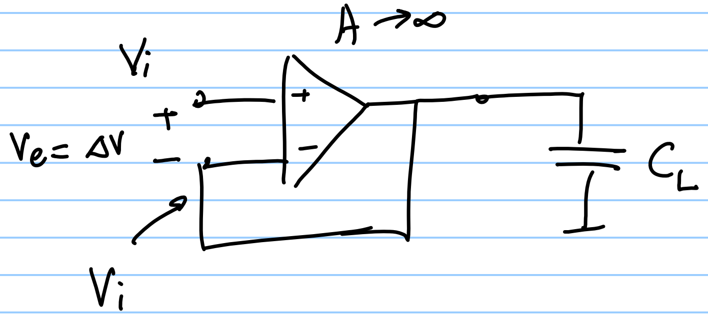
$$\frac{g_m R_o}{\sqrt{1 + \frac{\omega_u^2}{\omega_p^2}}} = 1$$

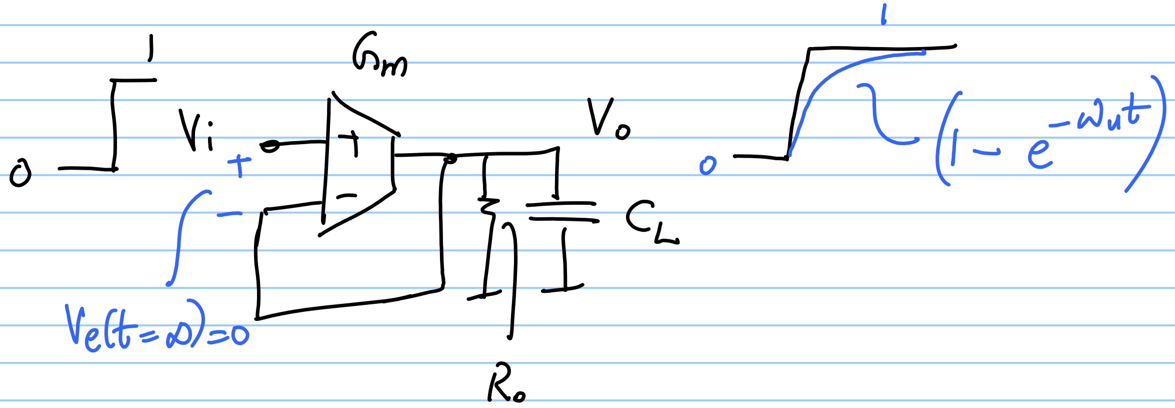
$$\angle A(s) = -\tan^{-1}\left(\frac{\omega}{\omega_p}\right)$$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$; if θ is very small $\Rightarrow$ $\cos\theta \approx 1$, $\sin\theta \approx \theta$



✗ ✓





$A(s)$ $\quad f$

$$C.L.G. = \frac{1}{f} \cdot \frac{A(s) \cdot f}{1 + A(s) \cdot f}$$

$$A(s) = \frac{G_m R_o}{1 + s/\omega_p}$$

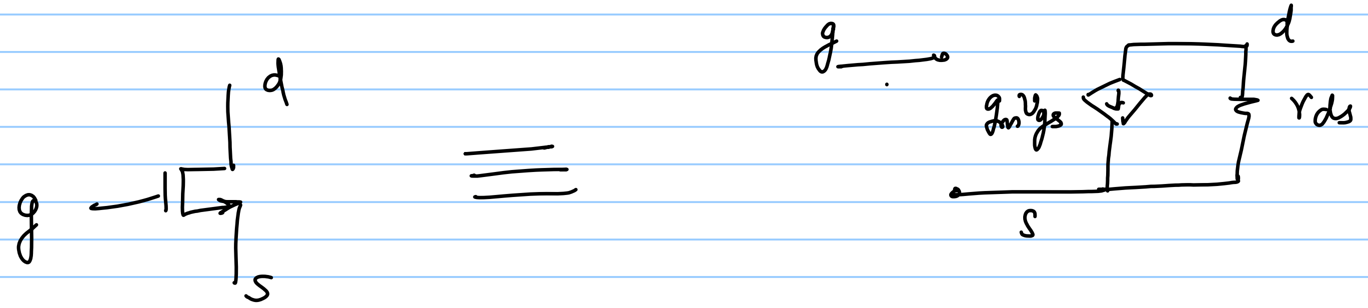
$f = 1$

$$CLG = \frac{G_m R_o / (1+s/\omega_p)}{1+ G_m R_o / (1+s/\omega_p)} = \frac{G_m R_o}{G_m R_o + 1 + \frac{s}{\omega_p}}$$

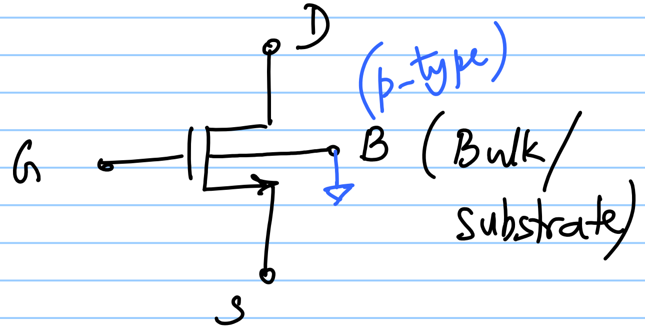
$$= \frac{G_m R_o / (1+G_m R_o)}{1+ \frac{s}{\omega_p (1+G_m R_o)}} \approx \frac{1}{1+\frac{s}{\omega_u}} \quad \text{if } G_m R_o >> 1$$

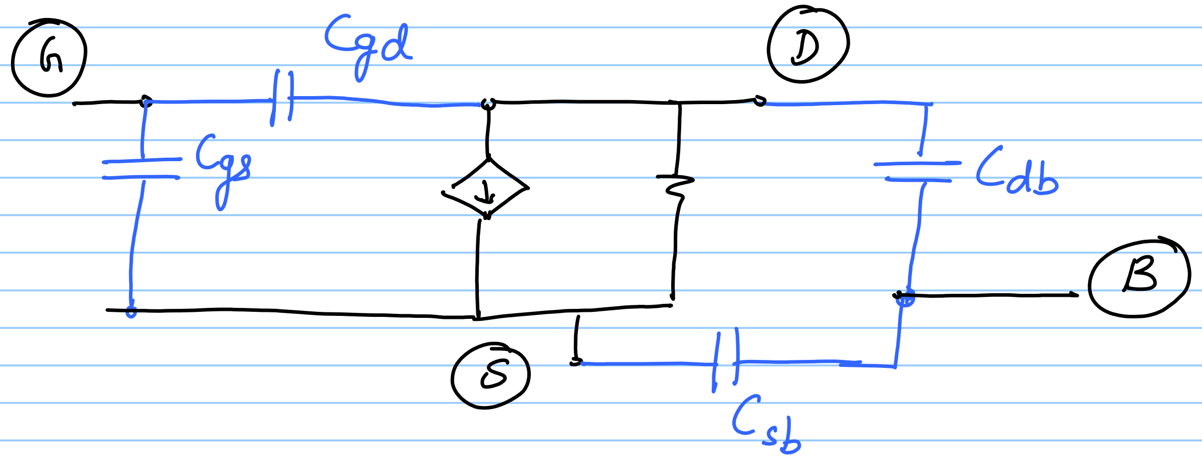
steady state error = 0

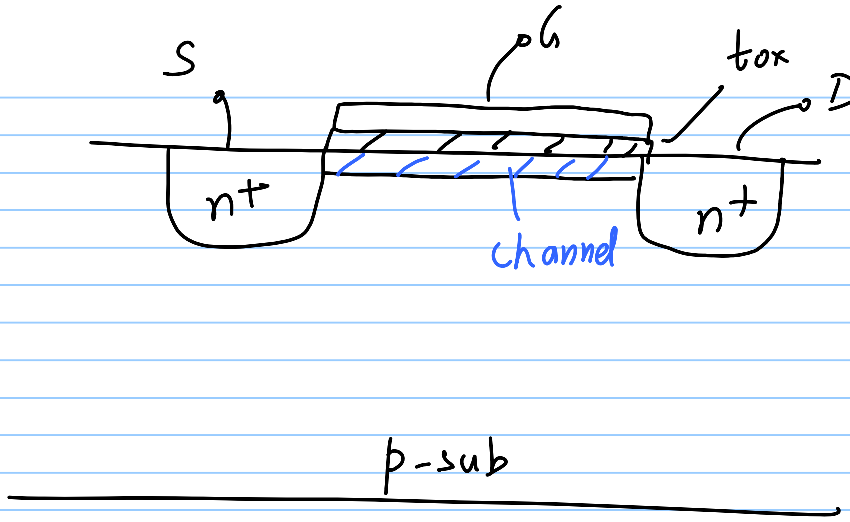
MOSFET capacitances

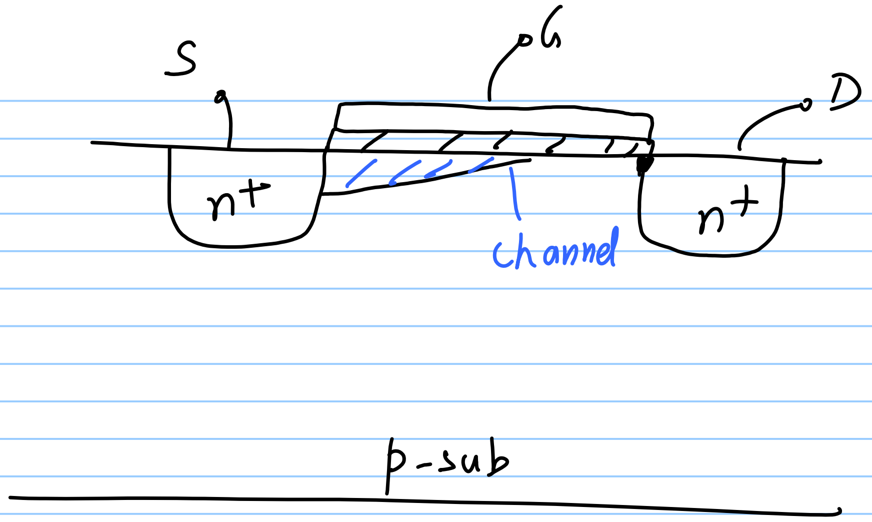


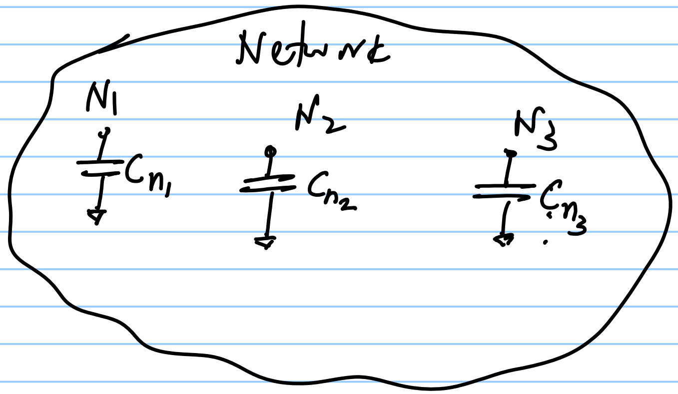
low frequency model



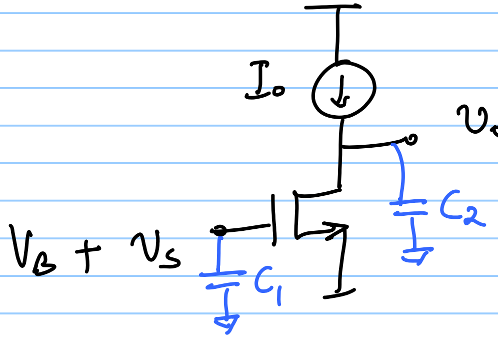




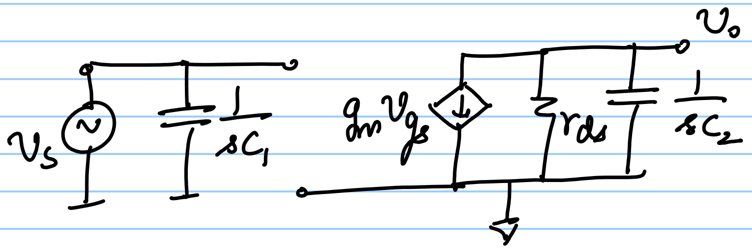




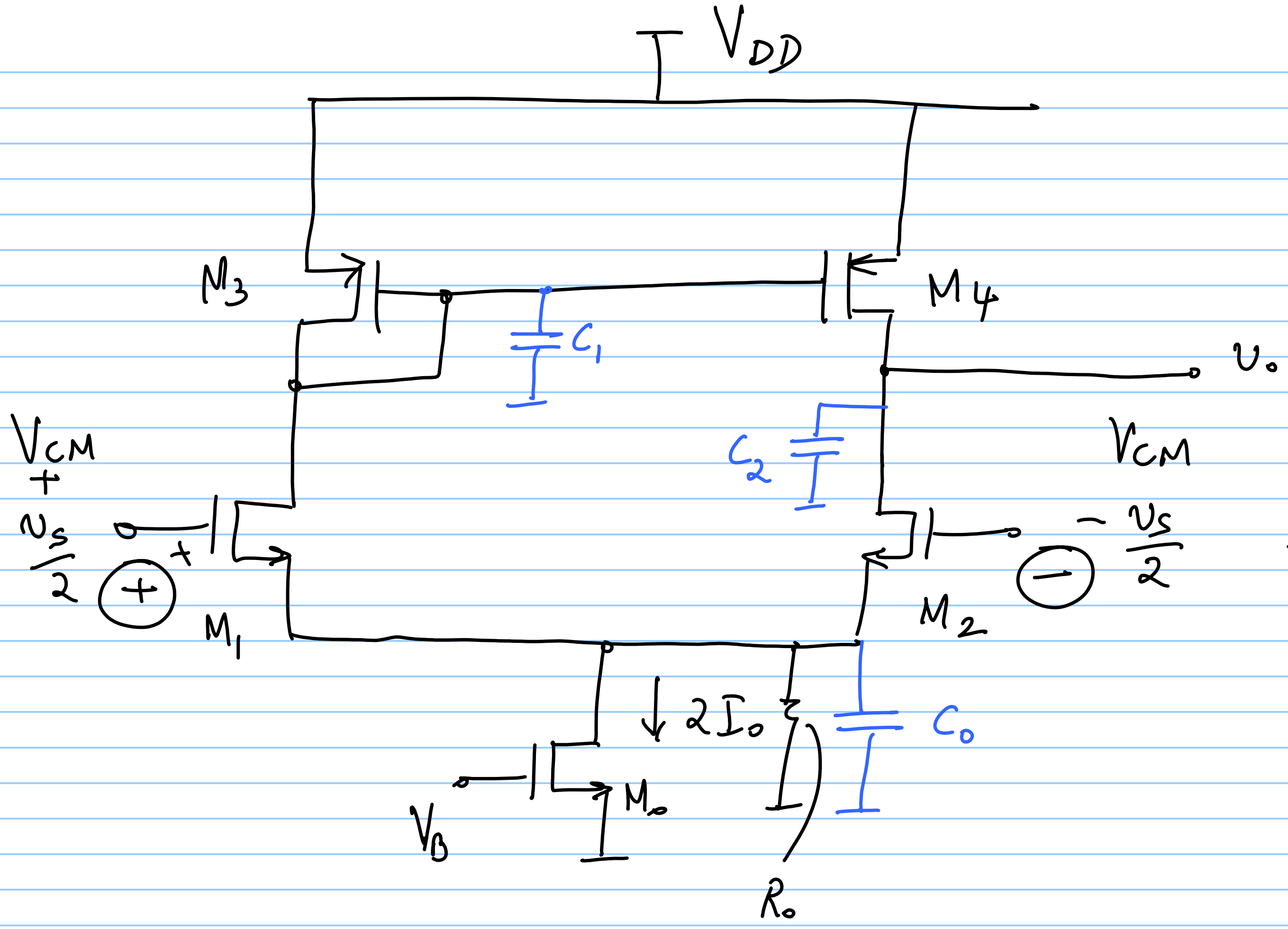
C_{gs}, C_{db} etc. are functions of W, L & C_{ox} and drain (or source) area



$$v_o = -g_m r_{ds} \cdot v_s$$



$$\frac{v_o}{v_s} = \frac{-g_m r_{ds}}{1 + s C_2 r_{ds}}$$



* C_o will not affect A_{dm}

* C_o will affect A_{cm} (degrades A_{cm} at high frequencies)

* at low freq:

$A_{cm} = 0$; $\therefore$ CMRR $= \infty$

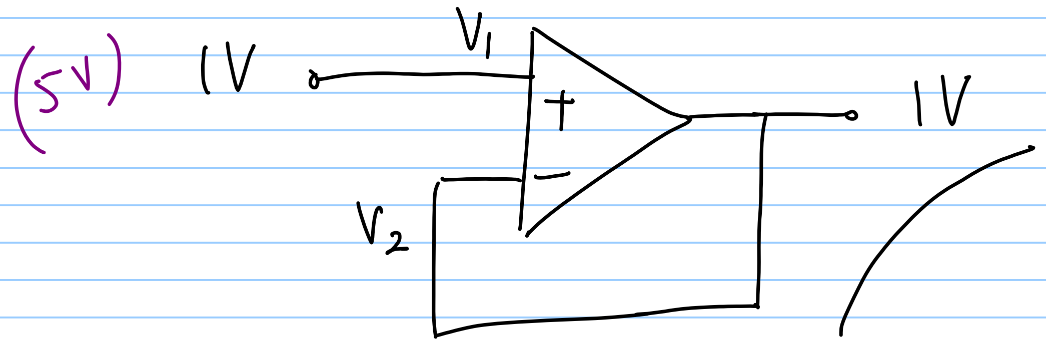
at high freq.

$A_{cm} \rightarrow A_{dm}$; CMRR $\rightarrow 1$

HW exercise

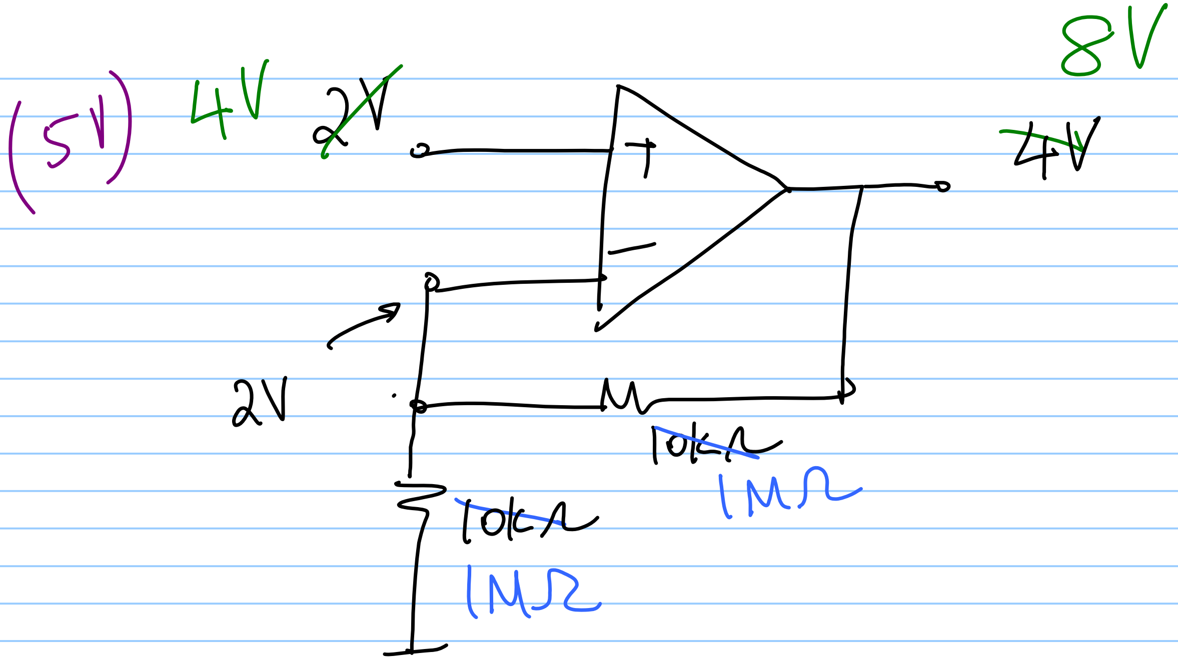
Differential gain $A(s)$

frequency response



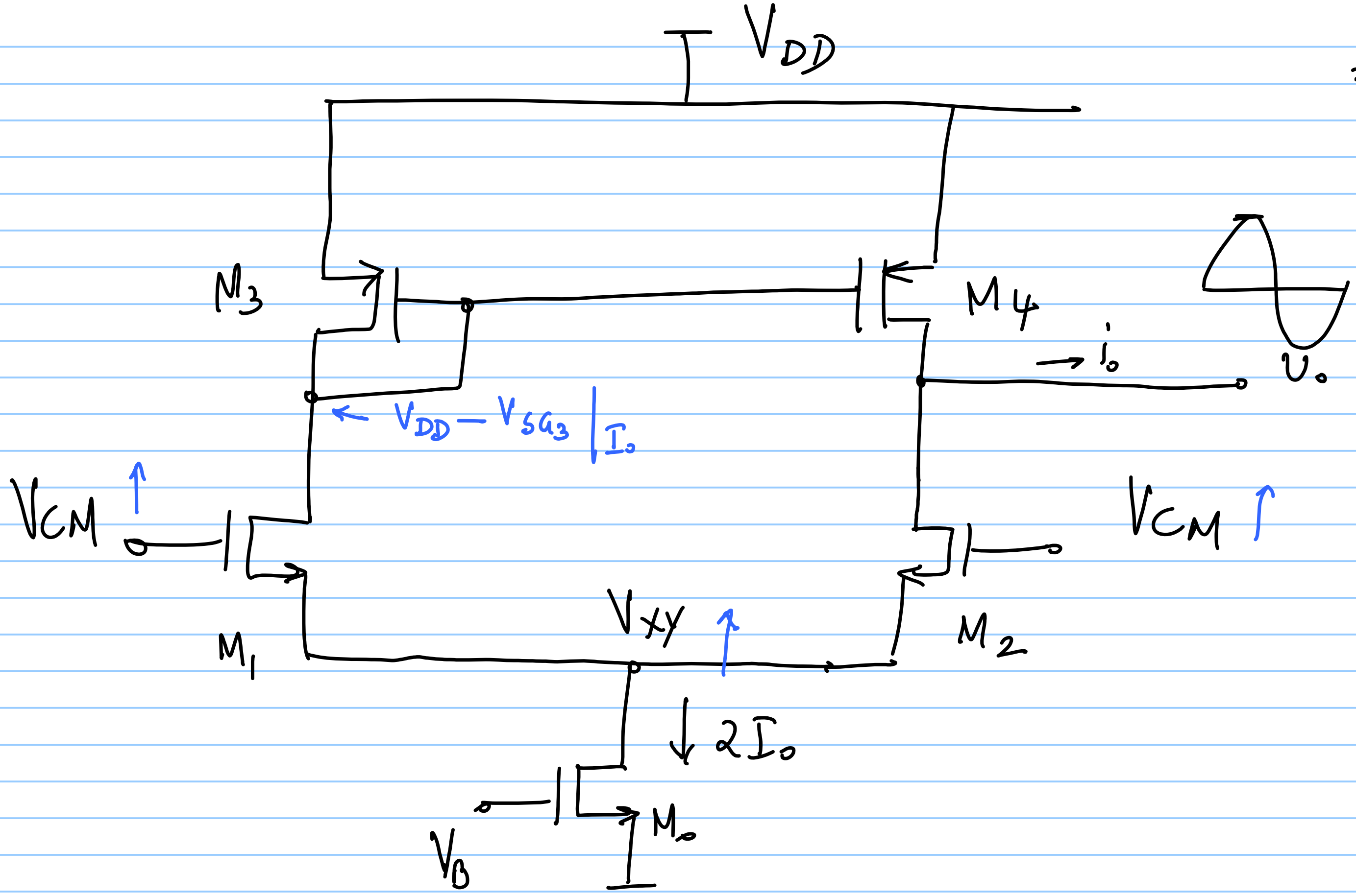
$V_{CM} = 1V$

$V_{dm} = 0V$



$V_{CM} = \cancel{2V}$ 4V

$V_{dm} = 0$



$* \downarrow V_{CM}$

$V_{xy} = (V_{CM} - V_{GS_1}) \downarrow$

at same rate till M_o goes into triode.

