2-8-12

Lec 3

$$\Delta V_S = \Delta I_D \cdot R_S + \Delta V_D$$

$$\Delta I_D = \frac{I_S}{V_t} \exp\left(\frac{V_{D_0}}{V_t}\right) \cdot \Delta V_D$$

$$\approx \frac{I_{D0}}{V_t} \cdot \Delta V_D$$

$$\Rightarrow \quad \Delta V_D = \frac{V_t}{I_{D_0}} \cdot \Delta I_D$$

$$\Delta V_S = \Delta I_D \cdot R_S + \Delta I_D \cdot \frac{V_t}{I_{D_0}}$$

$\Rightarrow$ Incremental quantities related linearly

\+ $I = f(V)$ can be linearised around (V_0, I_0) – OP

$\rightarrow$ any nonlinearity can be handled this way

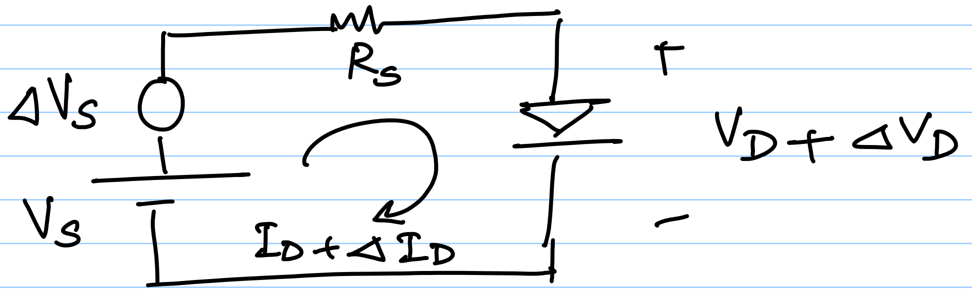
$\rightarrow$ $V = g(I)$ can also be used

e.g. $V_D = V_t \ln\left(\frac{I_D}{I_S} + 1\right)$

$$I_D' = I_{D_0} + \Delta I_D$$

expand V_D' in a taylor series

assumptions : ΔV_D is "small-signal"



* If V_s is changed by a small amount – i.e. the "signal" is small – the loop currents and node voltages also change by a small amount

$I_D \Leftrightarrow V_D \Leftrightarrow V_s \Rightarrow$ non-linear relation

$I_D' \Leftrightarrow V_D' \Leftrightarrow V_s' \Rightarrow$ " "

$\Delta I_D \Leftrightarrow \Delta V_D \Leftrightarrow \Delta V_s \Rightarrow$ linear relation

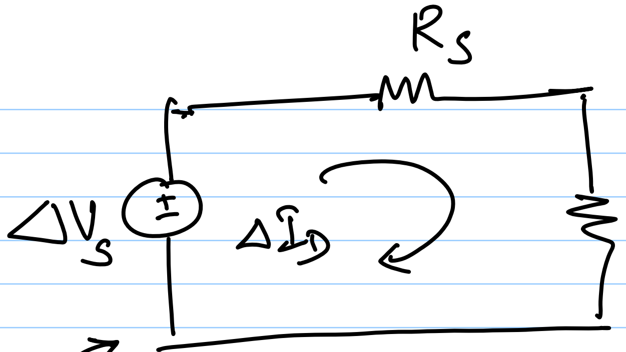
i.e. absolute voltage – current are still related by a NL eq'n.

BUT : incremental V & I are related linearly

Going back to diode network :

