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1,666,206

MODULATION SYSTEM

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Fig. 1

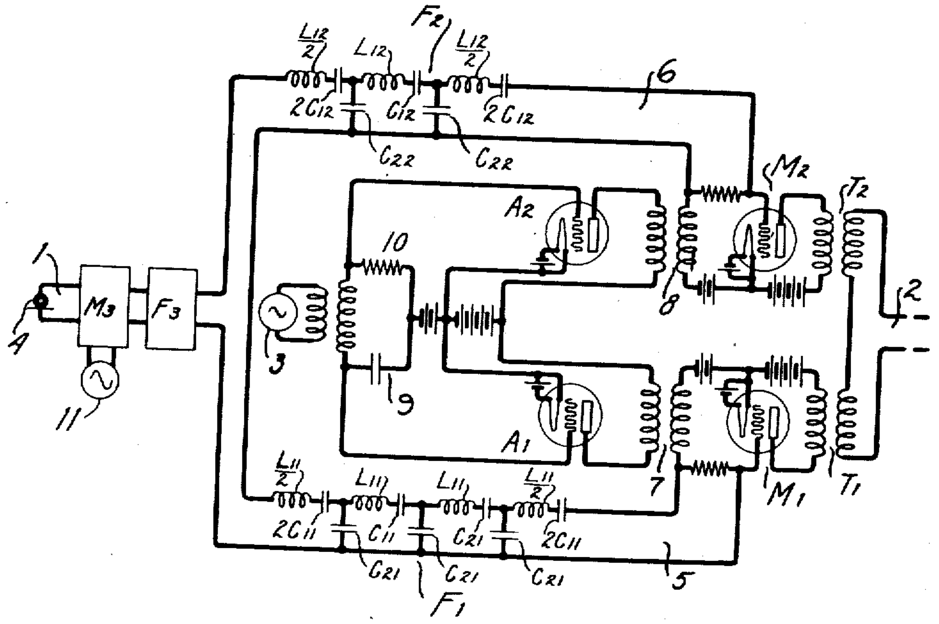
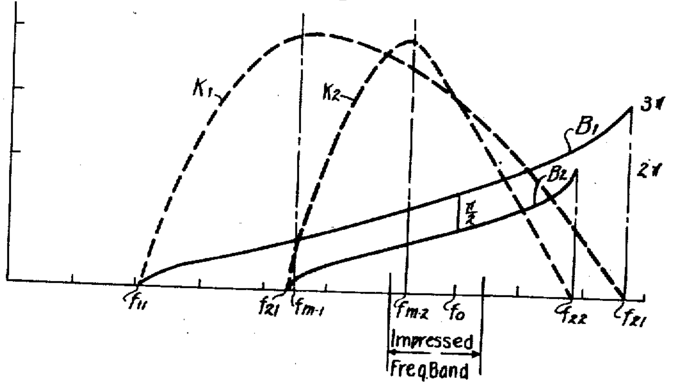


Fig. 2



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UNITED STATES PATENT OFFICE.

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MODULATION SYSTEM

Application filed January 15, 1925. Serial No. 2,483.

This invention relates to a modulation system and particularly to a modulation system for producing a modulated wave having a single side band.

The advantages accruing in the production and transmission of a modulated carrier wave having a single side band are now well understood. To name only two advantages, such an operation results in marked economy in usefully employed energy and a similar economy of frequency range.

Heretofore, it has been necessary first to generate a double side band modulated carrier wave and then to select from this wave, as by means of filters, a portion containing a single side band. Since the present invention relates only to the production of a single side band wave, without regard to the presence or absence of an unmodulated carrier component, a similar object will logically be assumed in considering the prior method. The problem, then, has been separate by this filter means, closely adjacent frequencies. The difficulty in accomplishing this is inversely proportional to the ratio of the frequency separation of the side bands to their mean frequency that is, to the carrier frequency. The difficulty accordingly increases with increasing carrier frequency.

Let it be assumed, as in the usual case, that the side bands in question are produced by simple modulation, that is, by a single modulating step, and that the modulating frequencies are among those commonly used in signal transmission. For these conditions and using the prior method of single side band production a practical limit of carrier frequency is reached, beyond which separation of the side band cannot be accomplished, which is less than the frequency ordinarily required for radio transmission. This limit has been found to be of the order of 30,000 cycles. Accordingly for the production of a single side band of a radio carrier wave simple modulation and selection is not sufficient.

In order to overcome this difficulty resort has been had prior to this invention to a method of multiple modulation. This method insures that for successive steps of carrier frequency increase, there will be a proportional increase in the spacing between the carrier and its modulated side bands so that there will be little difficulty in performing the necessary frequency separation after the last stage of modulation. However, such a method is involved and requires an elaborate and expensive outlay for circuits and apparatus.

It is an object of this invention to provide improved methods of and means for producing a modulated carrier wave having a single side band.

It is a further object of the invention to achieve the above object by methods and means which do not depend for their efficacy on a critical value of either carrier or side band frequency or on a particular relation of these frequencies.

The method involves the production of two pairs of side bands, using individual modulating circuits but the same carrier and modulating frequencies, and which pairs of side bands are so related that the phases of the current components in two side bands, one from each pair, are equal, and that the phases of the components in the remaining side bands are in opposition. The two pairs of side bands thus related are superposed with a resultant balancing out of one side band and a doubling of the amplitudes of the components in the other side band.

