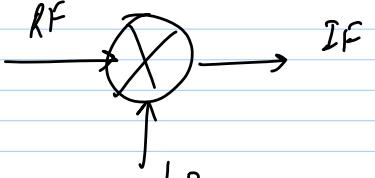


18-9-13

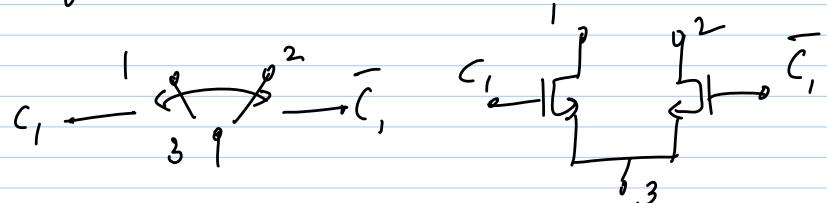
## Lec 20

### 3-port mixers

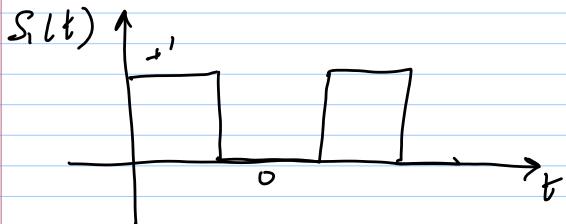


- \* ideally generate only IM component
- \* CMOS - good switches

### Single-balanced mixer (active)

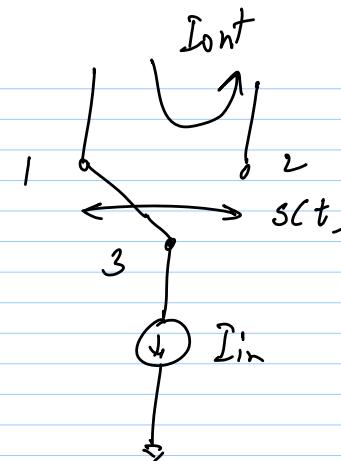


$$s_1(t) = 0.5 + 0.5 s(t)$$



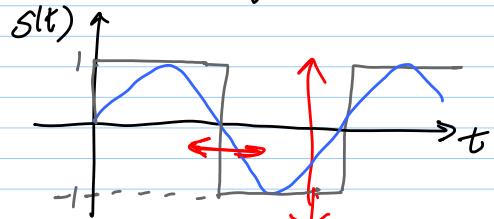
$$I_{\text{out}^+} = (I_{DC} + I_{RF} \cos \omega_{RF} t) \cdot \{0.5 + 0.5 s(t)\}$$

$$I_{\text{out}^+} = (I_{DC} + I_{RF} \cos \omega_{RF} t) \cdot \operatorname{sgn}(\sin \omega_{LO} t)$$



$$I_{\text{out}^+} = S(t) \cdot I_{\text{in}}$$

$$S(t) = \operatorname{sgn}(\sin \omega_{LO} t)$$



$$I_{\text{in}} = i_{RF} + I_{DC}$$

$$i_{RF} = I_{RF} \cos \omega_{RF} t$$

### Fourier Series of S(t)

$$S(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(n \omega_{LO} t) + b_n \sin(n \omega_{LO} t)]$$

$$\frac{a_0}{2} = 0$$

$a_n = 0$  because odd signal

$$b_n = \frac{2}{T_{LO}} \int_0^{T_{LO}} \operatorname{sgn}(t) \sin(n \omega_{LO} t) dt$$

$$\begin{aligned}
 &= \frac{4}{\pi} \int_{-\pi/2}^{\pi/2} \sin(n\omega_L t) dt \\
 &= \frac{4}{\pi} \cdot \frac{1}{n\omega_L} \left[ -\cos(n\omega_L t) \right]_{-\pi/2}^{\pi/2} \\
 &= \frac{2}{n\pi} \left[ 1 - \cos n\pi \right]
 \end{aligned}$$

$$b_n = \begin{cases} 0 & \text{if even } n \\ \frac{4}{n\pi} & \text{if odd } n \end{cases}$$

$$\textcircled{I} \Rightarrow \frac{4I_{DC}}{\pi} \left[ \sin \omega_L t + \frac{1}{3} \sin 3\omega_L t + \dots \right]$$