$$V_{CM_{min}} = V_B - V_{T_o} + V_{GS_1}$$

$$= V_{Dsat_o}\Big|_{2I_o} + V_{GS_1}\Big|_{I_o}$$

* ↑ V_{CM} : V_{G_1} increases

$$V_{D_1} = V_{DD} - V_{SG_3}\big|_{I_0} = \text{constant}$$

M_1 moves towards triode region

$$V_{CM_{max}} = V_{DD} - V_{SG_3}\big|_{I_0} + V_{T_1}$$

Input common-mode range of opamp

$$ICMR = \left\{ V_{CM_{min}} , V_{CM_{max}} \right\}$$

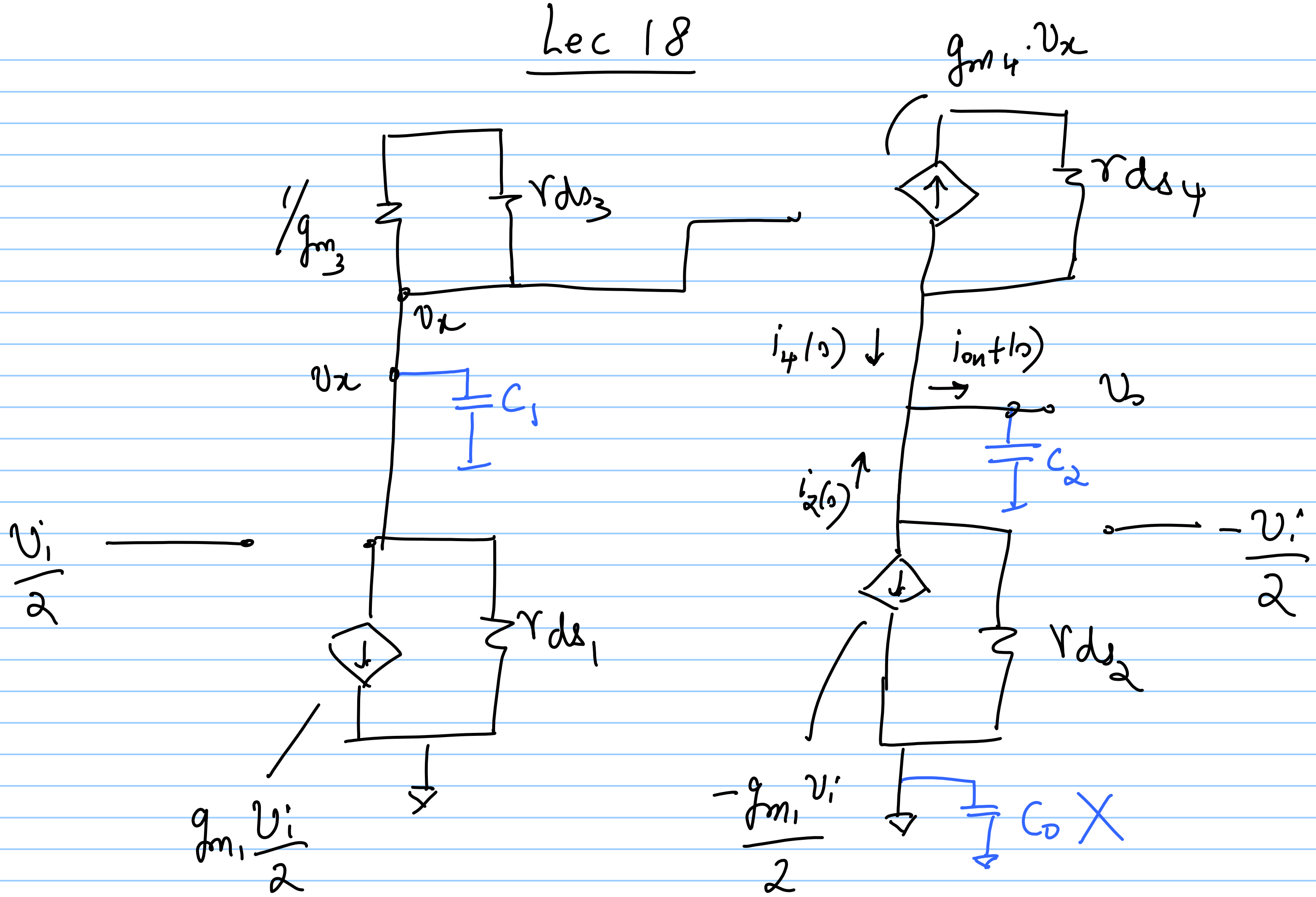
* Output swing limits: $V_{o_{max}} = V_{DD} - V_{SG_4} + V_T = V_{DD} - V_{SDsat_4}$

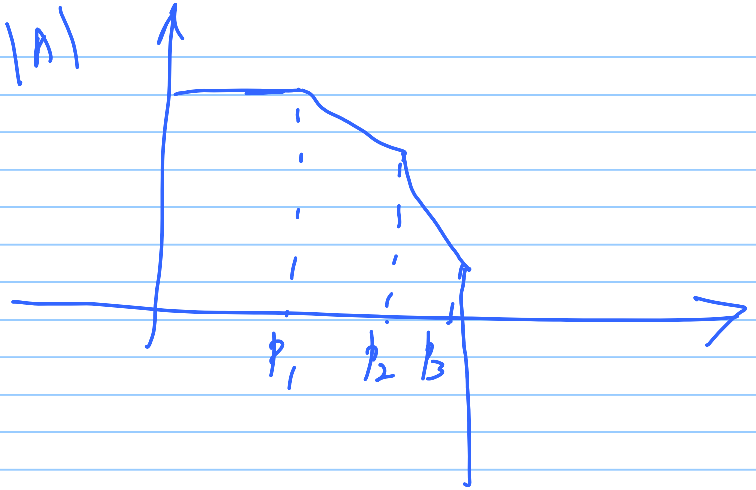
$$V_{o_{min}} = V_{CM} - V_T$$

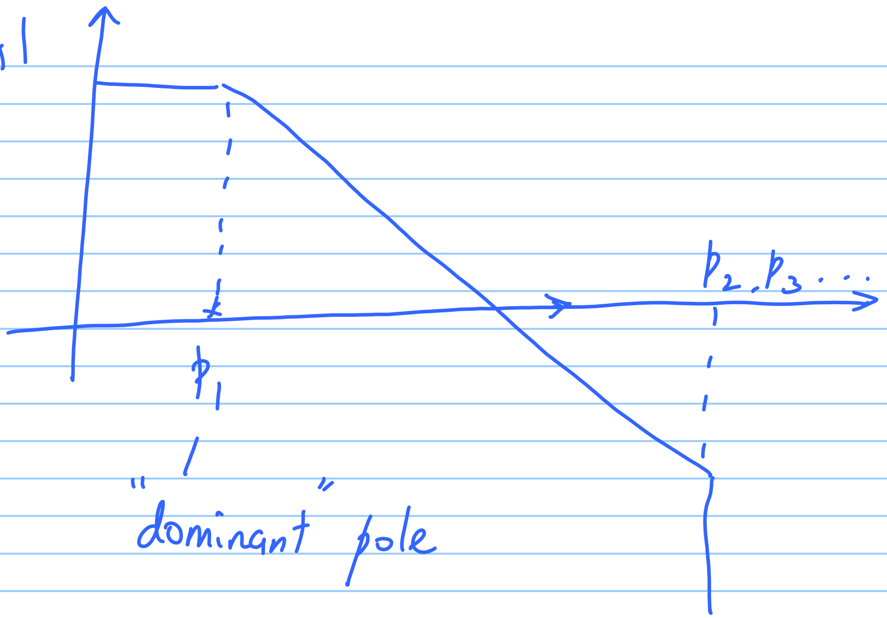
(assuming large gain)

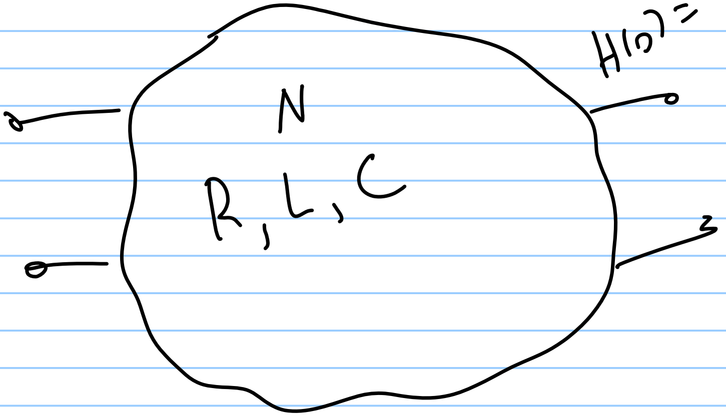
31/10/17

Lec 18







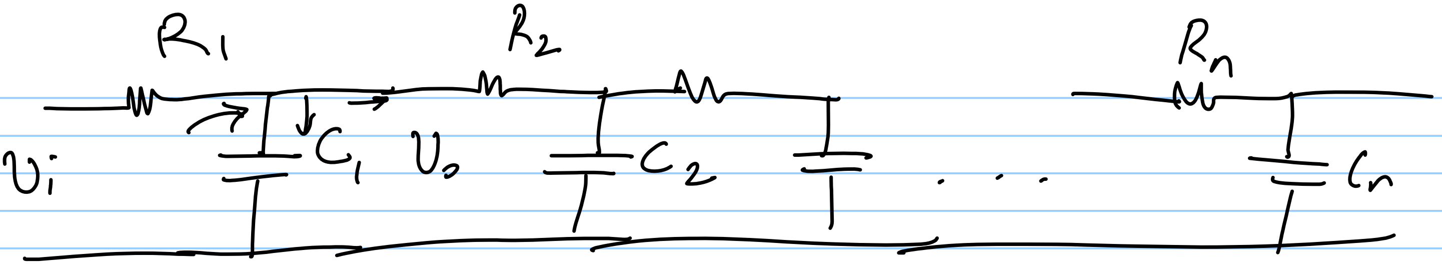


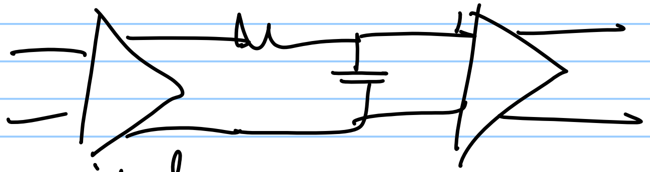
$H(s) = \frac{N(s)}{D(s)}$

\# of poles =

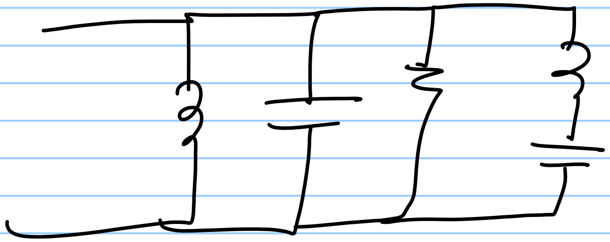
"order" of a system

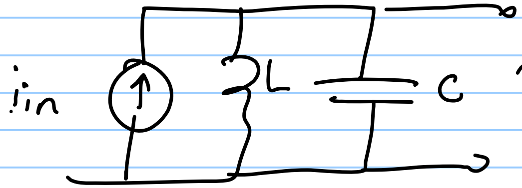
= order of $D(s)$

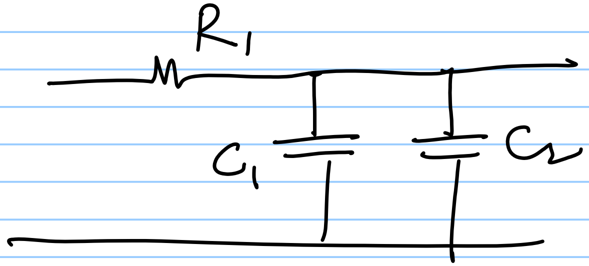


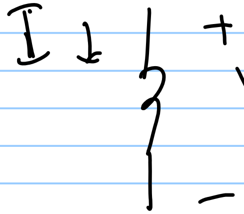


ideal
VCVS
gain=1



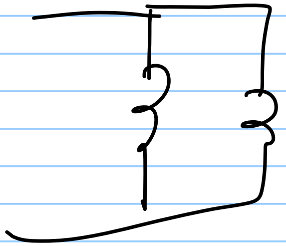


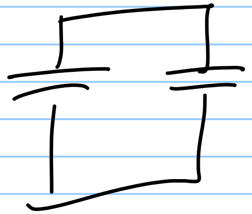


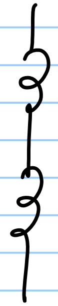


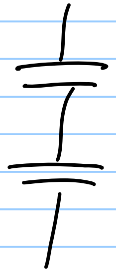
$$V = L \cdot \frac{dI}{dt} \Rightarrow V(s) = sL \cdot I(s)$$

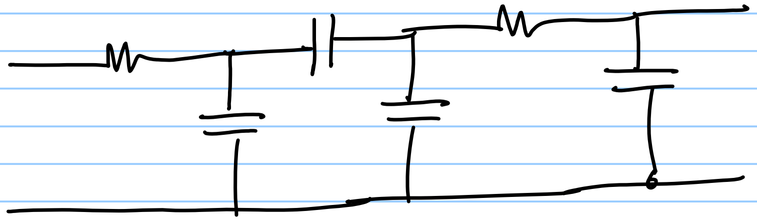
Order of system = # of reactive elements
(independent)











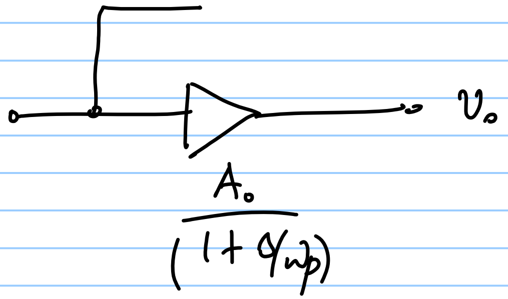
$$H(s) = A_o \cdot \frac{\left(1+\frac{s}{z_1}\right)\left(1+\frac{s}{z_2}\right)\cdots\left(1+\frac{s}{z_m}\right)}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)\cdots\left(1+\frac{s}{p_n}\right)}$$ m zeroes

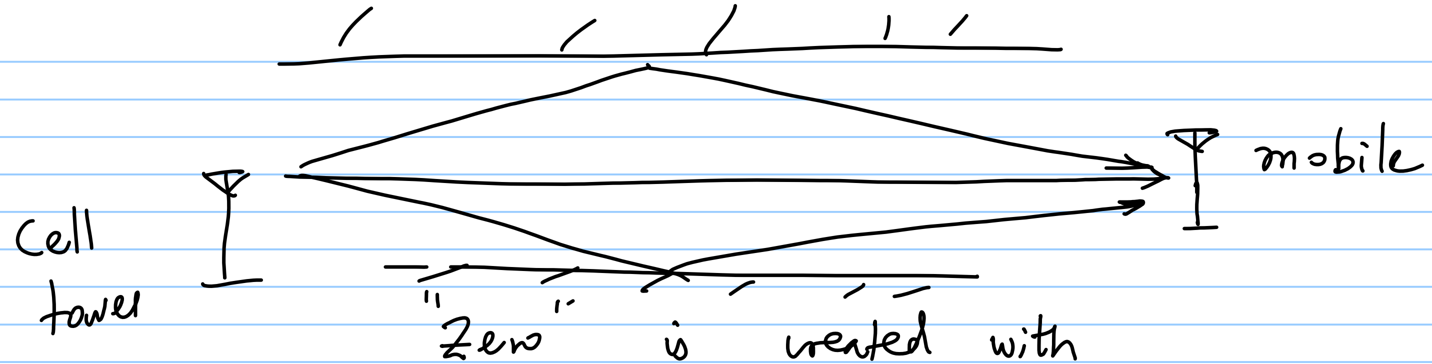
n poles

@ $\omega = -z_1, -z_2 \cdots -z_m$: $H(s) = 0$

@ $\omega = -p_1, -p_2 \cdots -p_n$: $H(s) = \infty$

$\frac{v_o}{v_i}(s) = 0$ @ any particular frequencies





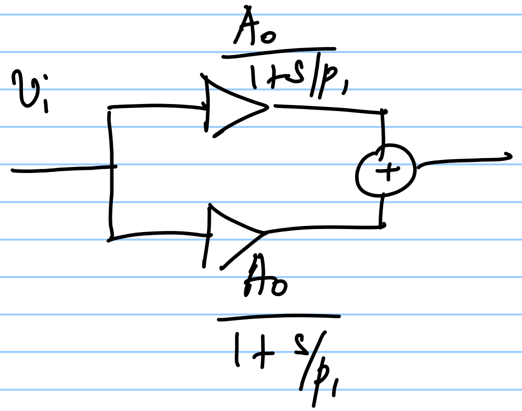
"destructive" multipath

freq. dependent

phase is freq. dep.

"Zero" is created with 2 or more paths with different phase shifts.

1)

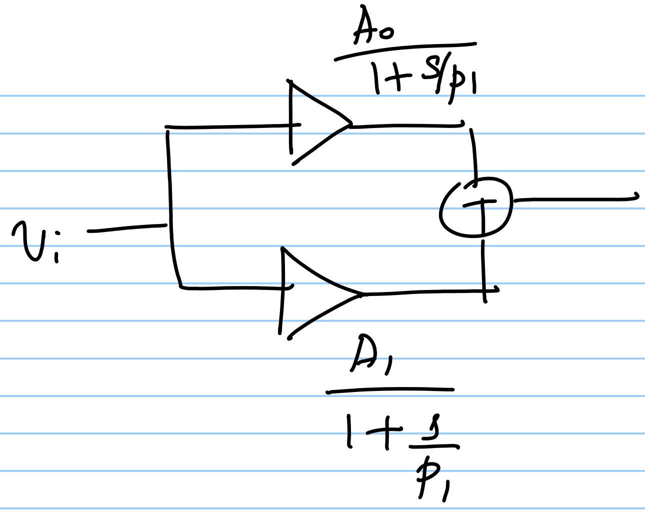


$$\frac{v_o}{v_i} = \frac{2A_o}{1+s/p_1}$$

No zero

ϕ in each path $= -\tan^{-1}\left(\frac{\omega}{p_1}\right)$

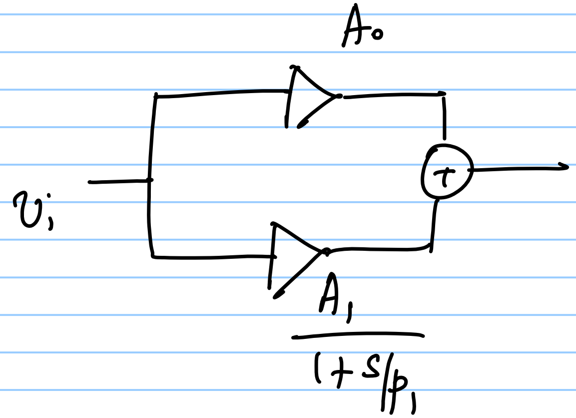
2)



$$\frac{v_o}{v_i} = \frac{A_0 + A_1}{1 + s/p_1}$$

No zero

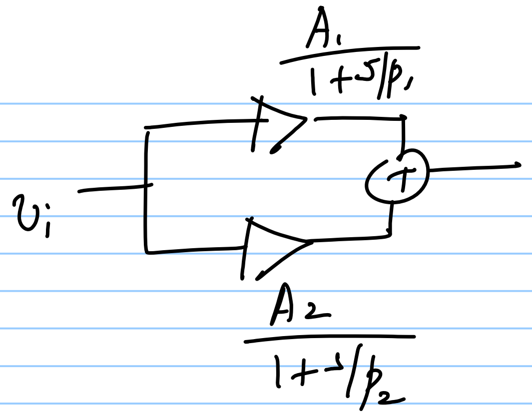
3)



$$\frac{v_o}{v_i} = A_0 + \frac{A_1}{1+s/p_1} = \frac{A_0 + A_1 + \frac{A_0 s}{p_1}}{1 + s/p_1}$$

$$= (A_0 + A_1) \cdot \frac{(1 + s \cdot \frac{A_0}{A_0 + A_1} \cdot \frac{1}{p_1})}{1 + s/p_1}$$

4)



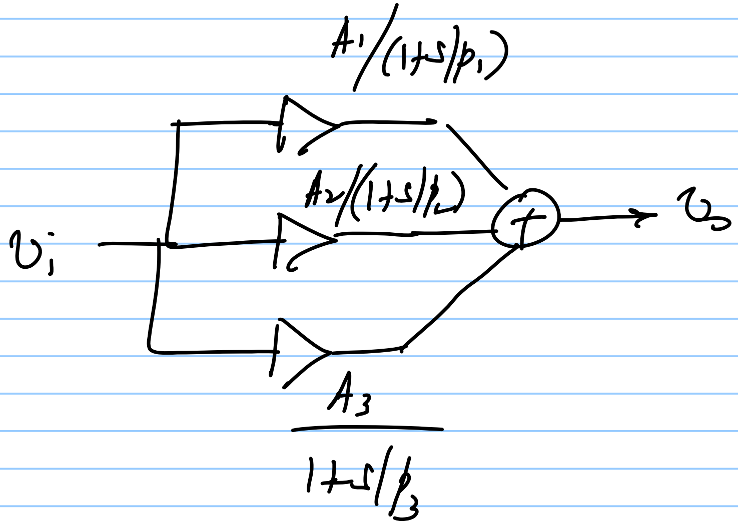
$$\frac{v_o}{v_i} = \frac{A_1}{1+s/p_1} + \frac{A_2}{1+s/p_2}$$

$$= \frac{A_1 + A_2 + \frac{A_1 s}{p_2} + \frac{A_2 s}{p_1}}{(1+s/p_1)\ (1+s/p_2)}$$

1 zero

2 poles

5)



3 poles

2 zeroes

For 1-stage opamp

$$\frac{v_o}{v_i}(s) = ?$$

$$v_x(s) = -g_{m_1}\frac{v_i}{2}\left[\frac{1}{g_{m_3}} \,\|\, \frac{1}{sC_1}\right] = -\frac{g_{m_1}\frac{v_i}{2}}{g_{m_3}} \cdot \left[\frac{1}{1+\frac{sC_1}{g_{m_3}}}\right]$$

$$i_4(s) = -g_{m_4} v_x(s) = g_{m_1}\frac{v_i}{2}\left[\frac{1}{1+\frac{sC_1}{g_{m_3}}}\right]$$

$$i_2(s) = g_{m_2}\frac{v_i}{2}$$

$$i_{out}(s) = i_2(s) + i_4(s) = g_{m_1}\frac{v_i}{2}\left[1 + \frac{1}{1+\frac{sC_1}{g_{m_3}}}\right]$$

$$i_{out}(s) = \frac{g_{m_1} v_i}{2} \left[\frac{2 + \frac{sC_1}{g_{m_3}}}{1 + \frac{sC_1}{g_{m_3}}} \right]$$

$$= g_{m_1} v_i \cdot \frac{1 + \frac{sC_1}{2g_{m_3}}}{1 + \frac{sC_1}{g_{m_3}}}$$