The necessary relation of phases for the current components in the two pairs of side bands is achieved by the use of certain critical phase relations between the carrier and modulating potentials that are impressed on the respective modulators. The relation may be satisfied, for example, by causing the phases of the carrier potentials for the two modulators to differ from each other by ninety electrical degrees and similarly causing the phases of the corresponding modulating potentials to differ by ninety degrees. This is on the assumption that second order modulation, that is, modulation resulting in side bands in which the frequencies are indicated by mp plus and minus q, and in which m equals one, is used in each modulating circuit, p and q indicating respectively the carrier and modulating frequencies. Other orders or methods of modulation resulting in side bands of the same type but in which *m* is greater than one, may be used, which may be the same or different in the two modulating circuits and which may require different relations of the phases of the impressed potentials. Of course, where other than second order modulation is used, in one or more of the modulating circuits, the frequencies of the impressed carrier potentials must be caused to have the proper harmonic relation to each other or to the desired frequency *mp* to insure that the frequencies will be the same for the two pairs of resultant side bands. Reference is made to U. S. application of Peterson, Serial No. 683,301, filed December 29, 1923 for a complete analysis of second and higher order modulation. The general solution of the method of the invention for producing single side bands as well as several particular solutions including the one specifically mentioned above (using second order modulation in both modulators) will be developed in the detailed description. It is a common characteristic of all of these solutions that it requires a uniform relative phase shift of all of the frequencies in the modulating wave.

There is obviously little difficulty in obtaining the corresponding relative phase shift in the carrier wave, which has only one frequency. The invention of an operative form of single side band modulation system has, however, waited upon the conception of an operative means for producing this uniform phase shift for a band of frequencies. Accordingly it is a still further object of the invention to provide methods of and means for achieving this result. The method of providing a uniform relative phase shift may be practised without regard to whether the bands of frequency between which the uniform phase shift is provided did or did not originally comprise portions of a single current, as in the specific application of the principle herein disclosed, and without regard to whether or whether not the corresponding frequency components in the two bands had originally the same phase.

The solution of the uniform phase shift problem proposed in this invention is of a kind which cannot well be treated otherwise than by a detailed mathematical demonstration. It must therefore suffice at present to state that it depends on the fact that for certain filter networks the phase shift per section varies nearly uniformly throughout the transmission range and that the slope of the phase shift-frequency curve is a function of the transmission range of the filter and of the number of sections. The modulating circuit is branched. Filters are included in the branch circuits. These filters differ in transmission band widths and in numbers of sections with the result that although in neither branch circuit does a uniform phase shift occur, a uniform relative phase shift occurs between the components transmitted by the filters in the two circuits.

The invention may now be better understood from the following detailed description thereof when read in connection with the accompanying drawing, in which Fig. 1 illustrates a preferred embodiment of the invention and in which Fig. 2 comprises certain characteristic curves drawn partially to scale and which are used in explaining the operation of the invention.

In the description of this system of the invention and its method of operation with reference to the drawing certain results will be assumed without proof, or at least without quantitative data. This omission will then be supplied by mathematical demonstration wherein the necessary conditions for effecting a uniform relative phase shift of a band of frequencies and their application in system for producing a single modulated side band will be developed.

Referring now to the drawing, the elements intermediate the circuits 1 and 2 cooperate to produce in circuit 2 a single side band of a carrier wave resulting from modulation of the wave from carrier source 3 by modulating waves from circuit 1. The conditions justifying a choice of frequencies for circuit 1 will be explained in more detail later.

The band of frequencies from circuit 1 after stepping up to a desired value by combination with a single frequency wave from source 11 in modulator M_3 is then transmitted through band pass filter F_3. The purpose of this filter, the reason for which will be made clear later, is to confine the frequencies to be later used in modulating in modulators M_1 and M_2 to as narrow a band as possible consistent with the grade of transmission desired. The stepping up operation is preparatory to the subsequent operations. Although, as will be explained later, this operation is conducive to greater accuracy in obtaining the desired uniform relative phase shift, it may be dispensed with under proper conditions without serious sacrifice. These conditions will be pointed out later. The currents passed by filter F_3 flow in branch circuits 5 and 6 which contain respectively band pass filters F_1 and F_2. In the particular branch arrangement illustrated, the terminations of the two filters are in series. An equivalent arrangement would result from putting these terminations in parallel, as is well understood. The currents in these branch circuits are impressed respectively on modulators M_1 and M_2 in which they are combined with carrier current from source 3. These carrier currents are amplified in amplifiers A_1 and A_2 respectively and impressed on the modulators through transformers 7 and 8.

In the particular circuit illustrated it is assumed that second order modulation occurs in modulators M_1 and M_2. For this particular case it is necessary, in order to obtain the requisite relation of phases in the resultant pairs of side bands, that the carrier waves impressed on the two modulators differ relatively from each other by ninety electrical degrees. This result is achieved by impressing currents from the source 3 on amplifiers A_1 and A_2 respectively across condenser 9 and resistance 10. The use of these two different types of potential impressing circuits insures the desired quadrature relation of impressed potentials.

It is also required, both for the assumed case and for other possible cases, that there be a ninety degree phase difference for each of the modulating frequencies in circuits 5 and 6. This is achieved by a proper choice of the number of filter sections and transmission ranges of filters F_1 and F_2. The particular theoretical relations governing the determination of these quantities will be taken up under a separate heading later.

