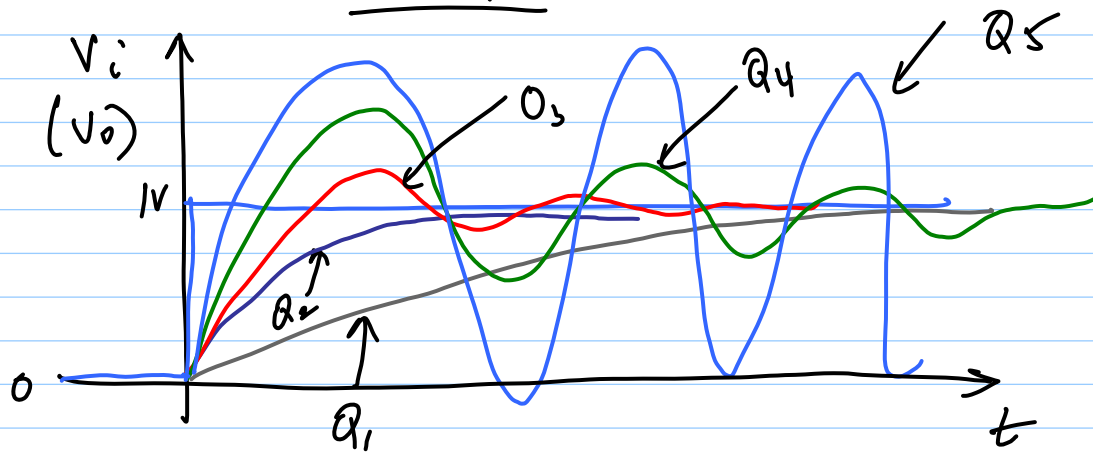


1-11-12

Lec 40



$$Q_1 < Q_2 < Q_3 < Q_4 < Q_5$$

$$Q_2 = 1/2, \quad Q_5 = \infty$$

↑
sinusoid that
does not die out

$$\underbrace{1 + \frac{s}{\omega_0 Q} + \frac{s^2}{\omega_0^2}}_{\text{gen. 2nd order}} \longleftrightarrow \underbrace{1 + \frac{2s}{A_{of} \omega_p} + \frac{s^2}{\omega_p^2 A_{of}}}_{\text{2-pole fwd. amp.}}$$

$$\omega_0 = \omega_p \sqrt{A_{of}}$$

$$Q = \frac{\sqrt{A_{of}}}{2}$$

trying to realise large $A_{of} \Rightarrow \underline{\underline{Q \text{ is very large}}}$

Not good enough

3rd - order system

$$A(s) = \frac{A_0}{\left(1 + \frac{s}{\omega_p}\right)^3}$$

$$= \frac{A_0}{1 + \frac{3s}{\omega_p} + \frac{3s^2}{\omega_p^2} + \frac{s^3}{\omega_p^3}}$$

$$CLG(s) = \frac{1}{f} \cdot \frac{A_0 f}{1 + A_0 f} \cdot \frac{1}{1 + \left[\frac{3s}{\omega_p} + \frac{3s^2}{\omega_p^2} + \frac{s^3}{\omega_p^3} \right] \frac{1}{1 + A_0 f}}$$

$\nearrow$
 $D(s)$

$$\text{Dr. } D(s) = 1 + \frac{3s}{\omega_p(1 + A_0 f)} + \frac{3s^2}{\omega_p^2(1 + A_0 f)} + \frac{s^3}{\omega_p^3(1 + A_0 f)}$$

$$D\left(\frac{s}{\omega_p}\right) = 1 + \frac{3s}{1 + A_0 f} + \frac{3s^2}{1 + A_0 f} + \frac{s^3}{1 + A_0 f}$$