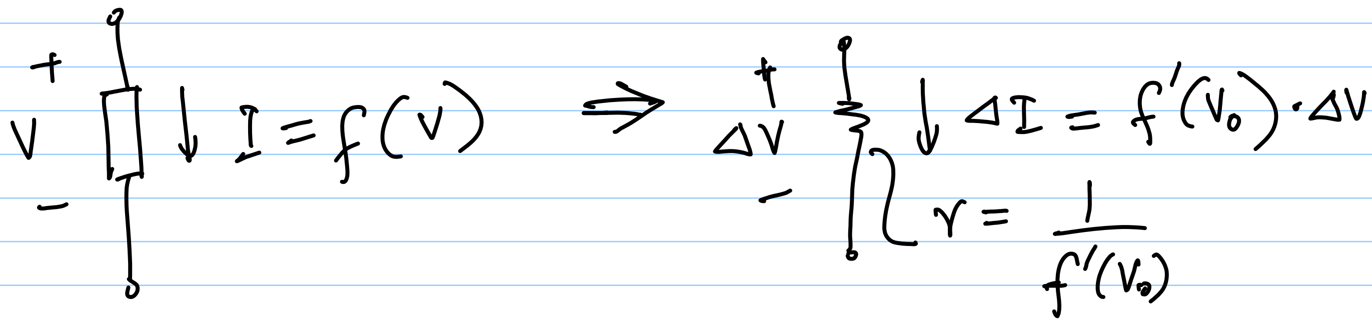
$$\Delta I_D = \frac{\Delta V_S}{R_S + \frac{V_t}{I_{D_0}}}$$

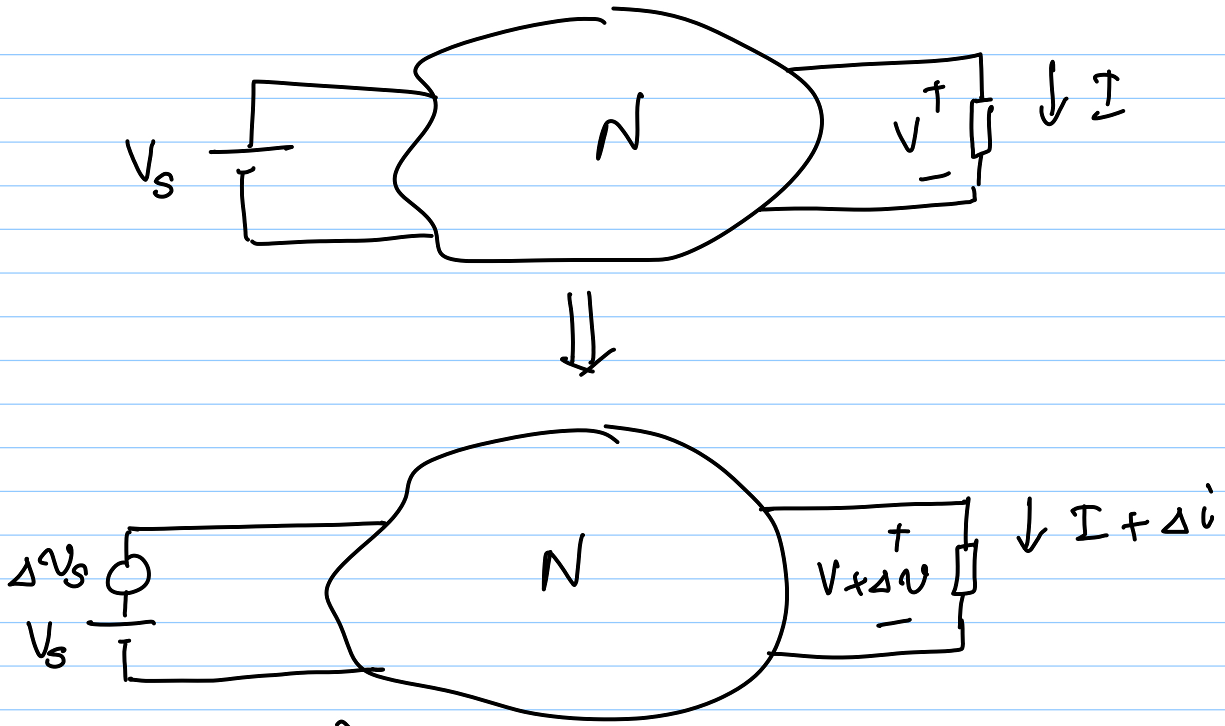
what kind of network has this loop eq'n?