The various elements of the circuit, including the amplifiers and modulators, are each old and of a conventional type. Accordingly, no further description of them is considered necessary.

Given the above relation of phases of the frequency components impressed on the two modulators, there will result in the output circuits of these modulators two pairs of side bands. Two side bands, one from each pair, will have the same phase. The phases of the components in the other two side bands will be opposed. Accordingly by superposing these two pairs of side bands in circuit 2, one side band of each pair may be balanced out and the other side bands arithmetically added. This results in the end desire, that is, the production of a single side band carrier modulated wave. Depending on the poling of transformers T_1 and T_2, the resultant side band may be the upper or the lower side band of the carrier wave. Besides the satisfaction of the precise phase relationship required as above, it is also necessary that the amplitudes of the components of the two pairs of side bands be equal. This would ordinarily result from using identical devices in the two symmetrical paths of the system. However, the elements may differ somewhat from each other and the amplitudes may, in that case, be made the same by adjustments of the amplifiers and modulators as in accordance with conventional practice.

Although it was assumed above that second order modulation is used in the two branches, the principles of the invention do not exclude the use of higher order modulation in the two branches or second order in one branch and a higher order in the other. However, in order that the resultant side bands may have the desired frequency, care must be taken that the carrier frequency of the wave from source 3 has a proper subharmonic relation to the desired frequency *mp* in the output circuit of a given modulator, if other than ordinary second order modulation is used in the modulator in question. Also, if side bands are produced of the type *mp* plus and minus *q* in which *m* is different in the cases of the two modulators, care must be taken that waves having the proper harmonic relation of frequencies are impressed on the two modulators. For example, if second order modulation is used in modulator M_2, and third order modulation is used in modulator M_1, it would be necessary to impress on modulator M_2 the first even harmonic of the frequency of the wave from source 3 and to impress on modulator M_1 the fundamental frequency of the wave from that source. In a system so operated the amplifier A_2 could be overloaded to function as a harmonic generator. For this specific system of modulation as well as for other modifications using other than second order modulation in each branch, a phase relation of the impressed frequencies different from that described must be used. The conditions that must be satisfied in each of these cases, or in the general case, will be explained in detail hereinafter.

Theoretical basis for the method of producing a uniform relative phase shift for a band of frequencies.

This analysis will be largely based on the expositions contained in the following publications: a paper by Campbell, entitled "Physical principles of the electric wave filter" in the November, 1922, number of the Bell System Technical Journal; a paper by Zobel, entitled "Theory and design of uniform and composite electric wave-filters" in the January, 1923, number of the above journal; and U. S. patent to Campbell No. 1,227,113, granted May 22, 1917. Certain of the formulæ which are stated herein without proof are proved in some one of the above publications; the others are well known to those versed in the art and their proofs are omitted from this specification as being readily available from standard sources.

In any iterative network, of which a wave filter is typical, the output current from a section is related to the input current to the section by a proportionality factor e^{A+jB} in which $A+jB$ is the propagation constant, A being the attenuation constant and B the phase angle (or phase constant).

For any ladder type filter, including the type disclosed in the drawing, the propagation constant

$$\Gamma = A + jB = \cosh^{-1}\left(1 + \frac{\gamma^2}{2}\right)$$

$$\gamma^2 = \frac{Z_1}{Z_2}.$$

Z_1 and Z_2 being respectively the series and shunt impedance per section.

For currents having frequencies within the transmission range, if the elements of the impedances have no resistance, $A=0$ that is, these currents are transmitted without attenuation. This condition may be closely approximated in practice with well designed filters.

For that case

$$B = \cos^{-1}\left(1 + \frac{\gamma^2}{2}\right). \quad (1)$$

It is proposed, starting with Equation (1), to first determine the phase angle-frequency characteristic of the type of filter disclosed in terms of the transmission range limiting frequencies and the frequencies f_o, at which the phase angle per section is $\frac{\Pi}{2}$.

For the type of filter disclosed it may be shown that

$$\gamma^2 = -4\frac{f^2 - f_1^2}{f_2^2 - f_1^2}$$

in which f is the given frequency and f_1 and f_2 are respectively the lower and upper cut-off frequencies.

Therefore

$$1 + \frac{\gamma^2}{2} = 1 - 2\frac{f^2 - f_1^2}{f_2^2 - f_1^2} = \frac{f_2^2 + f_1^2 - 2f^2}{f_2^2 - f_1^2} \quad (2).$$

The frequency f_o, at which the phase angle equals $\frac{\Pi}{2}$ may be found from Equation (1) as follows:

$$\cos B = 1 + \frac{\gamma^2}{2}$$

therefore, for the assumed phase angle

$$\cos\frac{\Pi}{2} = 1 + \frac{\gamma_o^2}{2} = 0$$

in which γ_o is the value of γ for frequency f_o.

$$\gamma_o^2 = -2$$

This value of γ_o^2 substituted in Equation (2) gives

$$f_o^2 = \frac{f_1^2 + f_2^2}{2}$$

Combining this equation with Equations (1) and (2)

$$B = \cos^{-1} 2\frac{f_o^2 - f^2}{f_2^2 - f_1^2}.$$

It follows from the standard expansion of the arc-cosine of an angle that

$$B = \frac{\Pi}{2} - 2\left\{\frac{f_o^2 - f^2}{f_2^2 - f_1^2}\right\} - \frac{2^3}{6}\left\{\frac{f_o^2 - f^2}{f_2^2 - f_1^2}\right\}^3 - 2^5\frac{3}{40}\left\{\frac{f_o^2 - f^2}{f_2^2 - f_1^2}\right\}^5. \quad (3)$$

Let $f_o^2 - f^2 = a$
and $f_2^2 - f_1^2 = b$

Note that a is a function of the frequency interval between the frequency f_o at which the phase angle is $\frac{\Pi}{2}$ and the variable frequency and that b is a function of the transmission range limits.