Zero @ $-\frac{2g_{m_3}}{C_1}$

pole @ $-\frac{g_{m_3}}{C_1}$

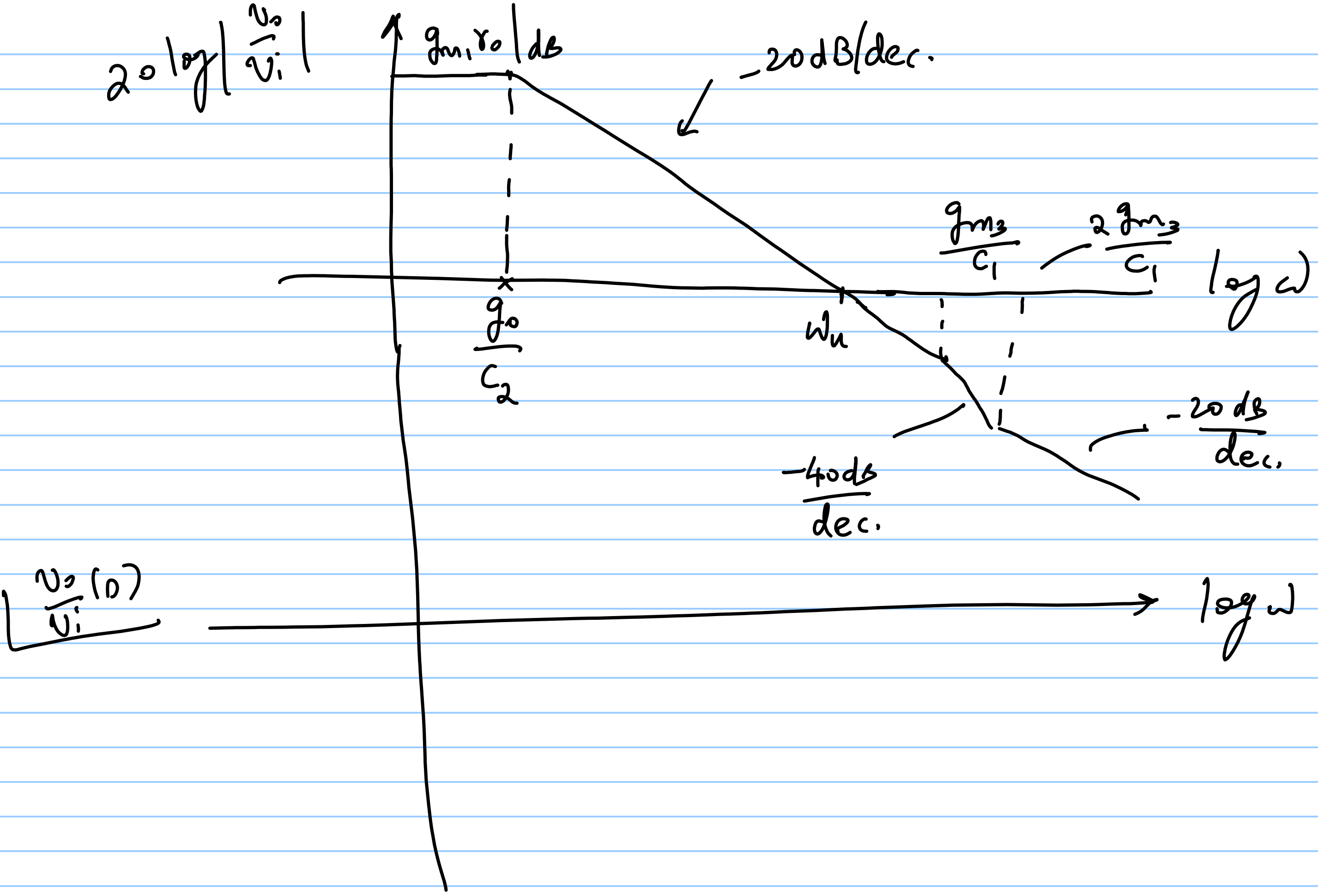
$$v_o(s) = i_{out}(s) \cdot \left(r_{ds_2} \| r_{ds_4} \| \frac{1}{sC_2} \right)$$

$r_{ds_2} \| r_{ds_4} = r_o$

$$= i_{out}(s) \cdot \frac{r_{ds_2} \| r_{ds_4}}{1 + sC_2(r_{ds_2} \| r_{ds_4})} = i_{out}(s) \cdot \frac{r_o}{1 + sC_2 r_o}$$

$$v_o(s) = \underbrace{g_{m_1} r_o}_{\text{DC gain (low-freq. gain)}} \cdot \underbrace{\frac{1 + \frac{sC_1}{2g_{m_3}}}{\left(1 + \frac{sC_1}{g_{m_3}}\right)\left(1 + \frac{sC_2}{g_o}\right)}}_{\text{gain freq. response}} \cdot v_i(s)$$

$$\left.\begin{array}{l} g_{m_3} >> g_o \\ C_2 >> C_1 \end{array}\right\} \text{Assumption}$$



HW: What happens at pole & zero freq. in bode plot?

$$A(s) = \frac{A_o}{1 + s/\omega_p} \quad ; \quad A(j\omega) = \frac{A_o}{1 + \frac{j\omega}{\omega_p}}$$

@ $\omega = \omega_p$: $|A(j\omega)| = \frac{1}{\sqrt{2}} A_o$

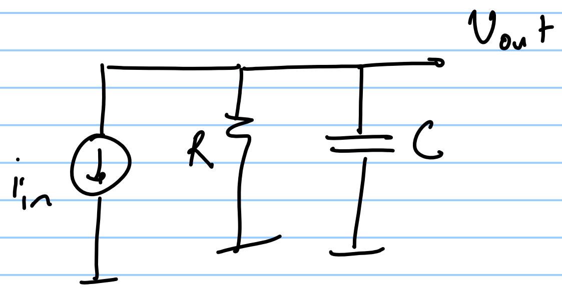
$$A(s) = A_o \left(1 + s/\omega_z\right)$$

@ $\omega = \omega_z$: $|A(j\omega)| = \sqrt{2} A_o$

7/11/17

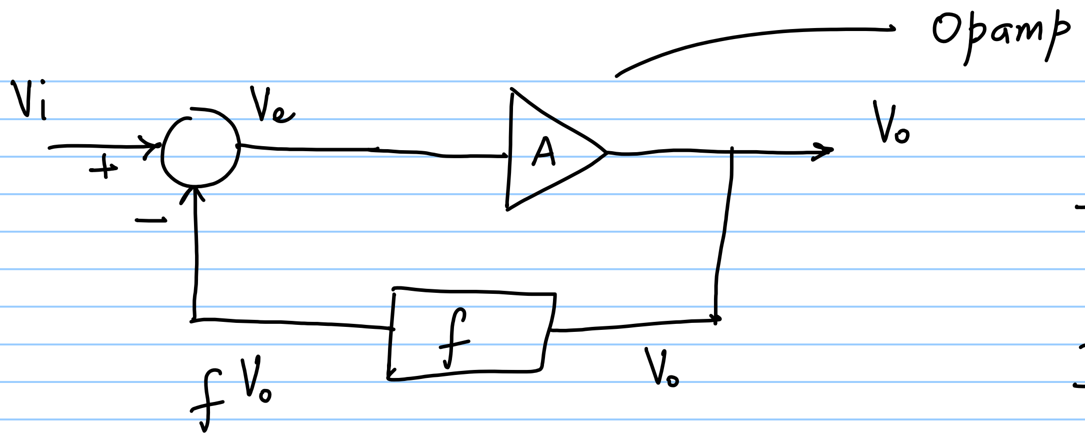
Lec 19

- * Every node has some capacitance to ground
- * Every node in signal path will contribute a pole



$$\frac{v_{out}}{i_{in}}(s) = \left(R \right) \cdot \frac{1}{1 + sRC}$$

- * # of poles affects negative feedback behaviour



$$\frac{V_o}{V_i} = \frac{1}{f} \cdot \frac{Af}{1+Af}$$

If $Af \rightarrow \infty$ $\{$normally $A \rightarrow \infty\}$
(loop gain)

$$\frac{V_o}{V_i} = \frac{1}{f} \quad \text{or} \quad V_o = \frac{V_i}{f}$$

normally $f < 1$, so $\frac{V_o}{V_i} > 1$

* $A = A(s)$

* $\frac{V_o}{V_i}(s) = \frac{1}{f} \cdot \frac{A(s) \cdot f}{1 + A(s) \cdot f}$ = Closed loop gain
CLG(s)

If A(s) has poles, what happens?

Negative feedback is operational when $A(s) \cdot f >> 1$
$\underline{\text{CLG}(s)}$ should be linear
$\omega << \omega_p(1 + A_o f)$

e.g. $A(s) = \dfrac{A_o}{1+ s/\omega_p}$ $\qquad LG = f \cdot A(s) = \dfrac{f\, A_o}{1+ s/\omega_p}$

$$CLG(s) = \frac{1}{f} \cdot \frac{f \cdot A_o/(1+s/\omega_p)}{1+ f \cdot A_o/(1+s/\omega_p)} = \frac{1}{f} \cdot \frac{A_o f}{1+A_o f} \cdot \frac{1}{1+ \dfrac{s}{\omega_p(1+A_o f)}}$$

$f A_o = 1000$

$\omega_p = (2\pi \cdot 10k)$ rad/s

pole of $LG = \omega_p = 2\pi \cdot 10k$ rad/s

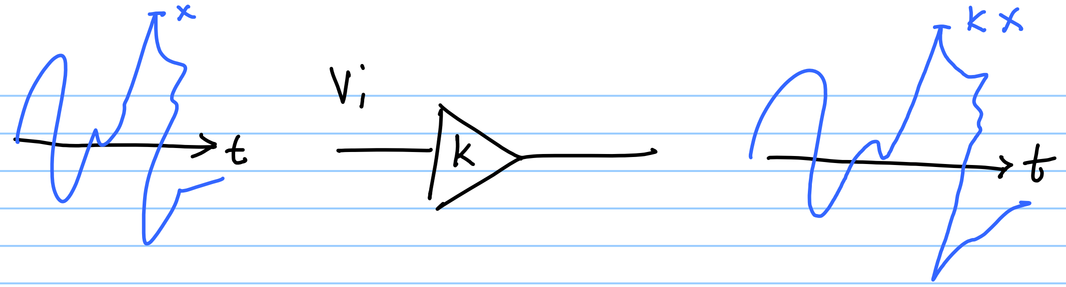
poles of $CLG = 2\pi \cdot 10k \cdot 1001$ rad/s.

<u>Stability</u>: bounded input

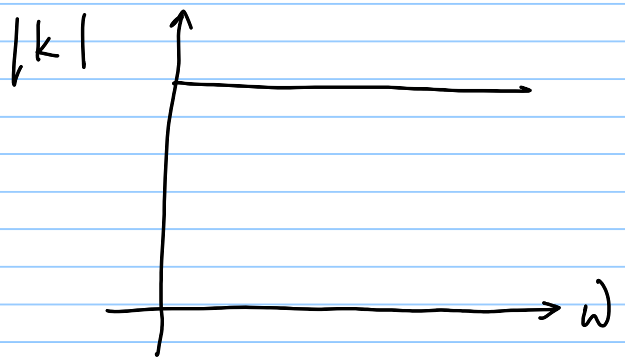
bounded output

Extreme case: output without input

We want "unconditionally stable" system

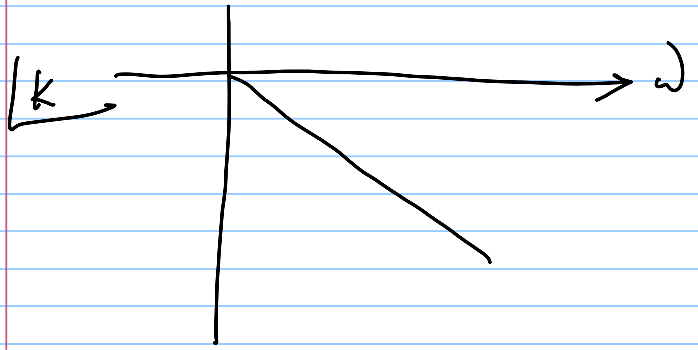


phase should be linearly related to frequency



group delay should be constant

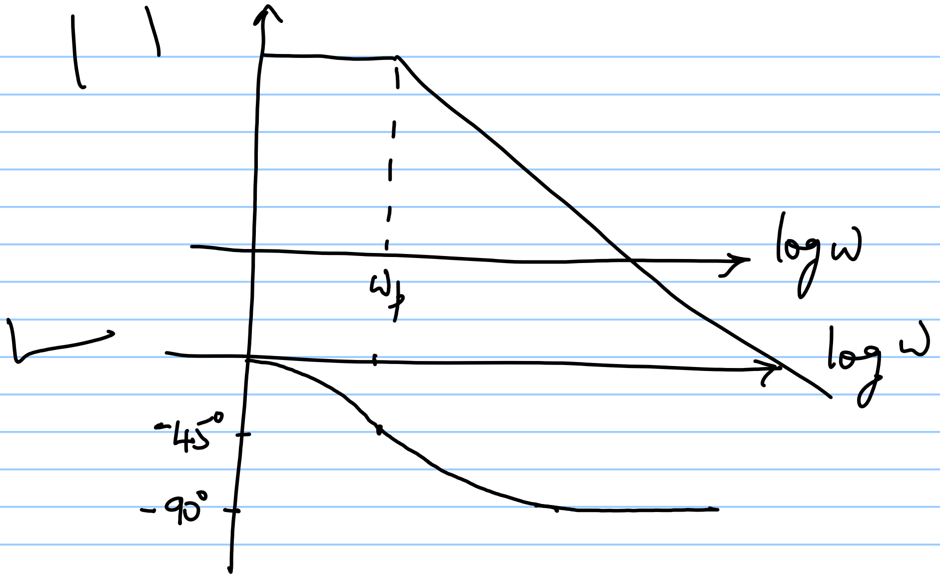
↳ delay of each freq. component of signal



$k \ , \ \tau$

$$V_i = A e^{j\omega t}$$

$$V_o = kA e^{j\omega(t+\tau)} = \underbrace{k}_{|k|} A e^{j\omega t} \cdot \underbrace{e^{j\omega\tau}}_{\angle k}$$



$$\frac{V_o}{V_i}(s) = \frac{1}{f} \cdot \frac{A(s) \cdot f}{1 + A(s) \cdot f}$$

$V_i = 0$

$V_o \neq 0$ is possible if

$$\boxed{1 + A(s) \cdot f = 0} \qquad 1 + LG(s) = 0$$

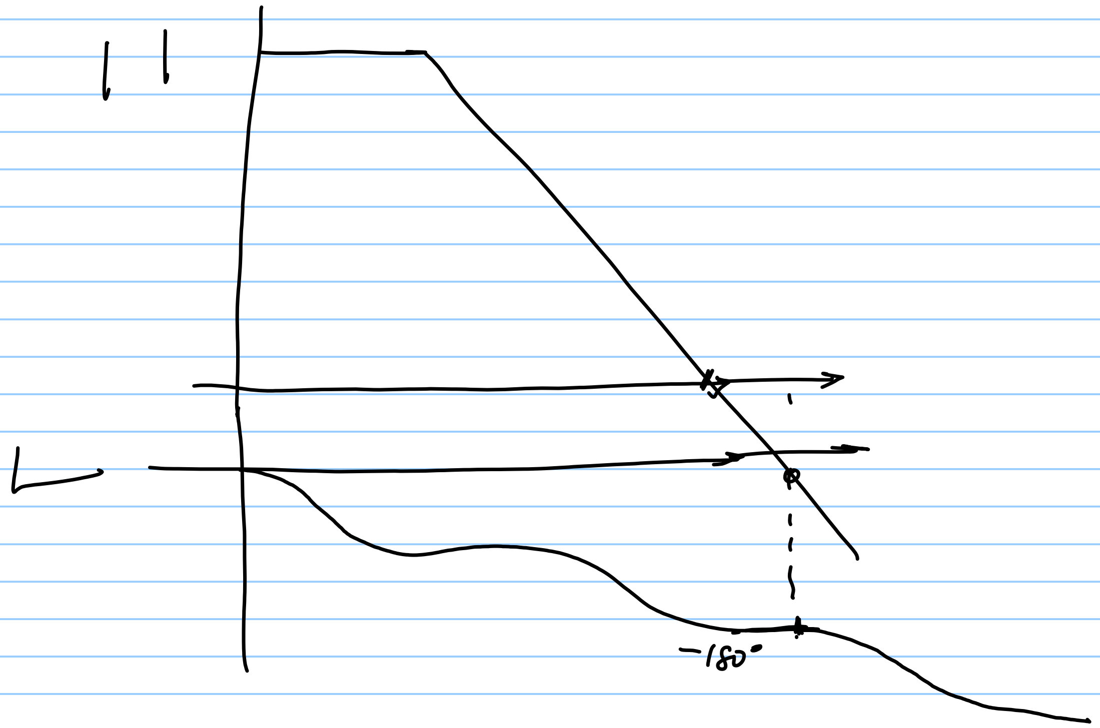
$f \cdot A(s) = -1$

$$\left\{ \begin{array}{l} |LG| = 1 \\ \angle LG = -180^\circ \end{array} \right)$$ Barkhausen Criteria for stability

If true, system can become unstable

For most systems, we use

at $\underline{\angle LG(s)} = -180°$, if $|LG(s)| \geqslant 1$ potential for instability



1) 1-pole system

$$A(s) = \frac{A_o}{1+s/\omega_p}$$

stability : a) poles in LHP

b) $\angle f A(s)$ can never go to -180°

@ $s \to \infty$ $\angle f A(s) = -90^\circ$

Unconditionally stable

We want large A_o

Easiest way to get large A_o is to cascade amplifiers

2) 2-pole system

$$A(s) = \frac{A_o}{(1+s/\omega_p)^2}$$

$$CLG(s) = \frac{1}{f} \cdot \frac{A_of}{1+A_of} \cdot \frac{1}{1+\frac{2s}{A_of\,\omega_p} + \frac{s^2}{\omega_p^2 \cdot A_of}} \longrightarrow D(s)$$

Stability: a) LHP poles

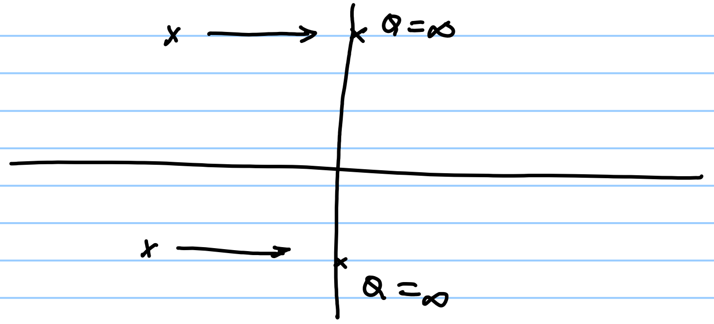
b) $\angle LG(s) = -180^\circ$ only at $\omega = \infty$

$\Rightarrow$ unconditionally stable

General 2nd order system

Denominator $D(s) = 1 + \frac{s}{\omega_0 \cdot Q} + \frac{s^2}{\omega_0^2}$

eq. to $s^2 + 2\zeta\omega_n s + \omega_n^2$



$$\frac{s}{\omega_0} = \frac{-1}{2Q} \pm j\sqrt{1 - \frac{1}{4Q^2}}$$

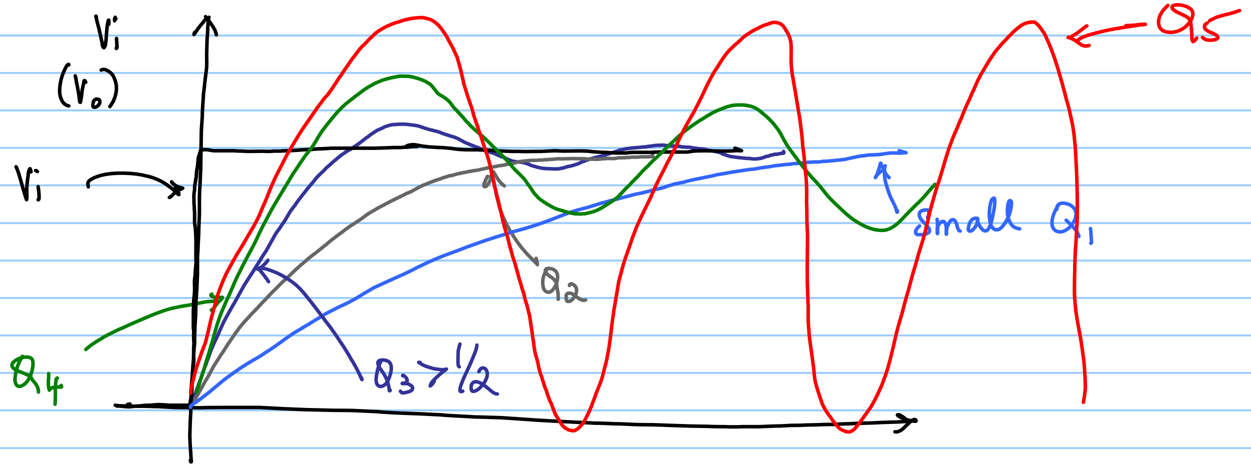
small Q : 2 real poles in LHP

$Q = \frac{1}{2}$: 2 co-incident poles

$Q > \frac{1}{2}$: complex conjugate poles

$Q = \infty$: poles on $j\omega$ axis

transient response to input step



$$Q_1 < Q_2 < Q_3 < Q_4 < Q_5$$

$$Q_1 < 1/2 \ , \ Q_2 = 1/2$$

$$Q_5 = \infty$$

$$1 + \frac{s}{\omega_0 \cdot Q} + \frac{s^2}{\omega_0^2} \longleftrightarrow 1 + \frac{2s}{A_0f\,\omega_p} + \frac{s^2}{\omega_p^2 A_0 f}$$

$$\omega_0 = \omega_p \sqrt{A_0 f} \quad ; \quad Q = \frac{\sqrt{A_0 f}}{2}$$

large gain ⇒ $A_0 f$ is large ⇒ Q is large ⇒ ringing takes a long time to settle

3) 3-pole system

$$A(s) = \frac{A_o}{\left(1 + s/\omega_p\right)^3}$$

$$= \frac{A_o}{1 + \frac{3s}{\omega_p} + \frac{3s^2}{\omega_p^2} + \frac{s^3}{\omega_p^3}}$$

$$CLG(s) = \frac{1}{f} \cdot \frac{A_o f}{1 + A_o f} \; \frac{1}{\underbrace{1 + \left[\frac{3s}{\omega_p} + \frac{3s^2}{\omega_p^2} + \frac{s^3}{\omega_p^3}\right] \frac{1}{1 + A_o f}}_{D(s)}}$$

8/11/17

Lec 20

$$D(s) = 1 + \frac{3s}{\omega_p(1+A_of)} + \frac{3s^2}{\omega_p^2(1+A_of)} + \frac{s^3}{\omega_p^3(1+A_of)}$$

$$x = \frac{s}{\omega_p}$$

$$D\left(\frac{s}{\omega_p}\right) = 1 + \frac{3x}{1+A_of} + \frac{3x^2}{1+A_of} + \frac{x^3}{1+A_of}$$

Roots of $D(x)$: $(1+A_of) + 3x + 3x^2 + x^3 = 0$

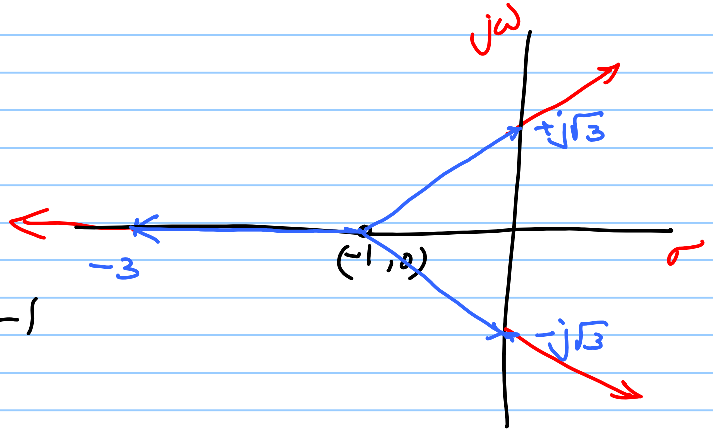
$$(1+x)^3 = -A_of$$

$$x = -1 + (-A_0 f)^{1/3} \rightarrow 3 \text{ solutions}$$

Example 1) $A_0 f = 8$

$$\left.\begin{array}{l} x_1 = -1 \quad -2 \\ x_2 = -1 \quad - 2e^{j2\pi/3} \\ x_3 = -1 \quad - 2e^{-j2\pi/3} \end{array}\right\} x = -3 , \pm j\sqrt{3}$$

poles are @ $\omega_p \cdot \{x_1, x_2, x_3\}$



2) $A_0 f = 0$: $x_1 = x_2 = x_3 = -1$

3) If $A_0 f > 8$: complex conjugate poles in RHP $\Rightarrow$ sinusoidal component with increasing envelope (BAD!)