Root of $(1 + A_0 f) + 3s + 3s^2 + s^3$

$$(1 + s)^3 = -A_0 f$$

$$s = -1 + (-A_f)^{1/3}$$

Example $A_f = 8$

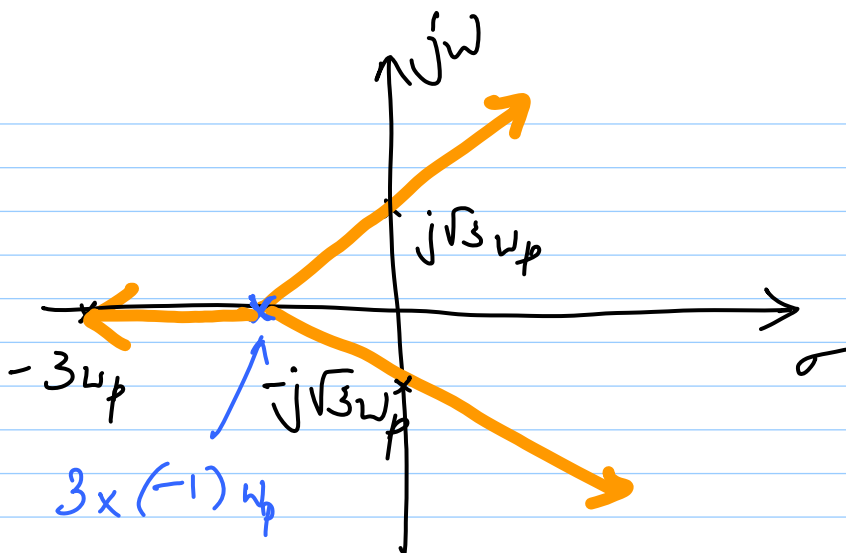
$$s_1 = -1 - 2 = -3$$

$$s_2 = -1 - 2 e^{-j2\pi/3}$$

$$s_3 = -1 - 2 e^{+j2\pi/3}$$

$$s = -3, \pm j\sqrt{3}$$

poles are @ $-3\omega_p, \pm j\sqrt{3}\omega_p$



$A_o f = 0 \Rightarrow$ all 3 roots lie @ (-1)

Sum of roots = $-3(\omega_p)$

huge
problem

$A_o f > 8 \Rightarrow$ complex conjugate roots
move into RHP

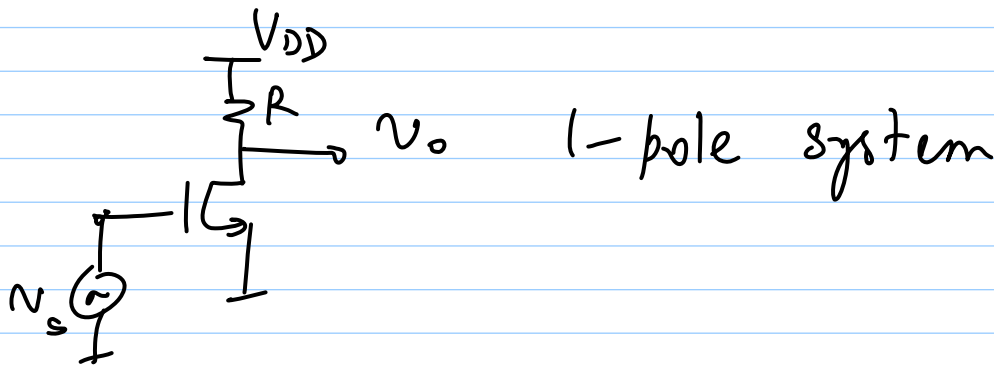
4th order $\Rightarrow$ will be worse

Solution 1

$$\left(\frac{A_0}{\left(1 + \frac{s}{\omega_{p1}}\right)^3} \right) \cdot \frac{\cancel{\left(1 + \frac{s}{\omega_{p1}}\right)^2}}{\left(1 + \frac{s}{\omega_{p2}}\right)^2}$$

$$\frac{\cancel{1 + s/\omega_{p1}}}{1 + s/\omega_{p2}}$$

$\omega_2 \approx \omega_p$



Solution 2:

1st order $\rightarrow$ low gain
 $\rightarrow$ unconditionally stable

2nd order $\rightarrow$ larger gain
 $\rightarrow$ technically stable

3rd order $\rightarrow$ v. large gain
 $\rightarrow$ unstable

make a higher order system look like a 1st order system

$$\frac{A_0}{(1 + s/\omega_p)^3} \longrightarrow \frac{A_0}{(1 + s/\omega_d)(1 + s/\omega_p)^3}$$

$$\omega_d \ll \omega_p$$

1) $\frac{A_0}{1 + s/\omega_d}$

2) $\frac{A_0}{(1 + s/\omega_p)^3}$

3) $\frac{A_0 \leftarrow}{(1 + s/\omega_d)(1 + s/\omega_p)^3}$