Substituting in Equation (3)

$$B = \frac{\Pi}{2} - 2\frac{a}{b} - \frac{2^3}{6}\frac{a^3}{b^3} - \frac{2^5 \times 3}{40}\frac{a^5}{b^5} - \ldots\ldots.$$

Suppose that there are n_1 and n_2 sections in the two filters F_1 and F_2 respectively, n_1 being the larger. The subscripts introduced after this time will refer to these respective filters.

Then

$$B_1 = n_1\frac{\Pi}{2} - 2n_1\frac{a}{b_1} - \frac{2^3}{6}n_1\frac{a^3}{b_1^3} - \ldots\ldots.$$

and

$$B_2 = n_2\frac{\Pi}{2} - 2n_2\frac{a}{b_2} - \frac{2^3}{6}n_2\frac{a^3}{b_2^3} - \ldots\ldots.$$

The quantity a has the same value for both filters if f_o is the same for both filters as will be assumed and as provided for in the practical methods of design to be explained later. The bs will have different values for filters of different transmission ranges, as will also be assumed for this case and as indicated by the use of different subscripts.

The relative phase shift will be

$$B_1 - B_2 = \frac{\Pi}{2}(n_1 - n_2) - 2a\left\{\frac{n_1}{b_1} - \frac{n_2}{n_2}\right\} - \frac{2^3}{6}a^3\left\{\frac{n_1}{b_1^3} - \frac{n_2}{b_2^3}\right\}$$

The end to be achieved is to make $B_1 - B_2$ in this equation uniform for all frequencies within the band impressed from filter F_3, to a close first approximation. This end would be achieved if the terms in the equation containing a (that is, those which are a function of f) were equal to zero. There would then result a uniform relative phase shift $(n_1-n_2)\frac{\Pi}{2}$. Since n_1 and n_2 are limited to integral values the shift would be limited to multiples of $\frac{\Pi}{2}$, that is, multiples of ninety electrical degrees. As a practical matter, since four ninety degree shifts, or a multiple thereof, would be the equivalent of a zero shift and since the other even multiple shifts could equally well be performed by transformers, an odd number of ninety degree shifts would be sought, which would require the two filters to differ by an odd number of sections. For use in the single side band production system described which requires a ninety degree shift, they would be caused to differ by one section. This condition will be assumed in the remaining part of the analysis.

It will be shown that the second term in the equation can be made equal to zero. Since the remaining terms, those involving the higher powers of a, have or can be made to have by proper choice of filters, exceedingly small values, the result is a very close proximation to the desired uniform relative shift.

The conditions accordingly are

$$n_1-n_2=1$$

and

$$\frac{n_1}{n_2}=\frac{b_1}{b_2} \text{ or } \frac{n_1}{n_1-1}=\frac{b_1}{b_2}.$$

The solution of these equations would determine the principal features of the filters to give the desired phase difference in so far as this difference is determined by the self-properties of the filters. However in a practical case the solution would require a consideration of the terminating impedances. For example if the terminating impedances, which are preferably resistances, are equal and if the filter impedances are nearly equal at the important frequencies the reflection effects will be substantially equal and will compensate each other. A trial solution will be made with certain constants more or less arbitrarily chosen and the phase shift and impedance curves will then be plotted for that case to check the accuracy of the solution.

In general it will be found best to choose f_o well within the band of impressed frequencies and to choose the transmission range of the narrower range filter at from three to four times the width of that band. The filter having the narrower range will be found to be the one which has the smaller number of sections, that is, filter F_2. The transmission range limits must conform, for each filter, to the relation derived above

$$f_o^2=\frac{f_1^2+f_2^2}{2}$$

or, for the respective filters,

$$f_o^2=\frac{f_{12}^2+f_{22}^2}{2} \text{ and } f_o^2=\frac{f_{11}^2+f_{21}^2}{2}. \quad (4)$$

If f_m equals the geometrical mean of the cut-off frequencies, so that

$$f_m^2=f_1\times f_2 \quad (5)$$

the above formulæ may be more conveniently used for determining the cut-off frequencies by substituting this value therein to give

$$f_o^2-f_{m2}^2=\frac{(f_{12}-f_{22})^2}{2} \text{ and } f_o^2-f_{m1}^2=\frac{(f_{11}-f_{21})^2}{2}.$$

To take an actual example, suppose that the band of impressed frequencies extends from 12,000 to 15,000 c. p. s. and that we choose f_o to equal 14,000 c. p. s. Let the narrower range have a width of 9,000 c. p. s., that is, three times the width of the impressed band.

Then

$$f_o^2-f_{m2}^2=\frac{(9000)^2}{2}$$

or

$$f_m^2=f_o^2-\frac{(9000)^2}{2}=(14000)^2-\frac{(9000)^2}{2}=12500^2.$$

Equations (4) and (5) can now be solved for f_{12} and f_{22} to give

$$f_{12}=8775$$

$$f_{22}=17775$$

Also $b_2=f_{22}^2-f_{12}^2=238\times10^6$.