$r_d = \frac{V_t}{I_{D_0}}$ = incremental resistance of diode @ I_{D_0}

Incremental network (linear)





$I = f(V)$

$I + \Delta i = f(V + \Delta v) \approx f(V) + f'(V) \cdot \Delta v$

$$\therefore \Delta i = f'(v) \cdot \Delta v$$

$$\frac{\Delta v}{\Delta i} = \text{resistance} = \frac{1}{f'(v)}$$

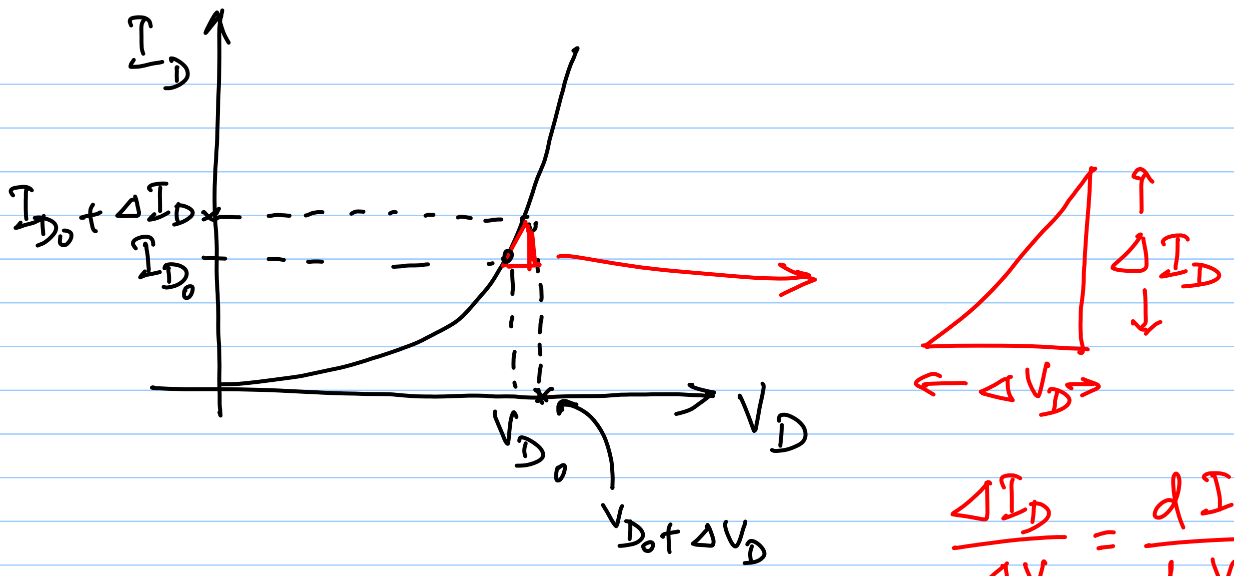
* $\frac{\Delta v}{\Delta i}$ depends on V, I etc.

* $I \Rightarrow$ Quiescent Current

  $\Delta i \Rightarrow$ incremental current

  $V \Rightarrow$ Quiescent Voltage

  $\Delta v \Rightarrow$ incremental voltage



$$\frac{\Delta I_D}{\Delta V_D} = \left.\frac{d I_D}{d V_D}\right|_{V_{D_0}}$$

$\frac{\Delta V_D}{\Delta I_D} =$ incremental resistance (or dynamic)

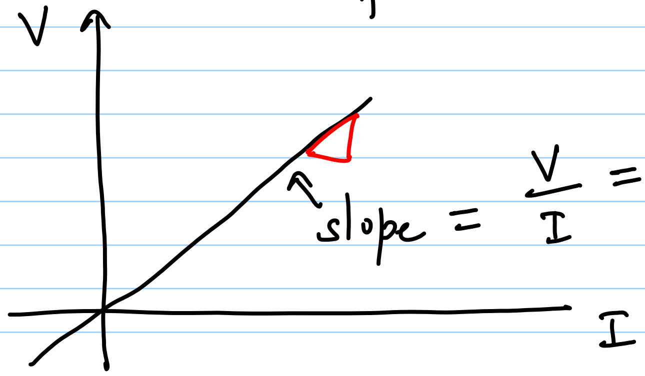
$\frac{\Delta I_D}{\Delta V_D} =$ " conductance

both depend on V_{D_0}, I_{D_0}

$\therefore (V_{D_0}, I_{D_0})$ is called the <u>operating point</u> of the nonlinear device

alternatively, if you want to find incremental resistance/ conductance of a NL device, you need to find the o.p. of the device