4) 4-pole system will be worse

Possible solutions

1) $A(s) = \dfrac{A_o}{(1+s/\omega_p)^3}$ → Add zeroes

$$\downarrow$$

$$\frac{A_o}{(1+s/\omega_p)^{\cancel{3}}} \times \frac{\cancel{(1+s/\omega_p)^2}}{(1+s/\omega_{p_2})^2}$$

← Still a 3rd order system

2) 1st order system $\rightarrow$ low gain
$\rightarrow$ unconditionally stable

2nd order system $\rightarrow$ moderate gain
$\rightarrow$ technically stable (but possible ringing)

3rd order system $\rightarrow$ large gain
$\rightarrow$ unstable if gain > 8

" Make higher order system look like 1st order system"

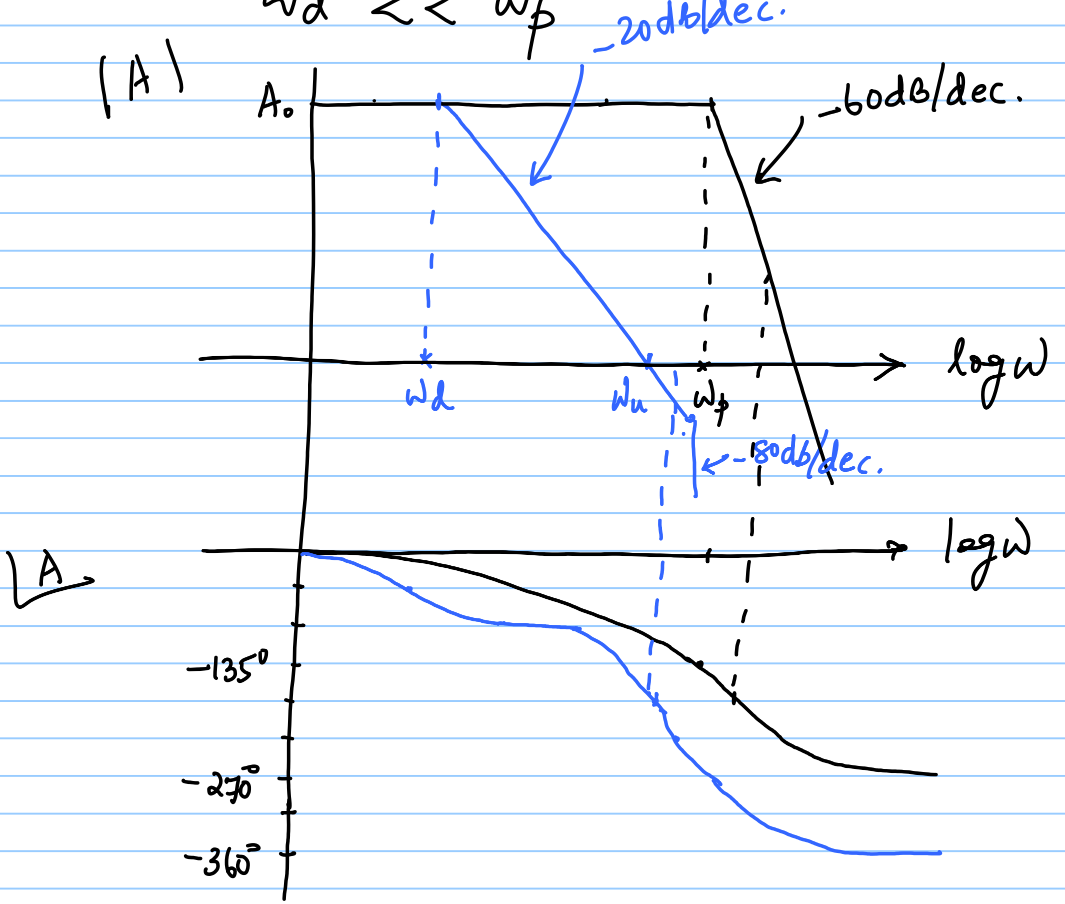
$$\frac{A_0}{(1+s/\omega_p)^3} \longrightarrow \frac{A_0}{(1+s/\omega_p)^3\left(1+\frac{s}{\omega_d}\right)}$$

Stability matters @ $\angle a(\omega) = -1$

$W_d << \omega_p$

W_d = "dominant" pole

"Dominant pole compensation"



Example :

$$LG(s) = \frac{A_0 f}{(1+s/\omega_p)^3} \longrightarrow \max(A_0 f) = 8$$

add a dominant pole $\omega_d = \frac{\omega_p}{1000}$

new $$LG(s) = \frac{A_0 f}{\left(1+\frac{s}{\omega_p}\right)^3 \left(1+\frac{1000s}{\omega_p}\right)}$$

roots of $LG(s) = -1$

$$\frac{A_0 f}{(1+s/\omega_p)^3 \left(1+\frac{1000s}{\omega_p}\right)} = -1$$

$\angle La = -180^\circ$

$$0 - 3\tan^{-1}\left(\frac{w_u}{w_p}\right) - \tan^{-1}\left(\frac{1000\,w_u}{w_p}\right) = -180^\circ$$

@ w_d : -45° from w_d $+ \; 0 = -45^\circ$

@ w_p : -90° from w_d $+ \; (-135^\circ) = -225^\circ$

$w_d < w_u < w_p$; $w_u >> w_d$ because $A_0 f >> 1$

$$-3\tan^{-1}\left(\frac{w_u}{w_p}\right) = -180^\circ + 90^\circ = -90^\circ$$

$$\tan^{-1}\left(\frac{w_u}{w_p}\right) = 30^\circ \Rightarrow$$ contribution from each w_p pole $= -30^\circ$

$$\frac{\omega_u}{\omega_p} = \frac{1}{\sqrt{3}}$$

Now, apply $|L_u(s)| = 1$

$$\left| \frac{A_o f}{\left(1 + j/\sqrt{3}\right)^3 \left(1 + j \cdot \frac{1000}{\sqrt{3}}\right)} \right| = 1$$

$$\frac{A_o f}{\left(\sqrt{1 + \frac{1}{3}}\right)^3 \left(\frac{1000}{\sqrt{3}}\right)} = 1 \Rightarrow A_o f = \frac{8000}{9} \approx 890$$

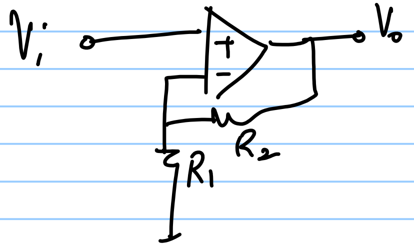
maximum value permissible

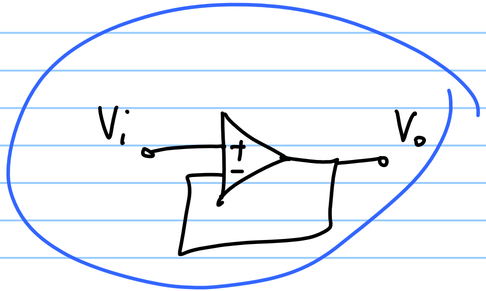
$$\frac{A_0}{(1+s/\omega_p)^3} \times \frac{(1+s/\omega_p)^2}{(1+s/\omega_{p_2})^2} = \frac{A_0}{(1+s/\omega_p)(1+s/\omega_{p_2})^2}$$

If $\omega_{p_2} \gg \omega_p \Rightarrow \omega_p$ becomes new dominant pole

" pole-zero compensation "

$LG = \underbrace{A_0}_{\text{opamp}} \cdot f$





worst-case : $LG =$ maximum $\Rightarrow f = 1$ (unity gain)

most general purpose opamps are unity-gain compensated.

Measures of Stability

1) Phase Margin: $180° + \angle LG(j\omega)$ at unity gain frequency

$LG = 0\,dB$
(mag. of 1)

e.g. $\angle LG(j\omega)$ @ $\omega_u = -135°$

$\Rightarrow PM = 45°$

$\{ \angle LG(j\omega)$ @ $\omega_u - (-180°)$ is calculated$\}$

2) Gain margin: $0\,dB - |LG(j\omega)|$ @ freq. where $\angle LG(j\omega) = -180°$

e.g. @ $\angle LG = -180°$, $|LG| = -20\,dB$

$\Rightarrow GM = 20\,dB$

Effect of zero

magnitude response : slope increases by $\frac{20dB}{dec} \times$ (# of zeroes)

phase response : depends on LHP or RHP zero

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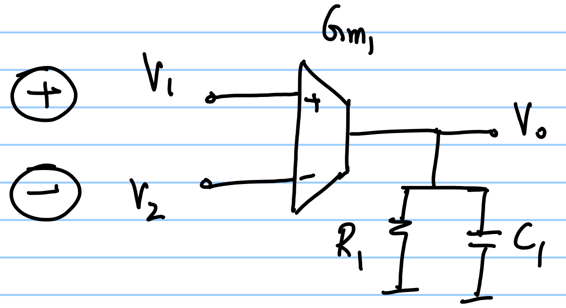


$$A(s) = \frac{A_o}{1 + sR_1C_1}$$

Single stage CMOS opamp = OTA

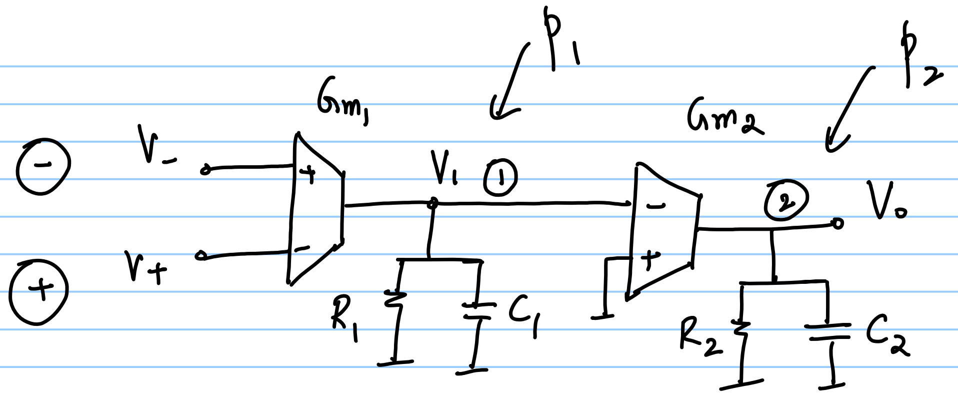
$R_1 = R_{out}$ of opamp (large)

$A_o = G_{m_1} R_1$

$C_1 =$ load cap + sum of all parasitic caps @ output node

$$\omega_{-3dB} = \frac{1}{R_1 C_1}$$

$$\omega_u = \frac{G_{m_1}}{C_1}$$



Gm_2 : large gain = $Gm_2 R_2$

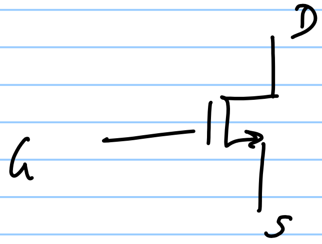
single ended input & output

⇒ Common source amplifier

$$A_o = A_1 \cdot A_2 = (Gm_1 R_1)(Gm_2 R_2)$$

$$A(s) = \frac{A_1}{1+\frac{s}{p_1}} \cdot \frac{A_2}{1+\frac{s}{p_2}}$$

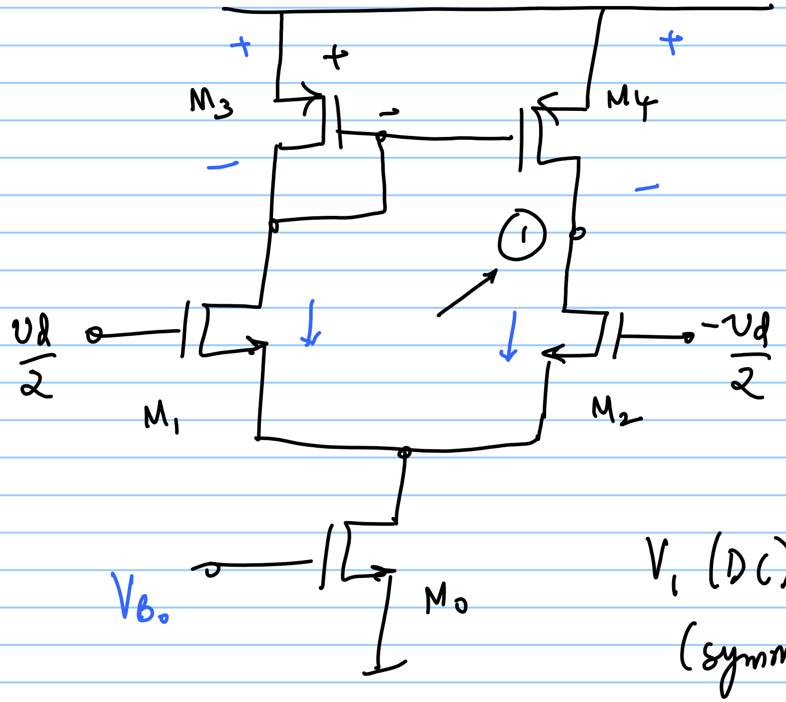
$$p_1 = \frac{1}{R_1 C_1} \quad ; \quad p_2 = \frac{1}{R_2 C_2}$$

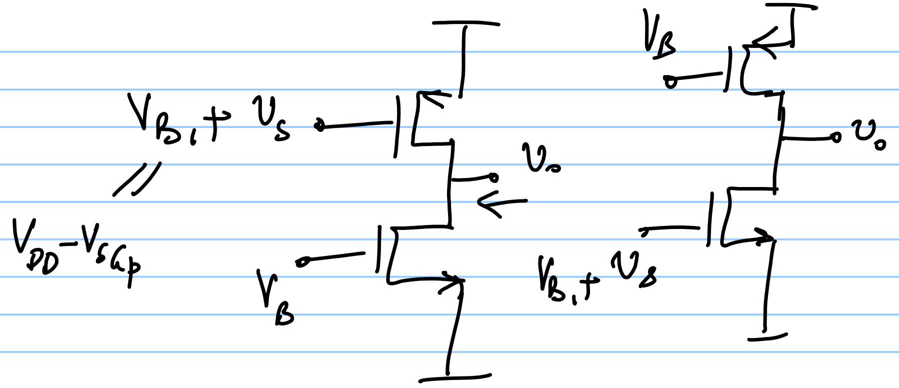


C_{gs}

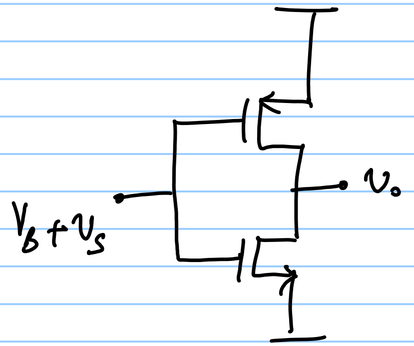
C_{db}, C_{sb}

C_{gd}





V_1 (DC) $= V_{DD} - V_{SG_3}$
(symmetry)

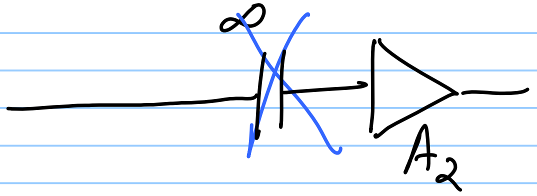


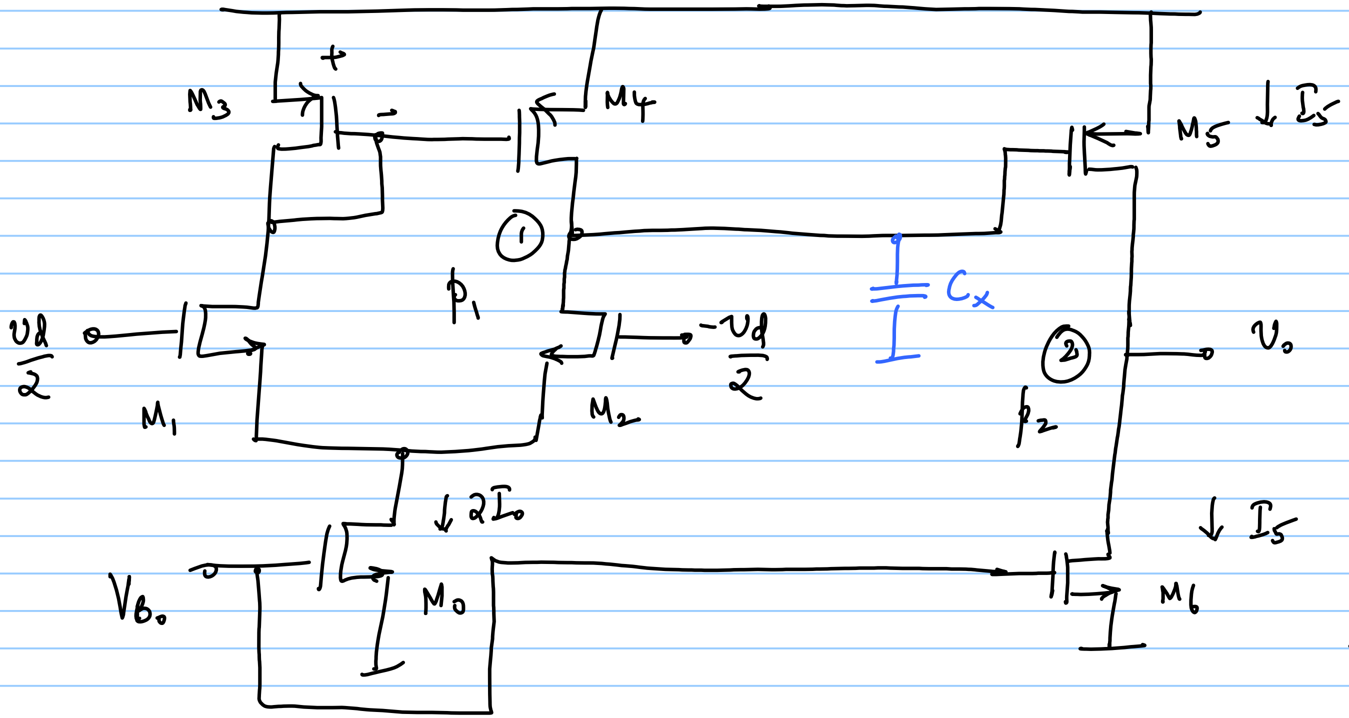
high gain
but
high cap

Bias voltage?

$R_1 = r_{ds_4} || r_{ds_2}$; $C_1 =$

$G_{m_1} = g_{m_1}$





$$V_{DD} - V_{SG_3}\Big|_{I_0} = V_{DD} - V_{SG_5}\Big|_{I_5}$$

$$V_{SG_3}\Big|_{I_0} = V_{SG_5}\Big|_{I_5}$$

$$V_{T_3} + \sqrt{\frac{2I_0}{\mu_p C_{ox}\left(\frac{W}{L}\right)_3}} = V_{T_5} + \sqrt{\frac{2I_5}{\mu_p C_{ox}\left(\frac{W}{L}\right)_5}}$$

$$\Rightarrow \boxed{\frac{I_0}{(W/L)_3} = \frac{I_5}{(W/L)_5}}$$

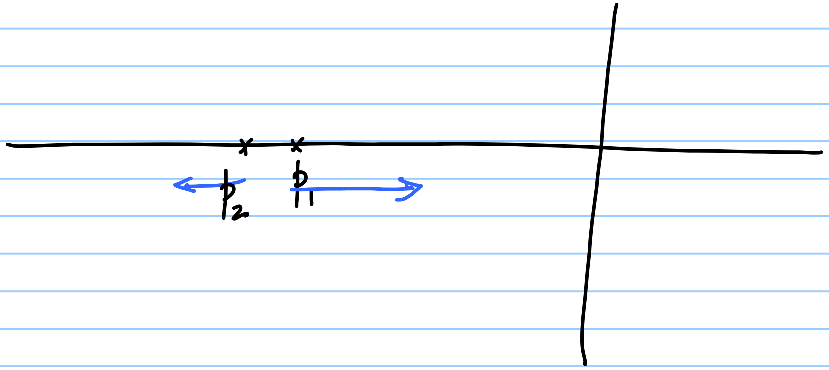
M_3, M_5 should have same "Current Density"

$$V_{GS_0} = V_{GS_6} \Rightarrow \boxed{\frac{2I_0}{(W/L)_0} = \frac{I_5}{(W/L)_6}}$$

$$L_3 = L_4 = L_5 \quad ; \quad L_0 = L_6$$

$$A_o = g_{m_1}(r_{ds_2} || r_{ds_4}) \cdot g_{m_5}(r_{ds_5} || r_{ds_6}) \longrightarrow \text{very large}$$

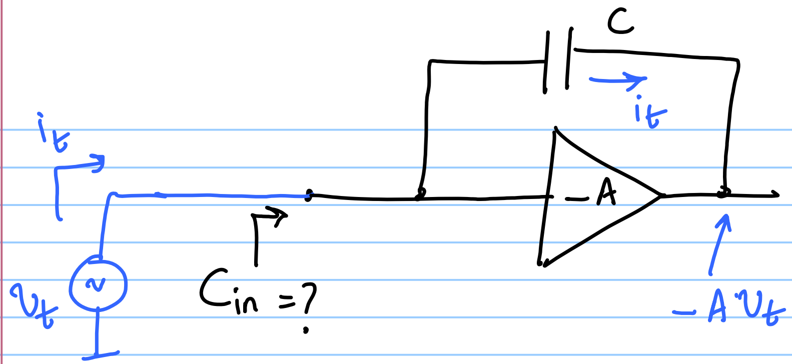
2 pole system: $Q = \frac{\sqrt{A_o f}}{2} = \text{large} \Rightarrow$ Ringing



$$p_1 = \frac{1}{R_1 C_1}$$

If $R_1 \uparrow \Rightarrow$ already as large as possible for A_o

$C_1 \uparrow \Rightarrow$ add C_x, but C_x is very large



$$\frac{v_t}{i_t} = \frac{1}{sC_{in}} \quad \text{or} \quad i_t = s\,C_{in}\cdot v_t$$