Let is be assumed that the narrower range filter F_2 has two sections and that filter F_1 has three sections.

Since, as explained above,

$$\frac{n_1}{n_2}=\frac{b_1}{b_2}$$

$$b_1=b_2\frac{n_1}{n_2}=\frac{3}{2}b_2=238\times10^6\times\frac{3}{2}=357\times10^6=f_{21}^2-f_{11}^2. \quad (6)$$

Since f_0 is the same for both filters

$$\frac{f_{21}^2+f_{11}^2}{2}=(14000)^2. \quad (7)$$

Solving equations (6) and (7) we obtain

$$f_{21}=19350 \text{ c. p. s.}$$

$$f_{11}=4180 \text{ c. p. s.}$$

The important features of the design of the two filters is determined by the values of the constants obtained above.

In order to check the appropriateness of the choice of quantities more or less arbitrarily chosen the curves of Fig. 2 have been plotted. In this figure the abscissæ are frequencies and the ordinates phase angles or impedances. The full line curves are the phase angle characteristics of the two filters F_1 and F_2 as indicated by the labels B_1 and B_2 respectively. The dashed lines are the corresponding impedance characteristics, the curves K_1 and K_2 corresponding respectively to filters F_1 and F_2. The frequency range of the impressed frequency band is indicated along the axis of abscissæ. Since the filters are of similar type it follows from the Campbell equations that equal phase angles per section correspond to equal impedances. Accordingly for the frequency at which the impedance curves cross, the phase angles per section are equal. But that frequency was chosen at which the individual phase angles were $\frac{\Pi}{2}$. It therefore corresponds to the frequency at which the phase difference is $\frac{\Pi}{2}$. The reflection effects will become reduced as the impedance characteristics become more nearly coincident over the range of the impressed frequency band.

The phase angle characteristics were determined from the above calculated values of the cut-off frequencies, knowing that, for the types of filters illustrated in Fig. 1, the phase angle varies from 0 to Π per section. The illustrated variation from linearity of these characteristics is in deference to the slight effects of the higher order terms in the phase angle-frequency equation. It is apparent that the two characteristics are generally parrallel and separated by $\frac{\Pi}{2}$. The nonlinearity does not appreciably affect the perellelism of the characteristics within the impressed frequency range for the reasons that this range is small relative to the transmission ranges of the filters and because it occupies a position near the center of those ranges. It is evident by a similar mode of reasoning that less favorable results would attend the use of a voice band itself, instead of a prepared, i. e., a stepped-up, voice band.

The impedance characteristics were determined from formulæ given by Campbell. It is evident that the choice of range to be occupied by the impressed band and the choice of filter taken for the first trial were fortunate from a standpoint of uniform relative phase shift as affected by reflection since the impedances are equal at frequency f_0 and nearly equal in that neighborhood and since this frequency f_0 was chosen within the impressed frequency band.

It can be demonstrated with the aid of the references noted at the beginning of this analysis that, with a given band of impressed frequencies, as the number of sections, and hence the slopes of the phase angle characteristics increase, the reflection effects due to the inequality of the impedances become less important. If a sufficient number of sections were used results comparable with those obtained in the example given could be obtained when using a voice band. In an actual case this would probably require filters having at least twenty sections and differing, as in the case given, by one section. It is apparent that the filter F_3 may be made to contribute to this result by eliminating as large a portion of the voice band at its lower edge as may be practically possible so as to permit as great a ratio of transmission range of the narrower filter to the range of the impressed frequency band as possible. It may contribute in a similar way, where a stepped-up band is used by limiting the width of this band to as small a limit as is practically possible for intelligible transmission.

To the extent that the signal band, for example, a voice band, could be used directly, the method would replace the more complicated and expensive method involving multiple successive modulation. Where the speech band width is too great to permit the direct use of the method, as perhaps in the practical case, the method as applied in the manner illustrated in Fig. 1, might still be useful as a step in side band elimination for very high carrier frequencies. For instance, suppose that it were attempted to build up a very high frequency single side band carrier modulated wave. Suppose that the final carrier frequency were so high that even after a preliminary step of modulation the finally produced side bands could not be separated by selective means. The method of this invention could be practiced on the modulated wave resulting from this preliminary step so as to produce the desired single side band, since the width of a side band resulting from the preliminary step of modulation as compared with its mean frequency would be sufficiently large for separation of the side bands by selective means. By the use of the prior method, an additional step of modulation, making three in all, would have to be used. In the operation of this modification of the invention, the system of the figure would accordingly replace the circuits required for the last two steps of modulation in a triple modulation method. For example, the method described in the U. S. patent to Espenschied 1,361,522, granted December 7, 1920 could be modified in this manner.

The physical principle of the method can now be stated very simply in terms of the necessary steps required to make the phase angle characteristics of two filters different and parallel. If the two filters had the same transmission range and had the same number of sections obviously the two characteristics would be coincident. As the first step in making them different and parallel an odd number of sections, for example one, may be added to one of the filters. This would separate the curves but would increase the slope of the curve corresponding to the longer filter so that the curves would not be parallel. To compensate for this increase of slope its transmission range would be increased to produce a decrease in slope. Obviously for any excess number of sections in one filter there would be a critical value of transmission range of the other filter, narrower than that of the first, which would tend to make the curves parallel.