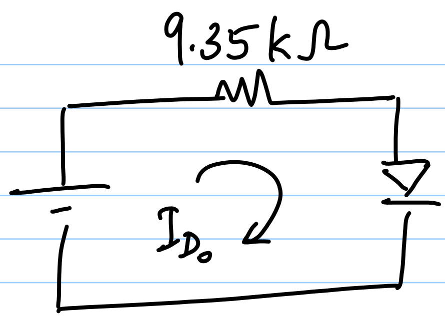
<u>R_s:</u>



$$\text{slope} = \frac{V}{I} = R_s = \frac{\Delta V}{\Delta I}$$

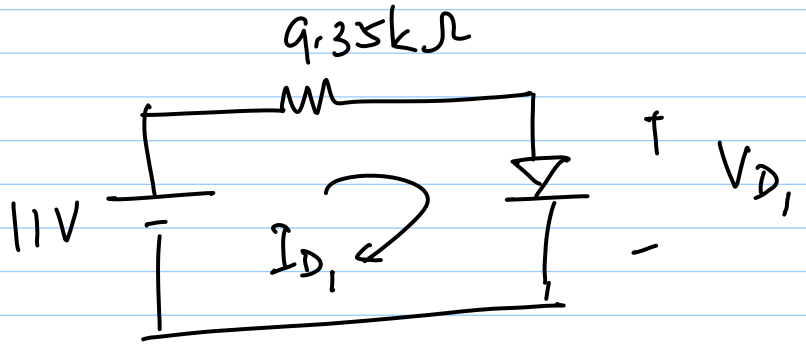
{i.e. incremental resistance also $= R_s$}

<u>e.g.</u>



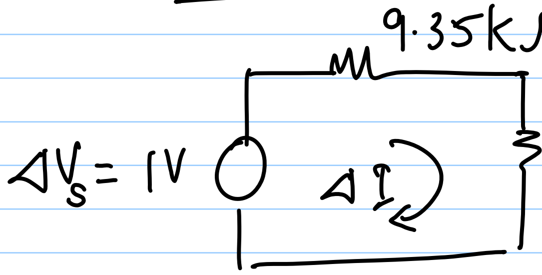
$V_{D_0} = 0.65V$

$I_{D_0} = 1mA$



$I_D, V_D = ?$

Incremental Equivalent Ckt



$$r_d = \frac{V_t}{I_{D_0}} = \frac{25mV}{1mA} = 25\Omega$$

$$\Delta I = \frac{1}{9.35k\Omega + 25} \approx 107\mu A$$

$$I_{D_1} = 1.107\ mA$$

If $V_s = 9V$, $I_{D_2} = ?$

$$\Delta V_s = -1V \Rightarrow \Delta I_D = -107\mu A$$

$$\Rightarrow I_{D_2} = 0.893mA$$

Next : is linear approx. valid ?

$$\Delta V_D = \Delta I_D \cdot r_d = 107\mu A \times 25$$

$$\approx 2.7mV$$

$$f''(V_{D_0}) = ?$$

$$I_D = I_s \exp\left(\frac{V_D}{V_t}\right)$$

$$f''(V_{D_0}) = \frac{I_s}{V_t^2} \exp\left(\frac{V_{D_0}}{V_t}\right) \approx \frac{I_{D_0}}{V_t^2}$$

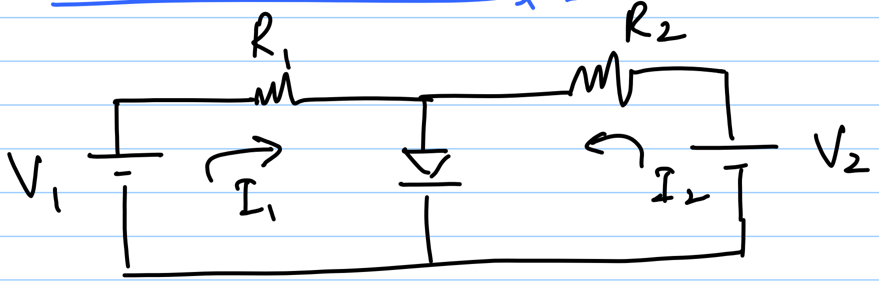
$$\text{second term} = \frac{f''(V_{D_0}) \cdot (\Delta V_D)^2}{2}$$

$$= \frac{I_{D_0}}{2V_t^2} \cdot (\Delta V_D)^2$$

$$I_D = I_{D_0} + \frac{I_{D_0}}{V_t} \cdot (\Delta V_D) + \frac{I_{D_0}}{2V_t^2} (\Delta V_D)^2 + \ldots$$