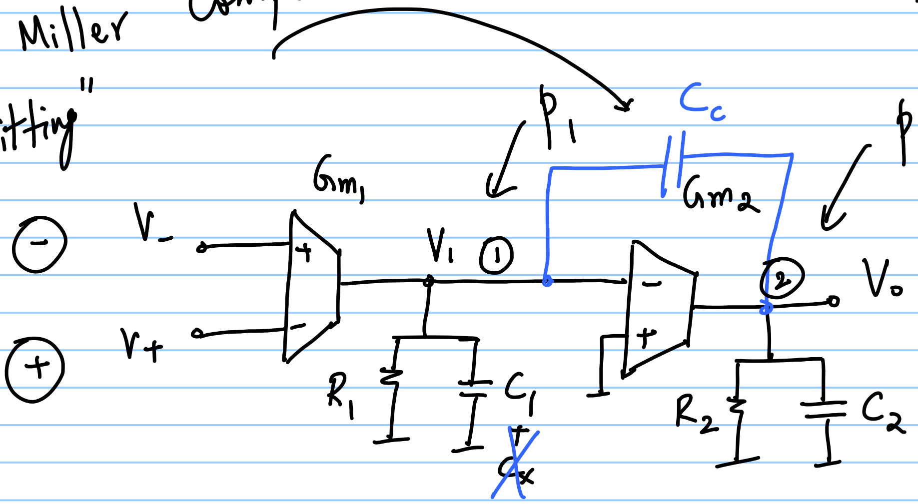
$$i_t = \left\{v_t - (-Av_t)\right\}\cdot sC$$

$$\frac{i_t}{v_t} = sC_{in} = sC\,[1+A] \Rightarrow \boxed{C_{in} = (1+A)C}$$

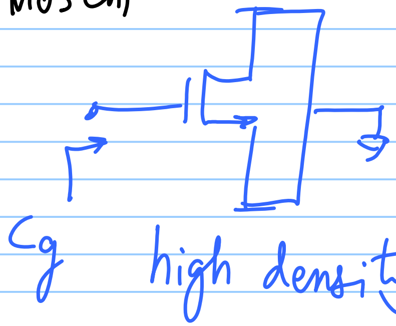
"Miller" Effect

"Miller" Compensation

"pole-splitting"

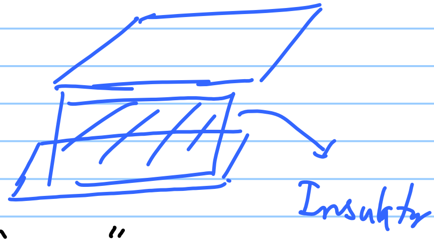


"MOS CAP"



high density

non-linear



"MIM" cap

high-k dielectric

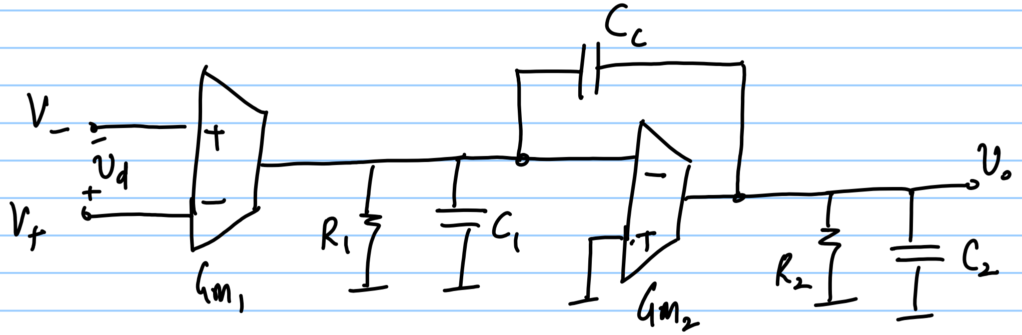
"MOM" Cap

$$\text{effective } C_x = \underbrace{(1+A)}_{G_{m_2} R_2 \; >>1} \cdot C_c$$

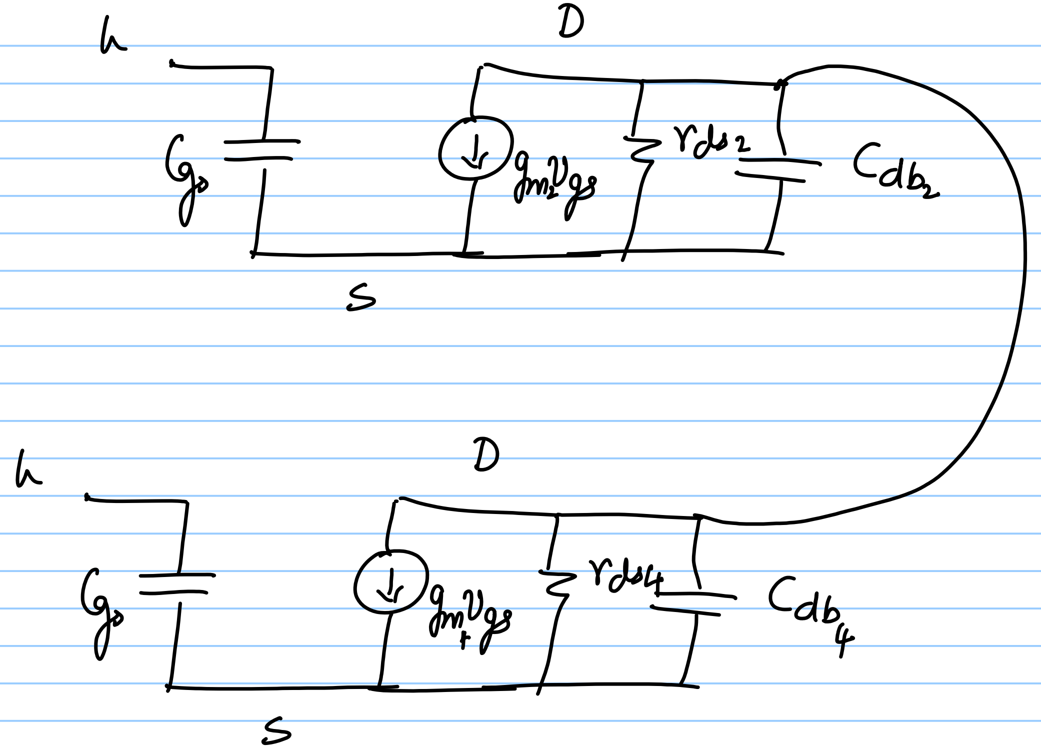
$$C_x \approx G_{m_2} R_2 C_c \quad >> C_{1_{orig}}$$

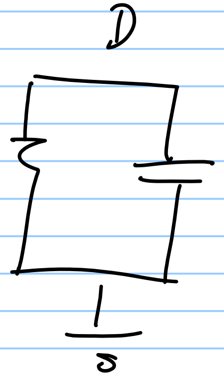
Two paths from ① to ② ⇒ through C_c & G_{m_2}

Expect a zero !!



$\frac{v_o}{v_d}(s) = ?$ HW exercise





OTA — VCCS

because Rout is large

1 Class : BJTs

3 classes : B.G ref + LDO

2 classes : Other opamps

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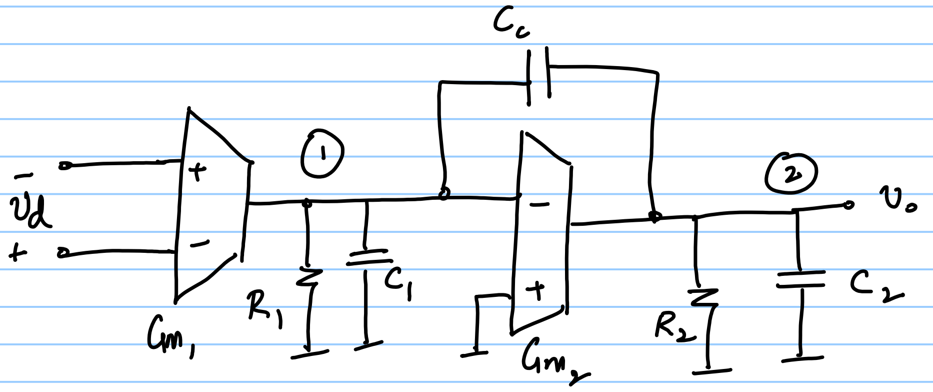
Lec 22



$$A_o = Gm_1 R_1 \cdot Gm_2 R_2$$

$$= \frac{Gm_1}{G_1} \cdot \frac{Gm_2}{G_2}$$

$$Z = + \frac{Gm_2}{C_c} \quad (\text{RHP zero})$$

$$\text{pole} = - \frac{\text{conductance}}{\text{capacitance}}$$

$$\frac{v_o}{v_d}(s) = A_o \frac{(1 - s/z)}{as^2 + bs + c}$$

p_1 & p_2 : roots of $ax^2 + bx + c = 0$

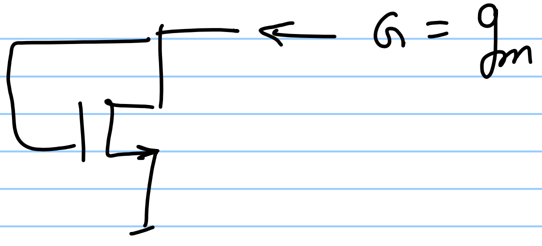
We know $p_1 << p_2$

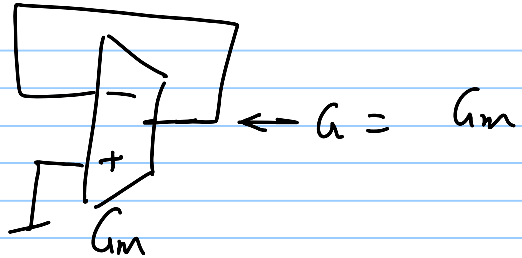
$$p_1 \approx \frac{-c}{b} \quad ; \quad p_2 \approx \frac{-b}{a}$$

$$p_1 = - \frac{G_1}{C_1 + (1 + Gm_2 R_2) C_c} + \dots$$

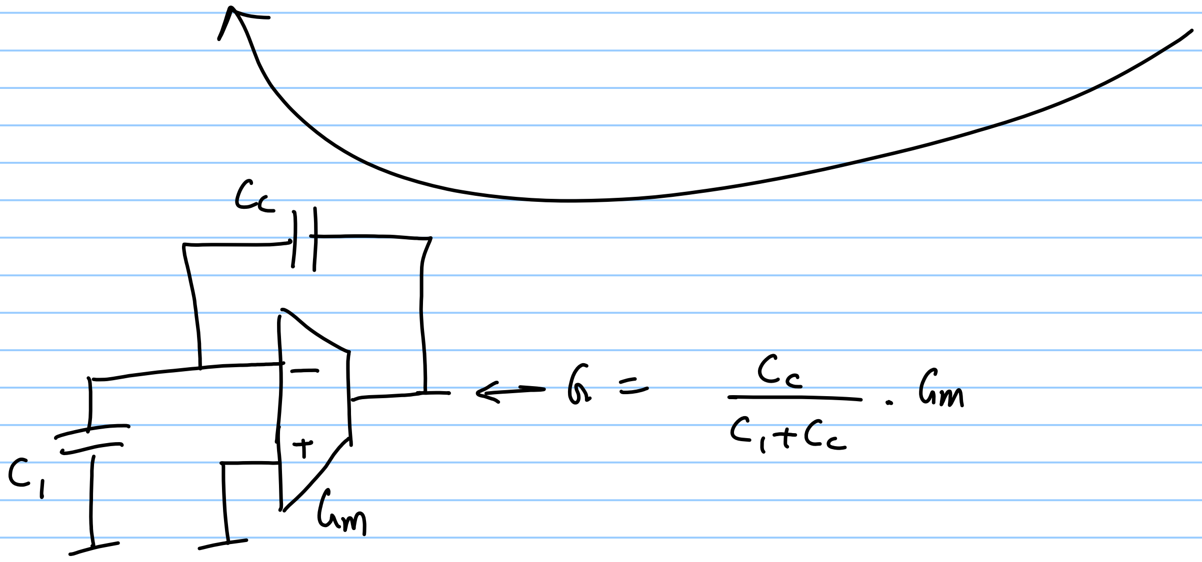
$$\approx - \frac{G_1 G_2}{Gm_2 C_c} \quad (\text{LHP})$$

$$p_2 \approx - \frac{G}{C} = - \frac{G_2 + \frac{C_c}{C_1 + C_c} \cdot Gm_2}{C_2 + \frac{C_c C_1}{C_c + C_1}} \quad (\text{LHP})$$

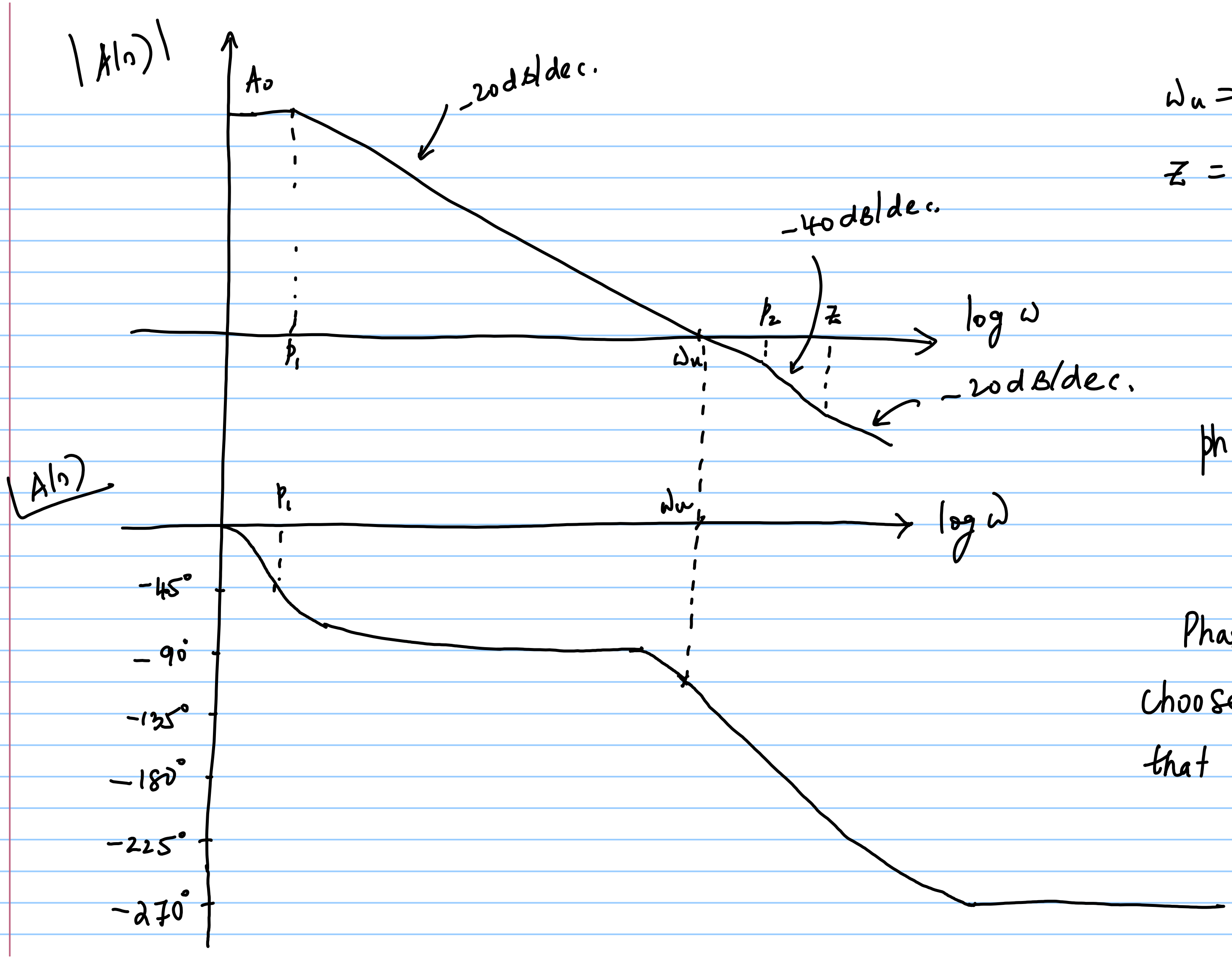




$$A(s) = A_0 \frac{\left(1 - s/z\right)}{\left(1 + \frac{s}{p_1}\right)\left(1 + \frac{s}{p_2}\right)}$$



$$\underline{\angle A(\omega)} = -\tan^{-1}\left(\frac{\omega}{z}\right) - \tan^{-1}\left(\frac{\omega}{p_1}\right) - \tan^{-1}\left(\frac{\omega}{p_2}\right)$$



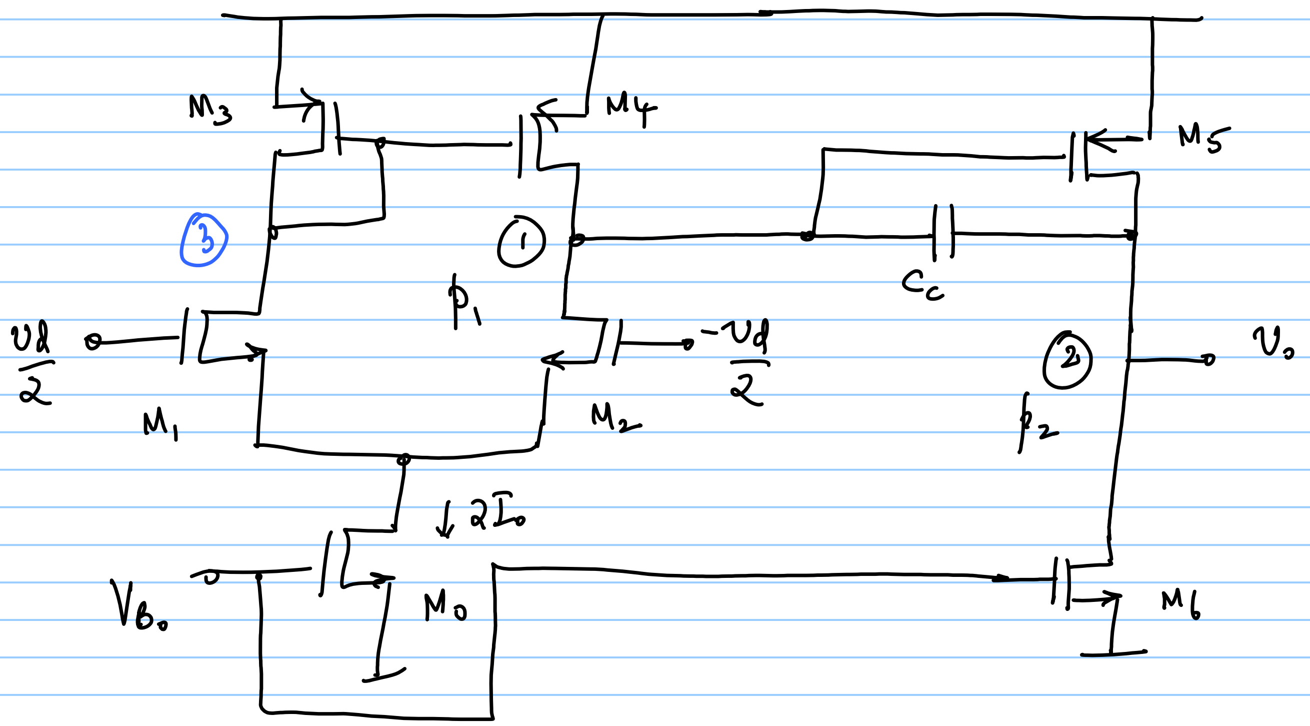
$$\left.\begin{aligned} \omega_u &= \frac{G_{m1}}{C_c} \\ z &= \frac{G_{m2}}{C_c} \end{aligned}\right\}$$

$$\text{phase @ } \omega_u = -90^\circ - \tan^{-1}\left(\frac{\omega_u}{p_2}\right) - \tan^{-1}\left(\frac{\omega_u}{z}\right)$$

$$\text{Phase margin} = 180^\circ + \text{phase@}\omega_u$$

choose positions of p_2 & z so that phase margin is adequate e.g. no phase impact from zero: $\frac{\omega_u}{z} << 1 \Rightarrow \omega_u << z$

$\Rightarrow G_{m_2} >> G_{m_1}$



$G_{m_1} = g_{m_1}$; $G_{m_2} = g_{m_5}$

C_1 = tot cap @ ①

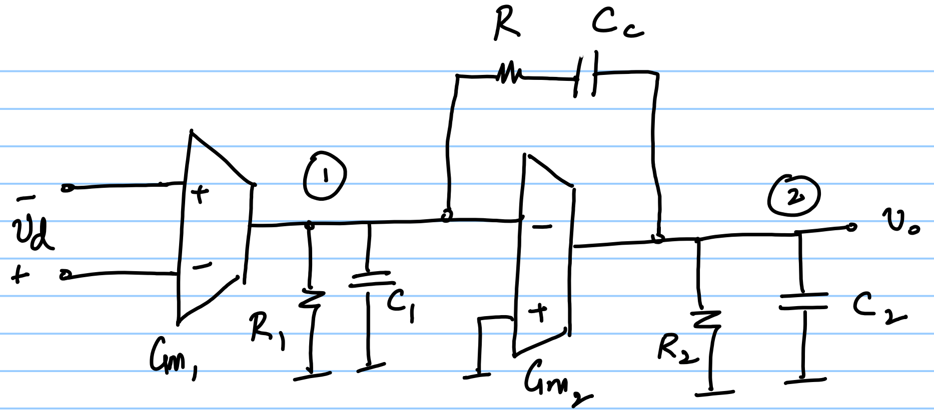
C_2 = " " " ②

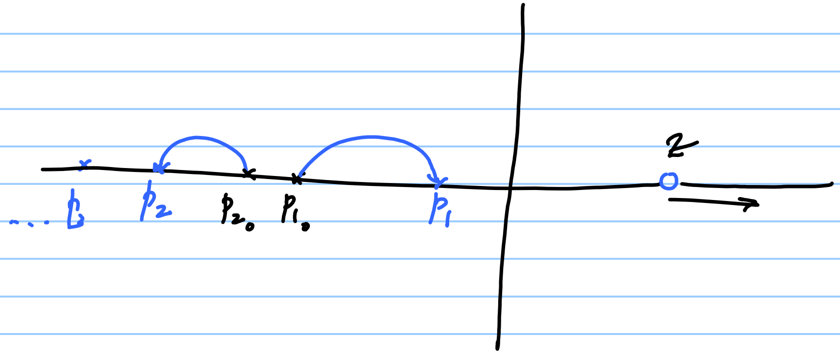
$R_1 = r_{ds_2} || r_{ds_4}$

$R_2 = r_{ds_5} || r_{ds_6}$

$p_3 = \frac{g_{m_3}}{C_3}$) p_3 & $z_2 >> \omega_u$

$z_2 = \dots$





pole-zero cancellation

RC milles compensation

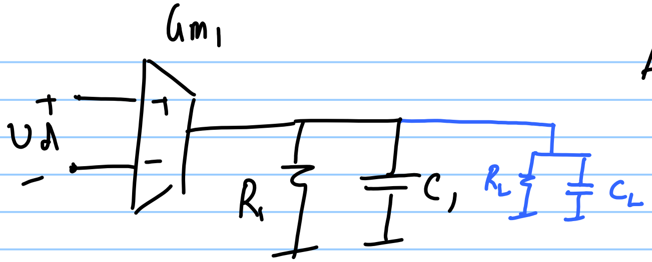
p_1 & p_2 are almost same as before

$p_3 > p_2$

$$Z = \frac{Gm_2}{C_c(1 - Gm_2 R)}$$

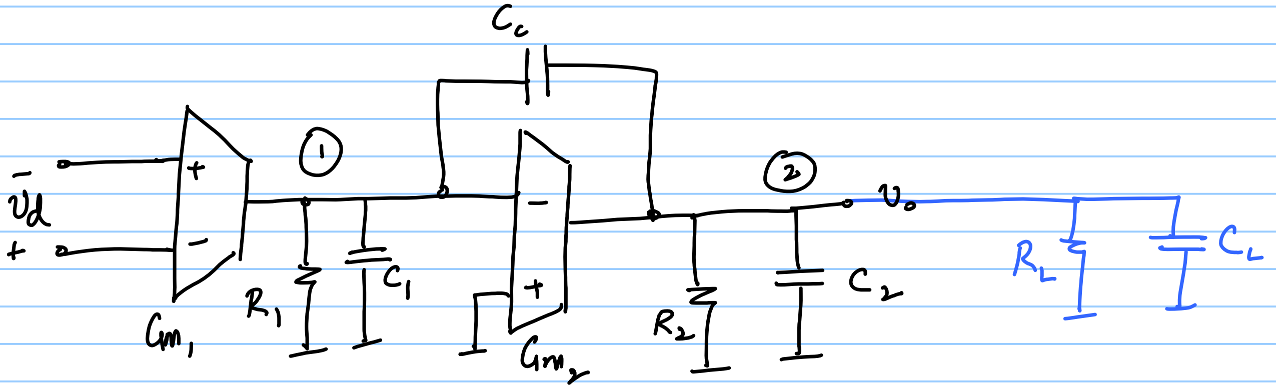
1) move $Z \to \infty$: set $Gm_2 R = 1 \Rightarrow R = \frac{1}{Gm_2}$