Theoretical basis for the method of producing a single side band by balance.

The presentation of this theory, will, in general, follow that used in United States patent to Carson 1,449,382, granted March 27, 1923. The treatment presented in United States application of Peterson 683,301 filed December 29, 1923 may be referred to with benefit in connection with the discussion of methods of modulation of other than the conventional second order.

According to the method used in the above mentioned patent, a statement of the potential (or current) resulting from modulation is obtained by substituting in the general equation of the type $y=ax+bx^2+cx^3$ . . . values of the simultaneously impressed voltages (or currents). Suppose that the input potentials impressed on a modulator of the present system are P cos $p_1 t$ and Q cos $q_1 t$ in which p_1 and q_1 equal respectively $2\pi p$ and $2\pi q$ in which p and q represent respectively the carrier and signal frequencies, and P and Q represent the corresponding maximum values. Accordingly x in the general equation equals P cos $p_1 t$+Q cos $q_1 t$. (No material change would result if an initial phase angle between the two impressed waves were assumed.)

This value of x should be substituted in the general equation. The first term, ax, yields merely amplified waves of the impressed frequencies p and q. The term bx^2 yields waves of frequencies $2p$, $2q$, $p+q$ and $p-q$ as is well known, the instantaneous values of the side bands $p+q$ and $p-q$ being represented by b P Q cos $(p_1+q_1)t$ and b P Q cos $(p_1-q_1)t$. The frequency significant portions of these equations result from the trigonometric expansion of the product cos $p_1 t$ cos $q_1 t$. If the other even power terms of the general equation are algebraically expanded by the binomial theorem and trigonometrically transformed as pointed out in pages (10) and (12) of the Peterson application above mentioned, it will be found that additional side bands having the same frequencies are obtained which are superposed upon those derived from the second power term of the general equation.

Similarly to the above, from the odd power terms of the general equation, third order side bands of frequencies $2p+q$ and $2p-q$ may be obtained. These frequencies result from the trigonometric expansions of the product cos 2 $p_1 t$ cos $q_1 t$. This is explained in pages (12) and (13) of the above mentioned application.

It could be shown in a manner similar to that used in the derivation of the above relations that side bands of other orders can be obtained from the general equation, even order side bands from the even power terms and odd order side bands from the odd power terms.

Assume now that the phases of the potentials impressed on the other modulator differ uniformly from those correspondingly impressed on the first modulator. Let the phase difference of the carrier frequencies equal B and the phase difference of the signaling frequencies equal C. Let the quantities cos $(p_1 t+\mathrm{B})$ and cos $(q_1 t+\mathrm{C})$ be used in place of cos $p_1 t$ and cos $q_1 t$ in the mathematical operations above.

Considering first the steps that would yield second order side bands, the frequency significant portions of the quantities indicating the instantaneous values of these components, resulting from the trigonometric expansion of the product

$$\cos(p_1t+\mathrm{B})\ \cos(q_1t+\mathrm{C}),$$

would equal $\cos[(p_1+q_1)t+\mathrm{B}+\mathrm{C}]$ and

$$\cos[(p_1-q_1)t+\mathrm{B}-\mathrm{C}]$$

They indicate side bands of frequencies $p+q$ and $p-q$ displaced in phase from those produced by the other modulator B+C and B−C degrees. In order that two side bands, one from each pair, will balance each other, and that the remaining side bands will add, the phase shifts B and C must each evidently have the value $\frac{\Pi}{2}$. Whether the upper or the lower side bands are balanced out is determined by the poling of the transformers T_1 and T_2. It is obvious that the result would be the same if one or both the phase shifts B and C were in the other direction from the one assumed, or if one or the other shift occurred with respect to the other modulator. In fact, the only condition that needs to be specified, so far as the phases of impressed potentials are concerned, is that there must be a uniform relative phase shift of the modulating potentials and a relative phase shift of the carrier potentials.

For the case of third order modulation in each modulator, similar relations would govern between 2B and C. That is, B must equal $\frac{\Pi}{4}$ and C must equal $\frac{\Pi}{2}$. For this modification, the original carrier having frequency p must of course be the first even sub-harmonic of the desired carrier which corresponds to the generated side bands, and B is the phase shift between the original carriers.

If second order modulation is used in one modulator and third order in the other, a carrier frequency $2p$ would be used in the second order modulator and the first even sub-harmonic thereof, p, in the third order modulator. The required relation of carrier frequencies for the two modulators could be obtained by using a harmonic generator in the circuit between the original carrier source of the second order modulator. Harmonic generation may be accomplished very simply by overloading the amplifier A_1 or A_2, as the case may be, in accordance with the method described in United States patent to Kendall 1,446,752 granted February 27, 1923. The relation of phase shifts would be identical for this case as for the case where third order modulation is used in both instances. In fact, since the resultant product is the significant element and not the process of producing it, the harmonic generation and the second order modulation could occur in a single modulating device by the process sometimes called "cascade modulation" and this process and third order modulation could be used interchangeably.

The whole process of single side band production detailed above may be generalized by stating the following rule which will be found to apply to all cases. Two pairs of side bands must be produced which must have the same fixed frequency characteristic mp, where the side band frequencies are indicated by $mp+q$ and $mp-q$, there must be a uniform relative phase shift of $\frac{\Pi}{2}$ between the frequencies q of the two modulating bands, and a resultant relative phase shift of $\frac{\Pi}{2}$ between the frequencies mp. When m is greater than one, and when second order modulation using the frequency mp is used for producing the side bands $mp+q$ and $mp-q$, this latter phase shift may be performed on the original carrier wave of frequency p or later on the wave of the frequency mp.