$\Rightarrow$ we need $\Delta V_D << 2V_t$ — quite valid

2.7mV — 50mV



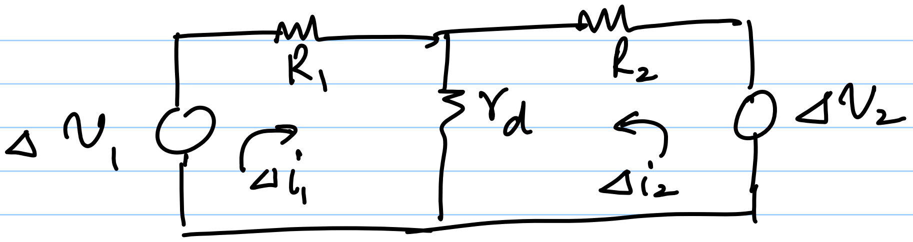
Use NL eqns to find out O.P.

e.g. $\frac{V_1 - 0.65}{R_1} = I_1$ & $\frac{V_2 - 0.65}{R_2} = I_2$

$$I_{D_0} = I_1 + I_2$$

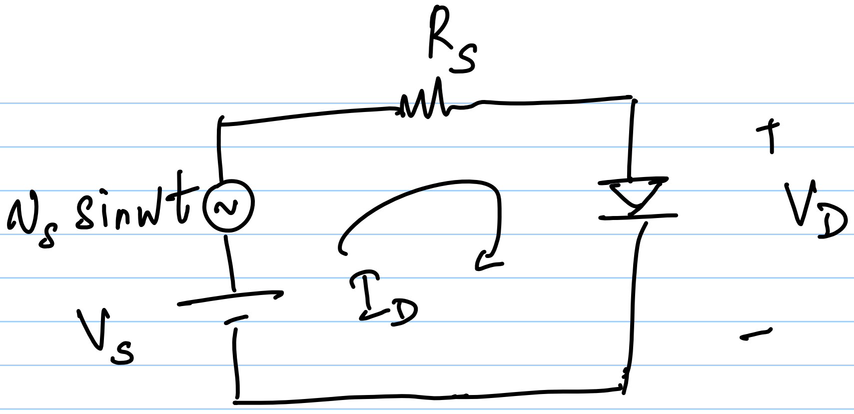
If $V_1 \rightarrow V_1 + \Delta v_1$ & $V_2 \rightarrow V_2 + \Delta v_2$

Incremental picture:



$$r_d = \frac{V_t}{I_{D_0}}$$

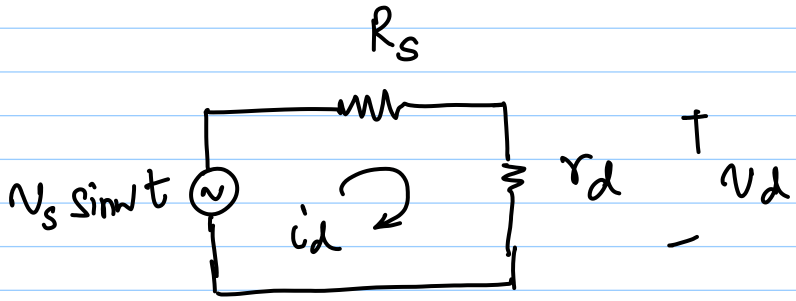
$$\Delta i_d = \Delta i_1 + \Delta i_2 \quad \ldots$$



If v_s is very small,

$I_D = I_{D_0} + i_d$

$V_D = V_{D_0} + v_d$



Incremental (or "small-signal") picture

$\Rightarrow$ can determine i_d & v_d (also sinusoids)