2) $Gm_2 R > 1 \Rightarrow$ LHP zero $\Rightarrow$ Use this to cancel p_2 $\Rightarrow$ p_3 becomes first non-dominant pole



$$A_o = Gm_1 R_1 \rightarrow Gm_1 R_L$$

* Cannot drive resistive loads (reduced DC gain A_o)

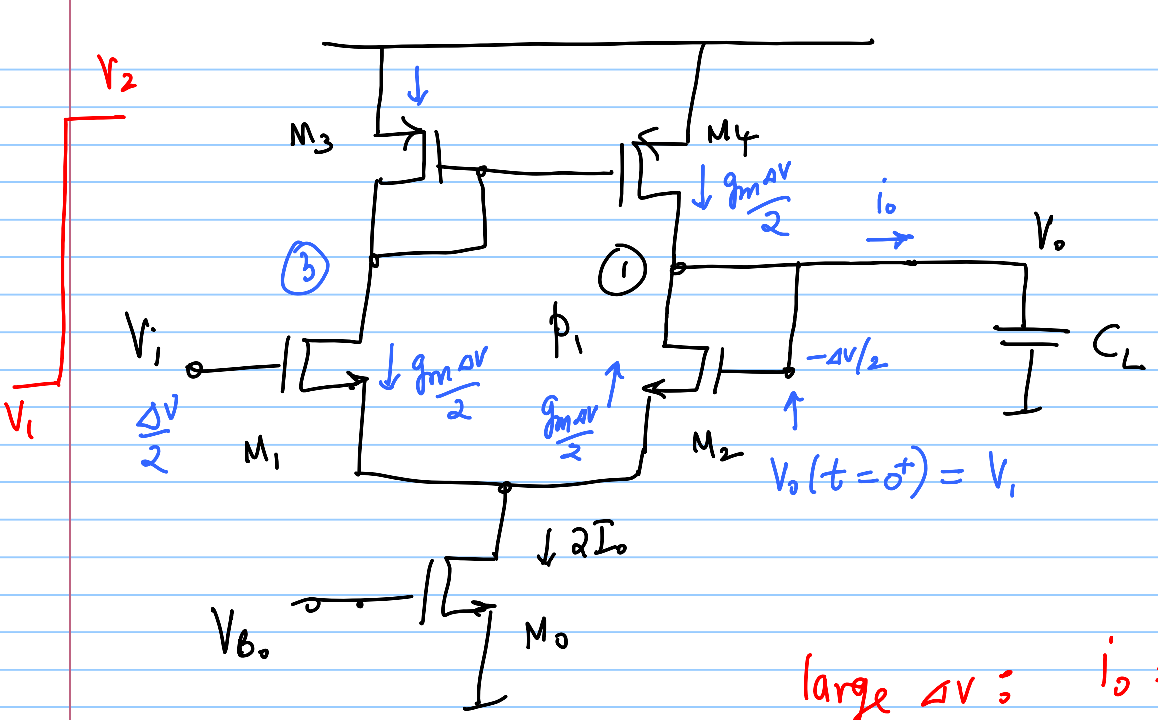


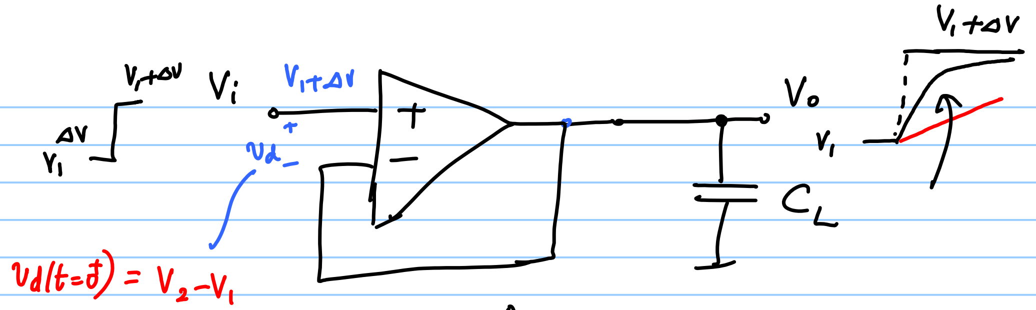
$$A_o = Gm_1 R_1 \; Gm_2 R_2$$

$$\downarrow$$

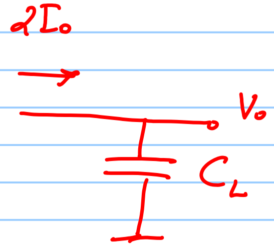
$$\underbrace{Gm_1 R_1}_{} \; Gm_2 R_L$$

DC gain from 1st stage





$$A = \frac{A_o}{1 + s/\omega_p}$$

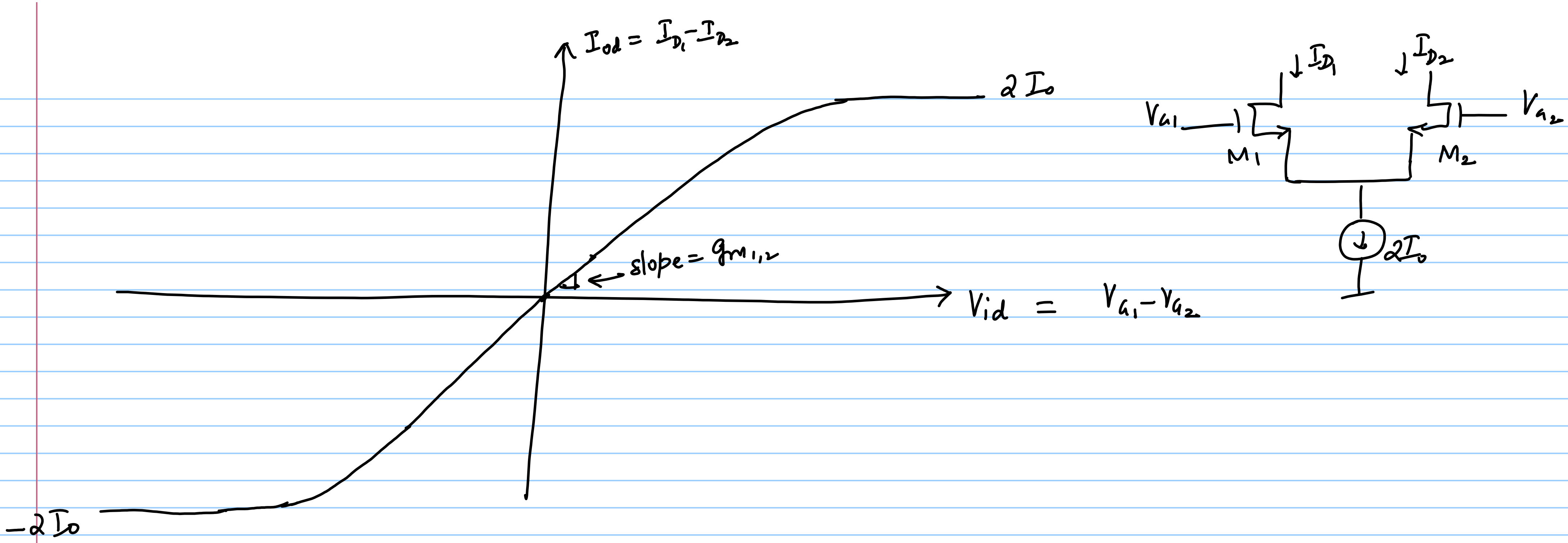


large ΔV : $i_o = 2I_o$

small ΔV } $i_o = g_m \Delta V$

$\Rightarrow V_o = \frac{2I_o}{C_L} \cdot t + V_1$ output increases linearly

"slew" rate $= \frac{2I_o}{C_L}$ V/s



$I_{od} = I_{D_1} - I_{D_2}$
$2 I_0$
slope = $g_{m1,2}$
$V_{id} = V_{G_1} - V_{G_2}$
$-2 I_0$
I_{D_1}
I_{D_2}
V_{G1}
V_{G_2}
M_1
M_2
$2I_0$

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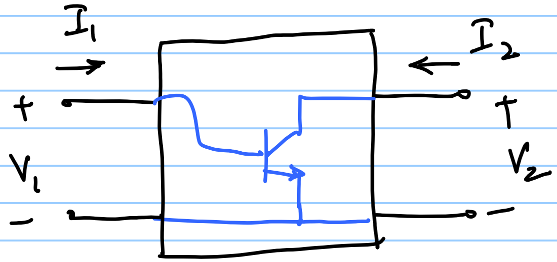
Lec 23

BJT "Bipolar Junction Transistor"



$I_1 = f(V_1, V_2)$

$I_2 = g(V_1, V_2)$

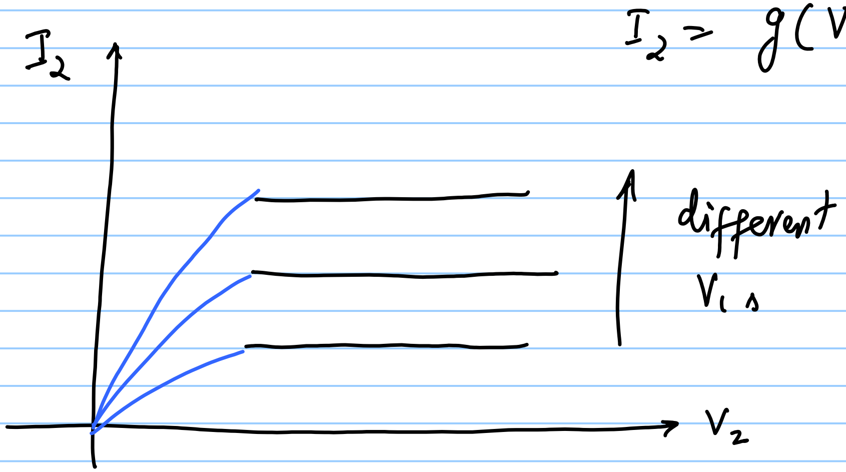
$y_{11} = \frac{\partial I_1}{\partial V_1}$ etc.

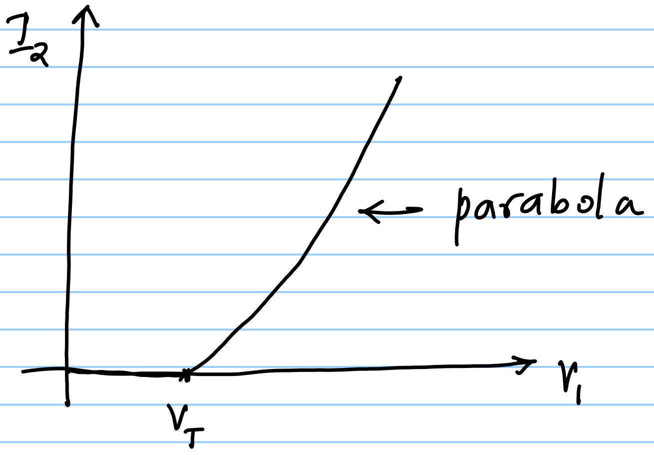
$y_{11} = 0 = y_{12} = y_{22}$

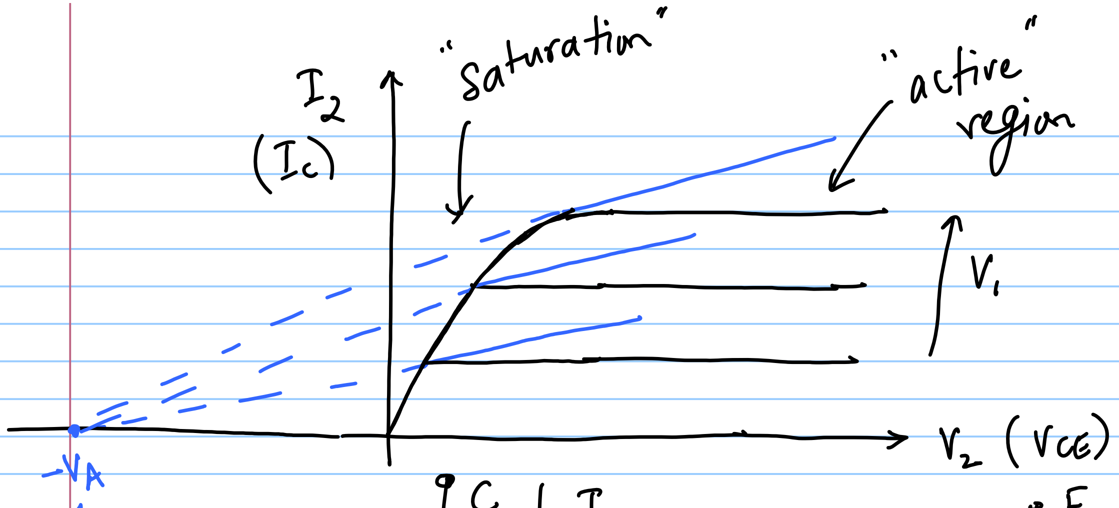
$y_{21} =$ large

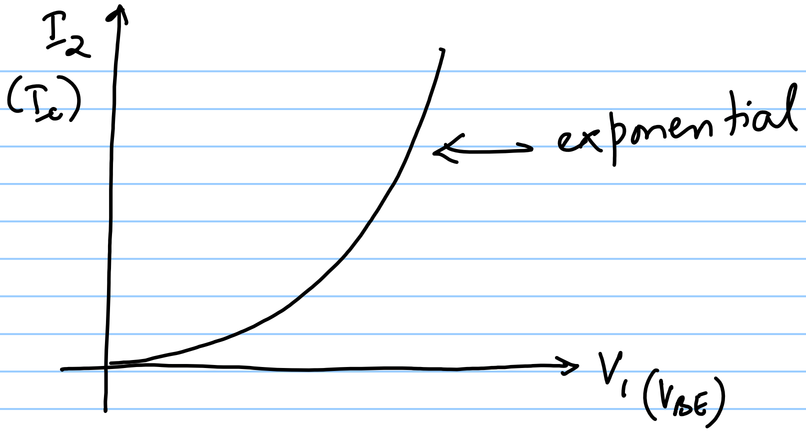
$I_1 =$ constant

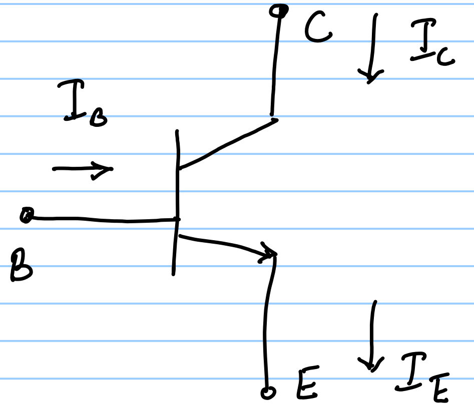
$I_2 = g(V_1)$



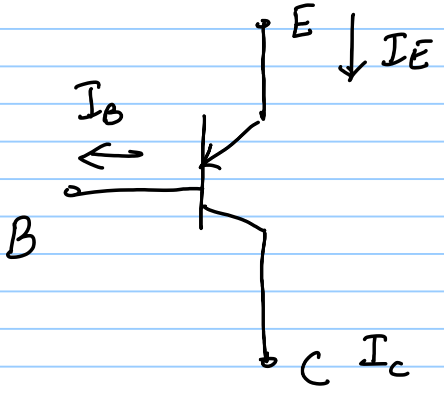








npn BJT



pnp BJT

$$I_B + I_c = I_E$$

$\beta = \frac{I_c}{I_B}$ should be as large as possible

~say 100 - 200

$$I_c = \frac{\beta}{\beta+1} I_E = \alpha I_E$$

↳ ~1

$$I_c = I_s \left\{ \exp\left(\frac{V_{BE}}{V_t}\right) - 1 \right\} \left\{ 1 + \frac{V_{CE}}{V_A} \right\}$$

$$\frac{kT}{q} \quad \text{thermal voltage} \approx 25.9\,mV \text{ @ } 300K$$

$V_{BE_{on}} \approx 0.7V$

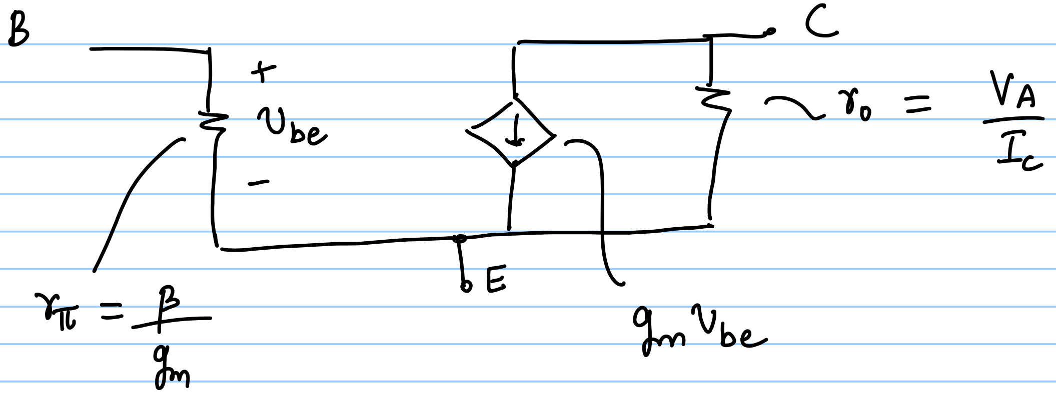
Small-signal parameters

$$y_{11} = \frac{\partial I_B}{\partial V_{BE}} = \frac{1}{\beta}\frac{\partial I_c}{\partial V_{BE}} = \frac{1}{\beta} \cdot I_s \cdot \frac{1}{V_t} \cdot \left\{ \exp\left(\frac{V_{BE}}{V_t}\right) \right\}$$

$$\approx \frac{I_c}{\beta V_t} \quad \text{because} \quad \exp\left(\frac{V_{BE}}{V_t}\right) >> 1 \quad \text{for } V_{BE} \sim 0.7V$$

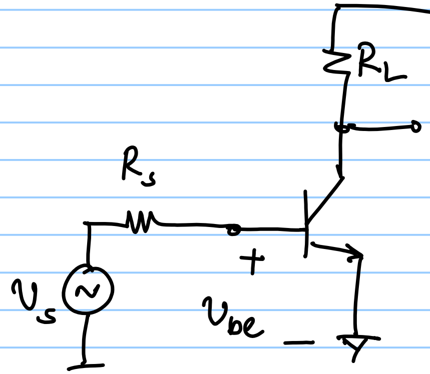
$$y_{12} = 0 \quad ; \quad y_{21} = \frac{\partial I_c}{\partial V_{BE}} \approx \underbrace{\frac{I_c}{V_t}}_{g_m} \quad ; \quad y_{22} = \frac{\partial I_c}{\partial V_{CE}} \approx \frac{I_c}{V_A}$$

$$\Rightarrow y_{11} = \frac{g_m}{\beta}$$



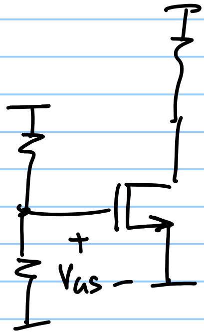
$$r_o = \frac{V_A}{I_C}$$

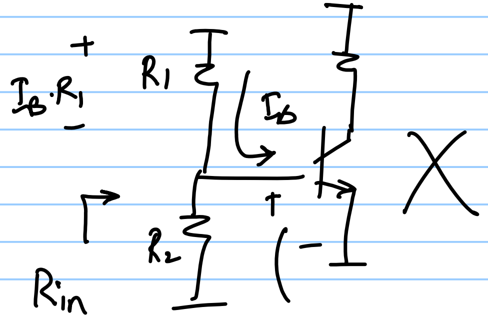
$$r_\pi = \frac{\beta}{g_m}$$



$$\frac{v_o}{v_s} = -g_m \cdot (R_L \| r_o) \cdot \frac{r_\pi}{r_\pi + R_s}$$

$$v_{be} = \frac{r_\pi}{r_\pi + R_s} \cdot v_s$$

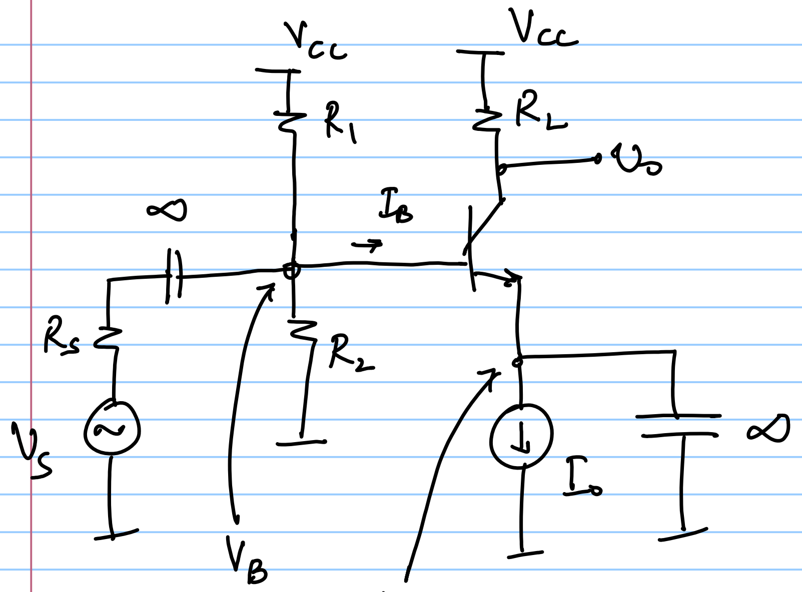




large
⇓
R_1 & R_2 large

1) $V_{BE_{on}} \sim 0.7\,V$

2) I_B flows through R_1



R_1 & R_2 should not be too large because of $I_B \cdot R_1$ drop

$r_\pi = \frac{\beta}{g_m}$ e.g. $g_m = 1mS$; $\beta = 200$

$r_\pi = 200k\Omega$

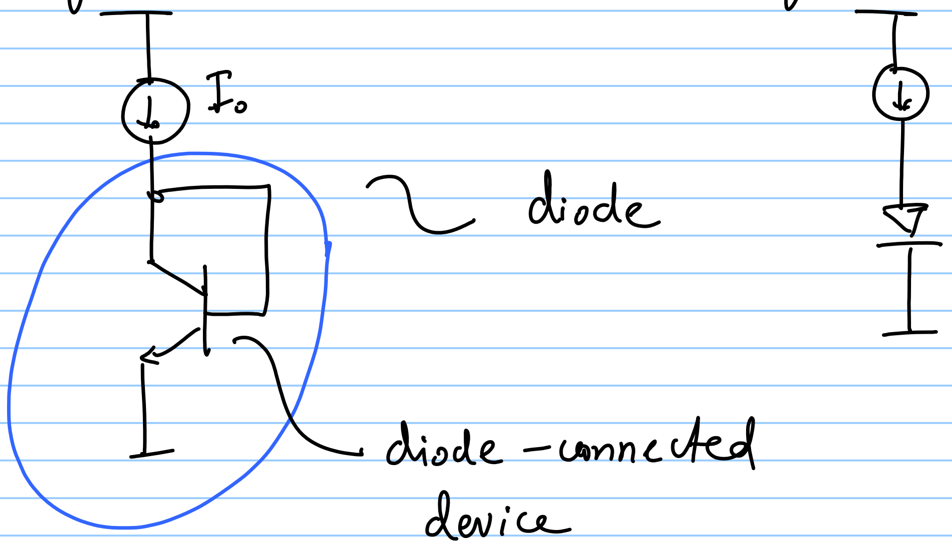
$V_B - V_{BE_{on}} = V_B - 0.7V > V_{min}$ for Current source