Since side bands of frequencies $mp+q$ and $mp-q$ may be produced by modulation of the $m+1$ order using a carrier p, or by second order modulation using a carrier mp, or in several other ways, a generic expression is required, which is equally descriptive of these methods. Consequently, the words "multiple" and "sub-multiple" will be used in certain of the claims in their generic senses to include both the single and plural multiples or sub-multiples, unless such an interpretation would be inconsistent with the context.

Although the invention has been illustrated as embodied in a specific form, it should be understood that this is illustrative of only one of many possible forms in which the system of the invention may be embodied and that the scope of the invention is not to be limited by the particular form illustrated but only by the appended claims.

What is claimed is:

1. The method of producing a uniform relative phase shift of the band of frequencies in a multi-frequency current which comprises transmitting individual portions of the current, each portion including all of the frequencies, by wave propagation to different electrical distances, whereby there tends to be produced a phase shift for each frequency in each portion, which shifts are greater for the currents transmitted the greater distance and which increase with increase of frequency, and causing the increase in shift with increase in frequency to be sufficiently less for the currents transmitted over the greater distance to produce an equal relative phase shift between the corresponding frequencies in the two portions.

2. The method of producing a uniform relative phase shift of the band of frequencies in a multi-frequency current which comprises propagating individual portions of the current, each portion including all of the frequencies, by wave motion for different periods of time so as to tend to produce phase differences between the corresponding frequencies in the two portions, the resultant phase difference being proportional to the frequency, and simultaneously compensating for this differential phase shift by causing the phase shifts of the frequencies in the portion which is propagated the greatest length of time to increase less rapidly with increase in frequency than in the other portion.

3. A system for producing a uniform relative phase shift of a band of frequencies comprising in combination, a source of frequencies constituting a band, two circuits branch from this source, a delay circuit in each branch adapted to transmit each of the frequencies from said source, each said delay circuit comprising means for delaying the transmission of the frequency components in proportion to the frequencies in said

band, means whereby said delay circuits tend to produce different delays for each frefrequency, said difference increasing with the frequency and means affecting said delay-frequency proportionality characteristic of said delay circuits whereby the tendency for said differences to increase with the frequency is compensated so as to result in a uniform difference, hence uniform phase shift.

4. The system of claim 3 in which the second means is adapted to produce a greater delay in one delay circuit than in the other and in which the last mentioned means is adapted to cause the delay to change more slowly with frequency increase in the delay circuit in which the greatest delay occurs than in the other delay circuit.

5. The method which comprises dividing the current of a wave comprising a band of frequencies into two portions each having all the frequency components of the wave and producing any desired phase shift, in one portion with respect to the other portion, which is substantially constant for all frequencies within the band and conjointly utilizing the resultant waves.

6. The method which comprises dividing a wave comprising a plurality of frequency components into a plurality of portions each containing all the frequency components and producing phase shifts of any desired amount, in some of the portions with respect to other portions, which are substantially constant for all of the plurality of frequency components and conjointly utilizing said components.

7. The method of producing a wave having the characteristics of a single side band of a modulated wave which comprises generating four modulated carrier side bands, two of which have opposing phase relation, and superimposing all of said side bands.

8. A system for producing a uniform relative phase shift of a band of frequencies, comprising in combination a source of frequencies constituting a band, two circuits branched therefrom, a filter in each branch whose transmission range comprises each of the frequencies from said source and in which the phase change for frequencies within this transmission range varies as a function of the frequency, said filters differing from each other by an odd number of sections, and the frequency transmission range of the filter having the greater number of sections being sufficiently greater than that of the other filter to produce a parallelism of their phase shift-frequency characteristic curves.

9. The system of claim 8 in which the phase change-frequency characteristic curve of each of said filters is substantially linear.

10. The system of claim 8 in which the phase shift-frequency characteristic curve of each of said filters is substantially symmetrical about the point corresponding to the mid-frequency of its transmission range.

11. The system of claim 8, in which the phase change-frequency characteristic curve of each of said filters is substantially linear and in which the band occupies a portion of the transmission range of each filter substantially in the center thereof.

12. The system of claim 8 in which the phase shift-frequency characteristic curve of each of said frequencies is substantially symmetrical about the point corresponding to the mid-frequency of the transmission range and in which the band occupies a portion of the transmission range of each filter substantially at the center thereof.

13. The system of claim 8 in which the phase change-frequency characteristic curve of each of said filters is substantially linear, is substantially symmetrical about the point corresponding to the mid-frequency of its transmission range and in which the band occupies a portion of the transmission range of each filter substantially at the center thereof.

14. The method of producing a single side band carrier modulated wave which consists in dividing a modulating wave having a plurality of frequencies q into two portions each having all of the frequencies of the wave, producing a 90° uniform relative phase shift between the components of corresponding frequency in said portions, generating a single frequency wave of frequency p, changing said single frequency wave to an otherwise similar carrier wave which has two portions whose phases differ by an amount that would correspond to a 90° phase difference between the same portions if each were converted to a wave having a frequency mp, utilizing a portion of said single frequency wave and one portion of the modulating wave to produce side bands of frequencies $mp+q$ and $mp-q$ in which m is an integer, including intermodulating the portion of the modulating wave with a carrier wave whose frequency is sub-multiple of the frequency mp, utilizing another portion of said single frequency wave and the other portion of the modulating wave to produce side bands of frequencies $mp+q$ and $mp-q$, and superposing the resultant pairs of side bands.