*Lo feedthrough*

*feedthrough of Lo harmonics*

$$\textcircled{II} \Rightarrow \frac{4I_{RF}}{\pi} \left[ \cos \omega_{RF} t \sin \omega_L t + \frac{1}{3} \cos \omega_{RF} t \sin 3\omega_L t + \dots \right]$$

$$s(t) = \frac{4}{\pi} \left[ \sin \omega_L t + \frac{1}{3} \sin 3\omega_L t + \dots \right]$$

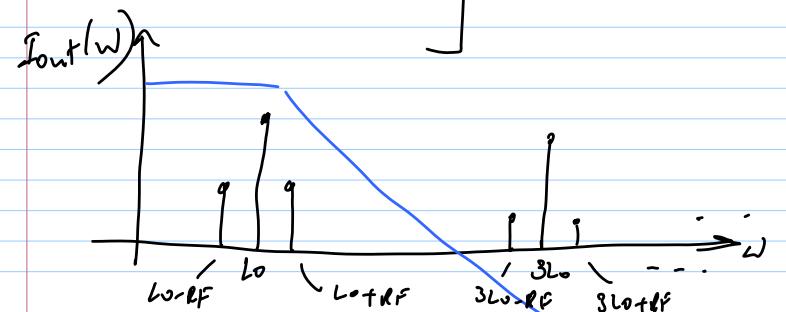
$$i_{out}(t) = \left( I_{DC} + I_{RF} \cos \omega_{RF} t \right) \cdot s(t)$$

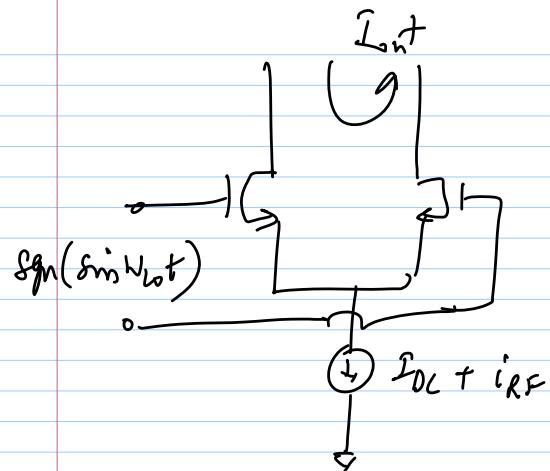
$$= I_{DC} \cdot s(t) + I_{RF} \cos \omega_{RF} t \cdot s(t)$$

\textcircled{I}

\textcircled{II}

$$\begin{aligned}
 &= \frac{2I_{RF}}{\pi} \left[ \sin(\omega_L - \omega_{RF}) t + \sin(\omega_L + \omega_{RF}) t \right. \\
 &\quad \left. + \frac{1}{3} \sin(3\omega_L - \omega_{RF}) t + \frac{1}{3} \sin(3\omega_L + \omega_{RF}) t \right. \\
 &\quad \left. + \dots \right]
 \end{aligned}$$