$r_\pi = \frac{\beta}{g_m}$ large because of large β

$$g_{m_{BJT}} = \frac{I_C}{\underbrace{V_t}_{25.9mV}}$$

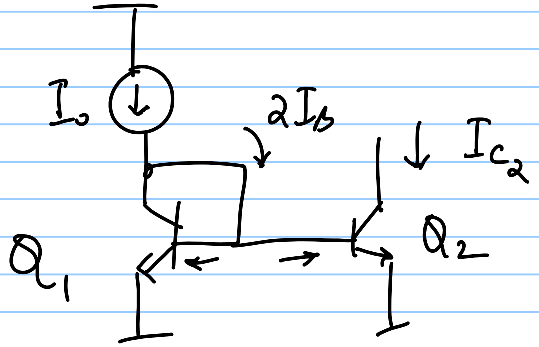
$$g_{m_{MOS}} = \frac{2 I_D}{(V_{GS}-V_T)} = \frac{I_D}{\underbrace{(V_{GS}-V_T)/2}_{100-200mV}}$$

for a given bias current, BJT gives more g_m i.e. more gain



Current Mirror

$Q_1 = Q_2$

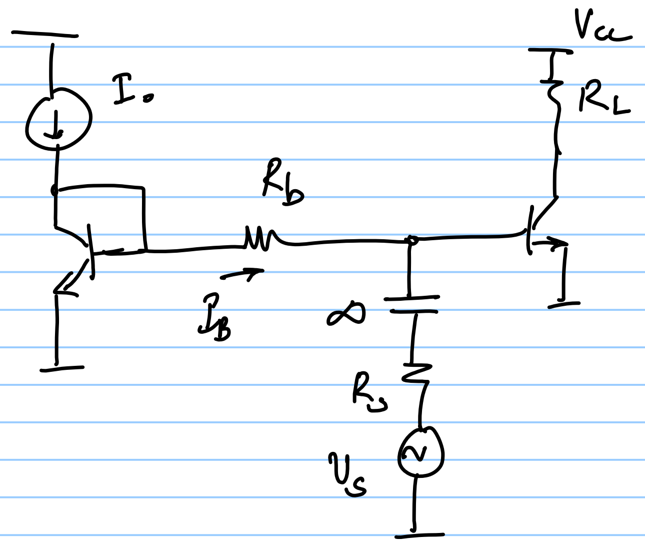


relate I_C to V_{BE}

$$V_{BE_1} = V_{BE_2} \Rightarrow I_{C_1} = I_{C_2} \Rightarrow I_{B_1} = I_{B_2}$$

$$I_{C_1} = I_o - 2I_B = I_o - \frac{2I_{C_1}}{\beta}$$

$$I_{C_1} = I_{C_2} = \frac{\beta}{\beta+2} I_o$$



Swing limits

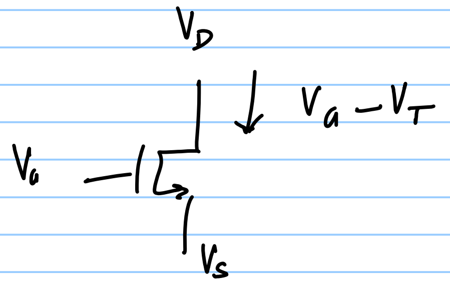
$I_c = 0$ cut off limit

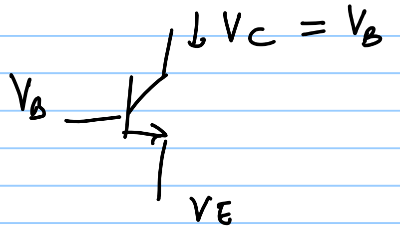
$V_{CE} \geq V_{CE_{sat}}$; $V_{CE_{sat}} \approx V_{BE_{on}} = 0.7V$

saturation limit

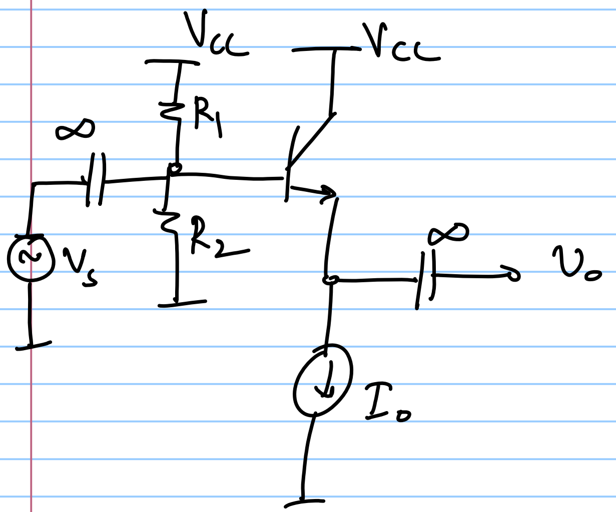
MOS: $V_{DS} \geq V_{DSat}$ for saturation region operation

$V_{GS} - V_T$





VCVS (CCA)

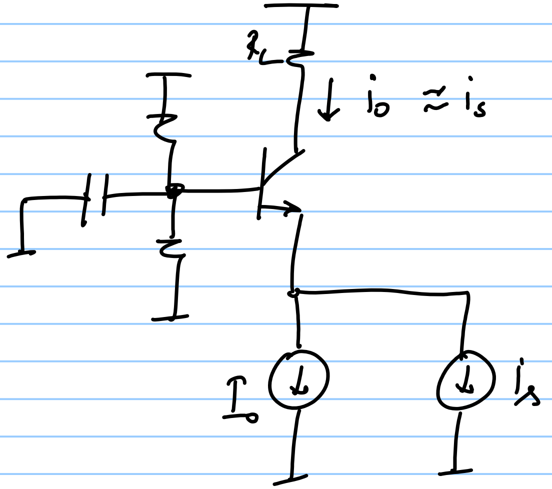


$$\frac{v_o}{v_s} \approx 1$$

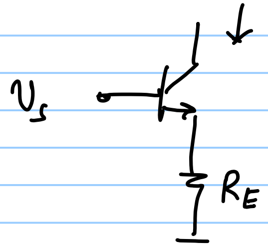
$$r_{out} = \frac{1}{g_m}$$

$$r_{in} = \text{high}$$

CCCS (CGA)



VCCS

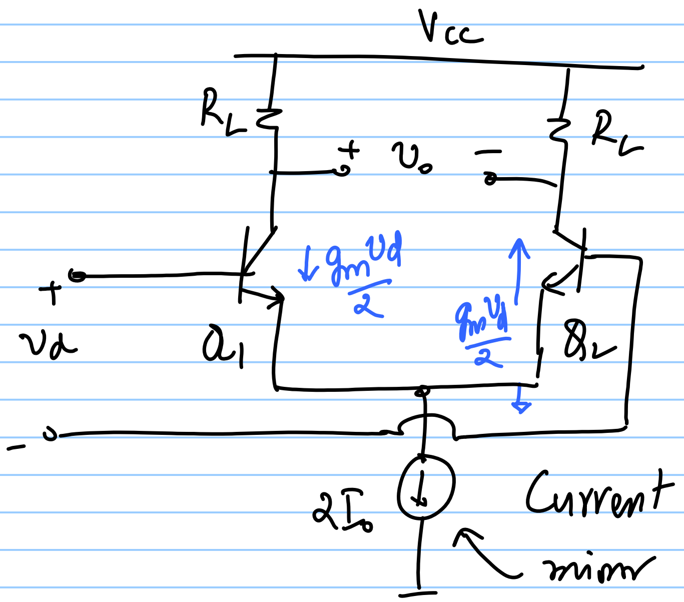


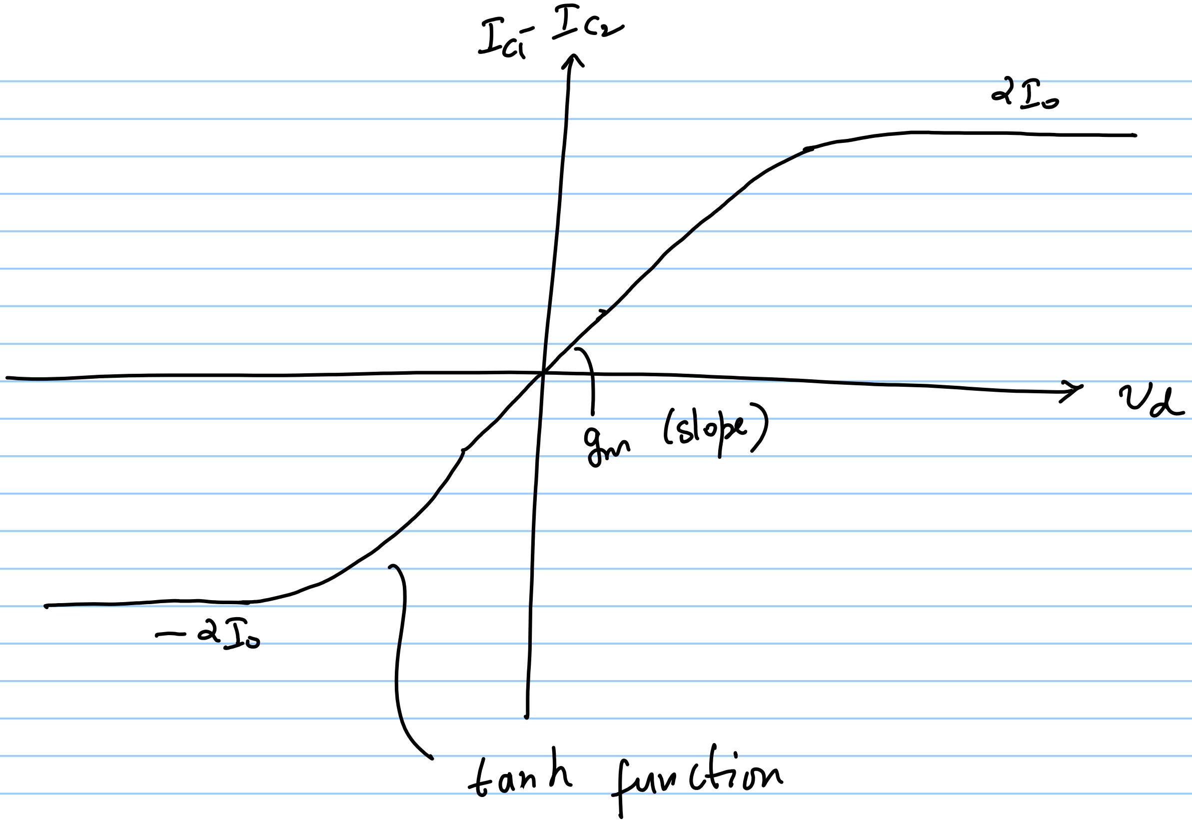
$$\downarrow i_o \approx \frac{1}{R_E} \cdot v_s$$

CCVS

HW

Differential Amplifier





"741" opamp

23/11/17

Lec 24

Bandgap Reference

V_{ref} independent of V_{DD}, T, Process etc.
→ Silicon bandgap

$$V_{ref} = \underbrace{\alpha_1 V_1}_{\text{tempco}} + \underbrace{\alpha_2 V_2}_{\text{tempco}}$$

$$\frac{\partial V_{ref}}{\partial T} = 0 \Rightarrow \alpha_1 \frac{\partial V_1}{\partial T} + \alpha_2 \frac{\partial V_2}{\partial T} = 0 \quad @ \ 300K$$

α_1 & α_2 have to be chosen such that this is true

V_{BE} or V_{Diode}

$$I_D = I_s \left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]$$

$$I_s = b \cdot T^{4+m} \exp\left(\frac{-E_g}{kT}\right)$$

E_g = BG of silicon ≈ 1.1eV

$$V_{BE} = V_t \ln\left(\frac{I_D}{I_s}\right)$$

$$V_t = \frac{kT}{q}$$

$$\frac{\partial V_{BE}}{\partial T} = \frac{\partial V_t}{\partial T} \cdot \ln\left(\frac{I_D}{I_s}\right) - \frac{V_t}{I_s} \cdot \frac{\partial I_s}{\partial T}$$

$$\longrightarrow V_t \frac{\partial}{\partial T} \left\{ \overset{0}{\cancel{\ln I_D}} - \ln I_s \right\}$$

$$\frac{\partial I_s}{\partial T} = b\,(4+m)\,T^{3+m} \exp\left(\frac{-E_g}{kT}\right) + b\,T^{(4+m)} \exp\left(\frac{-E_g}{kT}\right) \cdot \left(\frac{E_g}{kT^2}\right)$$

$$= \frac{4+m}{T} \cdot I_s + \frac{E_g}{kT^2} I_s$$

$$\frac{V_t}{I_s} \cdot \frac{\partial I_s}{\partial T} = (4+m) \cdot \frac{V_t}{T} + \frac{E_g}{kT^2} \cdot V_t$$

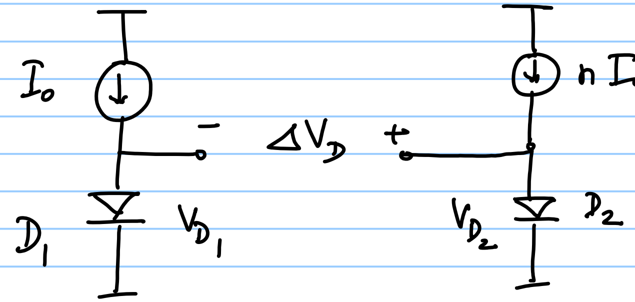
$$\frac{\partial V_{BE}}{\partial T} = \frac{V_t}{T} \ln\left(\frac{I_D}{I_s}\right) - (4+m) \cdot \frac{V_t}{T} - \frac{E_g}{kT^2} \cdot V_t$$

$$= \frac{V_{BE} - (4+m)V_t - E_g/q}{T} .$$

$$\left.\frac{\partial V_{BE}}{\partial T}\right|_{300k} \approx -1.5mV/K$$ negative tempco

$$V_t = \frac{kT}{q} \longleftarrow$$ positive tempco

BG ref. $= \alpha_1 V_{BE} + \alpha_2 V_t$



$D_1 = D_2 \Rightarrow I_{s_1} = I_{s_2} = I_s$

$$\Delta V_D = V_{D_2} - V_{D_1}$$

$$= V_t \ln\left(\frac{nI_o}{I_s}\right) - V_t \ln\left(\frac{I_o}{I_s}\right)$$

$$= V_t \ln(n) = \frac{kT}{q} \ln(n)$$ positive temp co

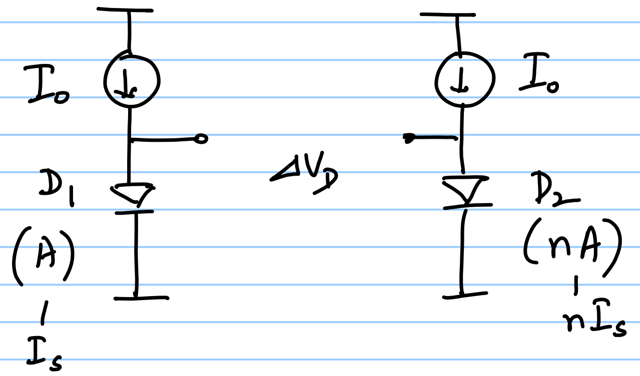
$$\frac{\partial \Delta V_D}{\partial T} = \frac{k}{q} \ln(n) \approx 0.086 \ln(n) \quad mV/K$$

$$V_{ref} = \alpha_1 V_1 + \alpha_2 V_2$$

$$= V_{D_1} + \Delta V_D$$

for 0 temp co @ 300k ⇒ set $\left.\frac{\partial V_{ref}}{\partial T}\right|_{300k} = 0$

$$\Rightarrow \ln(n) = \frac{1.5}{0.086} \approx 17.4$$

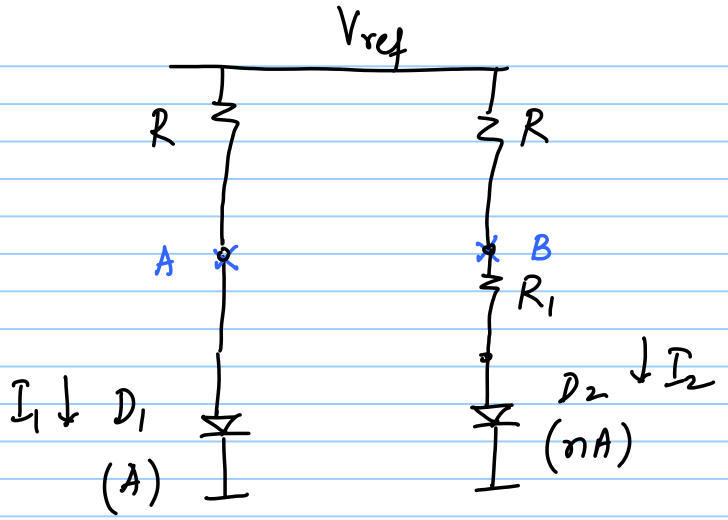


alternative implementation

1) Add V_{D_1} & ΔV_D

- ↳ generate a current $\propto \Delta V_D$
- ↳ pass current through resistor
- ↳ connect resistor in series with diode

2) Don't need current sources



If $I_1 = I_2 \Rightarrow V_{D_1} > V_{D_2}$

$$V_{D_1} = V_t \ln\left(\frac{I_D}{I_s}\right)$$

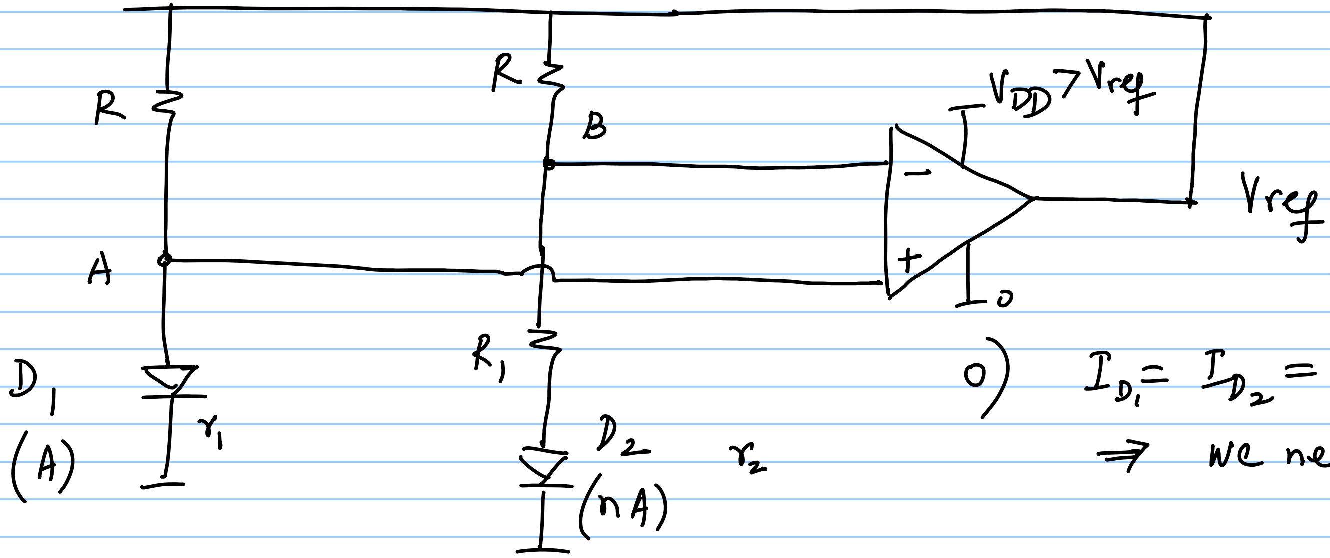
$$V_{D_2} = V_t \ln\left(\frac{I_D}{nI_s}\right)$$

If $V_A = V_B \Rightarrow V_{R_1} = V_{D_1} - V_{D_2} = \Delta V_D$

$$V_B = V_{D_2} + \Delta V_D$$

$V_A = V_B$ through opamp placed in −ve f.b. $\Rightarrow I_1 = I_2$

$$V_{ref} = V_{D_2} + I_2 (R + R_1) = V_{D_2} + \frac{\Delta V_D}{R_1}(R + R_1) = V_{D_2} + \Delta V_D \left(1 + \frac{R}{R_1}\right)$$



0) $I_{D_1} = I_{D_2} = 0$ is a valid state

⇒ we need a startup circuit

1) This ckt has +ve & −ve f.b.