15. The method of producing a single side band carrier modulated wave which consists of dividing a modulating wave having a plurality of frequencies q into two portions each having all of the frequencies of the wave, producing a 90° uniform relative phase shift between the frequencies in said portions generating a carrier frequency wave, dividing said carrier wave into two portions, producing a 90° relative phase

shift between the phases of said carrier wave portions, intermodulating each portion of said carrier wave with a portion of the modulating wave to produce side bands of said carrier wave, and superposing the resultant pairs of side bands whereby one side band of each pair is balanced out and the remaining side bands are added.

16. A system for producing a single side band carrier modulated wave, which comprises in combination a source of multi-frequency modulating waves, two circuits branched therefrom, means in said branched circuits for producing a uniform relative phase shift of frequencies in the respective branch circuits, a source of carrier frequency, means for deriving from said source two carrier waves each having a frequency of the wave from said source but differing from each other in phase by 90°, means for modulating each of said derived carrier waves with the output wave from each of said branch circuits to produce upper and lower side bands of each of said derived carrier waves, and means for superposing the resultant pairs of side bands, whereby one side band of each pair is balanced out and the remaining side bands are added.

17. The system of claim 16 in which the means for producing a uniform relative phase shift of the modulating frequency comprises a filter in each of said branch circuits, said filters differing from each other by an odd number of sections, the filter having the greater number of sections also having a sufficiently greater transmission range to compensate for the differential phase shift-frequency characteristic resulting from the use of a different number of sections.

18. The method of producing a composite signaling wave comprising producing two separate composite wave trains having at least two frequency components in common, producing a phase shift in at least one wave train which changes the phase relation between the corresponding components of the two trains, the phase shift difference between corresponding components of one frequency being the same as that for the other frequency or frequencies, and conjointly utilizing said wave trains.

19. A method of producing a single sideband carrier modulated wave which comprises generating a modulating frequency band, coverting said band into two portions which have the same frequency characteristics but in which there is between the components of the same frequencies occurring in the respective portions, a phase difference which is the same for all components, and utilizing such portions.

20. In combination, means for generating a band of frequencies, means for dividing said band of frequencies into two portions each of which includes all of the frequencies, means adapted to operate on said portions to cause any desired uniform relative phase displacement between corresponding frequency components thereof, and means to jointly utilize the resultant portions.

In witness whereof, I hereunto subscribe my name this 14th day of January A. D., 1925.

RALPH V. L. HARTLEY.

CERTIFICATE OF CORRECTION.

Patent No. 1,666,206. Granted April 17, 1928, to

RALPH V. L. HARTLEY.

It is hereby certified that error appears in the printed specification of the above numbered patent requiring correction as follows: Page 1, line 22, after the word "been" insert the word "to"; page 2, lines 67 and 68, italicize the word "relative"; page 3, line 127, strike out the division sign in the exponent and insert a plus sign; page 4, in the equation following line 106, for the denominator "n sub 2" read "b sub 2"; page 5, line 100, for the word "is" read "it"; page 6, lines 52 and 53, for the misspelled word "parrallel" read "parallel" and line 55, for "perellelism" read "parallelism"; page 7, line 23, for the word "would" read "could"; page 9, line 2, strike out the syllable "fre-"; and that said Letters Patent should be read with these corrections therein that the same may conform to the record of the case in the Patent Office.

Signed and sealed this 11th day of September, A. D. 1928.

M. J. Moore,
Acting Commissioner of Patents.

(Seal)

shift between the phases of said carrier wave portions, intermodulating each portion of said carrier wave with a portion of the modulating wave to produce side bands of said carrier wave, and superposing the resultant pairs of side bands whereby one side band of each pair is balanced out and the remaining side bands are added.

16. A system for producing a single side band carrier modulated wave, which comprises in combination a source of multi-frequency modulating waves, two circuits branched therefrom, means in said branched circuits for producing a uniform relative phase shift of frequencies in the respective branch circuits, a source of carrier frequency, means for deriving from said source two carrier waves each having a frequency of the wave from said source but differing from each other in phase by 90°, means for modulating each of said derived carrier waves with the output wave from each of said branch circuits to produce upper and lower side bands of each of said derived carrier waves, and means for superposing the resultant pairs of side bands, whereby one side band of each pair is balanced out and the remaining side bands are added.

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18. The method of producing a composite signaling wave comprising producing two separate composite wave trains having at least two frequency components in common, producing a phase shift in at least one wave train which changes the phase relation between the corresponding components of the two trains, the phase shift difference between corresponding components of one frequency being the same as that for the other frequency or frequencies, and conjointly utilizing said wave trains.

19. A method of producing a single sideband carrier modulated wave which comprises generating a modulating frequency band, coverting said band into two portions which have the same frequency characteristics but in which there is between the components of the same frequencies occurring in the respective portions, a phase difference which is the same for all components, and utilizing such portions.

20. In combination, means for generating a band of frequencies, means for dividing said band of frequencies into two portions each of which includes all of the frequencies, means adapted to operate on said portions to cause any desired uniform relative phase displacement between corresponding frequency components thereof, and means to jointly utilize the resultant portions.

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