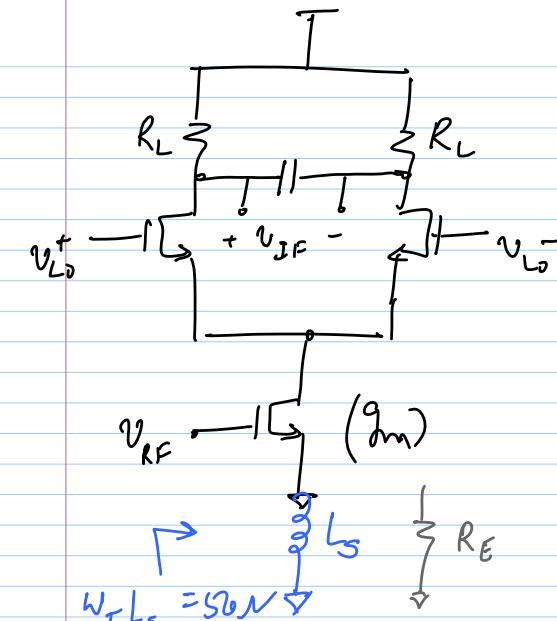




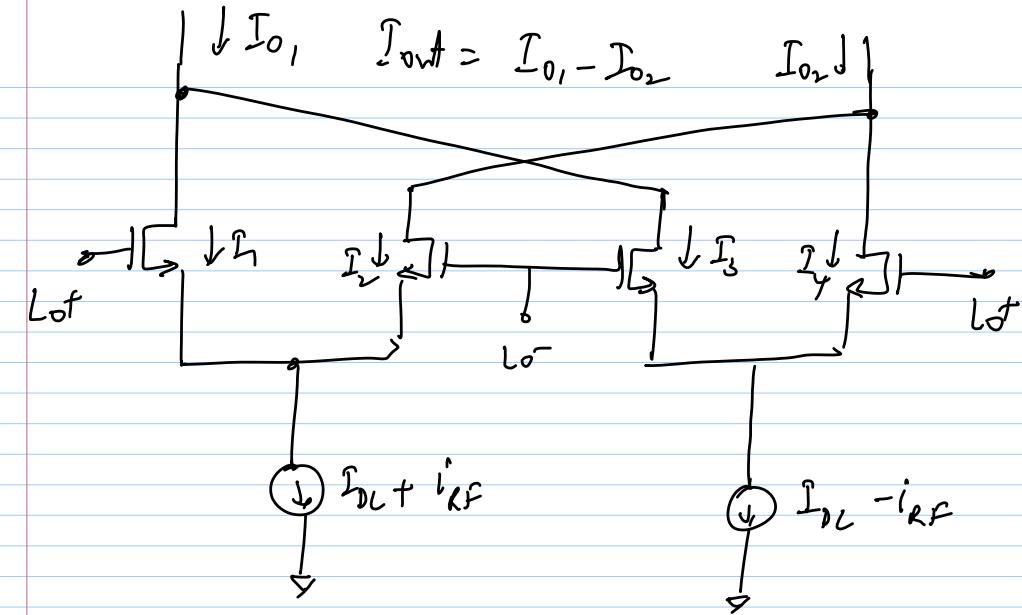
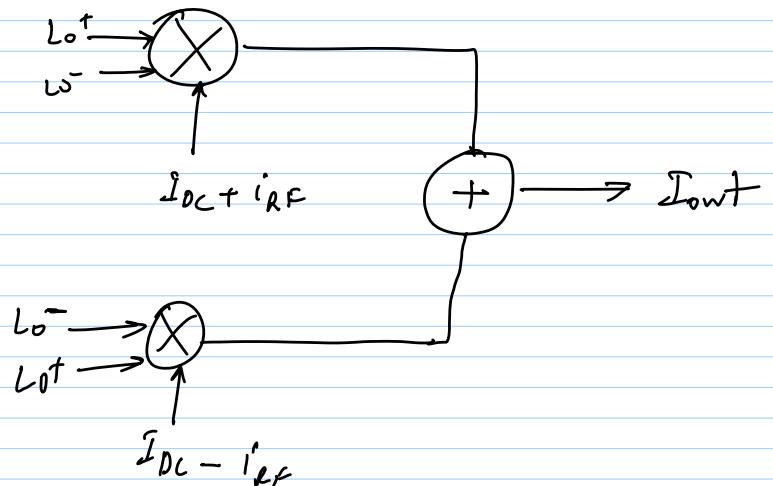
\* poor LO feedthrough  
 \* Input RF is in the form of current  
 $\rightarrow V-I$  conversion  
 non-linearity

$$* h_C = \frac{2}{\pi} g_m R_L$$

mixed output load



### Double-balanced mixer



Gilbert Cell Double-balanced Mixer

$$\begin{aligned}
 I_{\text{out}} &= (I_1 + I_3) - (I_2 + I_4) \\
 &= (I_1 - I_2) - (I_4 - I_3) \\
 &= \frac{4}{\pi} I_{\text{RF}} \left[ \sin(\omega_{\text{LO}} - \omega_{\text{RF}})t + \sin(\omega_{\text{LO}} + \omega_{\text{RF}})t \right. \\
 &\quad \left. + \frac{1}{3} (\sin(3\omega_{\text{LO}} + \omega_{\text{RF}}) + \dots) \right]
 \end{aligned}$$

\* Excellent LO-IF isolation (depends on matching)

\* Isolation:

- LO waveform
- LO derive mismatch
- RF " "
- L<sub>L</sub> mismatch