2) Ensure that −ve f.b. is stronger than +ve f.b.

$$v_A = \frac{r_1}{r_1 + R} \cdot V_{ref} \quad ; \quad v_B = \frac{r_2 + R_1}{r_2 + R_1 + R} \cdot V_{ref}$$

normally $r_1, r_2 << R, R_1 \Rightarrow v_B > v_A$

$$V_{ref} = V_{D_2} + I_2 (R_1 + R) = V_{D_2} + \frac{\Delta V_D}{R_1}(R_1 + R)$$

$$= \underset{\displaystyle\downarrow}{V_{D_2}} + \underset{\displaystyle\downarrow}{\frac{kT}{q}} \left[\ln(n) \right] \left[1 + \frac{R}{R_1} \right]$$

$$\text{tempco} \left. \right\} \quad -1.5\,mV/K \qquad 0.086 \left[\ln(n) \right] \left[1 + \frac{R}{R_1} \right]$$

$$\left. \frac{\partial V_{ref}}{\partial T} \right|_{300k} = 0 \quad \Rightarrow \quad \left(1 + \frac{R}{R_1} \right) \ln(n) = 17.4$$

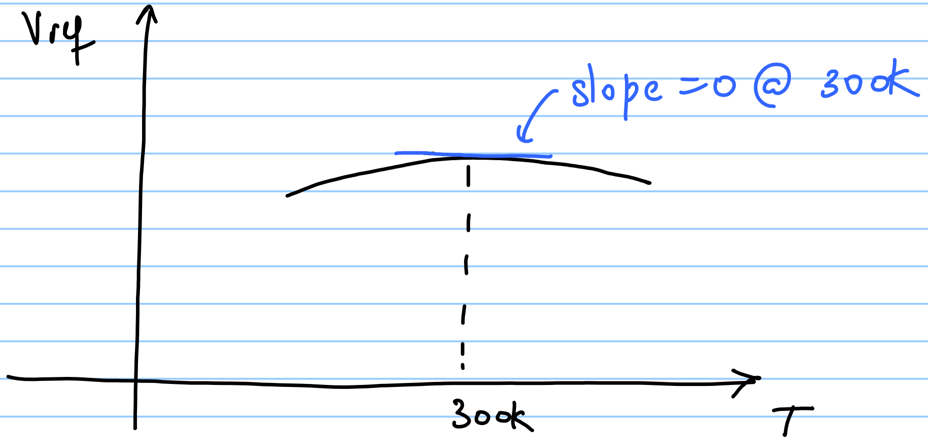
$$V_{ref} = V_{D_2} + \frac{kT}{q} \times 17.4 \quad \sim 1.25V$$

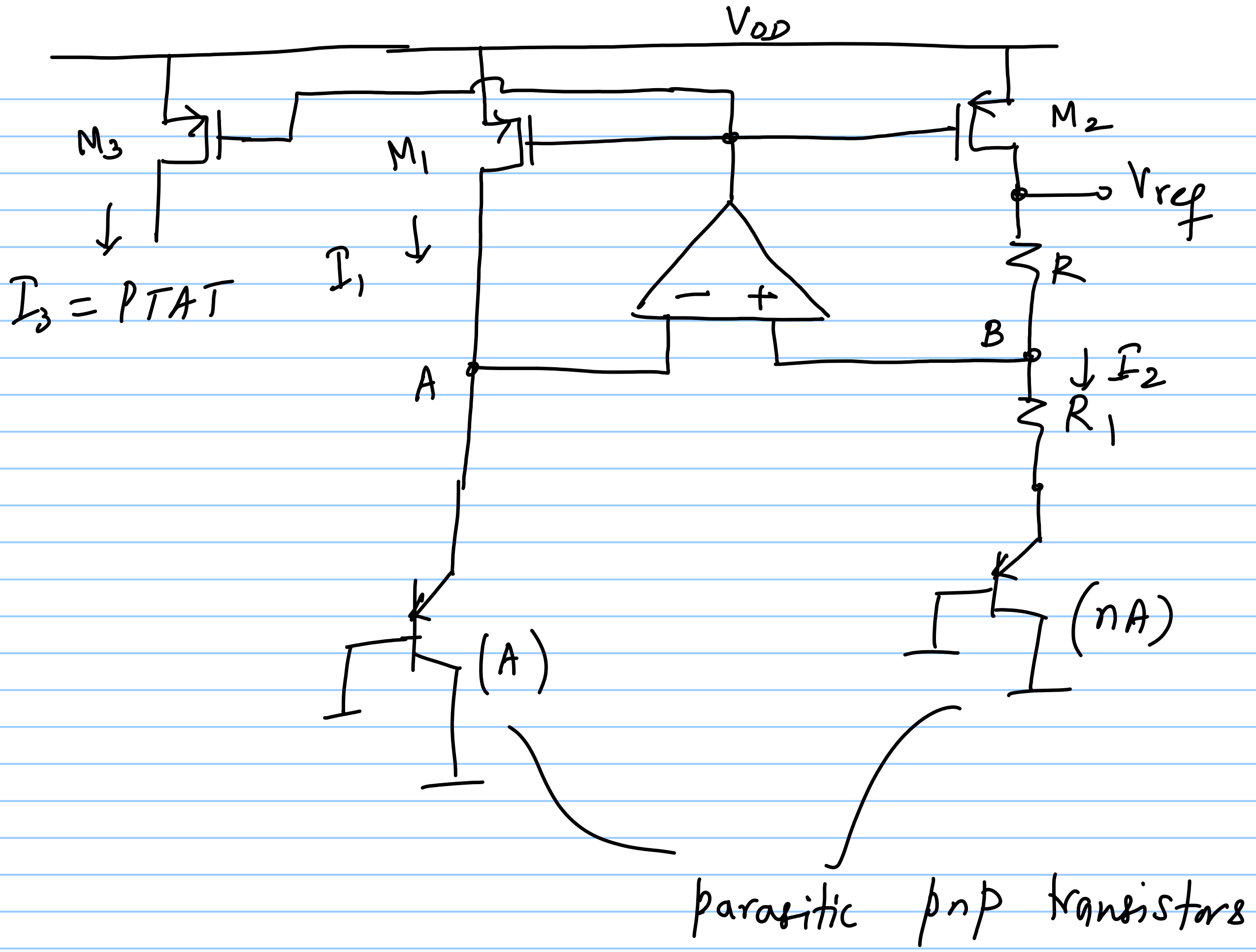
1) Tempco of resistors : choose R & R_1 to be same type of res.

$$\frac{R}{R_1} = \text{constant}$$

2) $I_1 = I_2 = \frac{\Delta V_D}{R} = \frac{kT}{q R_1} \cdot \ln(n) \quad \propto T$

"PTAT" current
"proportional to absolute temperature" $\Big\}$ Useful to bias circuits

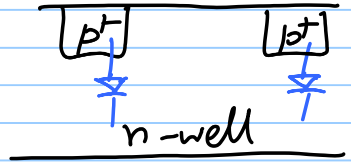




$M_1 = M_2$

$V_A = V_B$

$I_1 = I_2$ = PTAT current



→ make it behave as pnp

1) What happens if $V_{DD} < 1.25V$? "Fractional" BG

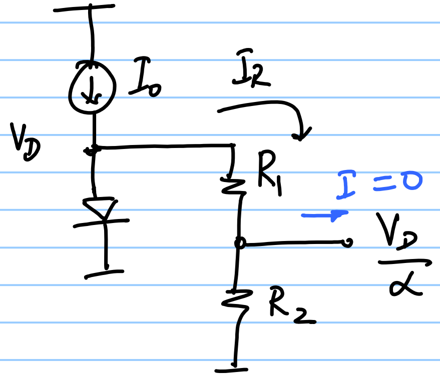
28/11/17

Lec 25

Fractional BG

$V_{ref} = V_D + \alpha \Delta V_D \approx 1.25V$ What if $V_{DD} < 1.25V$

$$\frac{V_{ref}}{\alpha} = \frac{V_D}{\alpha} + \Delta V_D$$

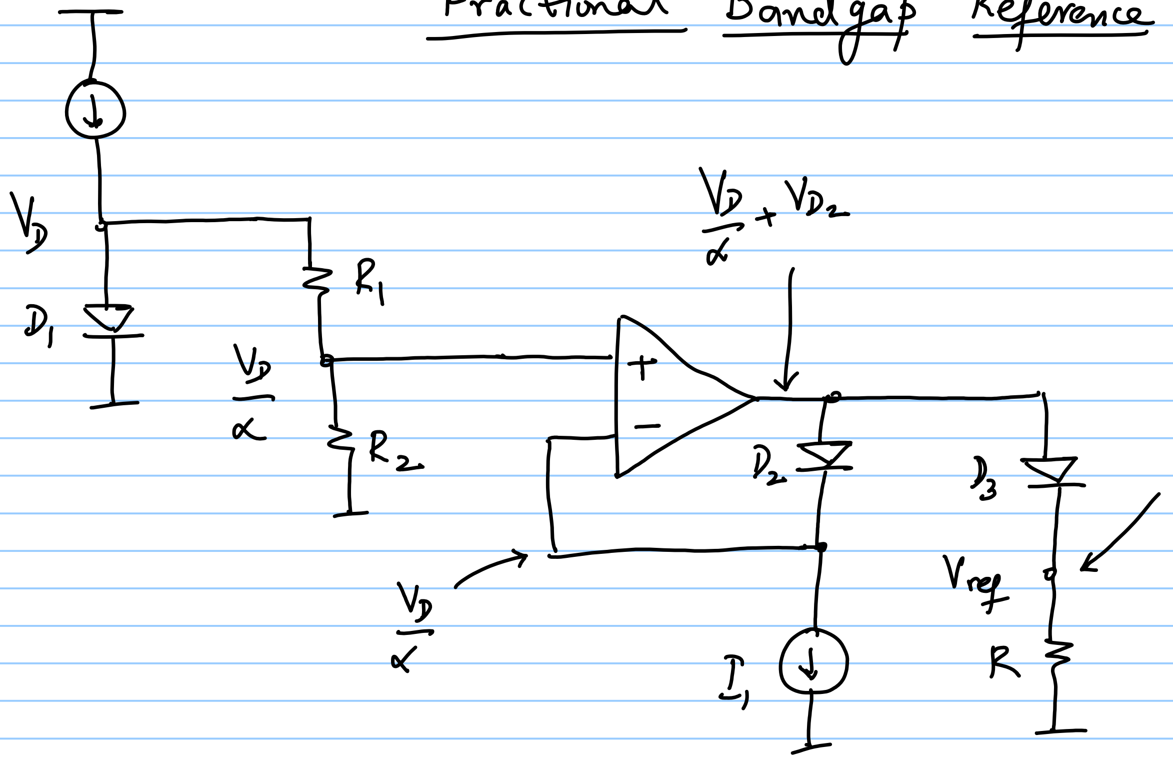


R_1 & R_2 : large so that $I_R << I_0$

$$\frac{V_D}{\alpha} = \frac{V_D \cdot R_2}{R_1 + R_2}$$

Fractional Bandgap Reference

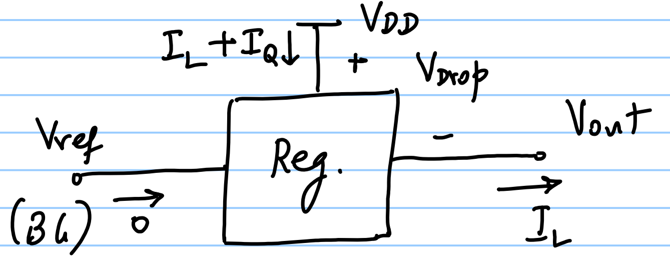
Conceptual Circuit



$$V_{ref} = \frac{V_D}{\alpha} + \left(V_{D_2} - V_{D_3}\right)$$

$$= \frac{V_D}{\alpha} + \Delta V_D$$

Voltage Regulators



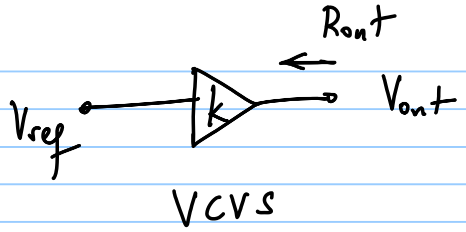
- * $V_{out} = V_{DD} - V_{Drop}$ — dropout voltage
- * V_{out} = constant
- * R_{out} = low
- * η = efficiency should be high

$$P_{out} = V_{out} \cdot I_L \; ; \; P_{in} = V_{DD}(I_L + I_Q)$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{V_{out}}{V_{DD}} \cdot \frac{I_L}{I_L + I_Q} = \frac{V_{out}}{V_{out} + V_{Drop}} \cdot \frac{I_L}{I_L + I_Q}$$

"LDO": Low Dropout Regulator

We want minimum V_{Drop} & I_Q for best efficiency



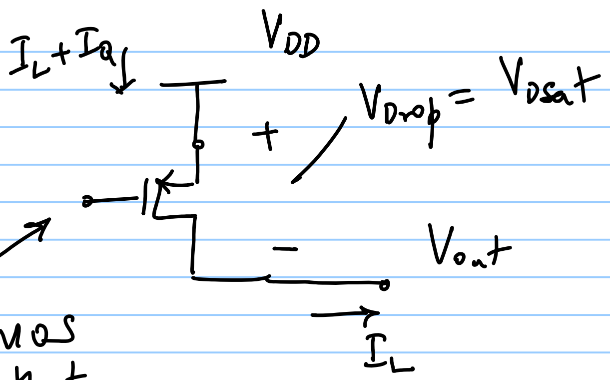
use negative feedback

- Load regulation : $\frac{\Delta V_{out}}{\Delta I_L}$ = small signal R_{out}
- Line regulation : $\frac{\Delta V_{out}}{\Delta V_{DD}}$

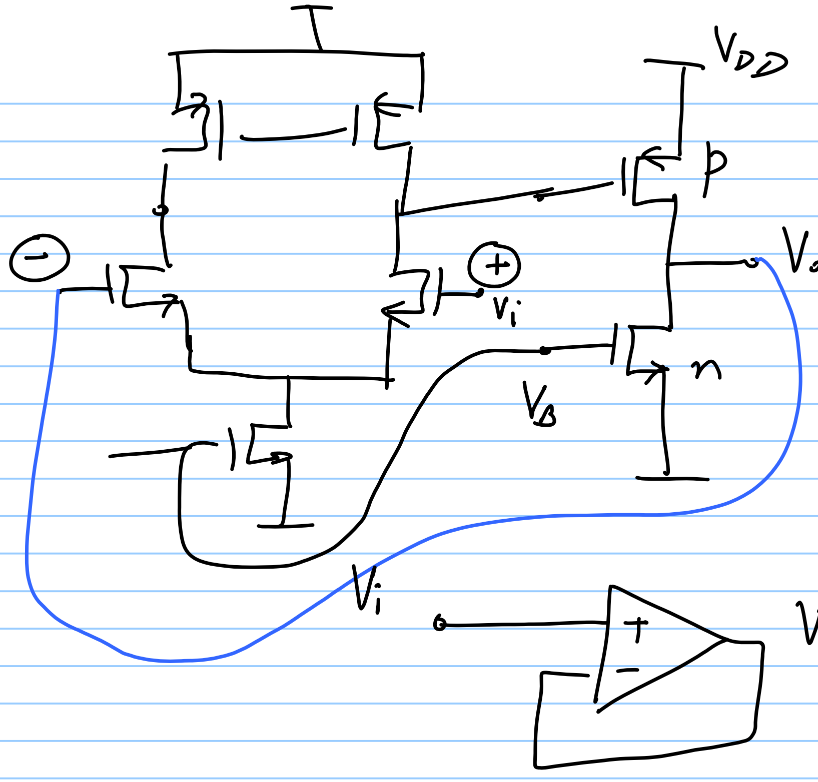
$\}$ we want both to be small

- step changes in I_L : change in V_{out} to be small
- Frequency response of feedback circuit
- $\left|\frac{V_{out}(f)}{V_{DD}(f)}\right| << 1$ (small-signal)

"Power Supply Rejection Ratio" or PSRR



Drive PMOS gate so that V_{out} = constant



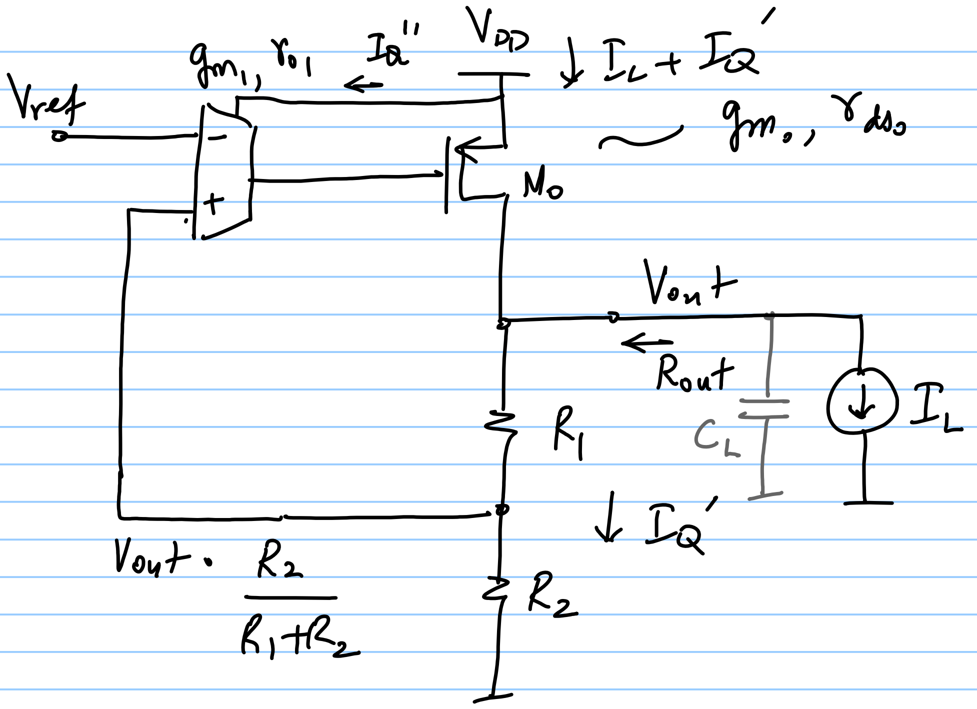
V_o max $V_{DD} - V_{Dsat_p}$

min V_{Dsat_n} $= V_B - V_{T_n}$

* Sense V_{out} (or a portion)

* Compare to V_{ref}

* Drive PMOS gate so that V_{out} = constant



R_1, R_2 : large (small I_Q)

$I_Q'' + I_Q' = I_Q$

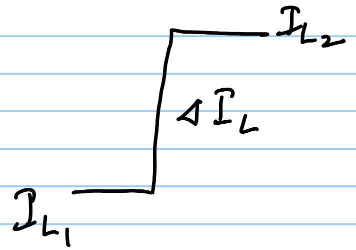
$r_{ds0} << R_1, R_2$

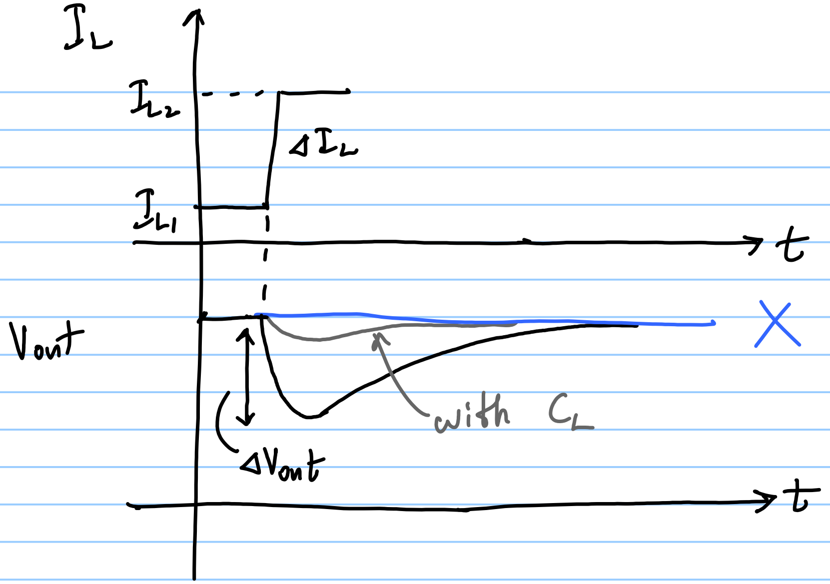
$$R_{out} = \frac{r_{ds0}}{1 + A_0}$$

$$A_0 = g_{m1} r_{o1} \cdot g_{m0} r_{ds0} \cdot \frac{R_2}{R_1 + R_2}$$

(dc gain of loop)

step I_L





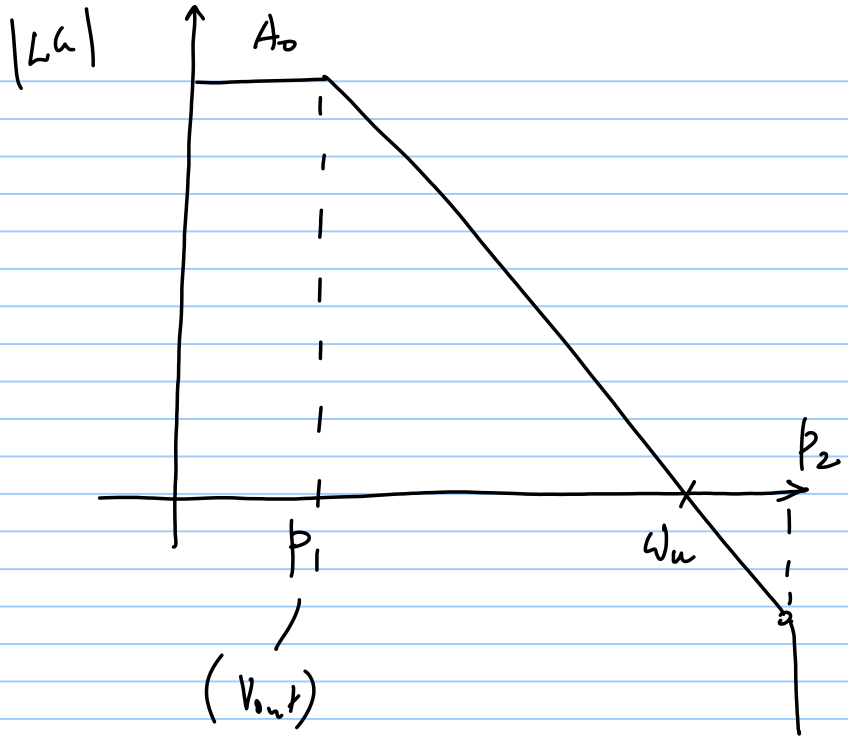
$\Delta Vout = \Delta I_L \cdot r_{ds_0}$

exact : $\Delta I_L \cdot \left(r_{ds_0} \| (R_1 + R_2) \right)$

{ can be large }

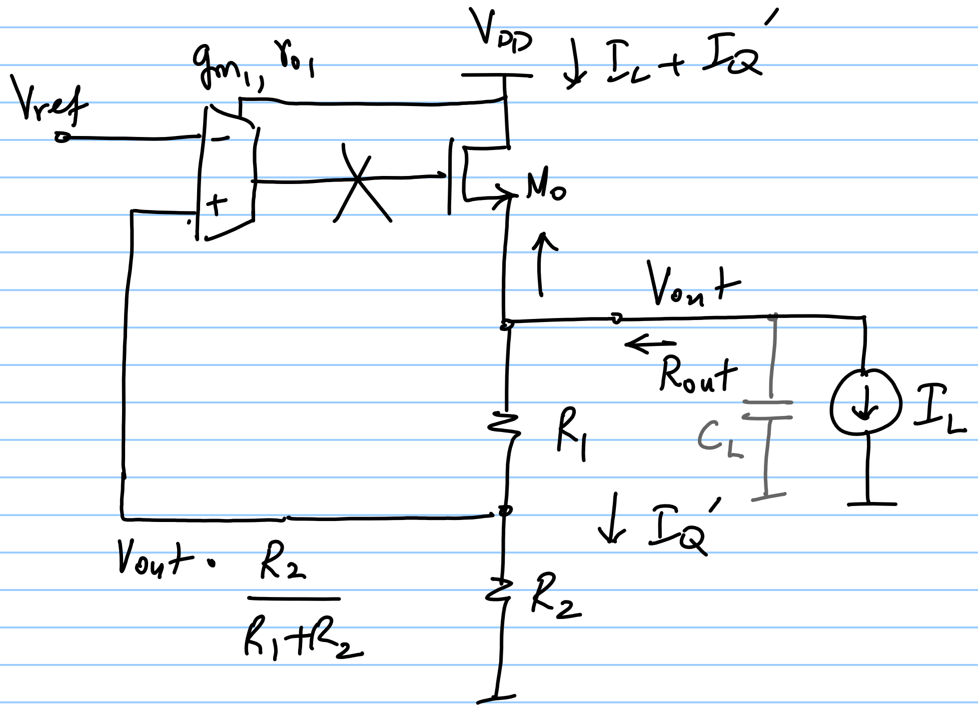
Add large cap. C_L at output node to deliver ΔI_L

Vout becomes the dominant pole node for freq. response



$p_1 : \frac{1}{C_L \cdot r_{ds_0}}$; $p_2 : \frac{1}{C_{gs_0} \cdot r_{o_1}}$

$\omega_u = p_1 \times A_0$



NMOS reg :

High BW

high Dropout